JPS 2022
Autumn Meeting

Presentation no.
14aH112-7

Multi species asymmetric simple exclusion process with impurity activated flips

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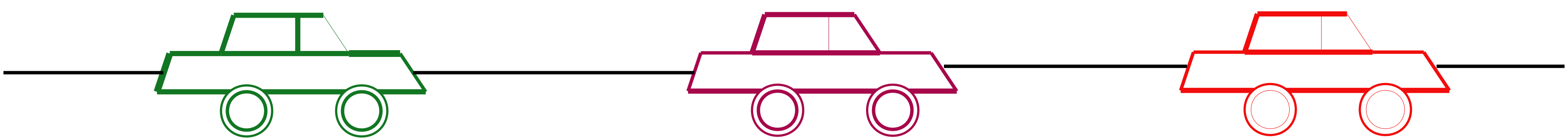
[*Reference*: *arXiv*:2205.03082 (2022)]

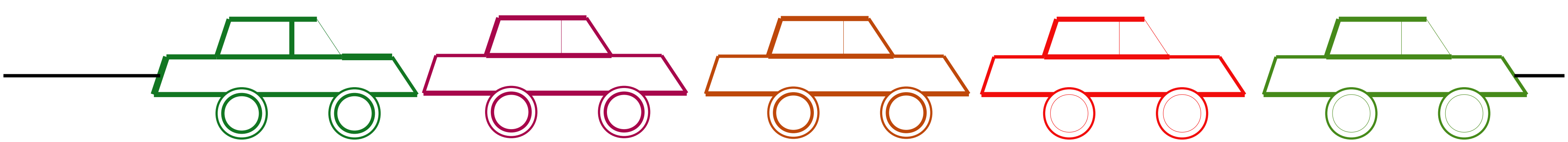
NON-EQUILIBRIUM SYSTEMS

- Transport phenomena

  net flow or current [absent in equilibrium]



- Phase transitions *(e.g. traffic jam)*



One general aim :

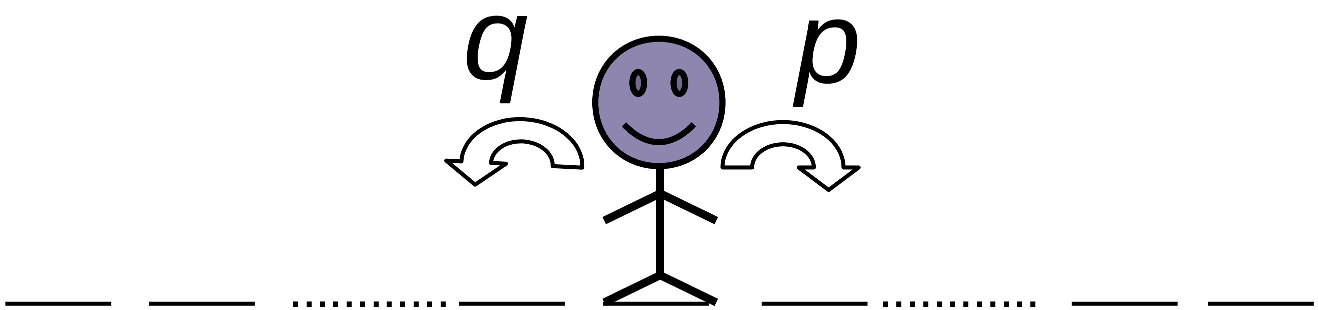
Understanding

COMPLEX NON-EQUILIBRIUM PHENOMENA

using

EXACTLY SOLVABLE TOY MODELS

A random walker



periodic lattice

STEADY STATE: all equally likely configurations

Generalization:

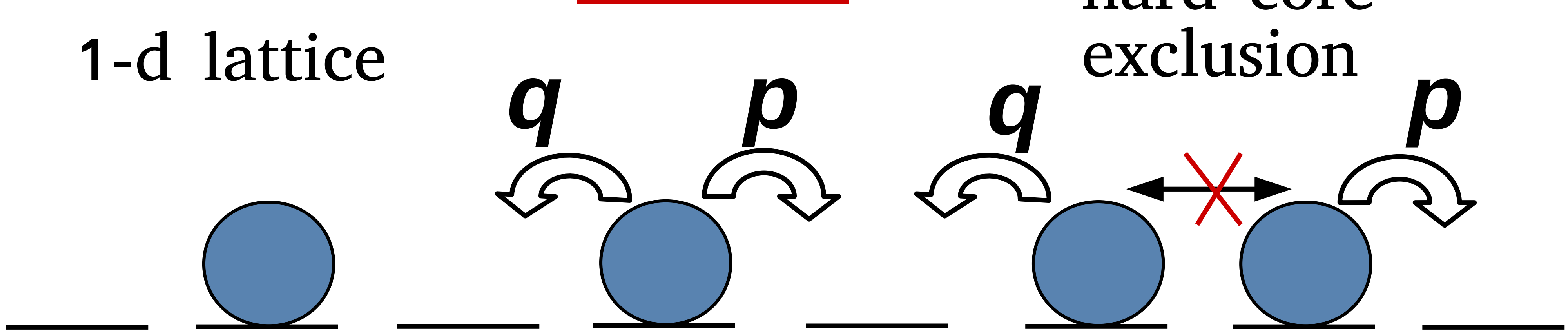
many interacting random walkers

⇓

Model:

Asymmetric Simple Exclusion Process

Asymmetric Simple Exclusion Process (ASEP)



- *Exactly solvable model: Matrix Product State*

  [*open boundary condition*]

