

# Running on liquid: Impact-induced hardening in dense suspensions

Y25.00002

Pradipto and H. Hayakawa, Phys. Rev. Fluids **6**, 033301 (2021).

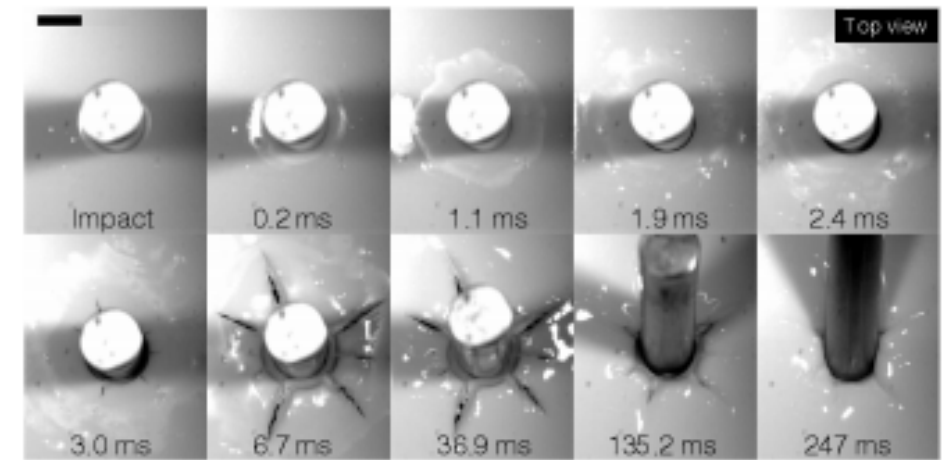
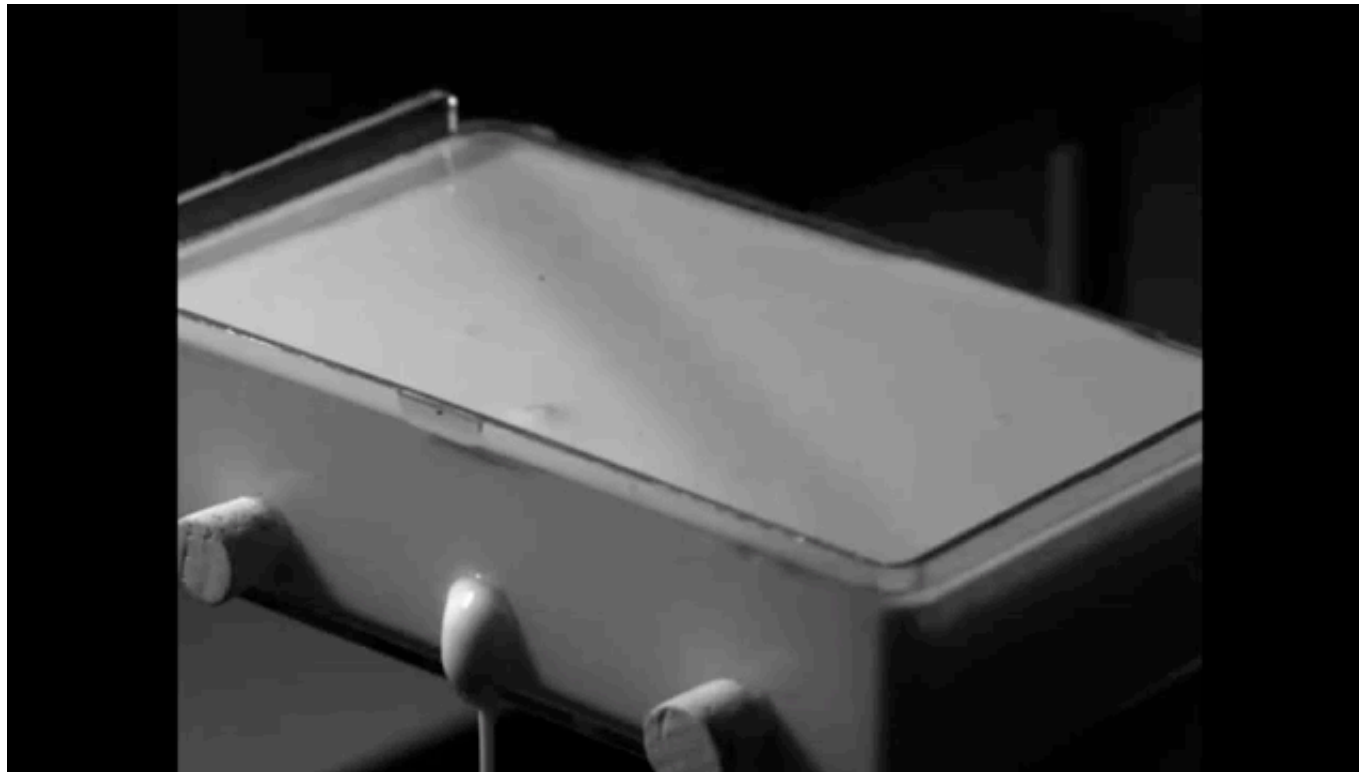
Pradipto and H. Hayakawa, Phys. Fluids **33**, 093110 (2021).

