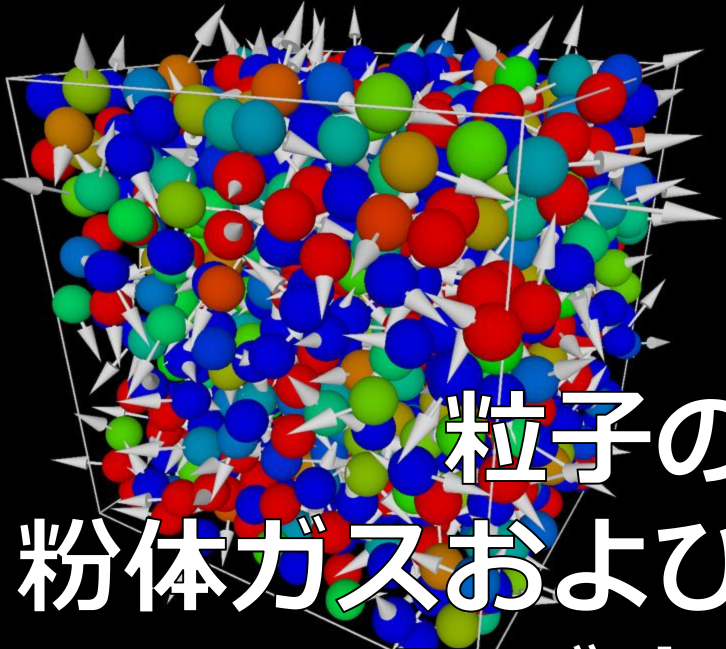
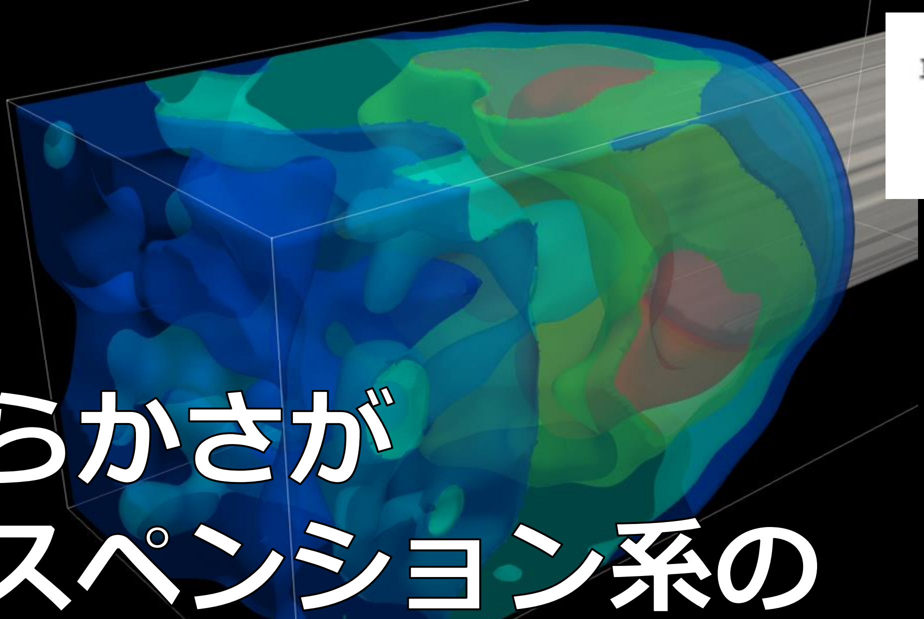
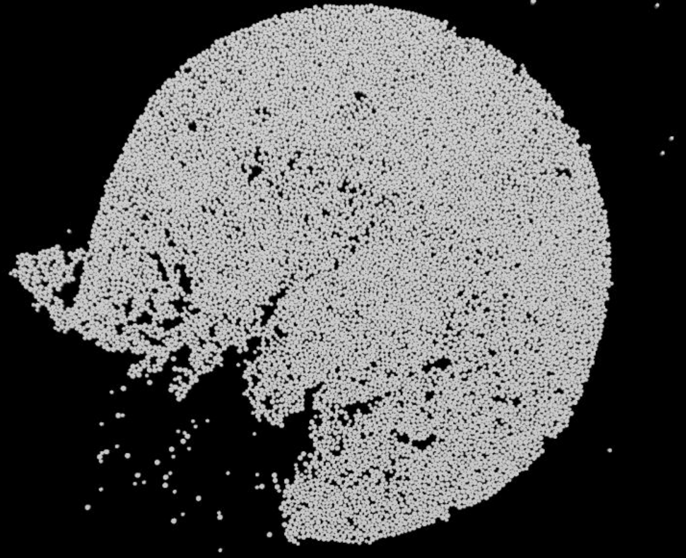
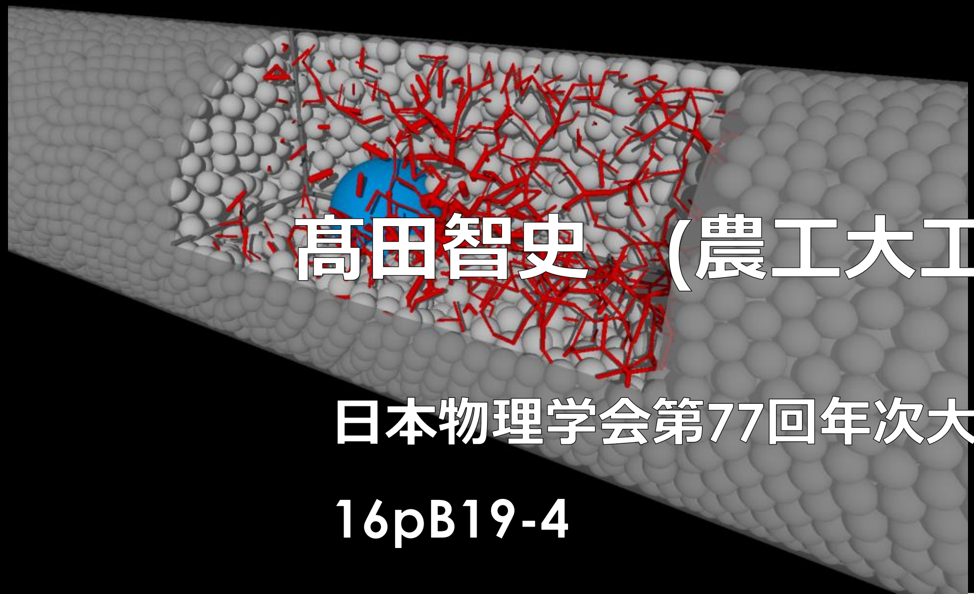




粒子の柔らかさが粉体ガスおよびサスペンション系のレオロジーに及ぼす影響についての理論的研究





髙田智史　(農工大工)

日本物理学会第77回年次大会(2022年)

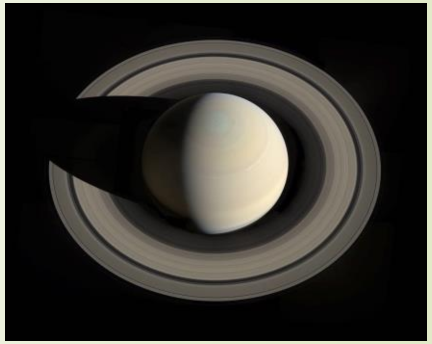
16pB19-4

http://web.tuat.ac.jp/~takada/

Granular Materials

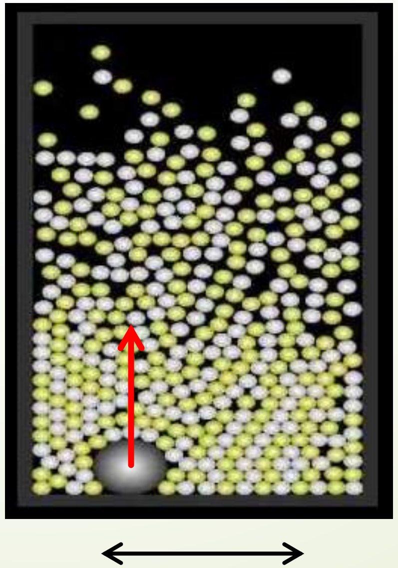
Macroscopic matters (μm~km)
(Quantum effects are negligible.)

Examples:
sand, toner particles, coffee, volcanic
PM2.5, Saturn's ring…



Behavior is different from
usual gas, liquid, or solid

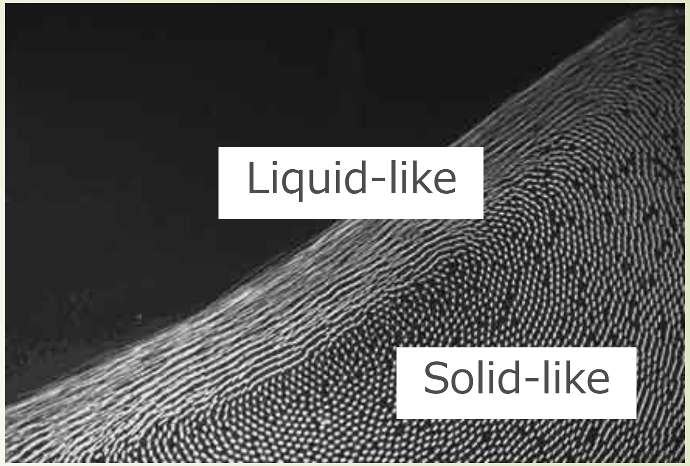
Example: Brazil nut effect
Larger particles move up.