- *Boundary induced phase transitions*

- *Applications: protein transport, traffic flow*

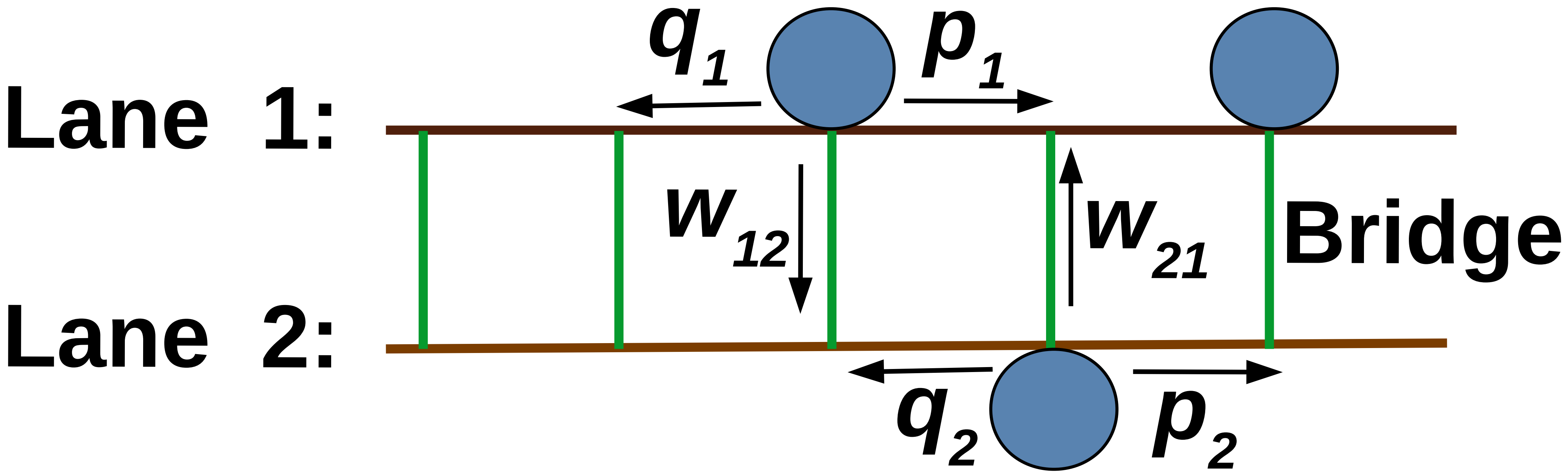
Multi-lane ASEP: [more realistic]

almost no exact solution with correlation between lanes

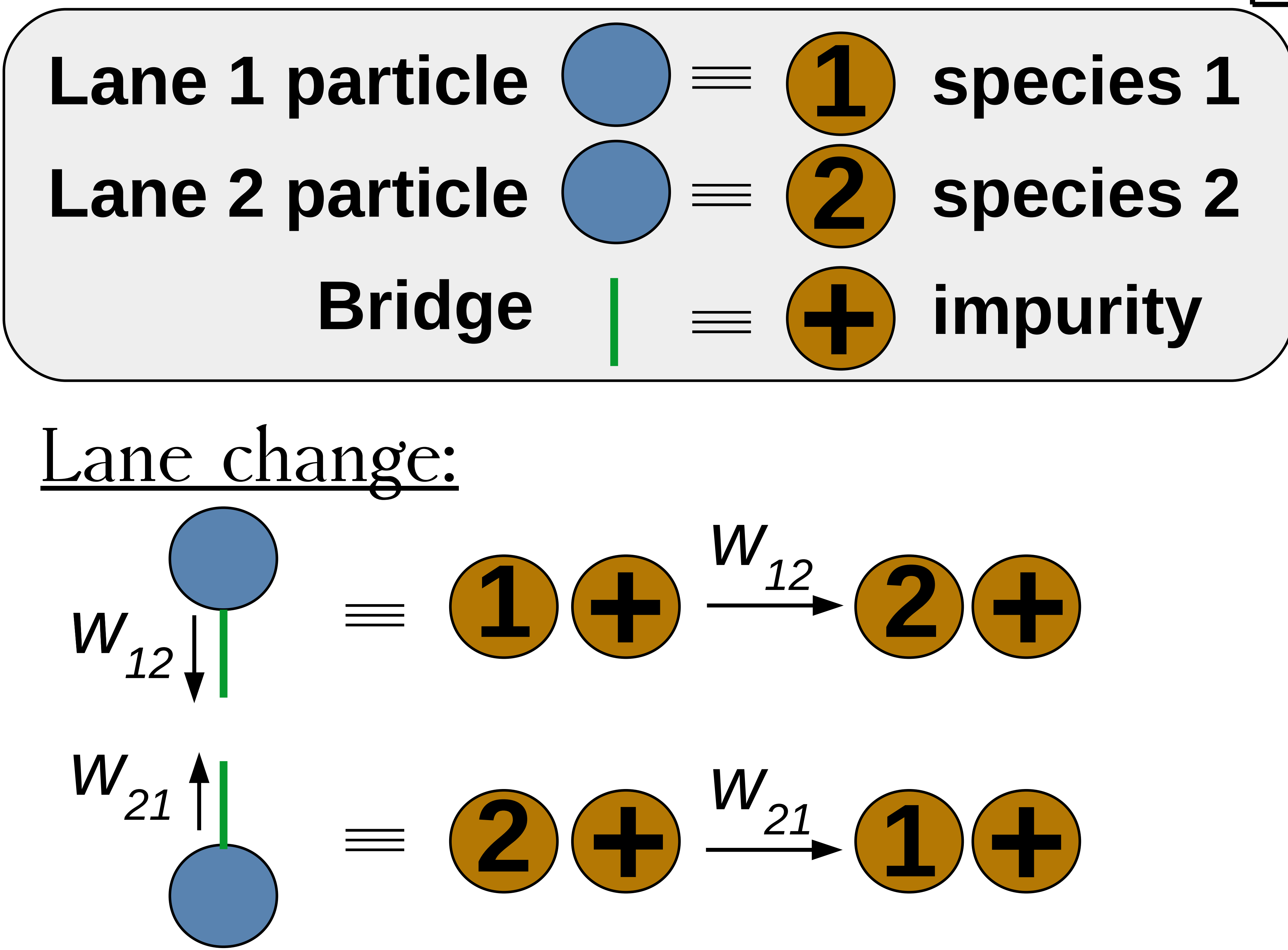
(*either mean field solution, or, exact solutions factorized over lanes*)

Question: Approximate mapping of
multi-lane ASEP
to
1-d model that allows exact solution ??