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**APS March meeting 2022**

**Y25 CONSTRAINT-BASED RHEOLOGY OF DENSE SUSPENSIONS AND GRANULAR MATERIALS II**

# Dense suspensions under impact



**Fracture** Roche et al, PRL 2013

Source: Itai Cohen Group on YouTube <https://youtu.be/hP88C-LgnE>

## Impact-induced hardening

Occurs on hard particles  
suspended in Newtonian solvent

Cannot be observed in fluids or  
granular particles alone

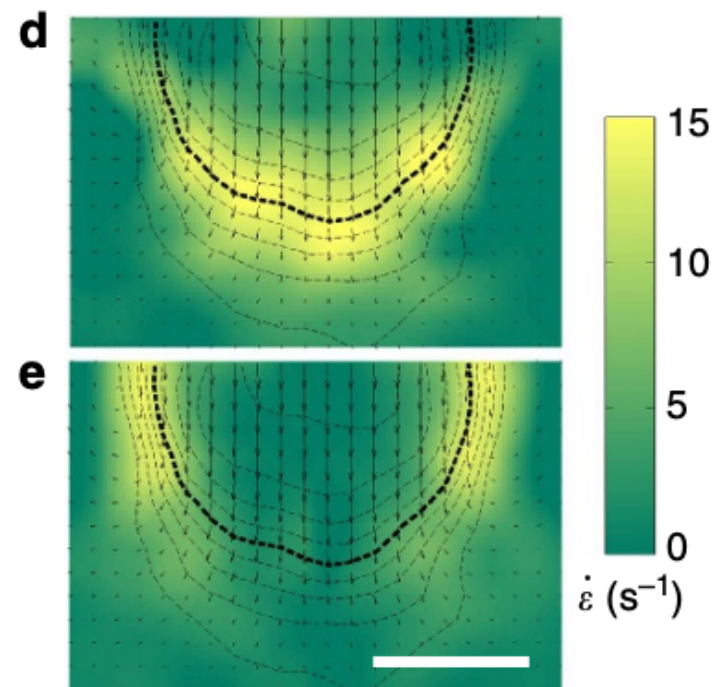
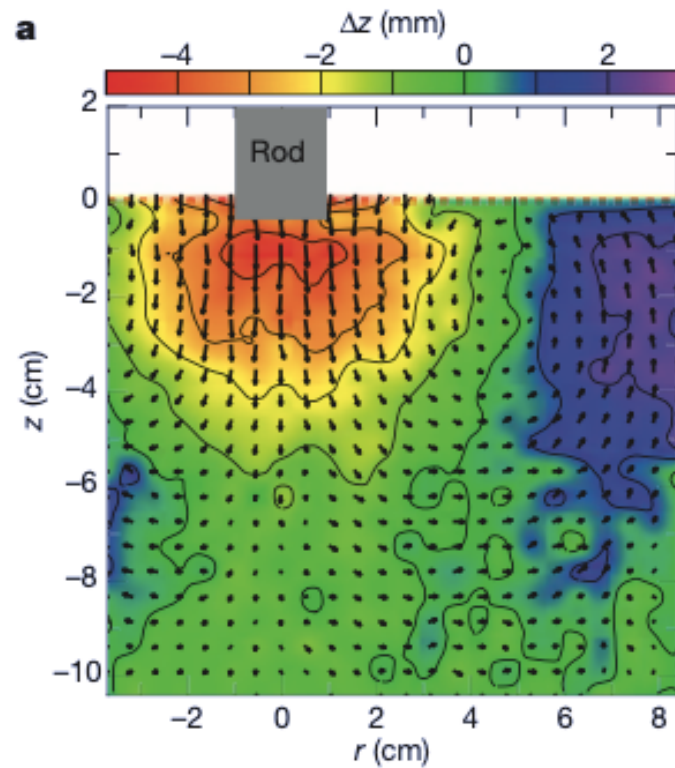
A person can run on top of  
dense suspensions

Physical explanations remain  
elusive

- Inherently far-from-equilibrium
- Highly dissipative —> transient
- **Can shear thickening (CST and DST) explain such strong solidification?**

# Dense suspensions under impact have stronger response than the one observed in shear thickening suspensions

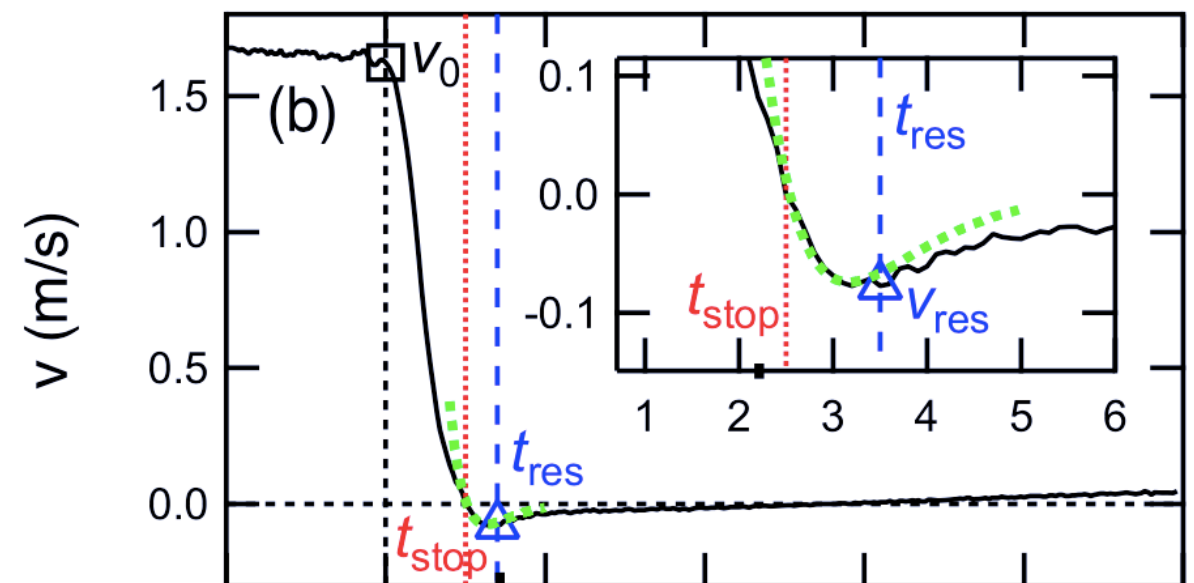
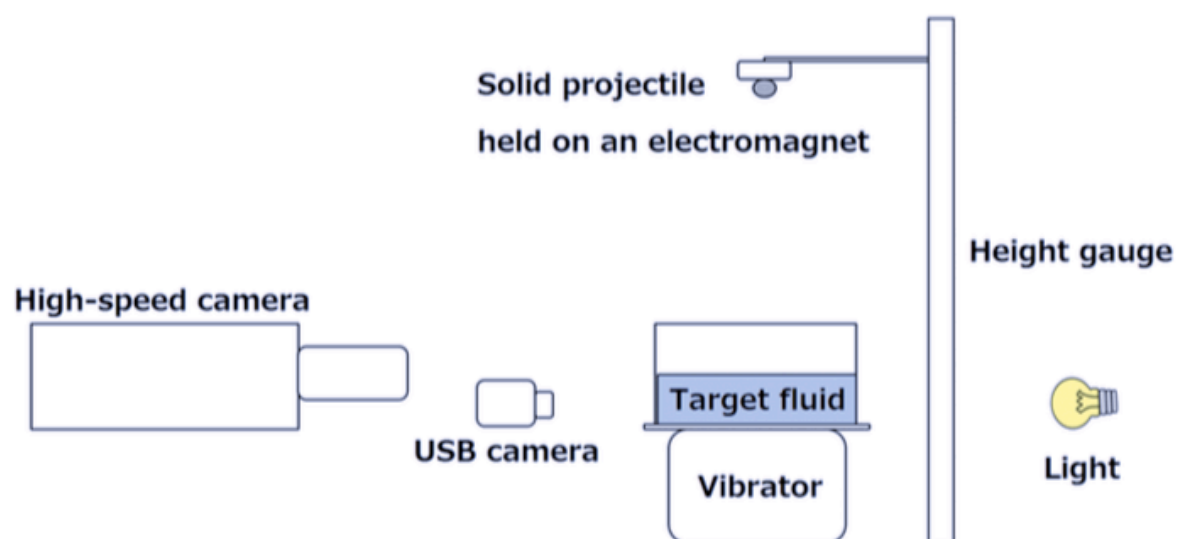
Waitukaitis and Jaeger, Nature **487**, 205 (2012) Han et al, Nat. Comms. 2016



## Dynamically Jammed Region

Localized and transient solid-like region beneath the impactor

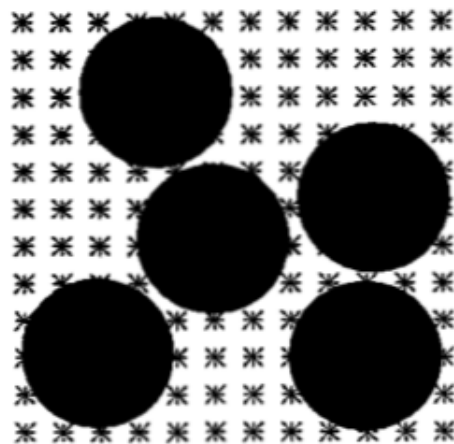
## Elastic rebound Egawa and Katsuragi, Phys. Fluids **31**, 053304 (2019)