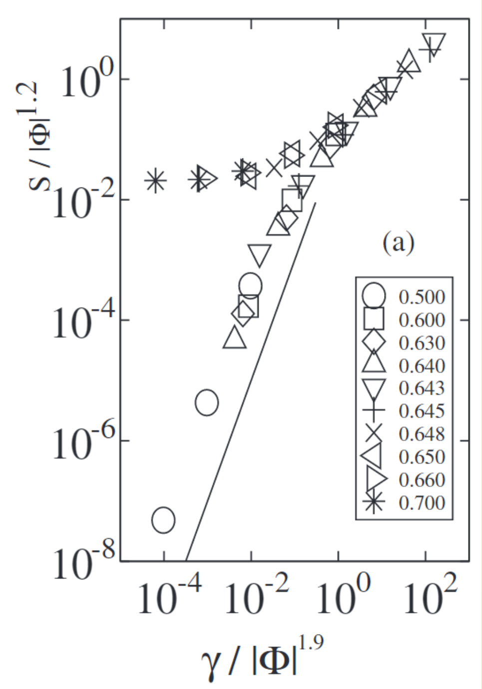
Flow of granular materials

- Surface flow of sand mountains
  - Only particles near the surface can flow (liquid-like).
  - Other particles do not move (solid-like).

  Coexistence of both states in one system.
- Existence of the jamming density φ_J
  - $\varphi < \varphi_J$: liquid-like response
  - $\varphi > \varphi_J$: solid-like response



Slope flow



T. Hatano, JPSJ **77**, 123002 (2008)

Viscosity

- Viscosity $\eta(\varphi) \equiv \frac{\sigma(\varphi)}{\dot{\gamma}}$: characterizes noneq. transport

  Example: Suspension

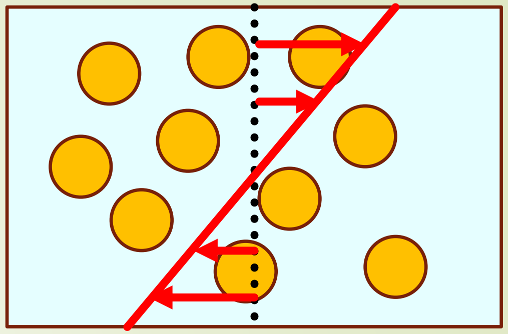
  Dilute case (Einstein, 1906): $\frac{\eta_s(\varphi)}{\eta_0} = 1 + \frac{5}{2}\varphi \quad (\varphi \leq 0.03)$

  Dense case (near jamming): $\frac{\eta_s(\varphi)}{\eta_0} = \left(1 - \frac{\varphi}{\varphi_m}\right)^{-2}$ (empirical)

- Shear rate dependent viscosity
  - Shear thickening (thinning):
    Viscosity becomes large (small)
    as $\dot{\gamma}$ increases.

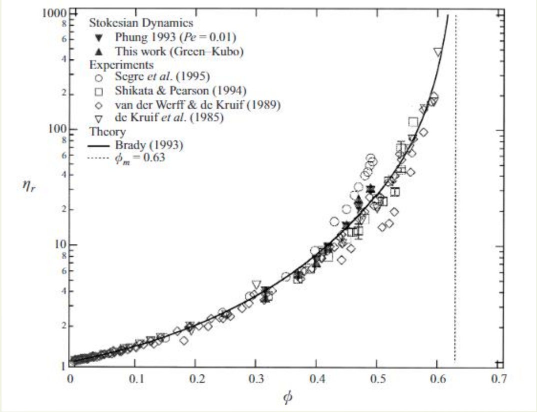
  Example: granular gas

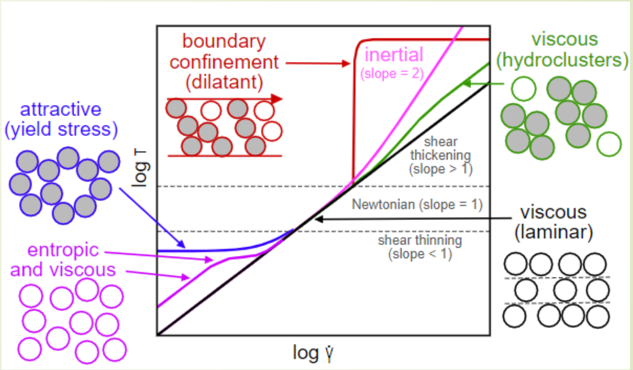
  $\eta \propto \dot{\gamma}$ (Bagnold scaling)



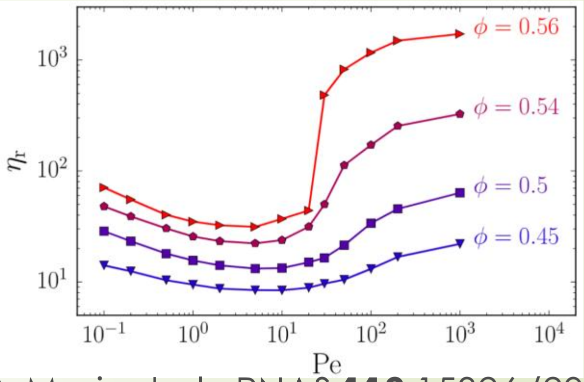
Solvent viscosity: η_0

Shear rate: $\dot{\gamma}$





Shear thickening



R. Mari, et al., PNAS **112**,15326 (2015)

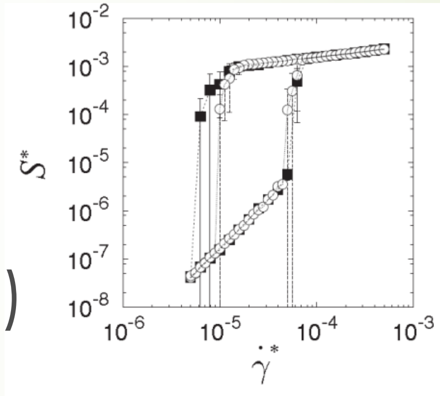
- Discontinuous shear thickening (DST): viscosity discontinuously increases.

  ⇒ DST is studied in many contexts and setups.

DST for dense systems (simulations)

- Mutual friction is important.

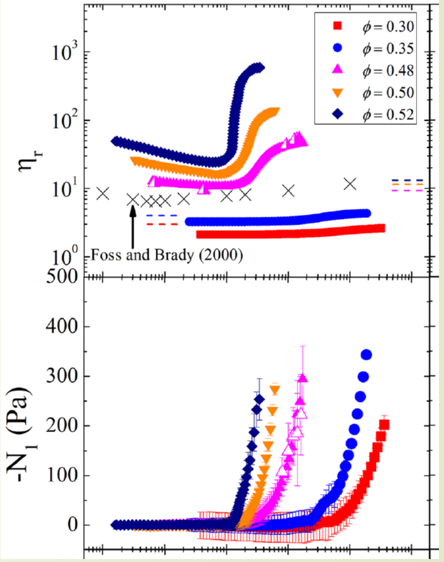
  M. Otsuki & H. Hayakawa, Phys. Rev. E **83**, 051301 (2011)
  R. Seto, et al., Phys. Rev. Lett. **111**, 218301 (2013)



DST for colloidal systems (experiments)

- Normal stress difference is also important.

  C. D. Cwalina & N.J. Wagner, J. Rheol. **58**, 949 (2014)

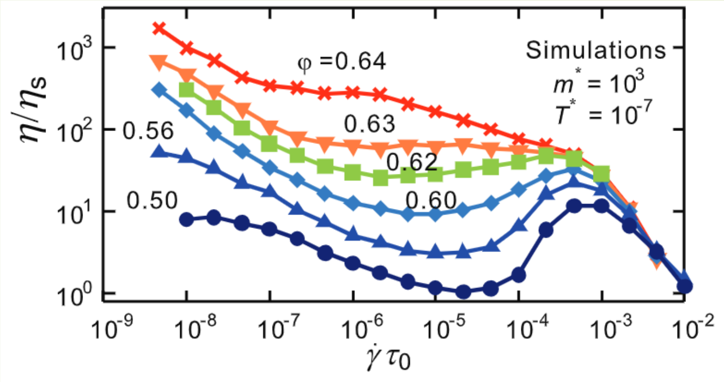


"Inertial effect" is often ignored for suspension. (overdamped)
If this is not ignored, system is called as
"inertial suspensions" (a model of aerosols or colloid)