Motivation: Two-lane traffic flow to 1-d ASEP with two species & impurity

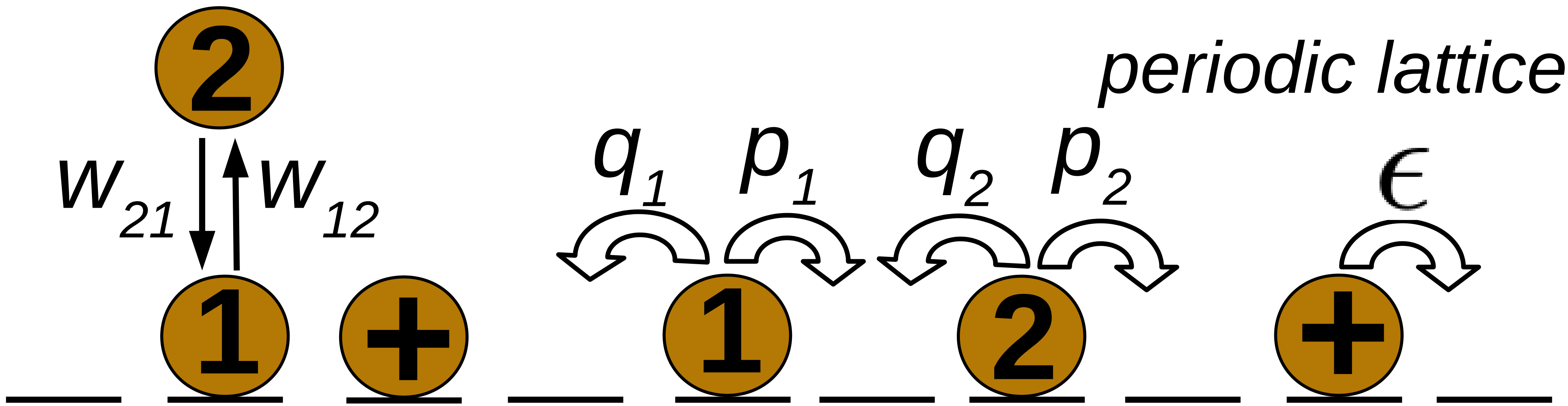


Question: Equivalent 1-d model ??



Lane 1 particle
≡
1
species 1
Lane 2 particle
≡
2
species 2
Bridge
≡
+
impurity
Lane change:
W_{12}
≡
1 +
W_{12}
2 +
W_{21}
≡
2 +
W_{21}
1 +

Multi species ASEP with impurity activated flips



(i) disordered

(ii) non-conserved

(iii) non-ergodic

Exact steady state and Observables ??

Steady state: Matrix Product Ansatz

component represented by matrix :

$\boxed{\textit{species "K"} \longrightarrow D_K}$ $\boxed{\textit{impurity} \longrightarrow A}$

$\boxed{\textit{vacancy} \longrightarrow E}$

Probability of any configuration:

$$P(\{s_i\}) \propto \mathrm{Tr}\left[\prod_{i=1}^{L} X_i\right],$$

$$X_i = E\,\delta_{s_i,0} + A\,\delta_{s_i,+} + \sum_{K=1}^{\mu} D_K\,\delta_{s_i,K}.$$

TASKS: (i) matrix algebra
(ii) matrix representations

RESULTS:

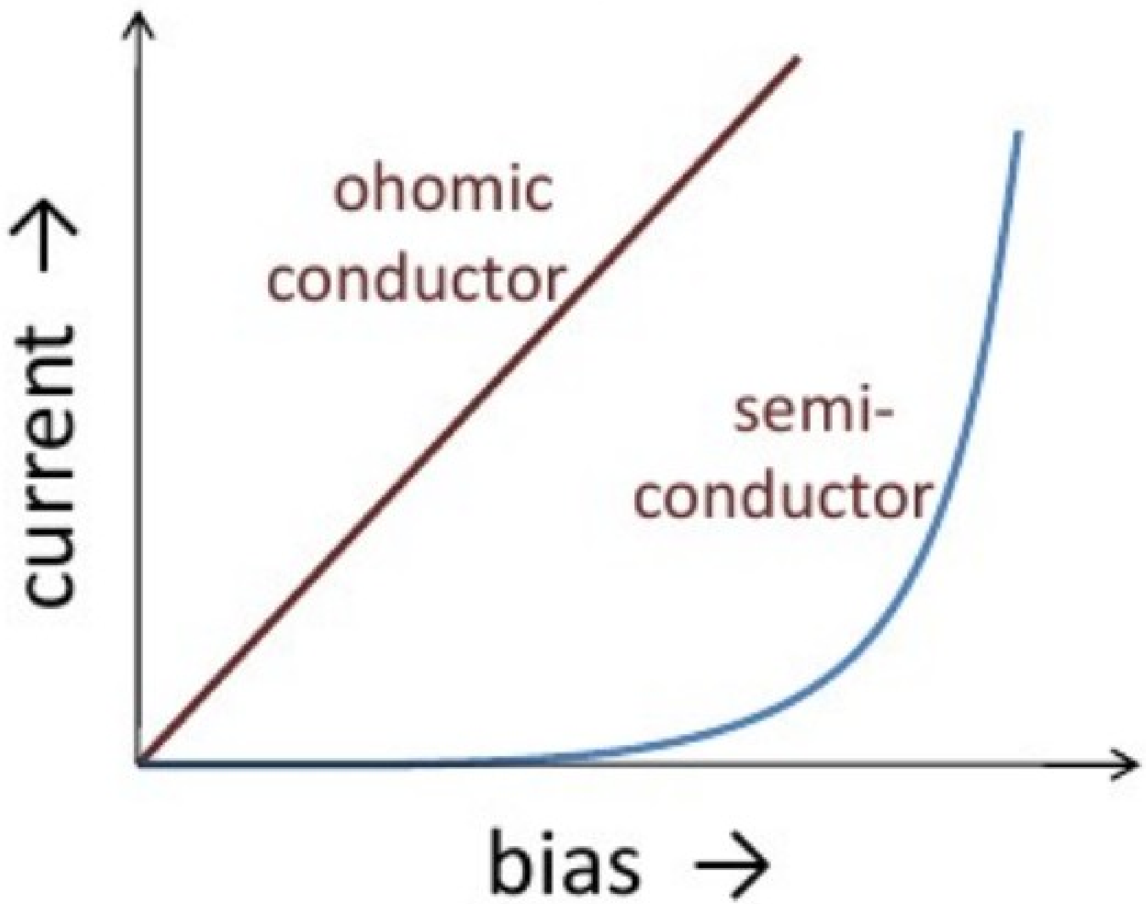
Totally asymmetric motion → Finite dimensional matrices

Partially asymmetric motion → Infinite dimensional matrices

[*arXiv*:2205.03082 (2022)]

Negative Differential Mobility

General notion:



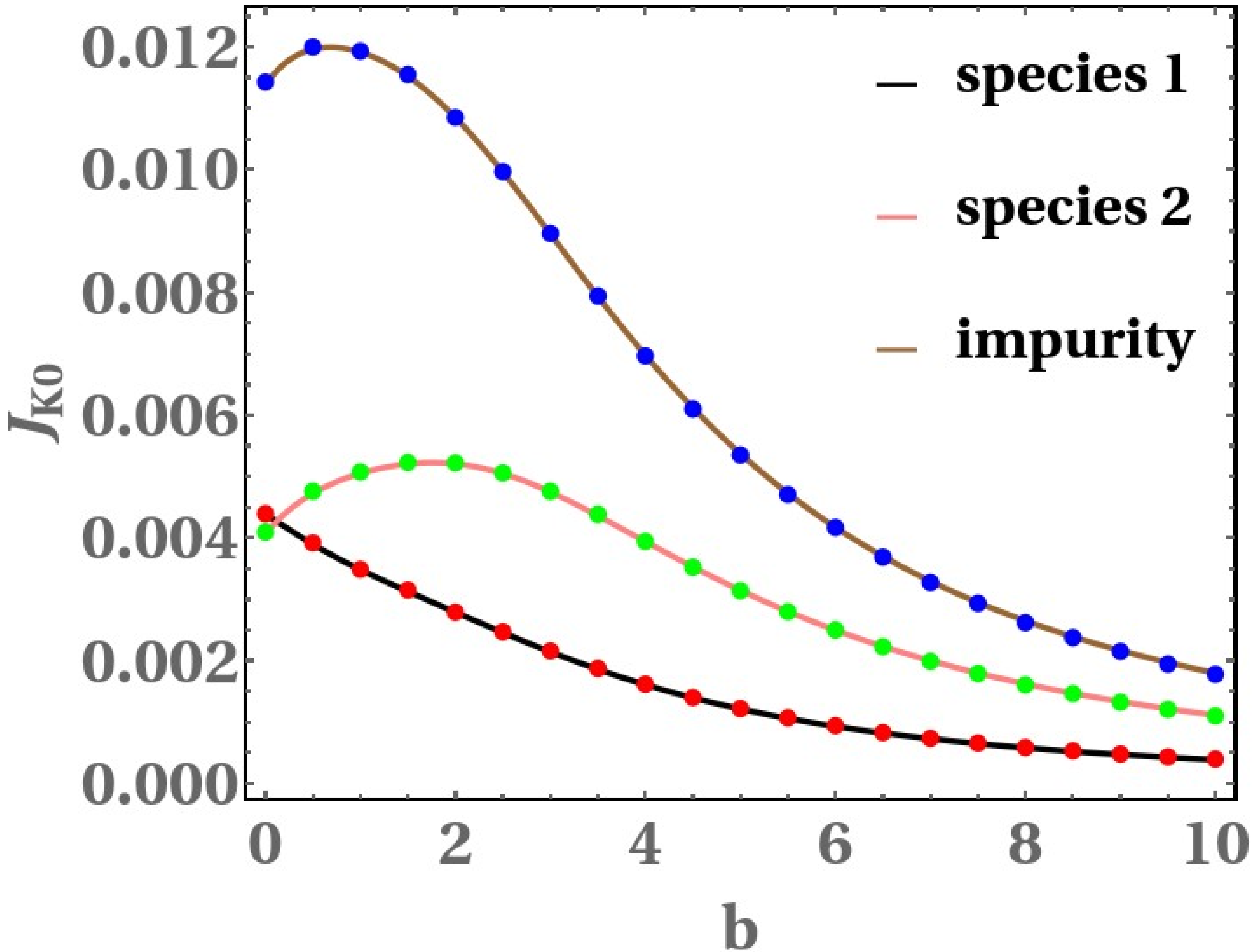
current increases with increasing bias

Our observation:

Decreasing current with increasing bias

Drift current of species "*K*": $\boxed{J_{K0}}$

Bias: $\boxed{\ln(p_1/q_1) = b}$ $\boxed{p_1 = 1, q_1 = e^{-b} \quad p_2 = \frac{1}{1+b^2} = q_2}$



Decreasing current with increasing bias

CONCLUSIONS

- Multi-lane traffic flow

  ⇓ *1-d approximation*

  Multi-species ASEP with impurity activated flips

  ⇓

  Exact matrix product steady state

- Motion and matrix dimension

Totally asymmetric

$\downarrow$

Finite dimensional

Partially asymmetric

$\downarrow$

Infinite dimensional

[*arXiv*:2205.03082 (2022)]

- $\boxed{\textit{Disordered}}$ $\boxed{\textit{Non-conserved}}$

  $\boxed{\textit{Non-ergodic}}$

- $\boxed{\textit{Negative differential mobility}}$

  [*arXiv*:2205.03082 (2022)]

★ $\boxed{\textit{Counter-flow induced clustering}}$

[*arXiv:2208.03297* (2022)]

THANK YOU

0+201**1**+2 $\longrightarrow$ 0+201**2**+2 *(accessible)*

0+20**1**1+2 $\not\longrightarrow$ 0+20**2**1+2 *(not accessible)*

only a subspace of the whole configuration space is accessible, for a given initial configuration