# Ingredients of our simulation

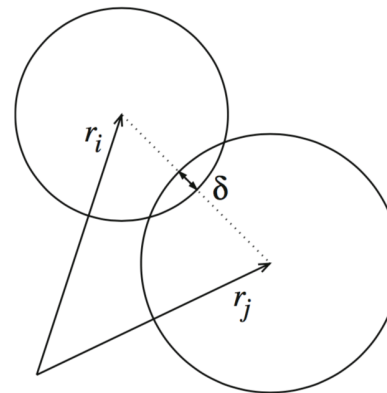
## Hydrodynamic interaction

LBM + Lubrication corrections



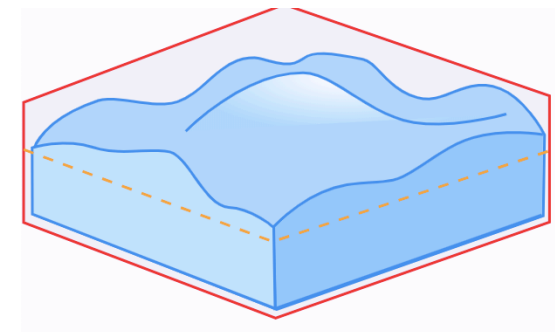
## Contact between particles

DEM

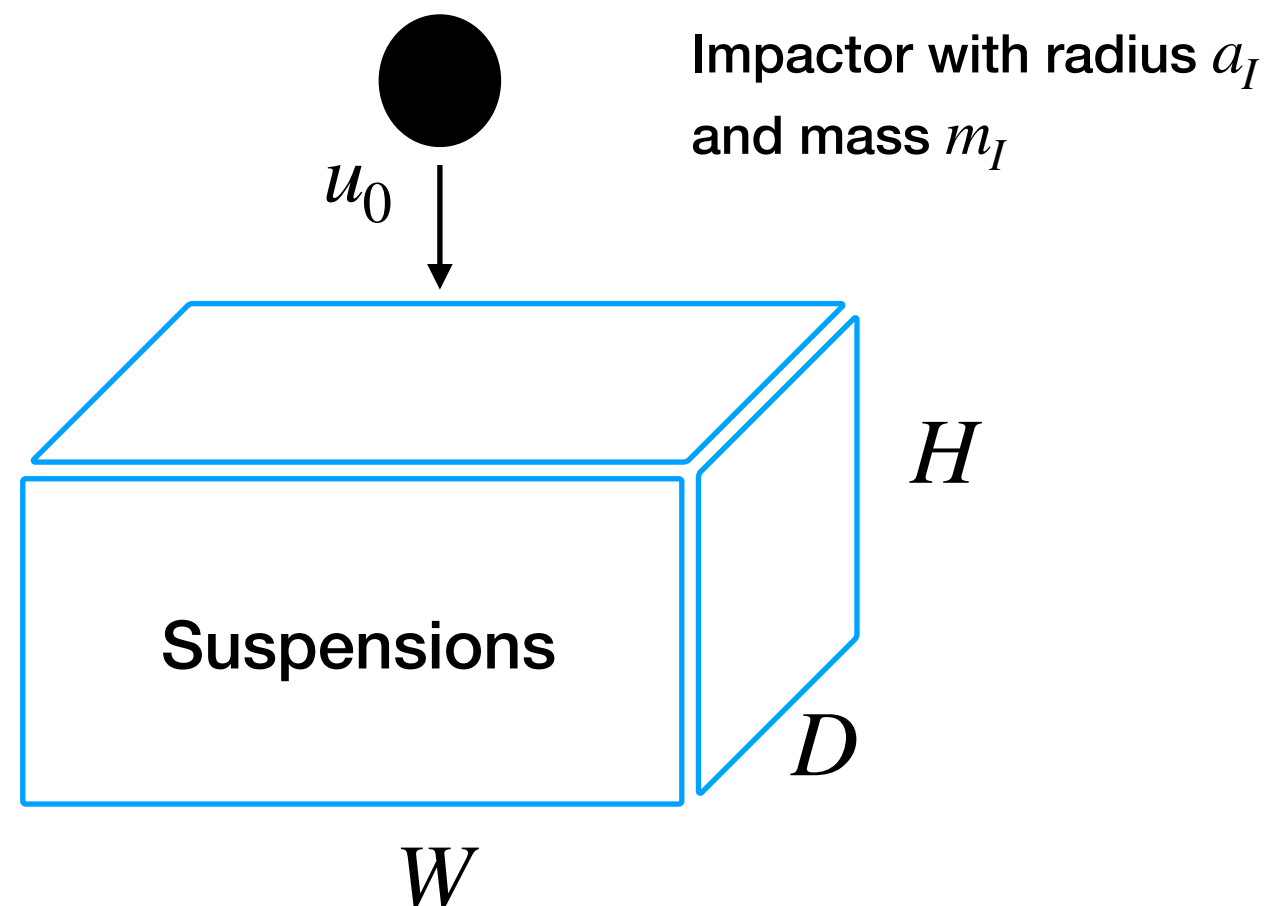


## Free surface

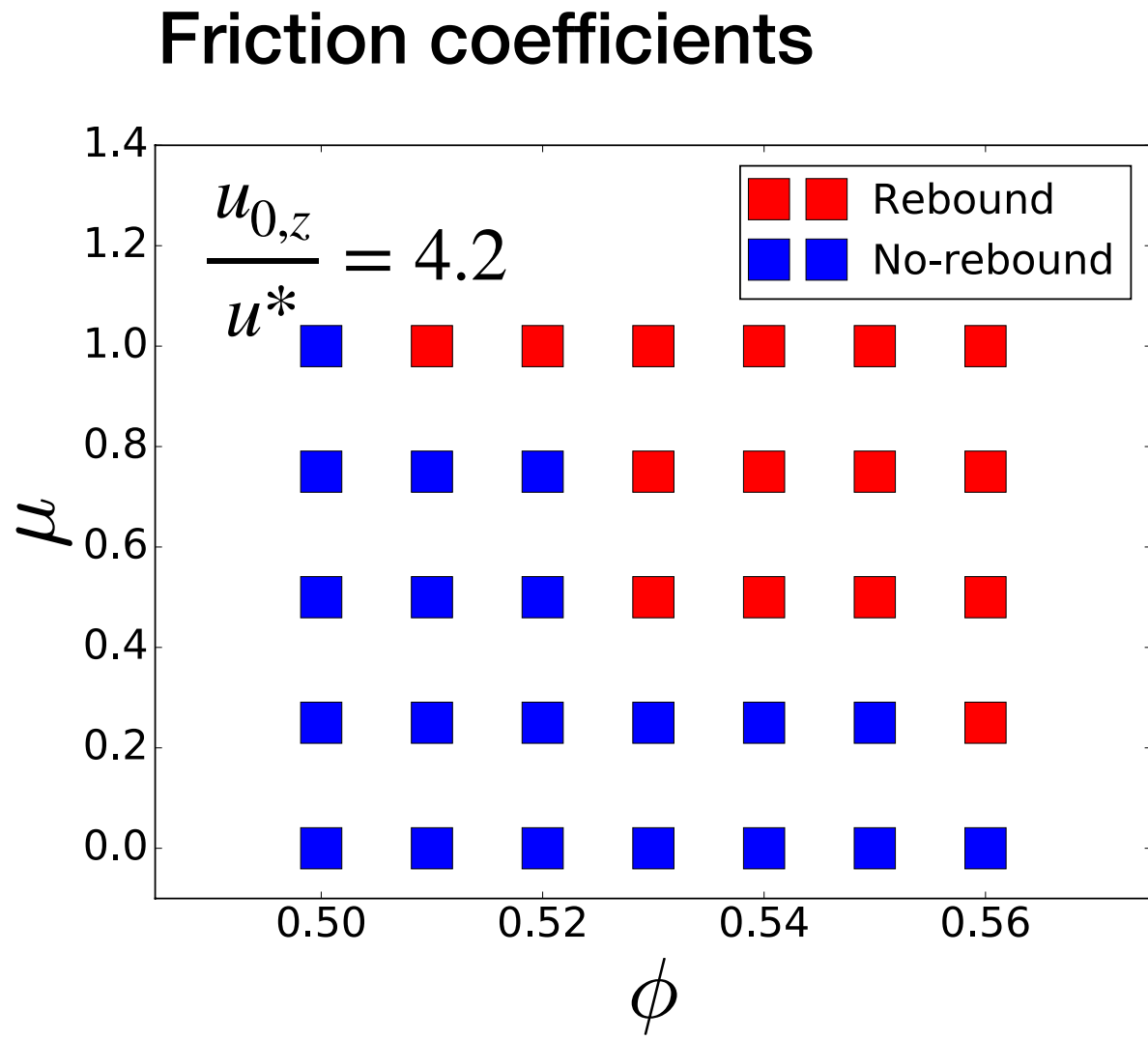
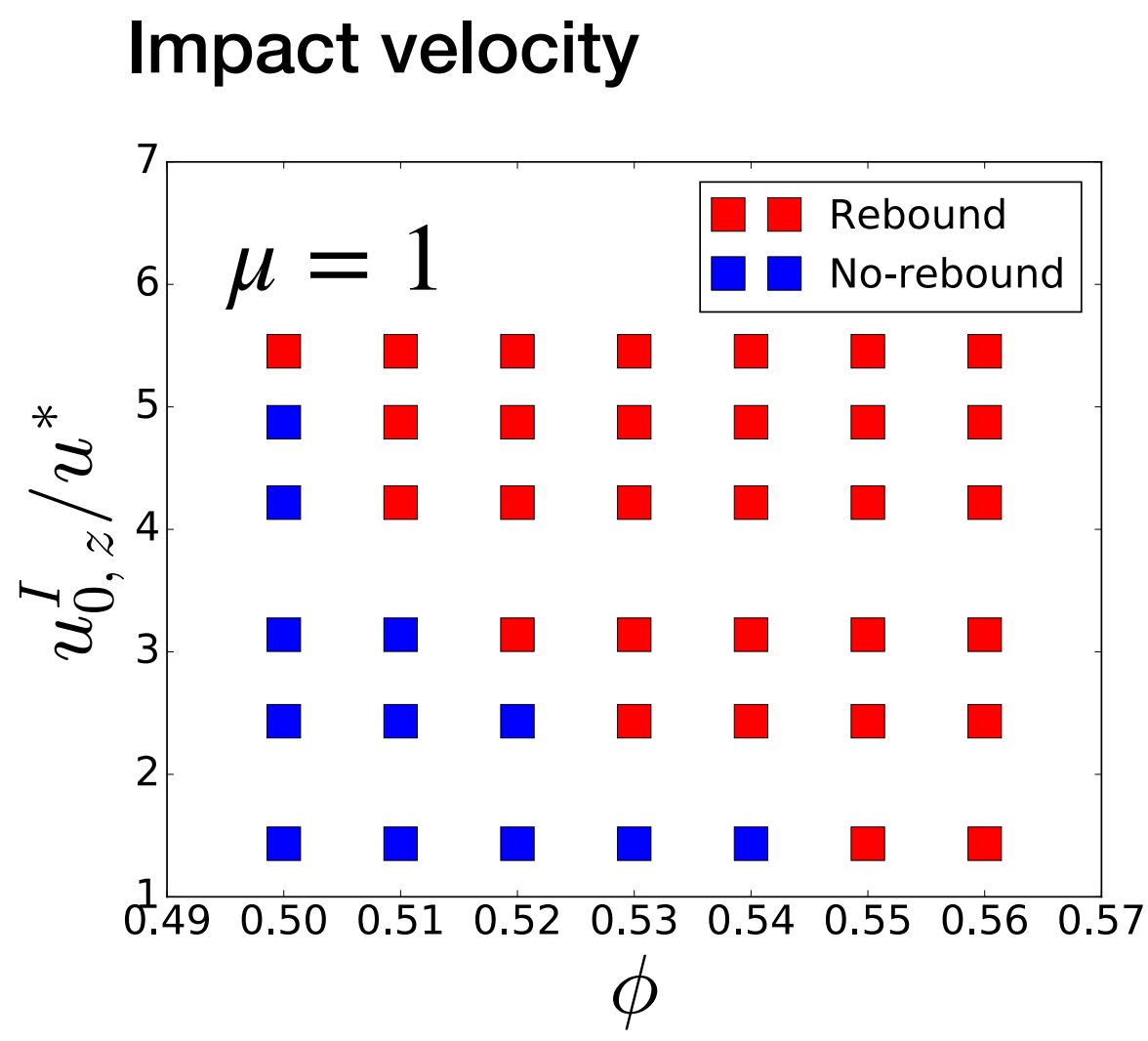
LBM



## Setup



*Rebound depends on impact velocity and frictional interactions between particles*