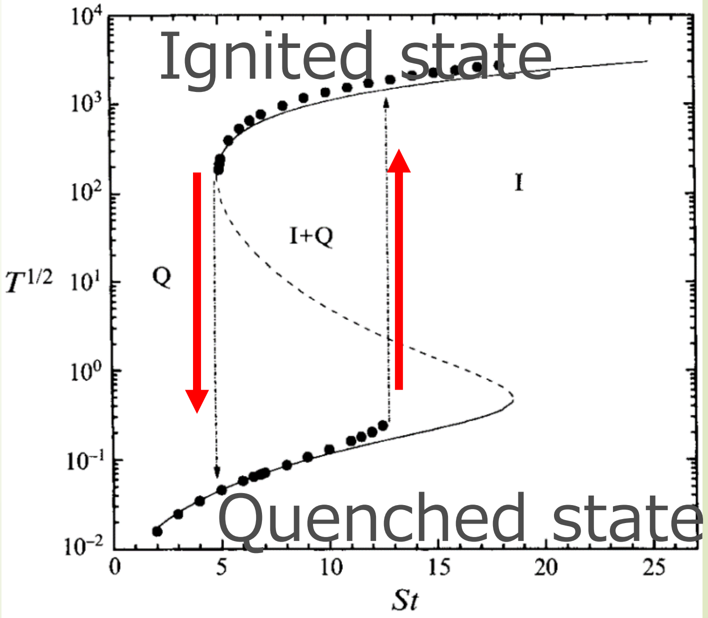
Previous studies of inertial suspension

System: frictionless, Stokes' drag

- Dense suspension (soft-core particles)
  ☞ Kawasaki, Ikeda, & Berthier, EPL (2014)
  - Thinning → thickening → thinning for $\varphi \lesssim 0.60$
  - No thickening for $\varphi \gtrsim 0.63$
  - ※ Only contact contribution

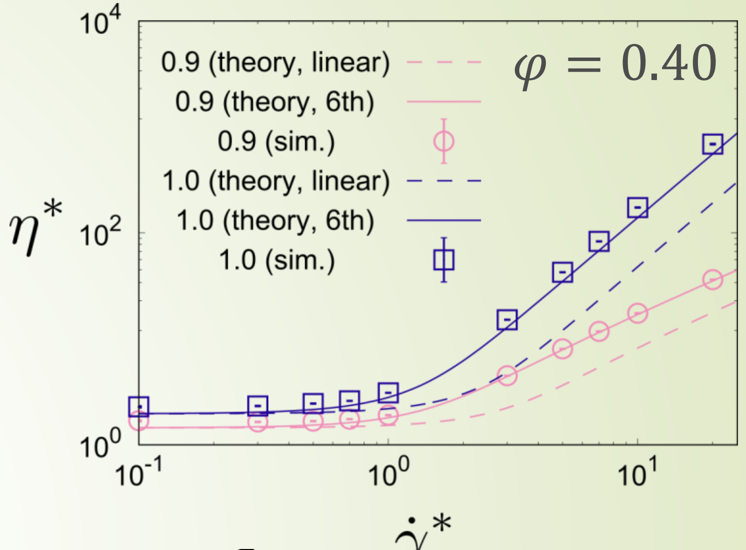


- Rheology of dilute gas-solid suspensions without thermal noise
  ☞ Tsao and Koch, JFM **296**, 211 (1995)
  Quenched-Ingnited transition
  DST-like transition for **temperature**
  but not for **viscosity**



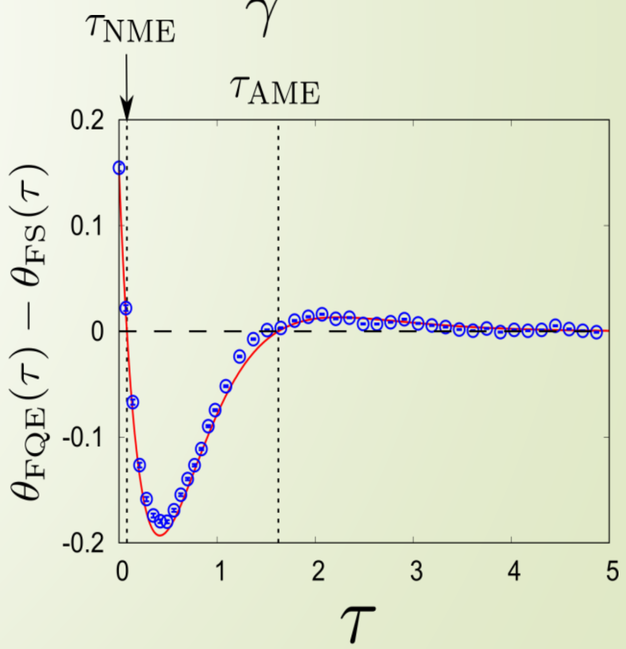
Our previous works

DST = discontinuous shear thickening
CST = continuous shear thickening



Kinetic theoretical approach considering the thermal noise (Hard-core system)

- Boltzmann-Enskog theory well describes monodisperse systems up to $\varphi \leq 0.50$.
- DST-like transition in the dilute system
  ☞ H. Hayakawa and S. Takada, PTEP **2019**, 083J01 (2019)
- DST-like to CST-like as the density ↗
  ☞ H. Hayakawa, S. Takada, and V. Garzó, PRE **96**, 042903 (2017)
- Mpemba effect in the relaxation process
  ☞ S. Takada, H. Hayakawa, and A. Santos, Phys. Rev. E **103**, 032901 (2021)



Motivation

We would like to know

Impact of softness of particles on the rheology

		Suspension		Granular system
		Hard-core	Soft-core	Soft-core
Theory & Sim.	Dilute	Hayakawa & Takada PTEP (2019)	Our study	Our study
Theory & Sim.	Moderately dense	Hayakawa, Takada, & Garzó PRE (2017), Takada, Hayakawa, Santos, & Garzó PRE (2020)	Our study	
Sim.	Dense		Kawasaki, Ikeda, & Berthier EPL (2014)	

Our theoretical tool: Kinetic theory

Why kinetic theory?

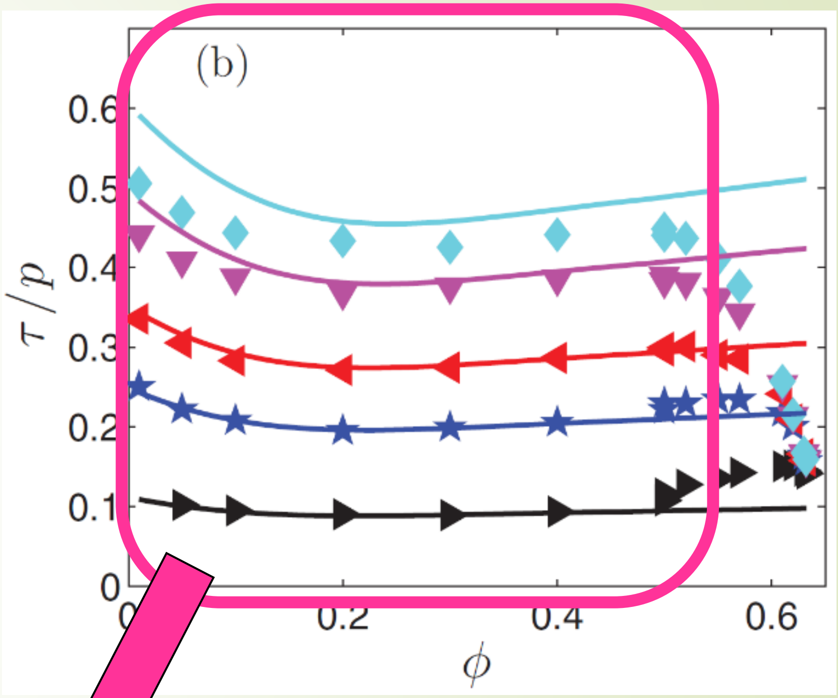
Kinetic theory of granular gases

- Ogawa; Savage and Jeffrey, 1978~1981
  Beginning of application of inelastic Boltzmann equation to granular system
- Brey *et al.*, 1998

  Application of Chapman-Enskog method to inelastic Boltzmann equation

  Transport coefficients for dilute system
- Garzó & Dufty, 1999

  Extension to finite density (Enskog equation)



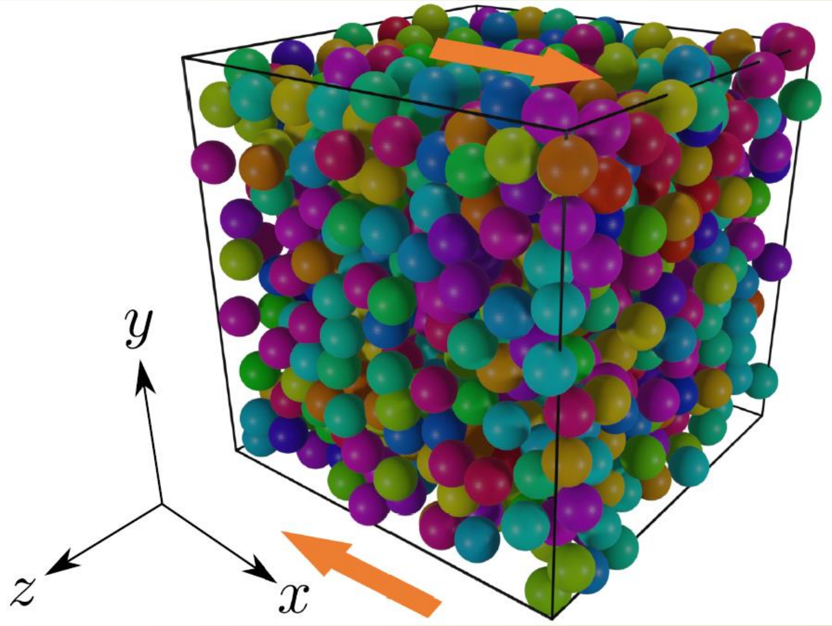
S. Chialvo and S. Sundaresan, Phys. Fluid. **25**, 0706503 (2013)

