Effective viscosity and elasticity of dynamically jammed region and their role in the hopping motion on dense suspensions

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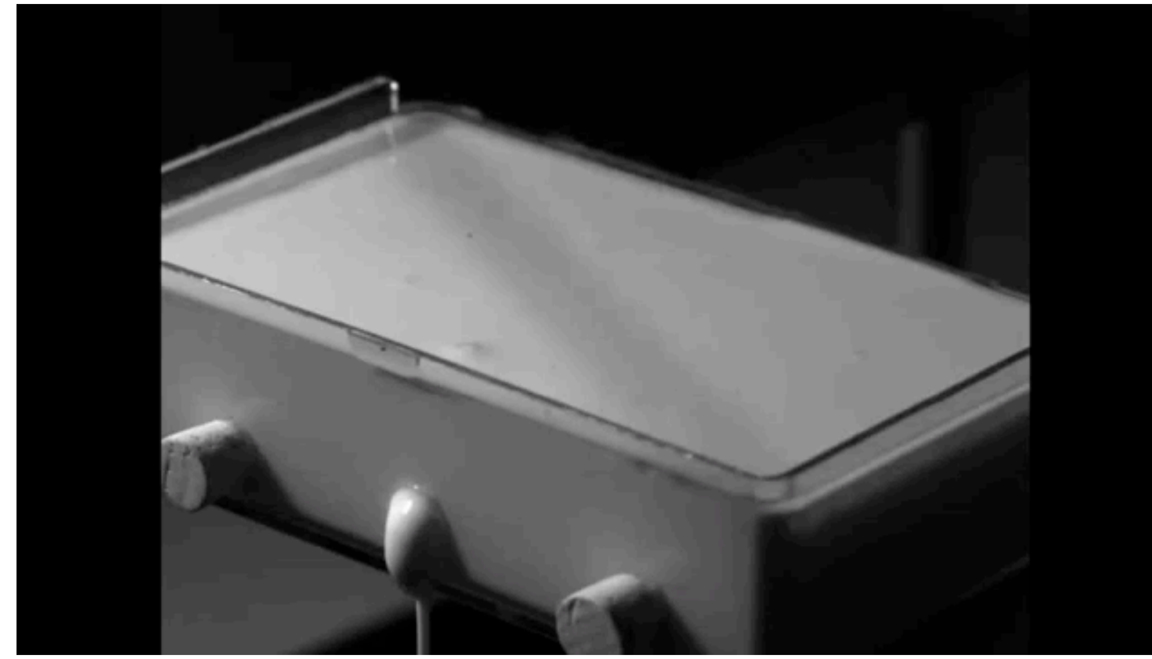
Introduction Dense suspensions under impact

Non-Brownian Suspensions

Mixture of macroscopic (1μm - 500μm) undissolved particles in a Newtonian solvent

+

Impact-induced hardening



source: https://www.youtube.com/watch?v=hP88C-_LgnE&list=PLVjilPzFTOLpxiwdwrFuPYSIBpr6jFm32

Able to run

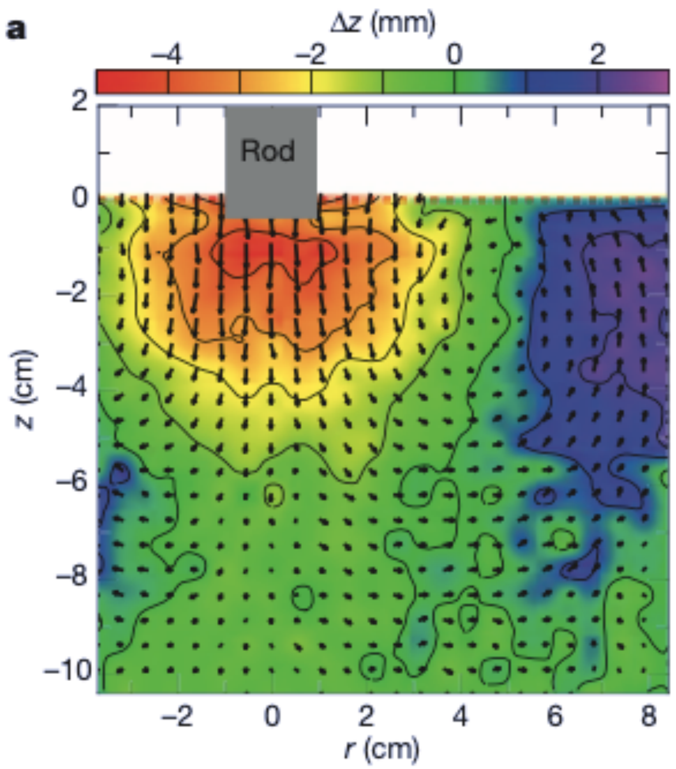
Brown, et. al., Rep. Prog. Phys 77, 046602 (2014).

Liquid body armor

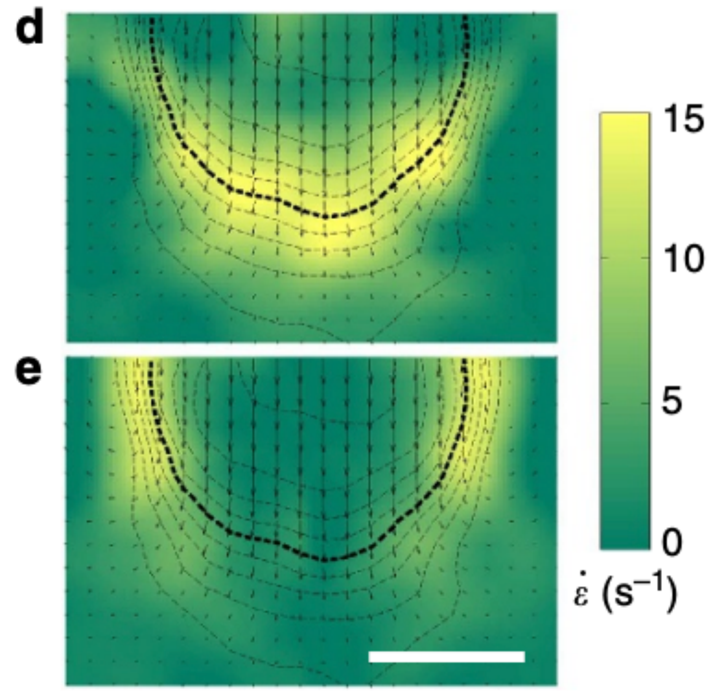
https://www.youtube.com/watch?v=L5Ts9lYZIDk