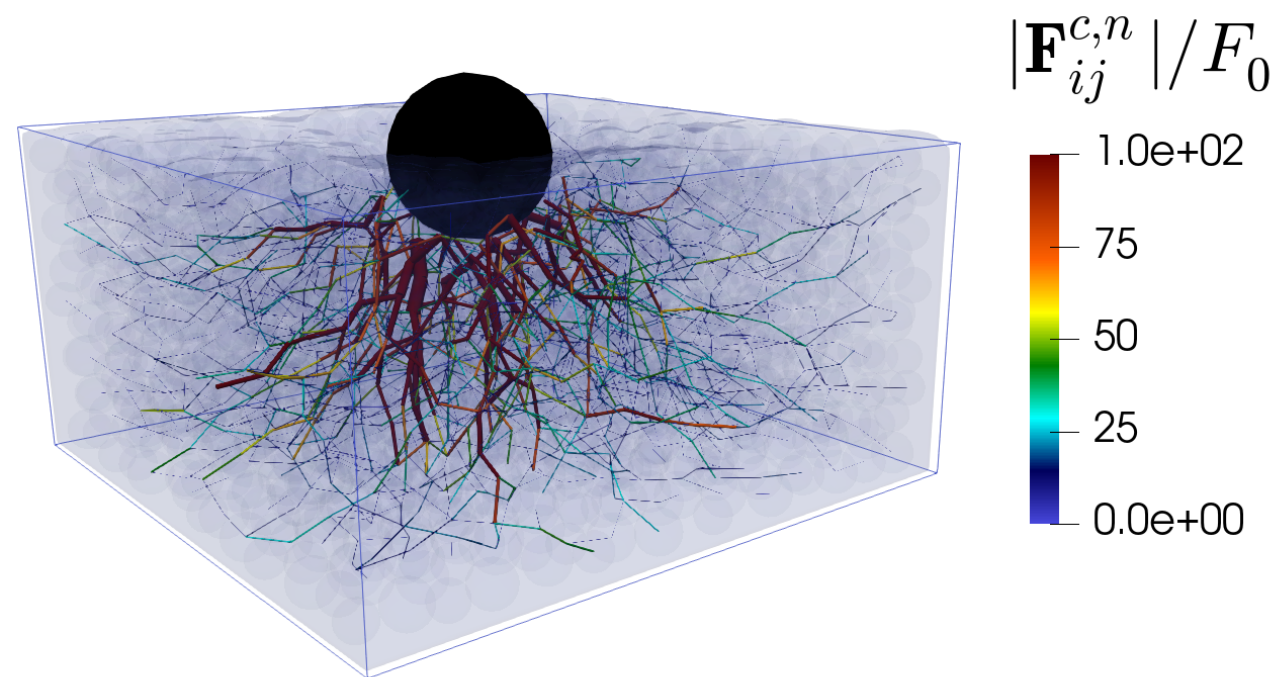
$$\phi = \frac{\text{Total solid particles volume}}{\text{Volume of the container}}$$

We can run on top suspension but we'll sink if we walk

Frictional interaction increases the contact duration between particles that leads to a stronger hardening



## Inside the hardening suspension

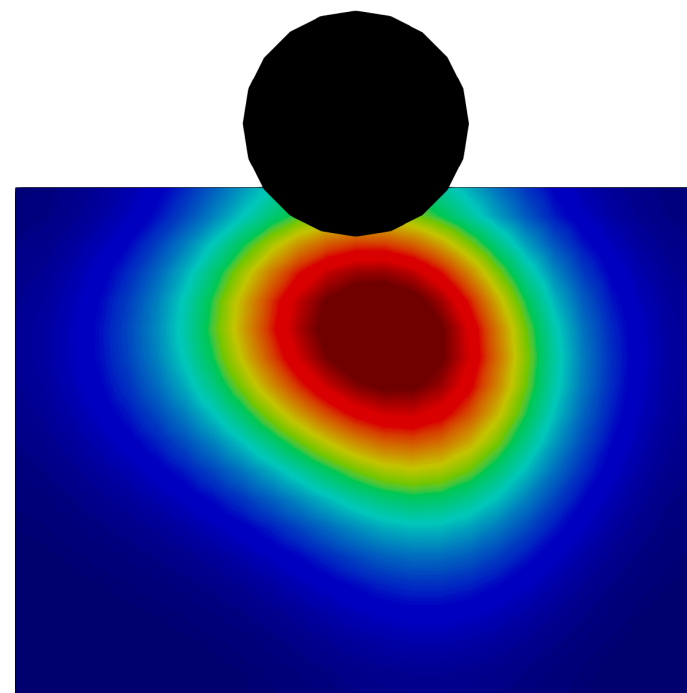


## Force chains

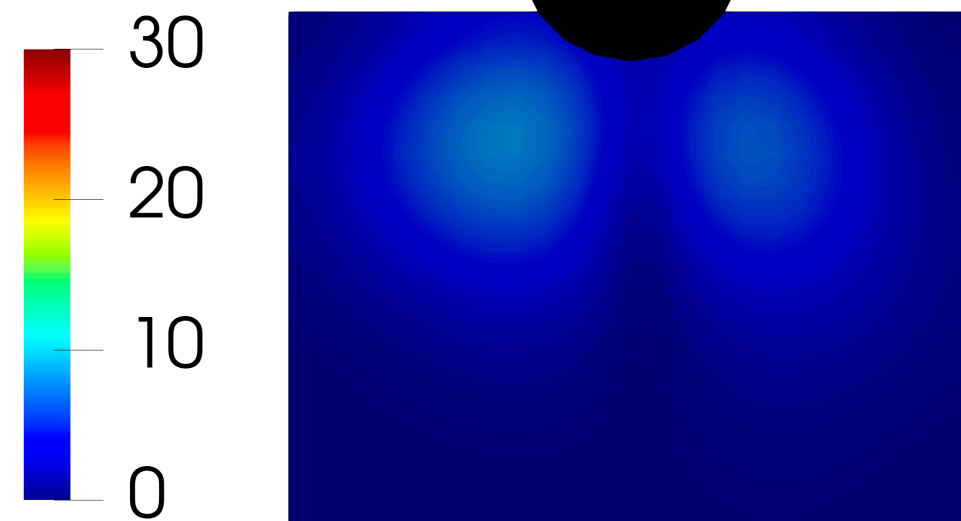
- Formed by contacting suspended particles
- Transmitting force from the impactor to the bottom boundary
- Sustain the impactor  $\rightarrow$  rebound

Coarse-grained fields J. Zhang, R. P. Behringer, I. Goldhirsch, Prog. Theor. Phys. Supp **184**, 16 (2010)