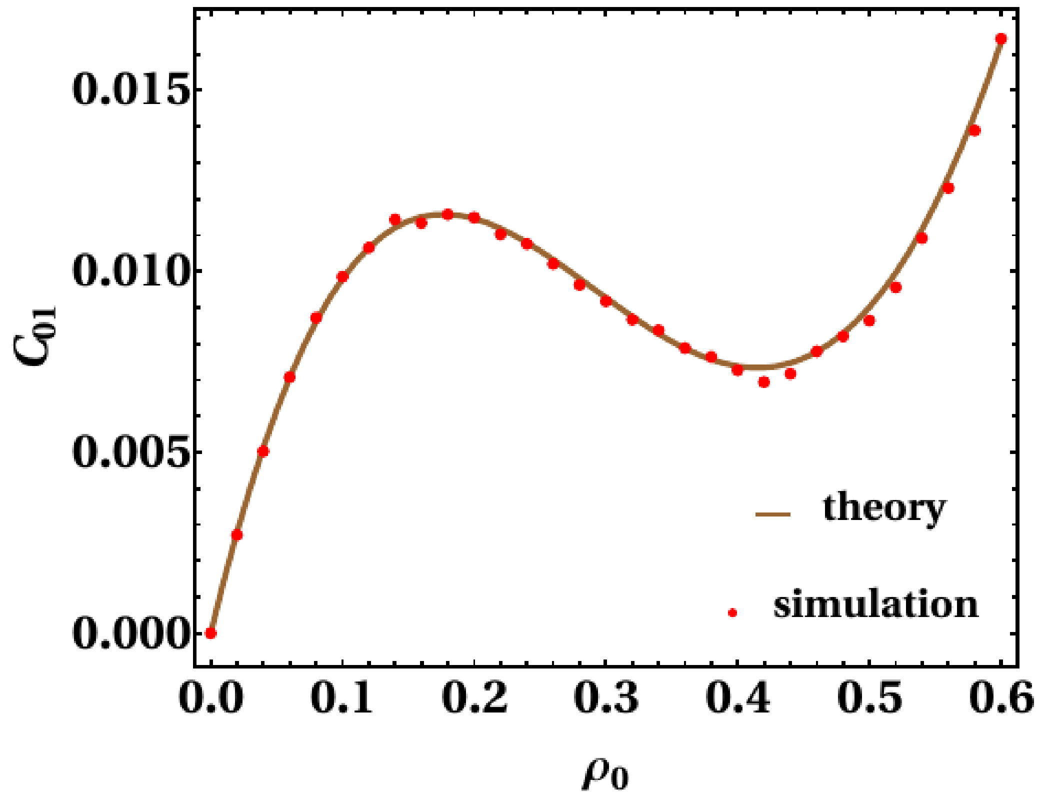
$\Downarrow$

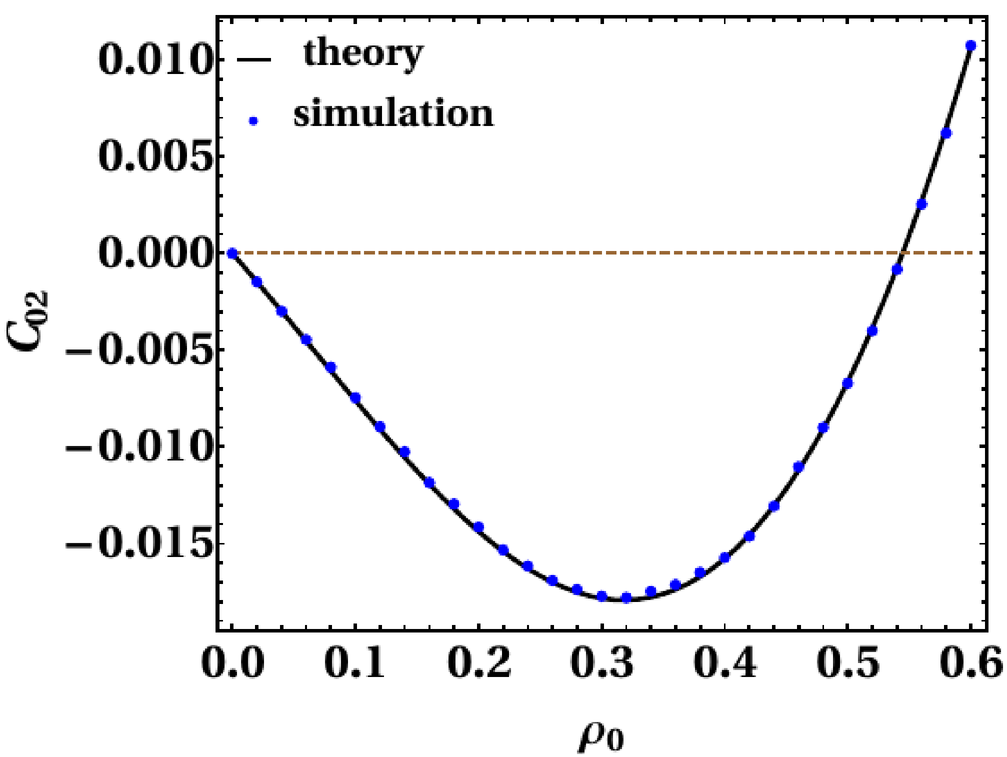
NON-ERGODIC

Two point correlations

two species case :

Non-monotonicity
(*increasing, then decreasing, then increasing again*)