Kinetic theory is a powerful tool to treat the system for $\phi < 0.5$

Model and setup (Suspension)

- System = Particle + Solvent
- Particle:
  monodisperse (mass m, diameter d)

$$\frac{d\boldsymbol{r}_i}{dt} = \frac{\boldsymbol{p}_i}{m} + \dot{\gamma} y_i \hat{\boldsymbol{e}}_x$$
$$\frac{d\boldsymbol{p}_i}{dt} = \sum_{j\neq i} \boldsymbol{F}_{ij}^{(\mathrm{el})} + \boldsymbol{F}_i^{\mathrm{H}} - \dot{\gamma} p_{i,y} \hat{\boldsymbol{e}}_x + \boldsymbol{\xi}_i(t)$$



Shear (shear rate: $\dot{\gamma}$)

Interparticle interaction = harmonic potential
$$\boldsymbol{F}_{ij}^{(\mathrm{el})} = -\frac{\partial U(r_{ij})}{\partial \boldsymbol{r}_i}, U(r_{ij}) = \frac{\varepsilon}{2}\left(1 - \frac{r_{ij}}{d}\right)^2 \Theta\left(1 - \frac{r_{ij}}{d}\right)$$

Hydrodynamic interaction
$\boldsymbol{F}_i^{\mathrm{H}} = -\zeta \boldsymbol{p}_i$ (Stokes' drag, $\zeta = 3\pi d \eta_0 / m$)

Noise term
(satisfies fluctuation-dissipation theorem)
Temperature T_{env}

Langevin model for suspensions

Langevin equation: $\frac{d\boldsymbol{p}_i}{dt} = \sum_j \boldsymbol{F}_{ij} - \zeta \boldsymbol{p}_i + m\boldsymbol{\xi}_i$

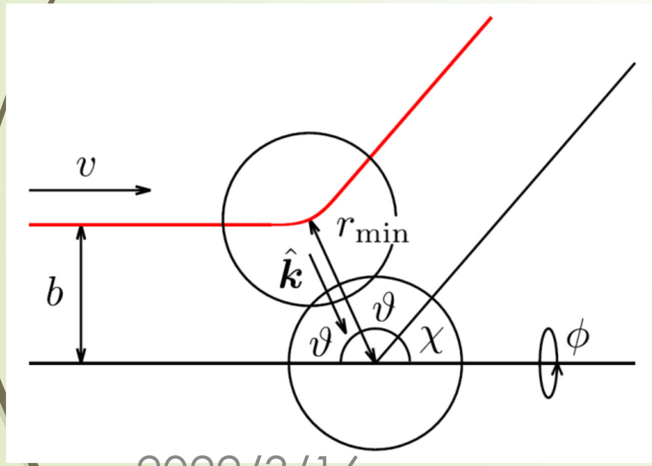
Boltzmann equation for the inertial suspension

$$\left(\frac{\partial}{\partial t} \boxed{- \dot{\gamma} V_y \frac{\partial}{\partial V_x}}\right) f(\boldsymbol{V}, t) = \boxed{\zeta \frac{\partial}{\partial \boldsymbol{V}} \cdot \left(\left[\boldsymbol{V} + \frac{T_{\mathrm{env}}}{m} \frac{\partial}{\partial \boldsymbol{V}}\right] f(\boldsymbol{V}, t)\right)} + \boxed{J(\boldsymbol{V}|f, f)}$$

shear — drag from the solvent — particle interaction

Collision integral: $J[\boldsymbol{V}|f, f] = \int d\boldsymbol{V}_2 \int d\widehat{\boldsymbol{k}} \sigma_{\mathrm{s}}(\chi, V_{12}) V_{12}$
$\times [f(\boldsymbol{r}_1, \boldsymbol{V}_1', t) f(\boldsymbol{r}_2, \boldsymbol{V}_2', t) - f(\boldsymbol{r}_1, \boldsymbol{V}_1, t) f(\boldsymbol{r}_2, \boldsymbol{V}_2, t)]$

$\sigma_{\mathrm{s}}(\chi, V_{12})$: collision cross section
determined from the scattering problem
Softness appears here.

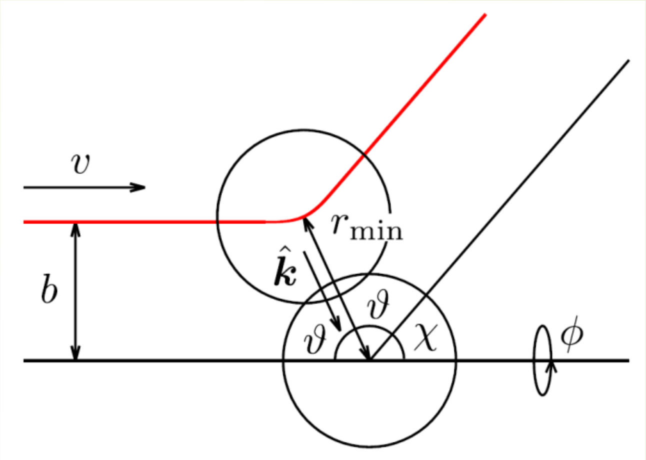


(Dimensionless) control parameters:
① Packing fraction: φ ② Shear rate: $\dot{\gamma}^* \equiv \dot{\gamma}/\zeta$
③ Particle softness: $\varepsilon^* \equiv \frac{\varepsilon}{m\sigma^2\zeta^2}$ ④ Env. temp.: $\xi_{\mathrm{env}} \equiv \sqrt{\frac{T_{\mathrm{env}}}{m}} \frac{1}{\zeta\sigma}$

Softness of particles

- Scattering analysis

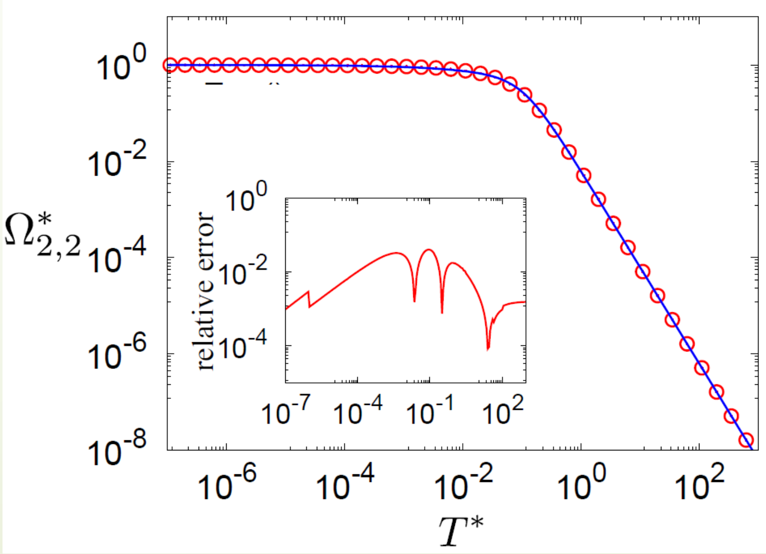
$$\theta = \int_0^{u_0} \frac{b\,du}{\sqrt{1-b^2u^2-\frac{4}{mv^2}U\left(\frac{1}{u}\right)}}$$
$$= \sin^{-1}\frac{b}{\sigma} + C_1F(\phi,\nu) + C_2\Pi(a;\phi|\nu) + C_3\tan^{-1}\gamma + C_4$$



Introduction of Omega integral:

$$\boxed{\Omega^*_{2,2}(T^*) \equiv \int_0^\infty dy\, y^7 e^{-y^2} \int_0^1 db^* b^* \sin^2\chi\,(b^*, 2y\sqrt{T^*})}$$

- The ratio of the coll. freq. of soft particles to that of hard-core particles
$$\Omega^*_{2,2} = \nu_{\mathrm{soft}}/\nu_{\mathrm{HC}}$$
- Low T: hard-core like ($\Omega^*_{2,2} \simeq 1$)
- High T: softer and softer ($\Omega^*_{2,2} \to 0$)



Enskog kinetic equation for the inertial suspension

$$\left(\frac{\partial}{\partial t} - \dot{\gamma} V_y \frac{\partial}{\partial V_x}\right) f(\boldsymbol{V}, t) = \zeta \frac{\partial}{\partial \boldsymbol{V}} \cdot \left(\left[\boldsymbol{V} + \frac{T_{\mathrm{env}}}{m} \frac{\partial}{\partial \boldsymbol{V}}\right] f(\boldsymbol{V}, t)\right) + J_{\mathrm{E}}(\boldsymbol{V}|f, f)$$

Evolution equation for the kinetic stress:

$$\frac{\partial}{\partial t} P_{\alpha\beta}^{k} + \dot{\gamma}\left(\delta_{\alpha x} P_{y\beta}^{k} + \delta_{\beta x} P_{y\alpha}^{k}\right) = -2\zeta\left(P_{\alpha\beta}^{k} - n T_{\mathrm{env}} \delta_{\alpha\beta}\right) - \Lambda_{\alpha\beta}$$

Kinetic stress: $P_{\alpha\beta}^{k} = m \int d\boldsymbol{V} V_\alpha V_\beta f(\boldsymbol{V}, t)$

Moment of the collision integral: $\Lambda_{\alpha\beta} = -m \int d\boldsymbol{V} V_\alpha V_\beta J(V|f, f)$

This equation is NOT closed!