Stress in normal direction  $\sigma_{zz}$

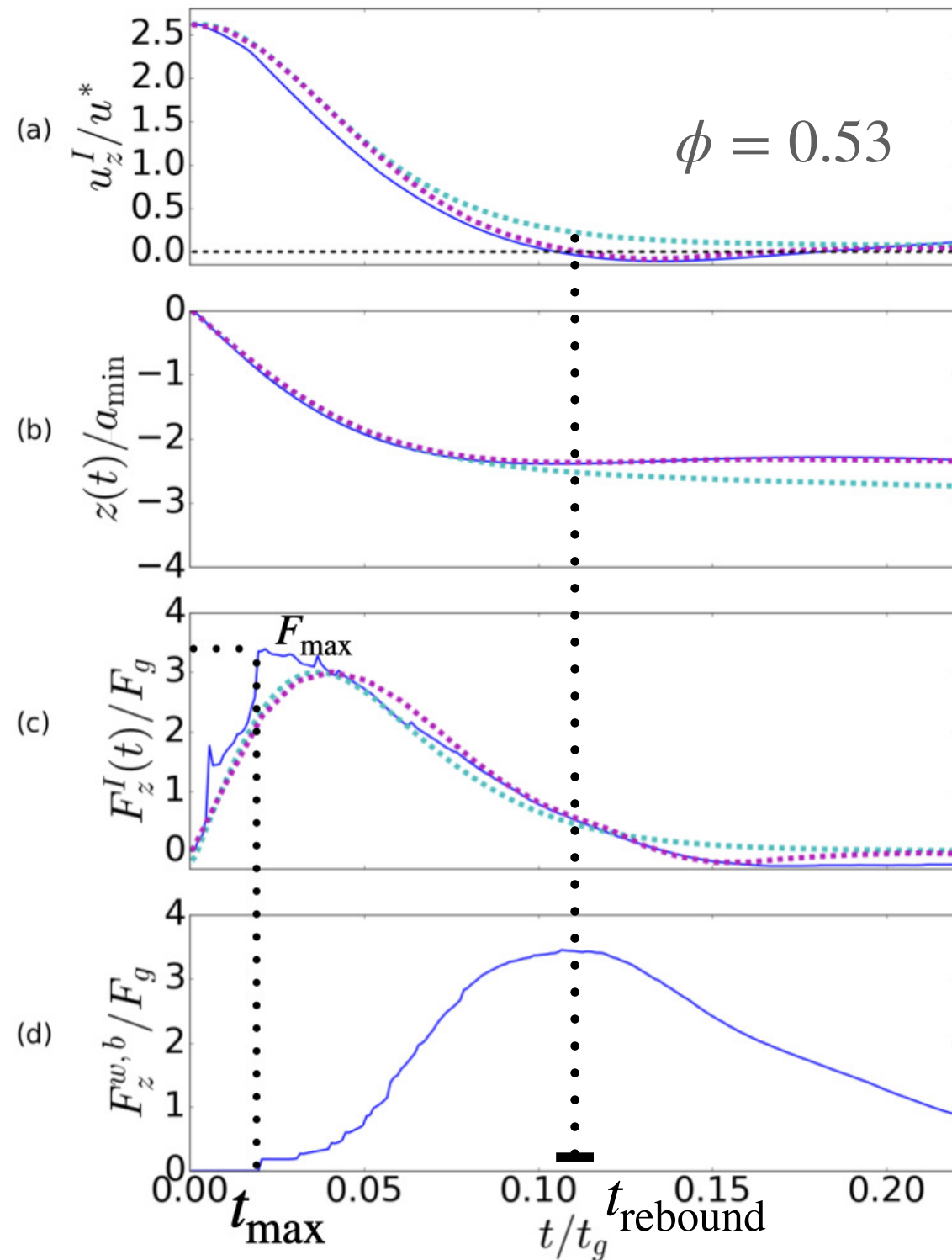


Stress in shear direction  $\sigma_{rz}$



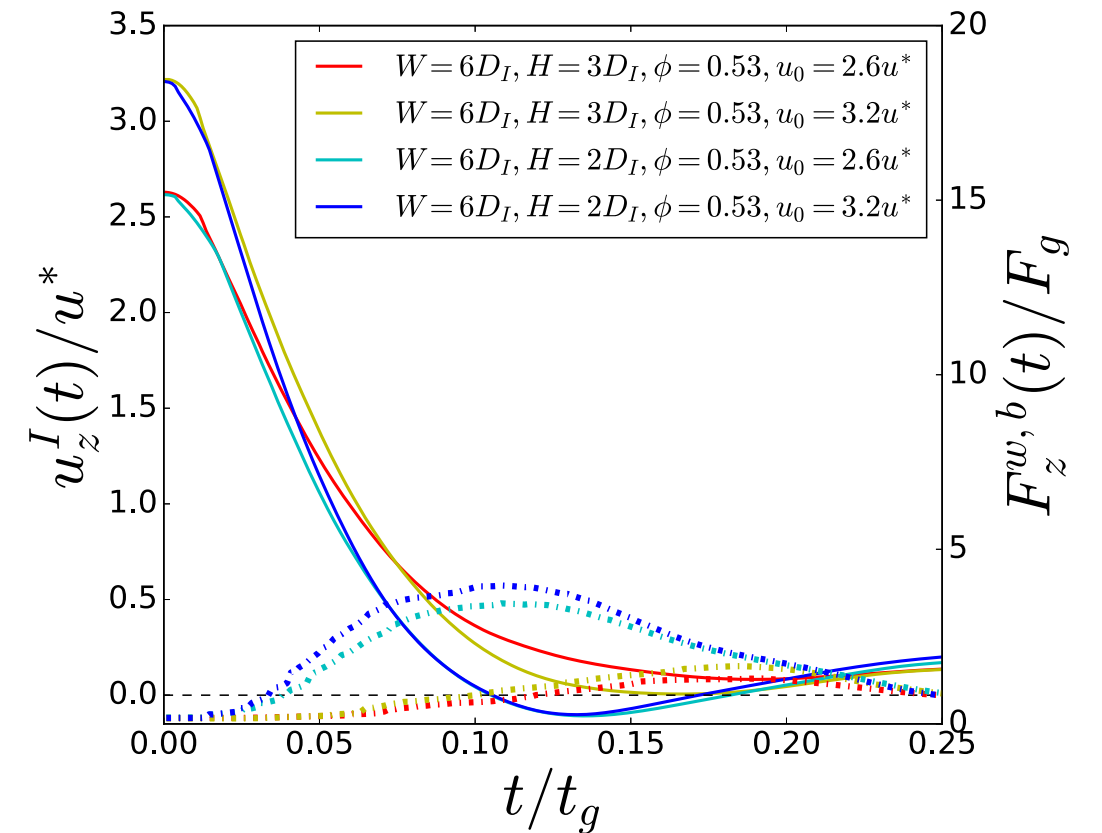
High normal stress instead of shear stress **c.f DST**

# Force on the impactor reach its peak before the rebound



Dyn. jammed region  
touch the bottom and  
starts accumulating

Dyn. jammed region  
reached its peak on the  
bottom and soften →  
impactor starts to sink



Rebound depends on the depth  
of the container

The origin of  $F_{\max}$  is the  
**floating** jammed region

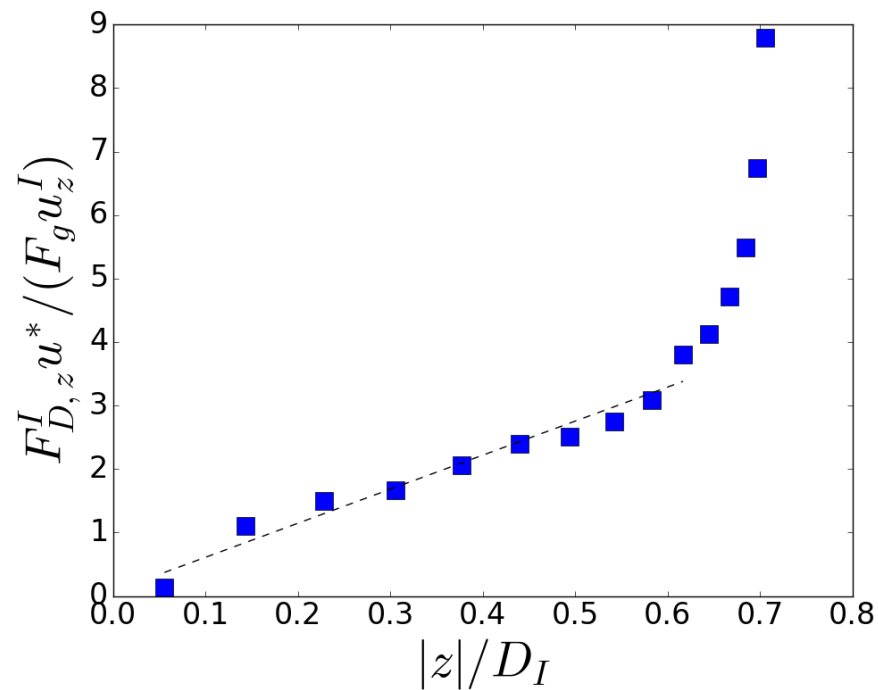
The origin of rebound is the  
transmission of force from the  
impactor to the bottom plate  
and vice versa