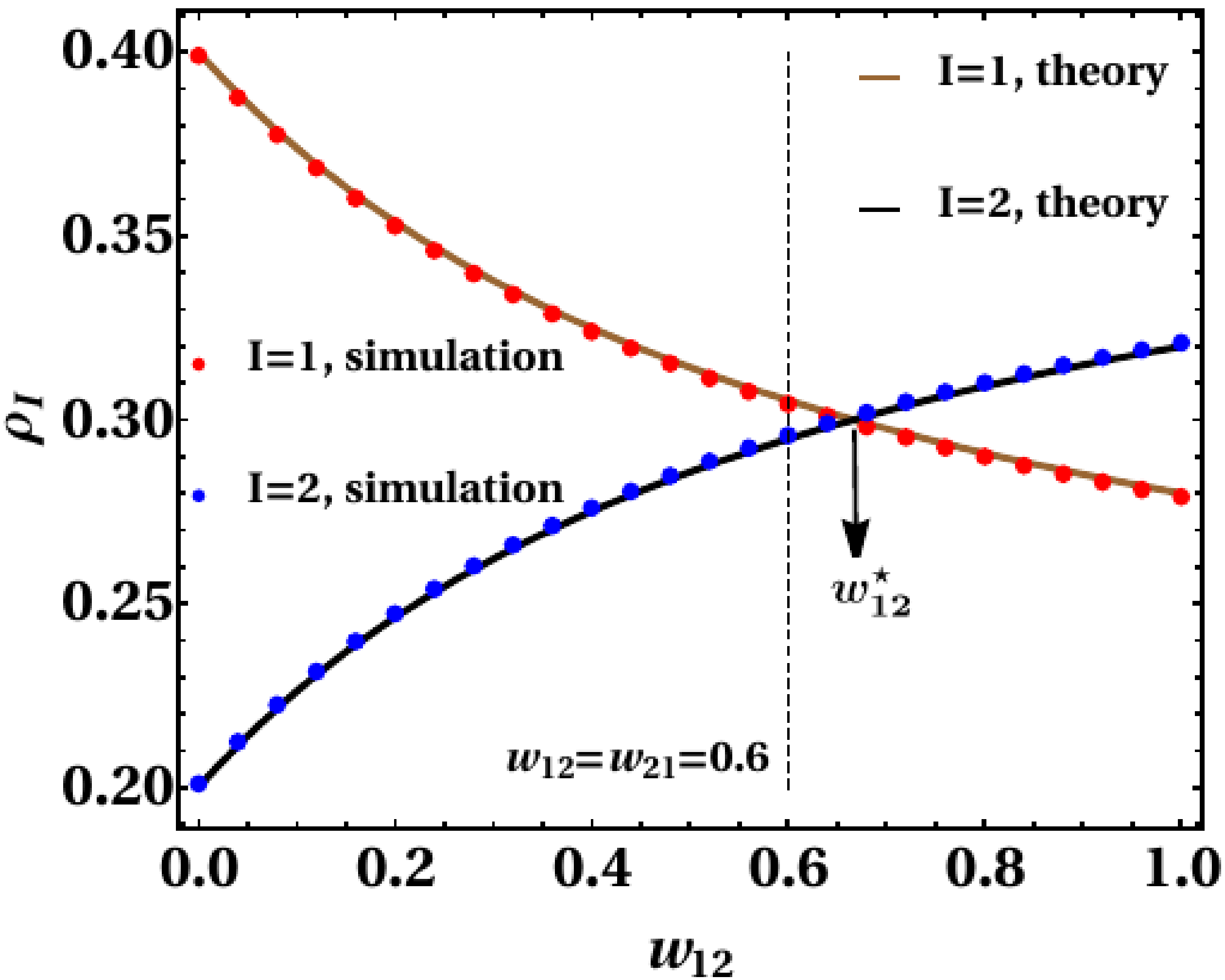




goes from negative to positive, with zero correlation at some intermediate density

Average species densities

Effect of drift on flip dynamics: two species case



$$w_{12}^{\star} = w_{21} \frac{p_1}{p_2} \frac{(p_2 - z_0)}{(p_1 - z_0)} \neq w_{21}$$

In the interval $(w_{12}^{\star} - w_{21})$

species with larger flip rate has larger density !

$$
\begin{array}{rcccl}
\text{drift (species)}: & I0 & \underset{q_I}{\overset{p_I}{\rightleftharpoons}} & 0I & I = 1, 2, ..., \mu \\
\text{drift (impurity)}: & +0 & \xrightarrow{\epsilon} & 0+ & \\
\text{flip}: & I+ & \underset{w_{KI}}{\overset{w_{IK}}{\rightleftharpoons}} & K+ & I, K = 1, ..., \mu
\end{array}
$$

$$
\boxed{\mathcal{M}_{i,i+1} \mathbf{X}_i \otimes \mathbf{X}_{i+1} = \tilde{\mathbf{X}}_i \otimes \mathbf{X}_{i+1} - \mathbf{X}_i \otimes \tilde{\mathbf{X}}_{i+1}}
$$

Matrix algebra:

$$
\begin{aligned}
p_K D_K E - q_K E D_K &= D_K \\
\epsilon A E &= A \\
\sum_{\substack{I=1 \\ I \neq K}}^{\mu} w_{IK} D_I A &= D_K A \sum_{\substack{I=1 \\ I \neq K}}^{\mu} w_{KI}
\end{aligned}
$$

Finite dimensional representations: Totally asymmetric motion

$$D_1 = d_1 \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad D_2 = d_2 \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad D_3 = d_3 \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$E = \begin{pmatrix} \frac{1}{\epsilon} & 0 & 0 & 0 \\ \frac{1}{p_1} - \frac{1}{\epsilon} & \frac{1}{p_1} & 0 & 0 \\ \frac{1}{p_2} - \frac{1}{\epsilon} & 0 & \frac{1}{p_2} & 0 \\ \frac{1}{p_3} - \frac{1}{\epsilon} & 0 & 0 & \frac{1}{p_3} \end{pmatrix} \quad A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{aligned} d_1 &= w_{21}w_{31} + w_{23}w_{31} + w_{32}w_{21} \\ d_2 &= w_{12}w_{32} + w_{13}w_{32} + w_{31}w_{12} \\ d_3 &= w_{13}w_{23} + w_{12}w_{23} + w_{21}w_{13} \end{aligned}$$

Infinite dimensional representations: Partially asymmetric motion

$$D_I = \begin{pmatrix} d_I^{1,1} & d_I^{1,2} & d_I^{1,3} & d_I^{1,4} & . & . \\ 0 & d_I^{2,2} & d_I^{2,3} & d_I^{2,4} & . & . \\ 0 & 0 & d_I^{3,3} & d_I^{3,4} & . & . \\ 0 & 0 & 0 & d_I^{4,4} & . & . \\ . & . & & & . & \\ . & . & & & & . \end{pmatrix}$$

$$d_I^{m,m+r} = \frac{(m)_r}{r!\,p_I^r} \left(\frac{q_I}{p_I}\right)^{m-1} d_I^{1,1}$$

$$I = 1, 2, 3$$

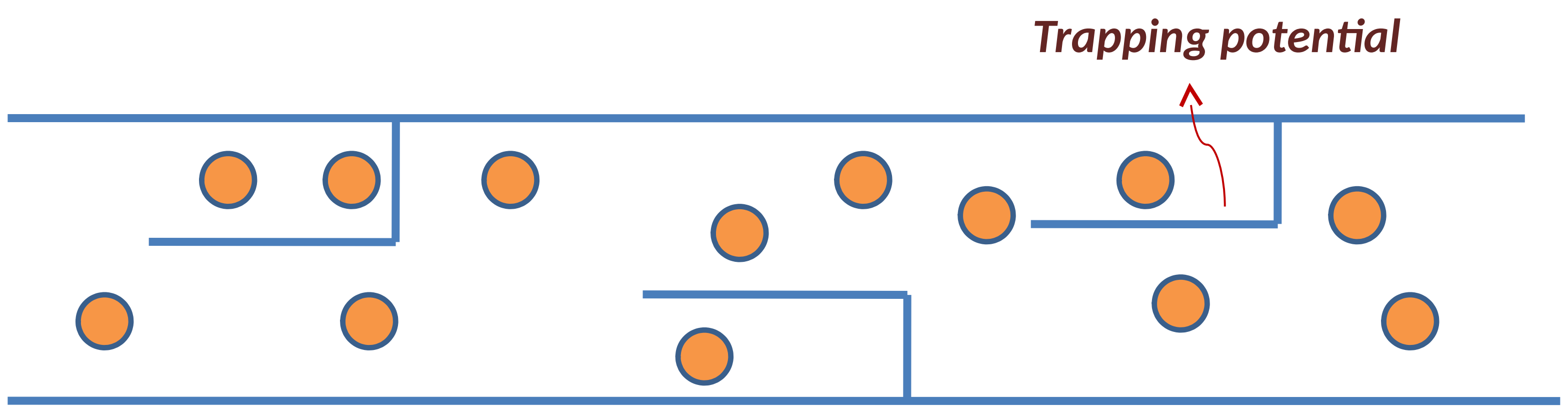
$$\begin{aligned} d_1^{1,1} &= w_{21}w_{31} + w_{23}w_{31} + w_{32}w_{21} \\ d_2^{1,1} &= w_{12}w_{32} + w_{13}w_{32} + w_{31}w_{12} \\ d_3^{1,1} &= w_{13}w_{23} + w_{12}w_{23} + w_{21}w_{13} \end{aligned}$$