Closure: Grad's moment method

$$f(\boldsymbol{V}) = f_{\mathrm{M}}(\boldsymbol{V})\left(1 + \frac{m}{2T}\left(\frac{P_{\alpha\beta}^{k}}{nT} - \delta_{\alpha\beta}\right) V_\alpha V_\beta\right)$$

Maxwellian distribution

$$f_{\mathrm{M}}(\boldsymbol{V}) = n\left(\frac{m}{2\pi T}\right)^{\frac{3}{2}} \exp\left(-\frac{mV^2}{2T}\right)$$

A set of dynamic equations:

$$\frac{d\theta}{d\tau} = -\frac{2}{3}\dot{\gamma}^*\Pi_{xy}^* + 2(1-\theta)$$

$$\frac{d\Delta\theta}{d\tau} = -2\dot{\gamma}^*\Pi_{xy}^* - (\nu^* + 2)\Delta\theta$$

$$\frac{d\Pi_{xy}^*}{d\tau} = \dot{\gamma}^*\left(\frac{1}{3}\Delta\theta - \theta\right) - (\nu^* + 2)\Pi_{xy}^*$$

Dimensionless quantities

$\theta = \frac{T}{T_{\mathrm{env}}}$: temperature

$\Delta\theta = \frac{\Delta T}{T_{\mathrm{env}}}$: anisotropic temperature

$\Pi_{xy}^* = \frac{P_{xy}^k}{nT_{\mathrm{env}}}$: kinetic shear stress

$$\nu^* = \frac{95}{5\sqrt{\pi}}\Omega_{2,2}^*\varphi\xi_{\mathrm{env}}\sqrt{\theta}$$

Then, the steady rheology is described by

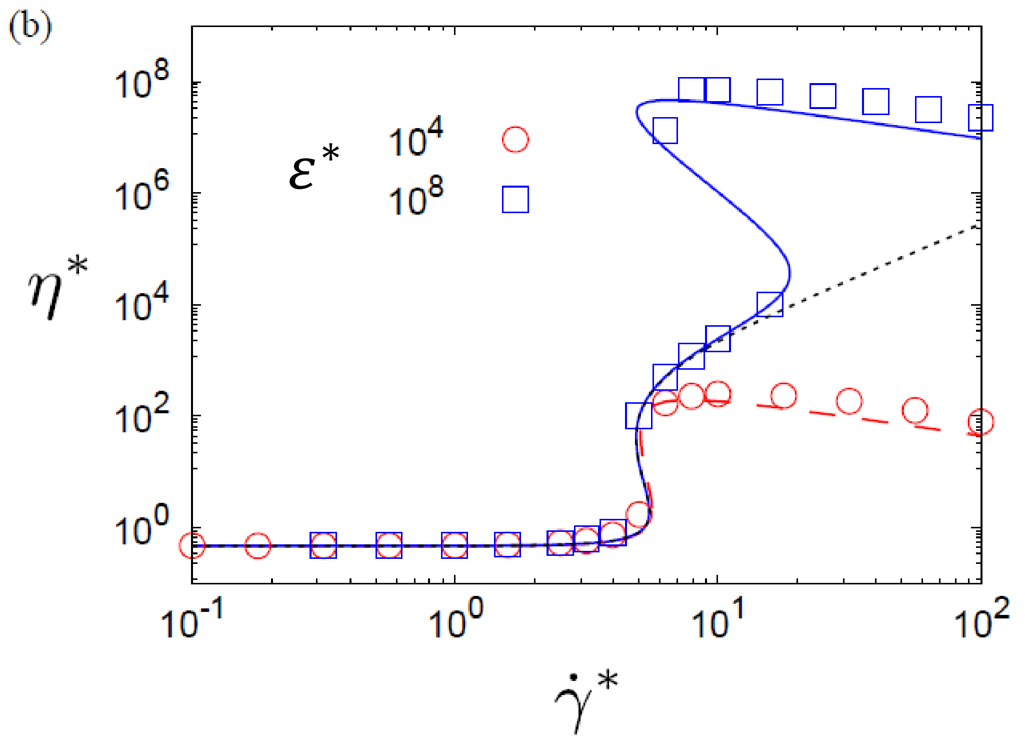
$$\dot{\gamma}^* = (\nu^* + 2)\sqrt{\frac{3(\theta-1)}{\nu^*\theta + 2}}$$

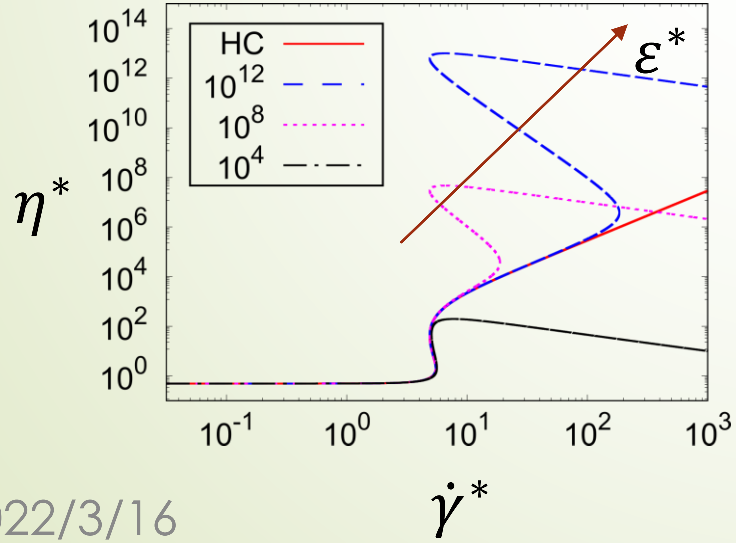
$$\eta^* = -\frac{\Pi_{xy}^*}{\dot{\gamma}} = \frac{\nu^*\theta + 2}{(\nu^* + 2)^2}$$

All quantities are written as a function of the temperature.

Steady rheology

- **DST-like behavior**
  even for soft system
- **2 step** DST-like behaviors
  for harder system
- **Shear thinning** in high shear regime
- Kinetic theory reproduces
  the sim. results.





- "Does this behavior survive even
  in denser situations?"

Extension to denser system

Enskog kinetic equation for the inertial suspension

$$\left(\frac{\partial}{\partial t} - \boxed{\dot\gamma V_y \frac{\partial}{\partial V_x}}\right) f(\boldsymbol{V},t) = \boxed{\zeta \frac{\partial}{\partial \boldsymbol{V}} \cdot \left(\left[\boldsymbol{V} + \frac{T_{\rm env}}{m}\frac{\partial}{\partial \boldsymbol{V}}\right] f(\boldsymbol{V},t)\right)} + \boxed{J_{\rm E}(\boldsymbol{V}|f,f)}$$

shear　　drag from the solvent　　particle interaction

Collision integral: $J_{\rm E}[\boldsymbol{V}|f,f] = \int d\boldsymbol{V}_2 \int d\hat{\boldsymbol{k}} \sigma_{\rm s}(\chi, V_{12}) V_{12} \left[f^{(2)}(\boldsymbol{r}, \boldsymbol{r} + r_{\rm min}\hat{\boldsymbol{k}}, \boldsymbol{V}_1'', \boldsymbol{V}_2'', t) - f^{(2)}(\boldsymbol{r}, \boldsymbol{r} - r_{\rm min}\hat{\boldsymbol{k}}, \boldsymbol{V}_1, \boldsymbol{V}_2, t)\right]$

$\sigma_{\rm s}(\chi, V_{12})$: collision cross section, $f^{(2)}(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{V}_1, \boldsymbol{V}_2, t) \simeq g_0 f(r_1, V_1, t) f(r_2, V_2, t)$: decoupling approx.