# Phenomenology Floating + force chains model

$$m_I \frac{d^2 z_I}{dt^2} = -m_I \tilde{g} + \overset{\text{Viscous}}{3\pi\eta_{\text{eff}} \dot{z}_I |z|} + \overset{\text{Elastic}}{n(t)k_n z_I}$$

## From floating jammed region

Drag on the impactor is proportional to its depth before completely immersed

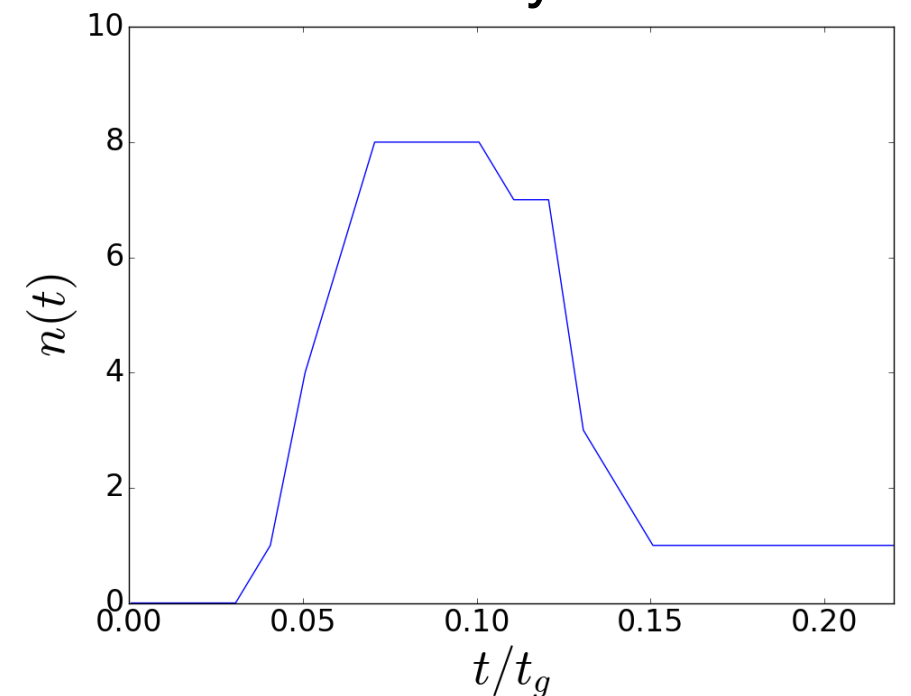


From analytical solution of floating model in  $u_0 \gg 1/\eta_{\text{eff}}$  one can obtain

$$F_{\text{max}} \propto u_0^{\frac{3}{2}} \text{ and } t_{\text{max}} \propto u_0^{-\frac{1}{2}}$$

## From force chains

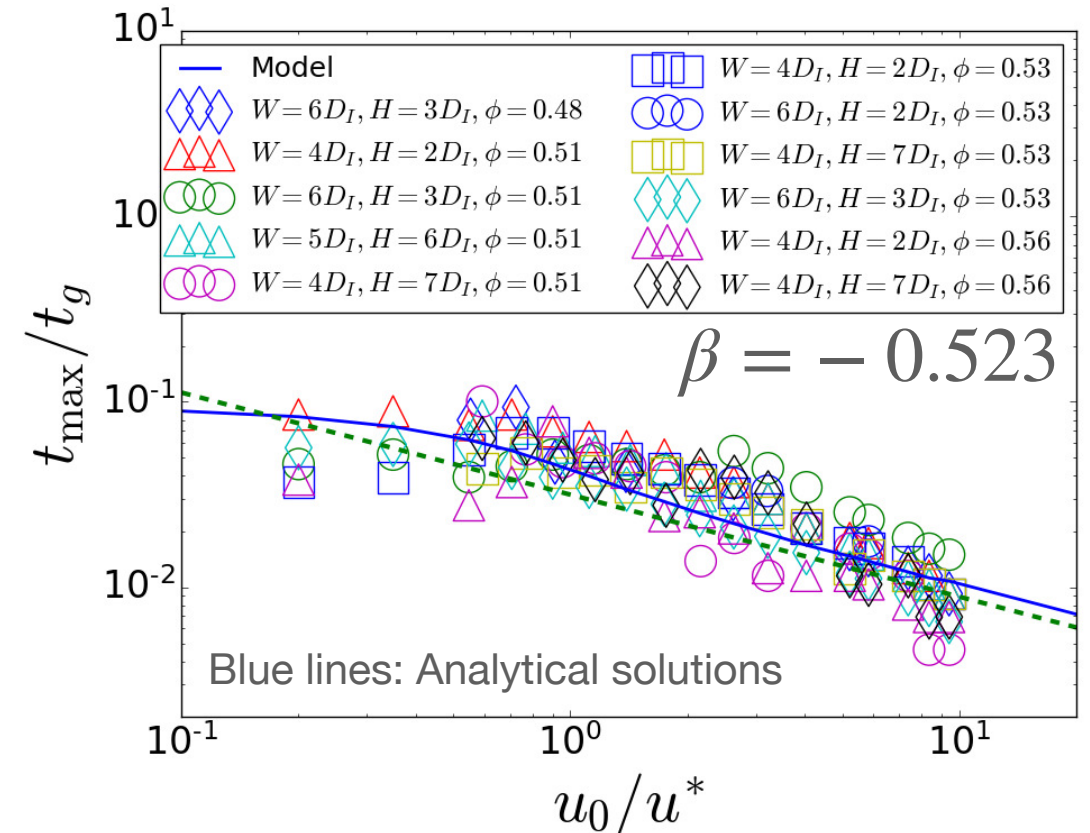
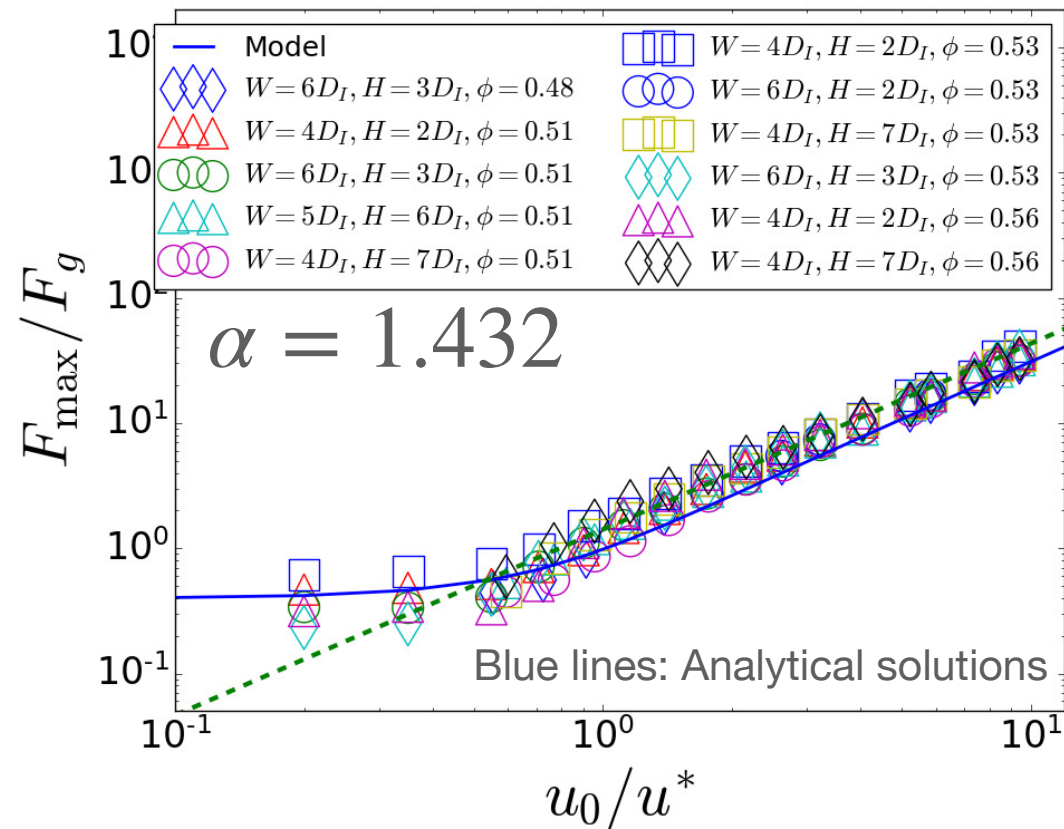
$n(t)$   $\longrightarrow$  Number of percolating force chains from the impactor to the bottom boundary