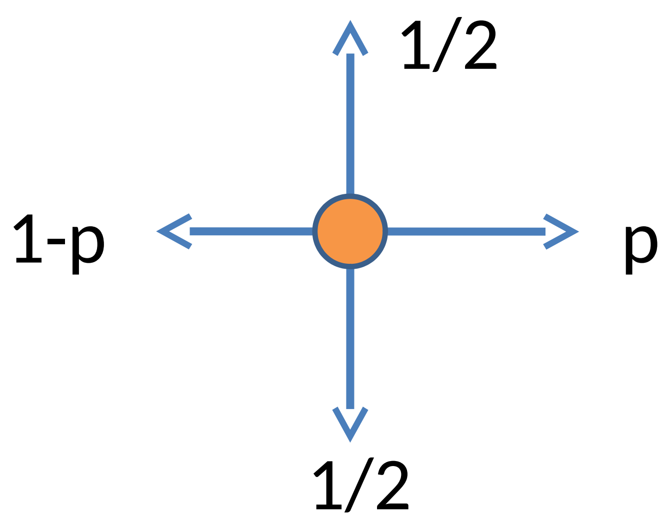
$$E = \begin{pmatrix} 0 & 0 & 0 & 0 & . & . \\ 1 & 0 & 0 & 0 & . & . \\ 0 & 1 & 0 & 0 & . & . \\ 0 & 0 & 1 & 0 & . & . \\ 0 & 0 & 0 & 1 & & \\ . & . & & & . & \\ . & . & & & & . \end{pmatrix} \quad A = \begin{pmatrix} 1 & \frac{1}{\epsilon} & \frac{1}{\epsilon^2} & \frac{1}{\epsilon^3} & . & . \\ 0 & 0 & 0 & 0 & . & . \\ 0 & 0 & 0 & 0 & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \end{pmatrix}$$

Negative differential mobility

With increasing bias, if current decreases, then we have negative differential mobility