Closure: Grad's moment method

$$f(\boldsymbol{V}) = f_{\rm M}(\boldsymbol{V})\left(1 + \frac{m}{2T}\left(\frac{P^k_{\alpha\beta}}{nT} - \delta_{\alpha\beta}\right) V_\alpha V_\beta\right)$$

Maxwellian distribution

$$f_{\rm M}(\boldsymbol{V}) = n\left(\frac{m}{2\pi T}\right)^{\frac{3}{2}} \exp\left(-\frac{mV^2}{2T}\right)$$

$$\frac{\partial \theta}{\partial \tau} = -\frac{2}{3}\dot\gamma^* \Pi^*_{xy} + 2(1-\theta) - \frac{1}{3}\Lambda^*_{\alpha\alpha},$$

$$\frac{\partial \Delta\theta}{\partial \tau} = -2\dot\gamma^* \Pi^*_{xy} - 2\Delta\theta - \delta\Lambda^*_{xx} + \delta\Lambda^*_{yy},$$

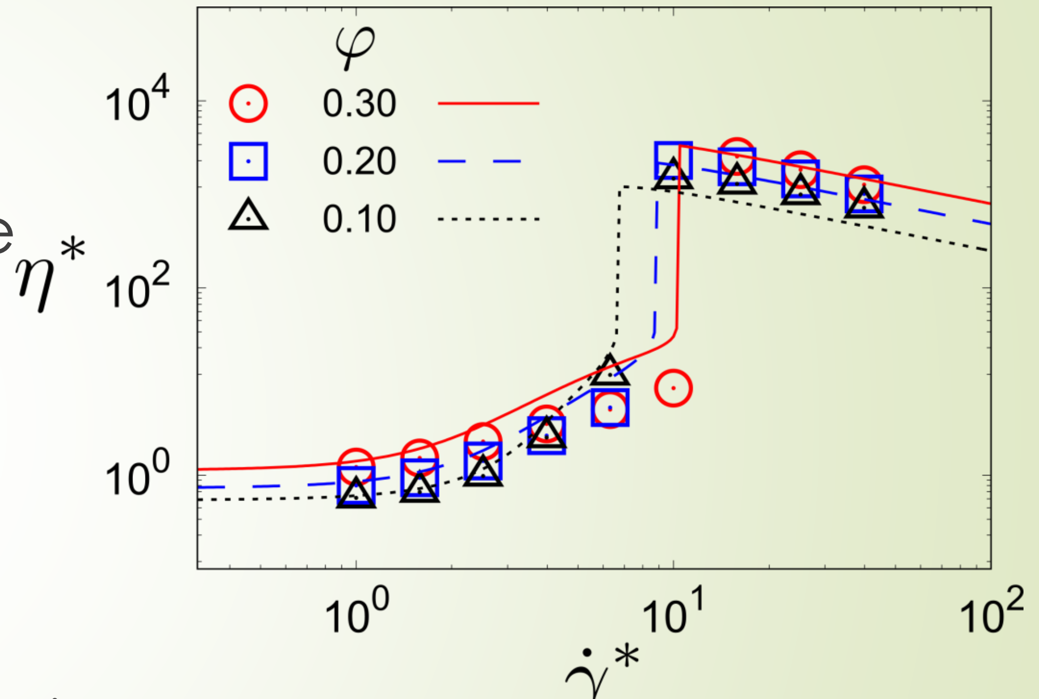
$$\frac{\partial \delta\theta}{\partial \tau} = -2\dot\gamma^* \Pi^*_{xy} - 2\delta\theta - 2\delta\Lambda^*_{xx} - \delta\Lambda^*_{yy},$$

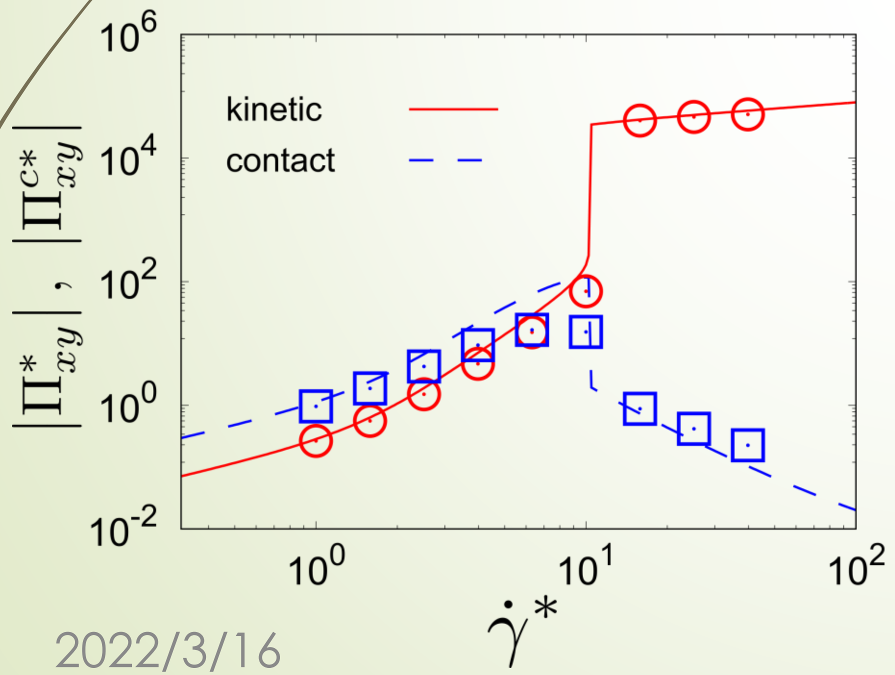
$$\frac{\partial \Pi^*_{xy}}{\partial \tau} = -\dot\gamma^*(\theta + \Pi^*_{yy}) - 2\Pi^*_{xy} - \Lambda^*_{xy},$$

Denser system

Parameters:
$\varphi = 0.10, 0.20, 0.30$
$\varepsilon^* = 10^4, \xi_{\mathrm{env}} = 1.0$

- **DST-like behavior survives** even for finite density.
  (⇔ CST-like for hard-core system)
- **Shear thinning** in high shear regime
- Kinetic theory reproduces the sim. results.





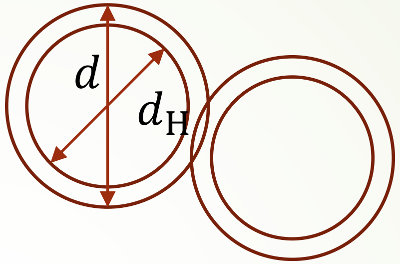
For large $\dot{\gamma}$,
Kinetic contribution ≫ Contact contribution

$$\left(P^k_{xy} = \frac{1}{V}\sum_i m v_{i,x} v_{i,y} \gg P^c_{xy} = \frac{1}{V}\sum_i \sum_{j\neq i} x_{ij} f_{ij,y} \right)$$

⇒ Inertia plays a role in this DST-like behavior.
(= cannot be ignored)

Stokes' + lubrication model

- More realistic hydrodynamic interaction
  Stokes' + lubrication model:
  $\boldsymbol{F}_i^{\mathrm{H}} = -\sum_j \overleftrightarrow{\zeta_{ij}} \boldsymbol{p}_j$
  $\overleftrightarrow{\zeta_{ij}}$ has nondiagonal components



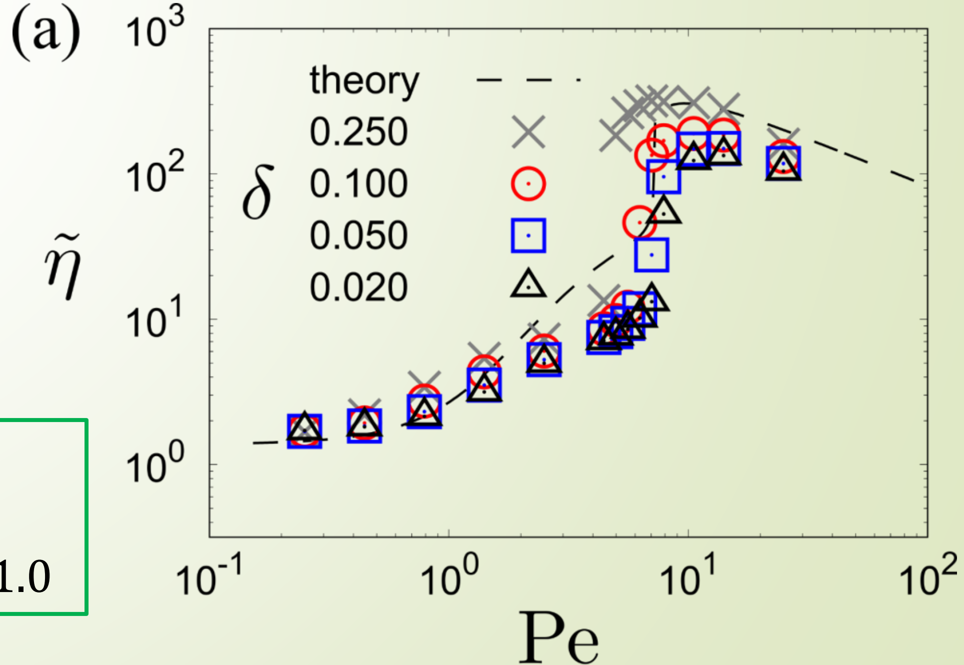
$$\zeta_{ij,\alpha\beta} = \begin{cases} \frac{3\pi d\eta_0}{m}\delta_{\alpha\beta} + \sum_{k\neq i}\frac{1}{m}A^{(1,1)}_{ik,\alpha\beta}\Theta(r_c - r_{ik}) & (i=j) \\ -\frac{1}{m}A^{(1,1)}_{ij,\alpha\beta}\Theta(r_c - r_{ij}) & (i\neq j) \end{cases}$$

$A^{(1,1)}_{ij,\alpha\beta}$: function of $\hat{\boldsymbol{k}} \equiv \boldsymbol{r}_{ij}/|\boldsymbol{r}_{ij}|$ ($\boldsymbol{r}_{ij} \equiv \boldsymbol{r}_j - \boldsymbol{r}_i$)
(☞ Kim & Karrila, "Microhydrodynamics")
$r_c \equiv d_{\mathrm{H}} + \lambda$: cutoff length ($\lambda = 0.25d$)

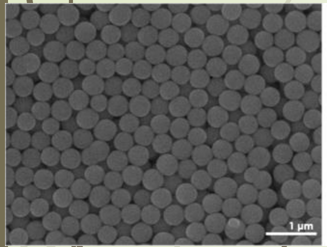
- Scaled viscosity $\tilde{\eta} \equiv \eta/\eta_1$ against the Peclet number $\mathrm{Pe} \equiv \frac{3\pi\eta_0 d^3}{4T_{\mathrm{env}}}\dot{\gamma}$
  $\eta \equiv P_{xy}/\dot{\gamma}, \eta_1 = \eta_0\left(1 + \frac{5}{2}\varphi + 4\varphi^2 + 42\varphi^3\right)$:
  η_1: Empirical expression of
  the apparent viscosity in the low shear limit

- **DST** occurs at $\mathrm{Pe} \simeq 10$.