# Model vs simulations

## Power-law relationships between $F_{\max}$ , $t_{\max}$ , and $u_0$



- **Crossover** from low  $u_0$  to high  $u_0$  regime

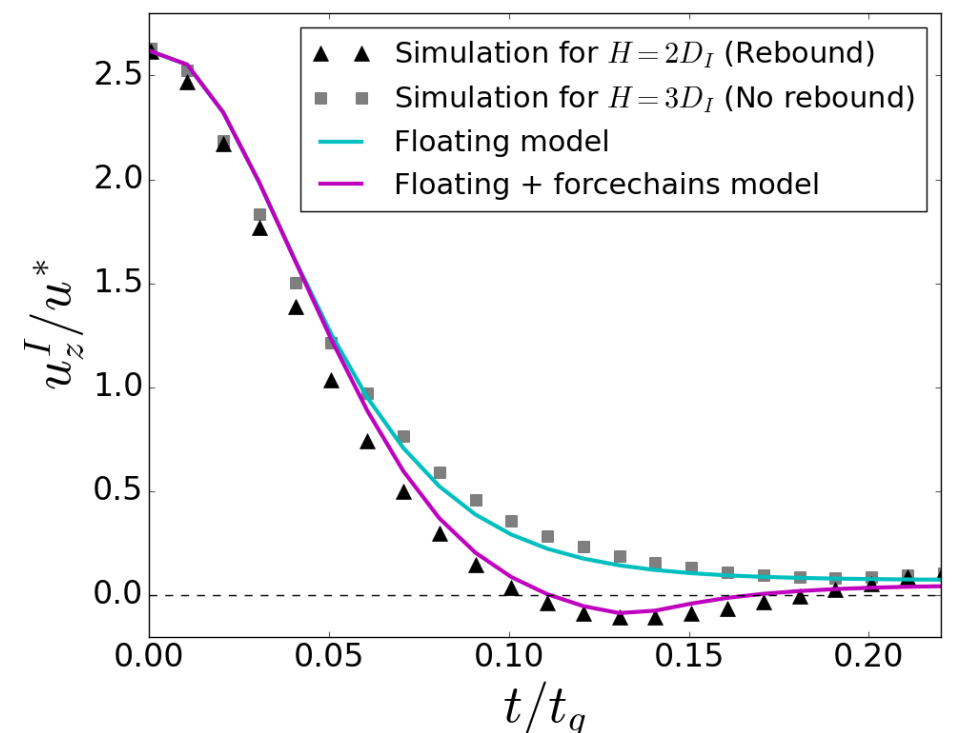
- High  $u_0$  regime:  $F_{\max} \propto u_0^\alpha$  and  $t_{\max} \propto u_0^\beta$

- Also observed in experiments

Brassard, et. al, JFM **923**, A38 (2021)

- Independent of system size

- Elastic term from percolating force chains is necessary to recover rebound



## Conclusions

The strength of the impact-induced hardening depends on volume fraction, impact velocity, and the frictional contact between particles

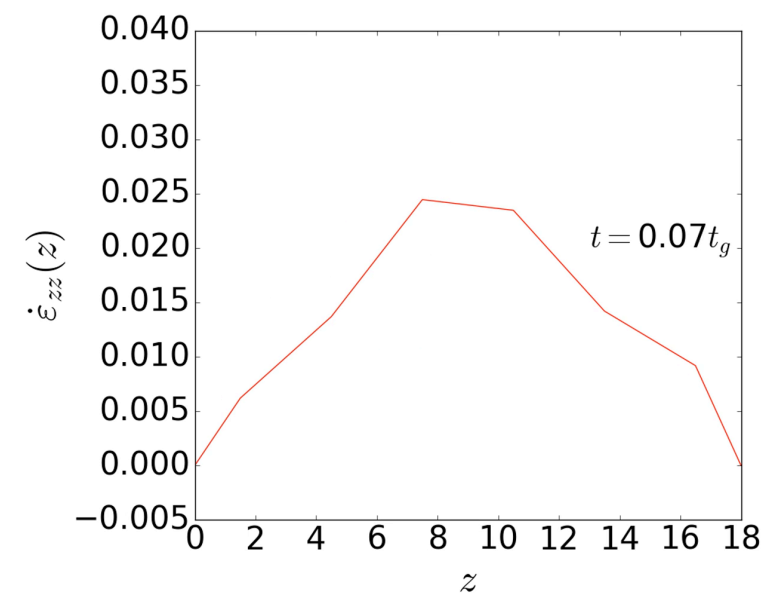
High normal stress (not shear stress) is observed in the dynamically jammed region

Our phenomenology shows that the power-law relationship between  $F_{\max}$ ,  $t_{\max}$ , and  $u_0$  arises solely from the viscous process

Elastic rebound is originated from elastic force due to the percolating force chains

## Future prospects

Dynamics and effective elasticity of the jammed region



Simulations of actual running model e. g Raibert hopper on dense suspensions