- *non-interacting particles*





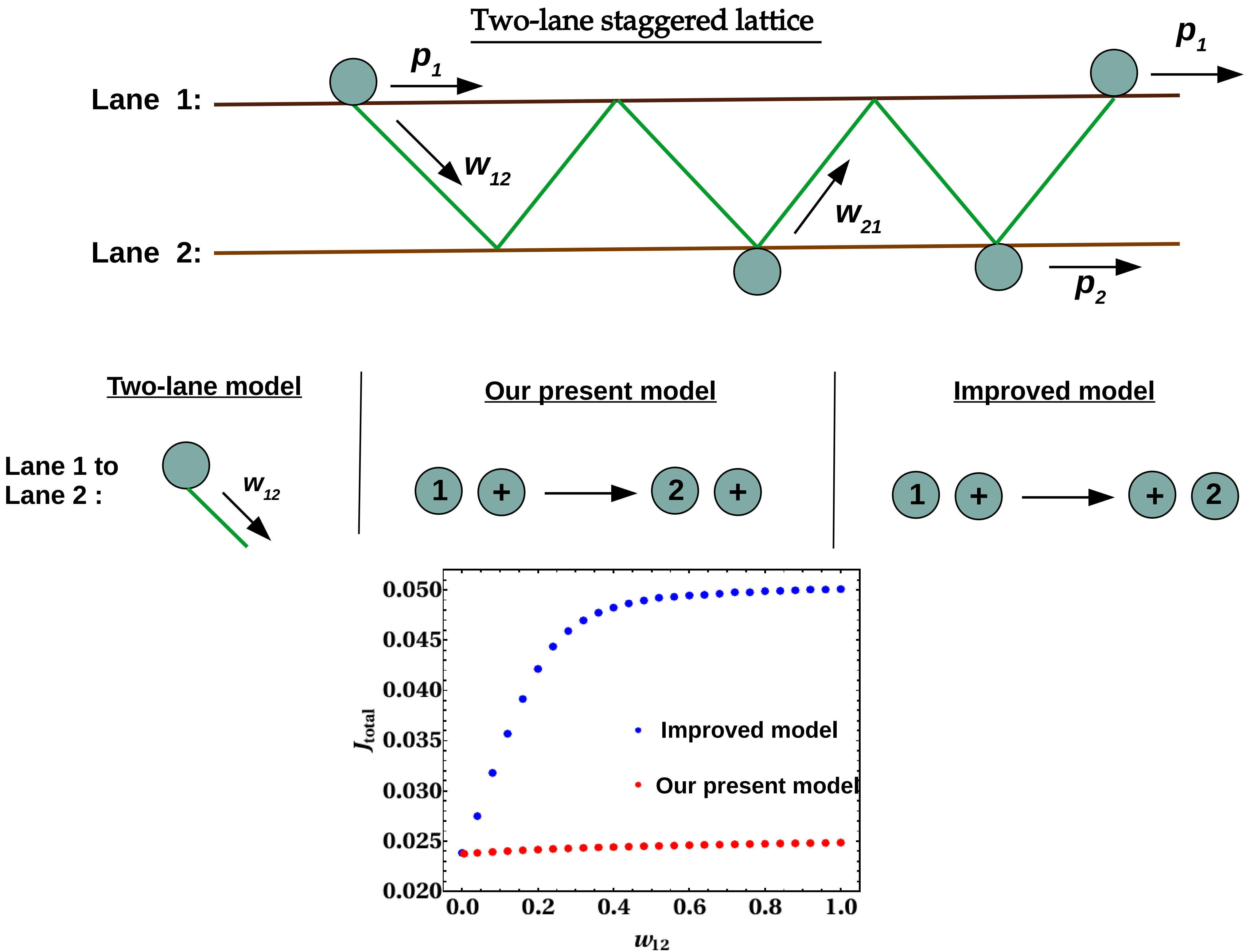
(a) **p=1/2** ***J=0***

(b) **p=0.51** , only few particles get trapped ,
positive response

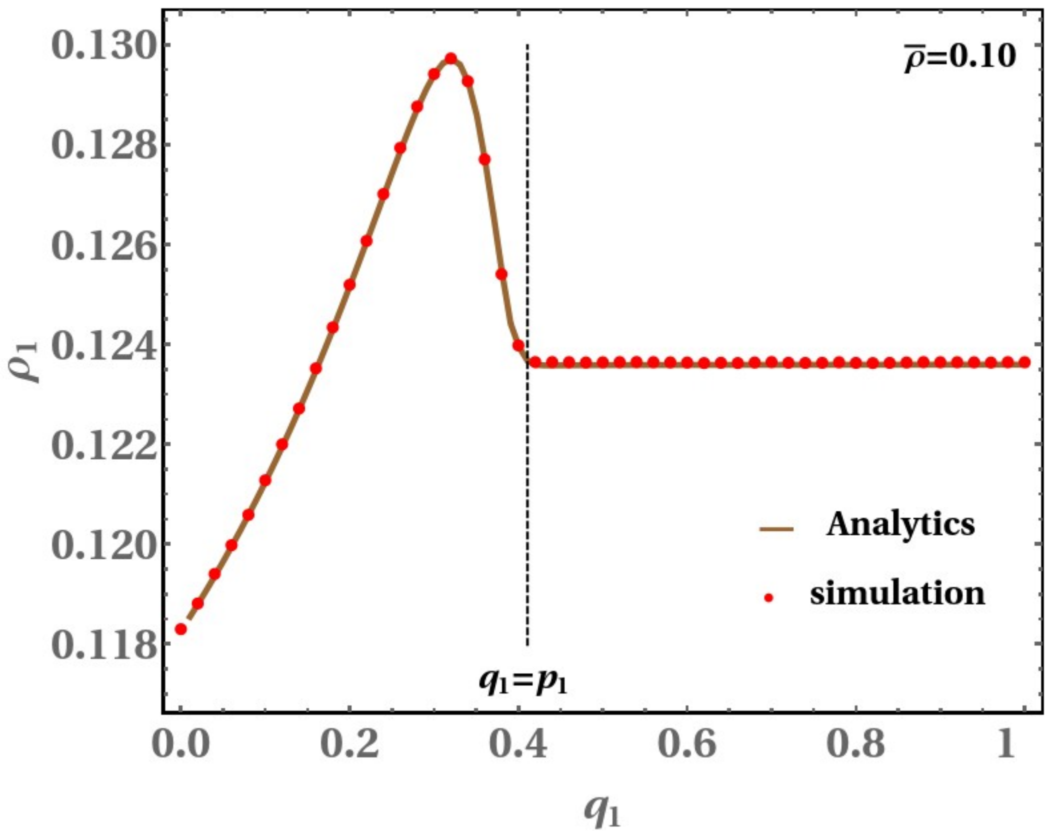
(c) **p=0.95** , most of the particles get trapped,
negative response

[Baerts *et.al.* , Phys. Rev. E 88, 052109 (2013)]

Mechanism: some kind of trapping that decreases dynamical activity


Two-lane staggered lattice
Lane 1:
Lane 2:
p_1
p_1
p_2
w_{12}
w_{21}
Two-lane model
Our present model
Improved model
Lane 1 to
Lane 2 :
w_{12}
1
+
2
+
1
+
+
2
J_{total}
w_{12}
0.050
0.045
0.040
0.035
0.030
0.025
0.020
0.0
0.2
0.4
0.6
0.8
1.0
Improved model
Our present model

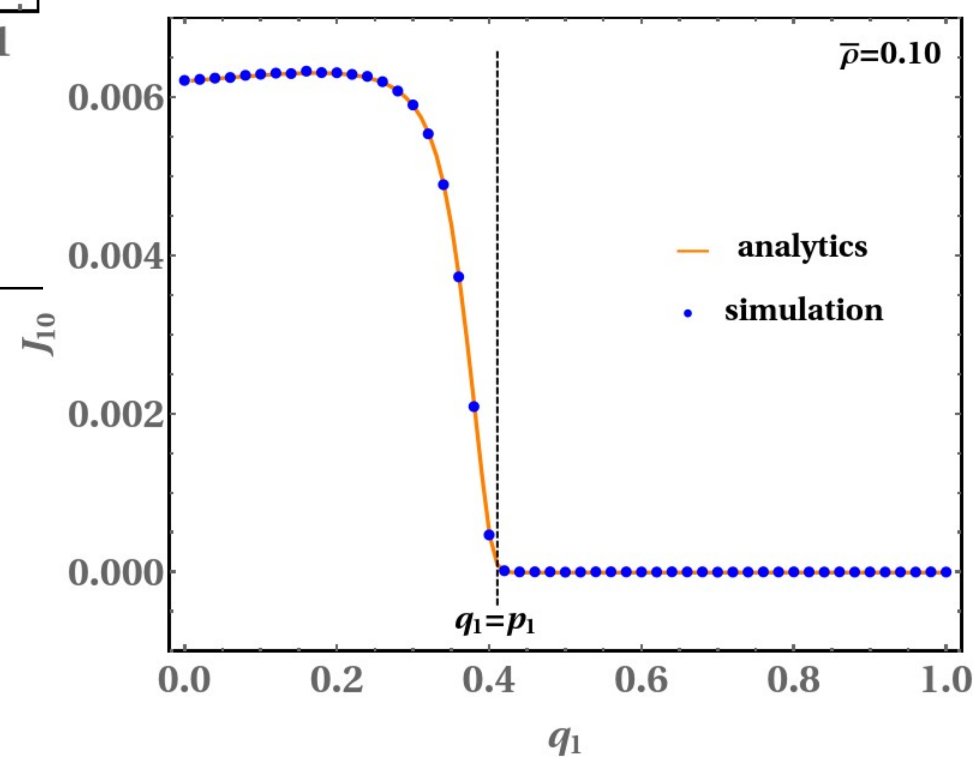
average density of species 1



- $q_1 < p_1$: non-monotonic density
- $q_1 > p_1$: constant density

Two different phases

average drift current of species 1



- $q_1 < p_1$: finite current (abrupt fall near q_1=p_1)
- $q_1 > p_1$: vanishing current

Two different phases,
*q_1>p_1, almost no drift, possible **clustering***