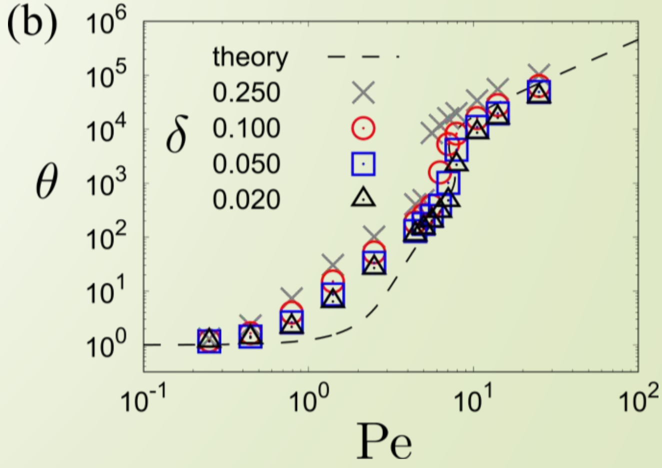
Parameters:
$\varphi = 0.30$
$\varepsilon^* = 10^4, \xi_{\mathrm{env}} = 1.0$



Discussion: Estimation

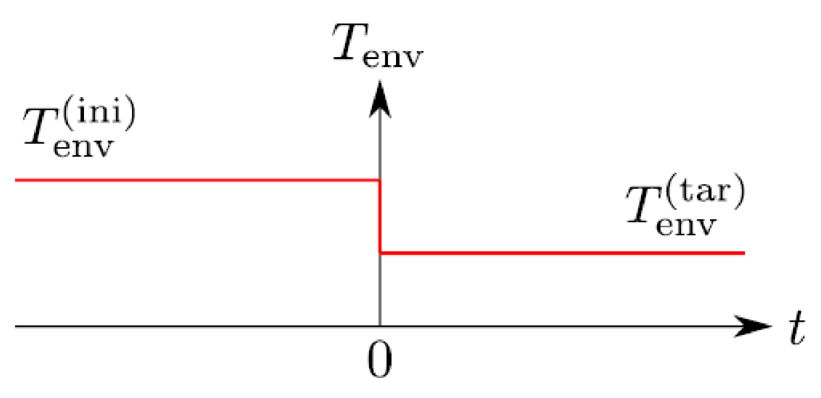


- Aerosol
  $d \sim 10^{-5}\ \mathrm{m}, \rho \sim 1\ \mathrm{g/cm^3}, E \sim 10\ \mathrm{GPa} \Rightarrow m \sim 10^{-12}\ \mathrm{kg}$
  Viscosity of air: $\eta_0 \sim 10^{-5}\ \mathrm{Pa \cdot s}$
  DST takes place at $\dot{\gamma}_\mathrm{c} \sim 10^3\ 1/\mathrm{s} \Rightarrow$ shear speed $10\ \mathrm{m/s}$ if $L = 1\ \mathrm{cm}$
- Colloid
  $d \sim 10^{-6}\ \mathrm{m}, \rho \sim 1\ \mathrm{g/cm^3}, E \sim 1\ \mathrm{GPa} \Rightarrow m \sim 10^{-14}\ \mathrm{kg}$
  Viscosity of water: $\eta_0 \sim 10^{-3}\ \mathrm{Pa \cdot s}$
  DST takes place at $\dot{\gamma}_\mathrm{c} \sim 10^4\ 1/\mathrm{s} \Rightarrow$ shear speed $10^4\ \mathrm{m/s}$ if $L = 1\ \mathrm{cm}$
- Kinetic temperature becomes 10^2 times larger.
  Is it possible to achieve this?
  $\Rightarrow$ This will open for all researchers.



Case for nonlinear drag

- Question:
  What happens if the drag coefficient has velocity dependence?
- By solving scattering problem, the drag coeff. becomes $\zeta(v) = \zeta_0(1 + \gamma v^2)$.
- Change the env. Temperature at $t = 0$.
- No external force



Velocity distribution func. deviates from the Maxwellian in the relaxation process.

Detect this deviation in terms of a_2

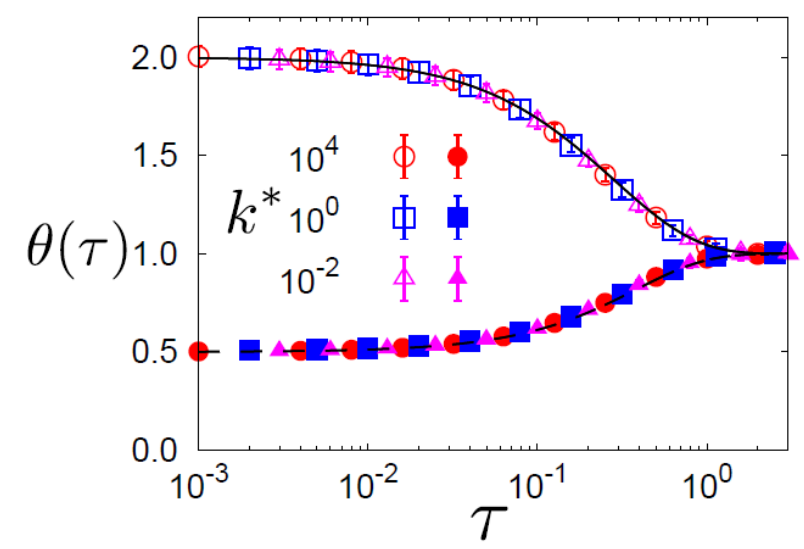
$$\tilde{f}(\boldsymbol{c}) = \tilde{f}_{\mathrm{M}}(\boldsymbol{c})\left(1 + a_2\left(\frac{1}{2}c^4 - \frac{5}{2}c^2 + \frac{15}{8}\right)\right), \tilde{f}(\boldsymbol{c}) = \frac{v_{\mathrm{T}}^3}{n} f(\boldsymbol{v}), c \equiv \frac{v}{v_{\mathrm{T}}}, v_{\mathrm{T}} \equiv \sqrt{\frac{2T}{m}}$$

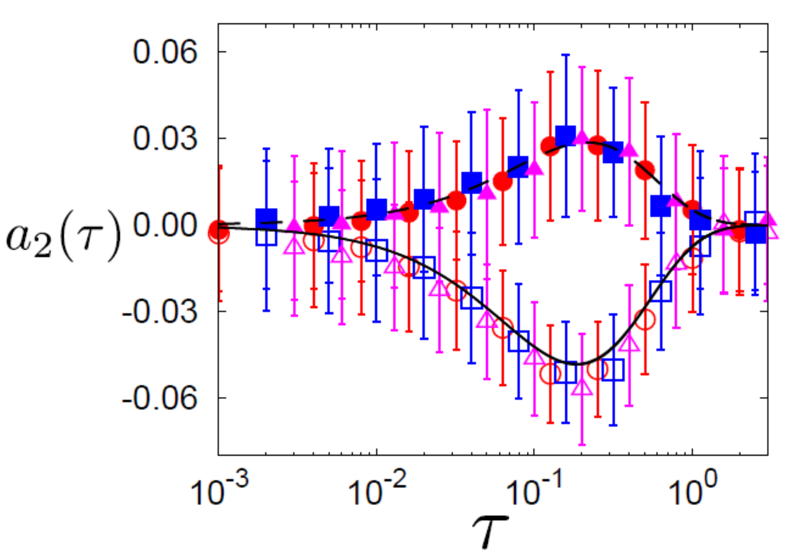
$$\frac{\partial}{\partial t} f(\boldsymbol{v}) - \frac{\partial}{\partial \boldsymbol{v}} \cdot \left[\zeta(v) \left(\boldsymbol{v} + \frac{k_B T_{\rm env}}{m} \right) f(\boldsymbol{v}) \right] = J[\boldsymbol{v}|f, f],$$

$$\frac{d\theta}{d\tau} = -2(\theta - 1)(1 + 5\gamma\theta) - 10\theta^2 a_2,$$

$$\frac{da_2}{d\tau} = -8\gamma(\theta - 1) - \left[\frac{4}{\theta} - 8\gamma + 44\gamma\theta + \frac{64}{5\sqrt{\pi}} \varphi \xi_{\rm env} \sqrt{\theta} \Omega^*_{2,2} \left(\frac{\theta}{k^*} \right) \right] a_2.$$

$$\theta \equiv \frac{T}{T_{\rm env}^{\rm (tar)}}, \quad \theta_0 \equiv \frac{T_{\rm env}^{\rm (ini)}}{T_{\rm env}^{\rm (tar)}}, \quad \tau \equiv \zeta_0 t.$$





Deviation has a peak at a certain time.
Softness is not important in the relaxation process.
Good agreement with the simulation.

Extension to granular system

- Our theory itself can treat only elastic collision.
- Assumption: collisions occur instantaneously. (Hard-core like)

$$\boxed{\begin{aligned} \boldsymbol{v}_1' &= \boldsymbol{v}_1 - \frac{1+e}{2}(\boldsymbol{v}_{12}\cdot\hat{\boldsymbol{k}})\hat{\boldsymbol{k}} \\ \boldsymbol{v}_2' &= \boldsymbol{v}_2 + \frac{1+e}{2}(\boldsymbol{v}_{12}\cdot\hat{\boldsymbol{k}})\hat{\boldsymbol{k}} \end{aligned}}$$

Boltzmann equation

$$\left(\frac{\partial}{\partial t} - \dot{\gamma}V_y\frac{\partial}{\partial V_x}\right) f(\boldsymbol{V},t) = J(\boldsymbol{V}|f,f)$$

Softness

Collision integral: $J[\boldsymbol{V}|f,f] = \int d\boldsymbol{V}_2 \int d\hat{\boldsymbol{k}}\sigma_s(\chi, V_{12})V_{12}$

$$\times\left[\frac{1}{e^2}f(\boldsymbol{r}_1,\boldsymbol{V}_1',t)f(\boldsymbol{r}_2,\boldsymbol{V}_2',t) - f(\boldsymbol{r}_1,\boldsymbol{V}_1,t)f(\boldsymbol{r}_2,\boldsymbol{V}_2,t)\right]$$

Hard-core like treatment

Boltzmann equation

$$\left(\frac{\partial}{\partial t} - \dot{\gamma} V_y \frac{\partial}{\partial V_x}\right) f(\boldsymbol{V}, t) = J(\boldsymbol{V}|f, f)$$

Evolution equation for the kinetic stress:

$$\frac{\partial}{\partial t} P^k_{\alpha\beta} + \dot{\gamma}\left(\delta_{\alpha x} P^k_{y\beta} + \delta_{\beta x} P^k_{y\alpha}\right) = -\nu\left(P^k_{\alpha\beta} - nT\delta_{\alpha\beta}\right) - \lambda n T \delta_{\alpha\beta}$$

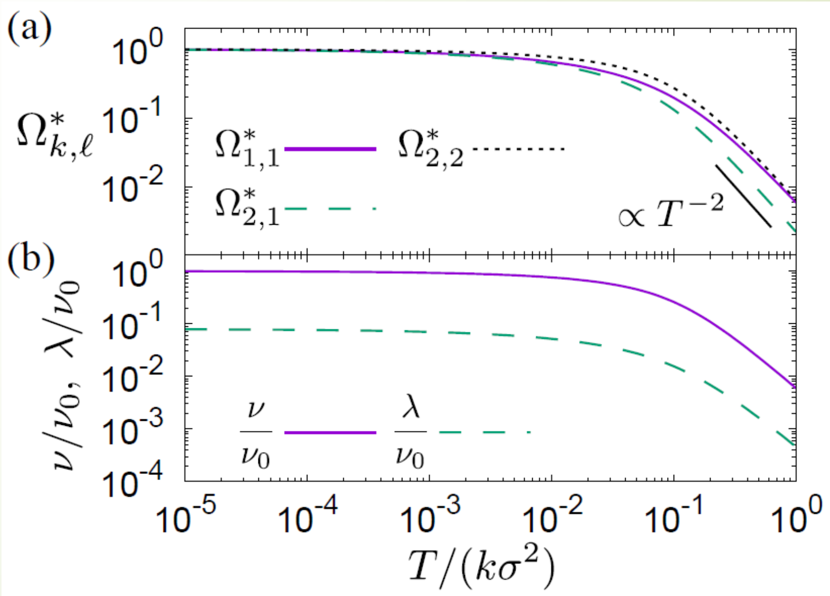
Kinetic stress: $P^k_{\alpha\beta} = m\int d\boldsymbol{V} V_\alpha V_\beta f(\boldsymbol{V}, t)$

$$\frac{\lambda}{\nu_0} = \frac{5}{12}(1 - e^2)\Omega^*_{1,1}(T^*),$$

$$\frac{\nu}{\nu_0} = \frac{1+e}{4}[2(1-e)\Omega^*_{2,1}(T^*) + (1+e)\Omega^*_{2,2}(T^*)],$$

$$\nu_0 = \frac{16}{5} n\sigma^2 \sqrt{\frac{\pi T}{m}},$$

$$\Omega^*_{k,\ell} = C_{k,\ell} \int_0^\infty dy\, y^{2k+3} e^{-y^2} \int_0^1 db^*\, b^* \times \{1 - (-1)^\ell \cos^\ell [2\theta(b^*, 2y\sqrt{T^*})]\},$$



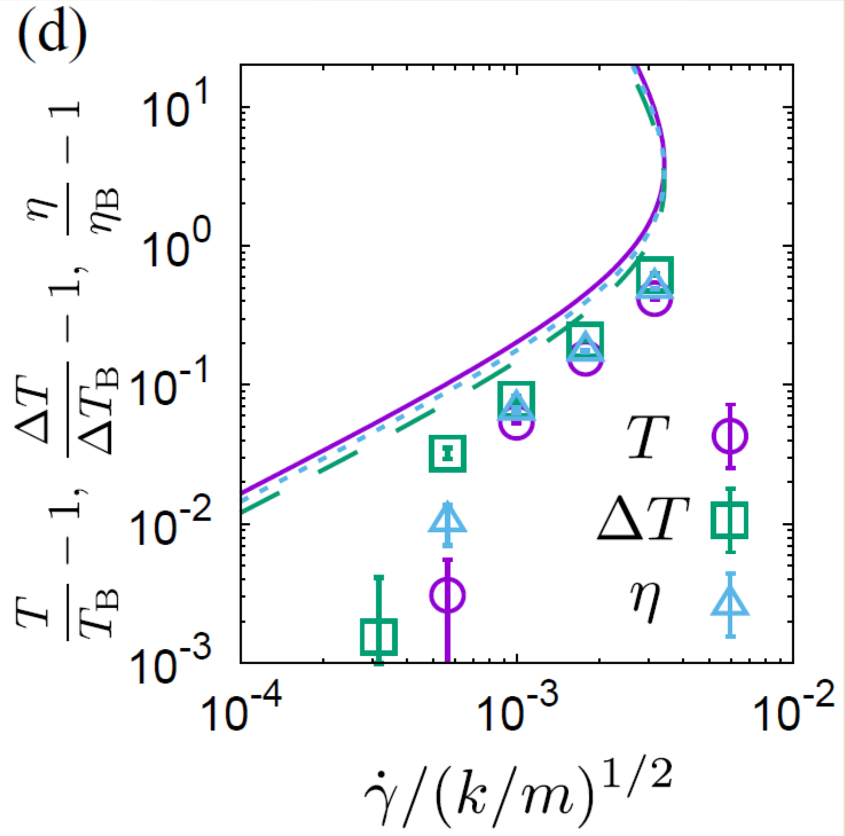
Results:

- Deviations from the Bagnold scaling

$$T_{\mathrm{B}} = \frac{5\pi(2+e)}{432(1-e)(1+e)^2(3-e)^2}\frac{1}{\varphi}m\sigma^2\dot{\gamma}^2,$$

$$\Delta T_{\mathrm{B}} = \frac{25\pi(2+e)}{432(1+e)^2(3-e)^3}\frac{1}{\varphi^2}m\sigma^2\dot{\gamma}^2,$$

$$\eta_{\mathrm{B}} = \frac{5(2+e)}{72(1+e)^2(3-e)^3}\sqrt{\frac{5(2+e)}{3(1-e)}\frac{1}{\varphi}\frac{m}{\sigma}}\dot{\gamma}.$$



- Agreement with simulations for $\dot{\gamma}^* \lesssim 10^{-3}$
- No solution for $\dot{\gamma}^* \gtrsim 4\times 10^{-3}$
  Assumption does not hold for larger $\dot{\gamma}$
  Any idea? ⇒ No.

Summary

- We have developed the kinetic theory of soft-core suspension and granular gases.
- Suspension:
  - Softness induced DST-like behaviors for frictionless system
  - DST-like behaviors can occur twice.
  - Second DST-like behavior survives even for finite density.
  - Deviation from the Maxwellian
    if the drag coeff. has vel. dependence.
- Granular gas:
  - Deviation from the Bagnold scaling
  - No solution for larger shear rate