Introduction Dense suspensions under impact

Waitukaitis and Jaeger, Nature **487**, 205 (2012)



Han et al, Nat. Comms. 2016



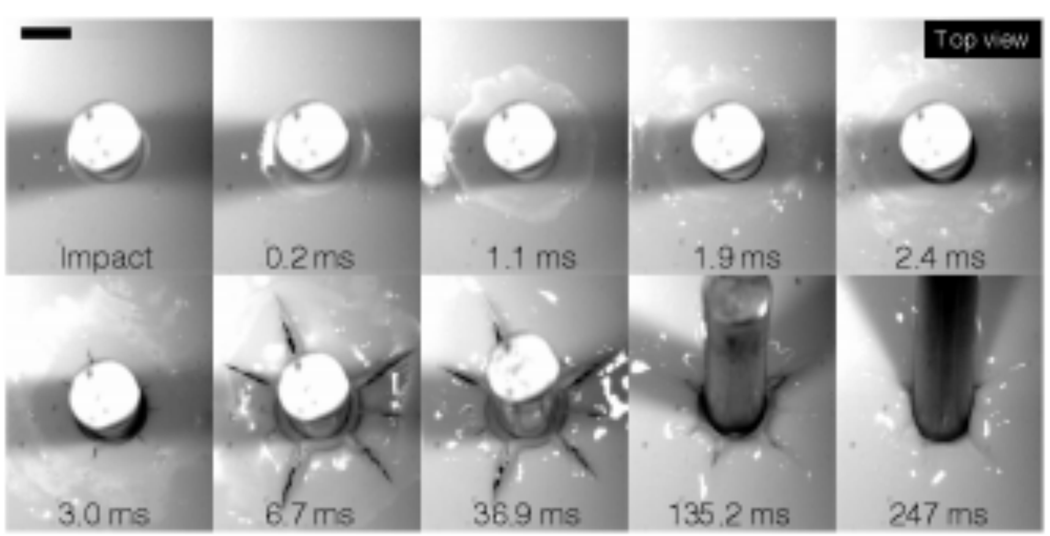
Dynamically Jammed Region (DJR)

Localized and transient solid-like region beneath the impactor

Boundary → peak of strain rate

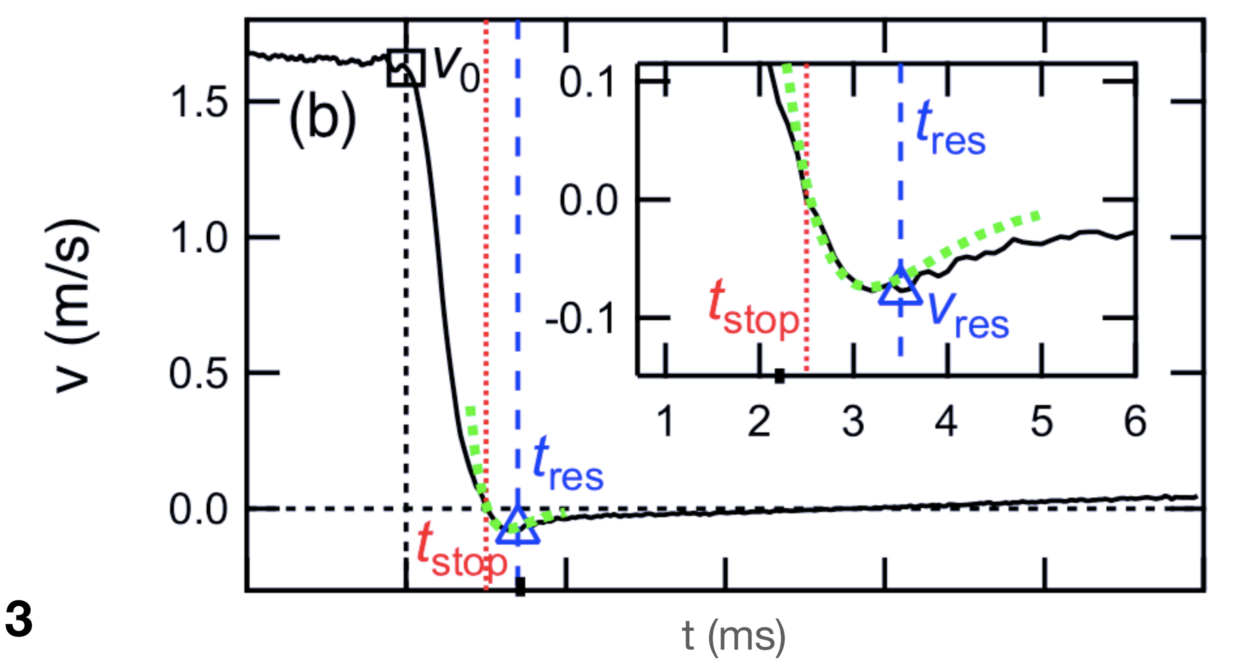
Fracture

Roche et al, PRL **110**, 148304 (2013)



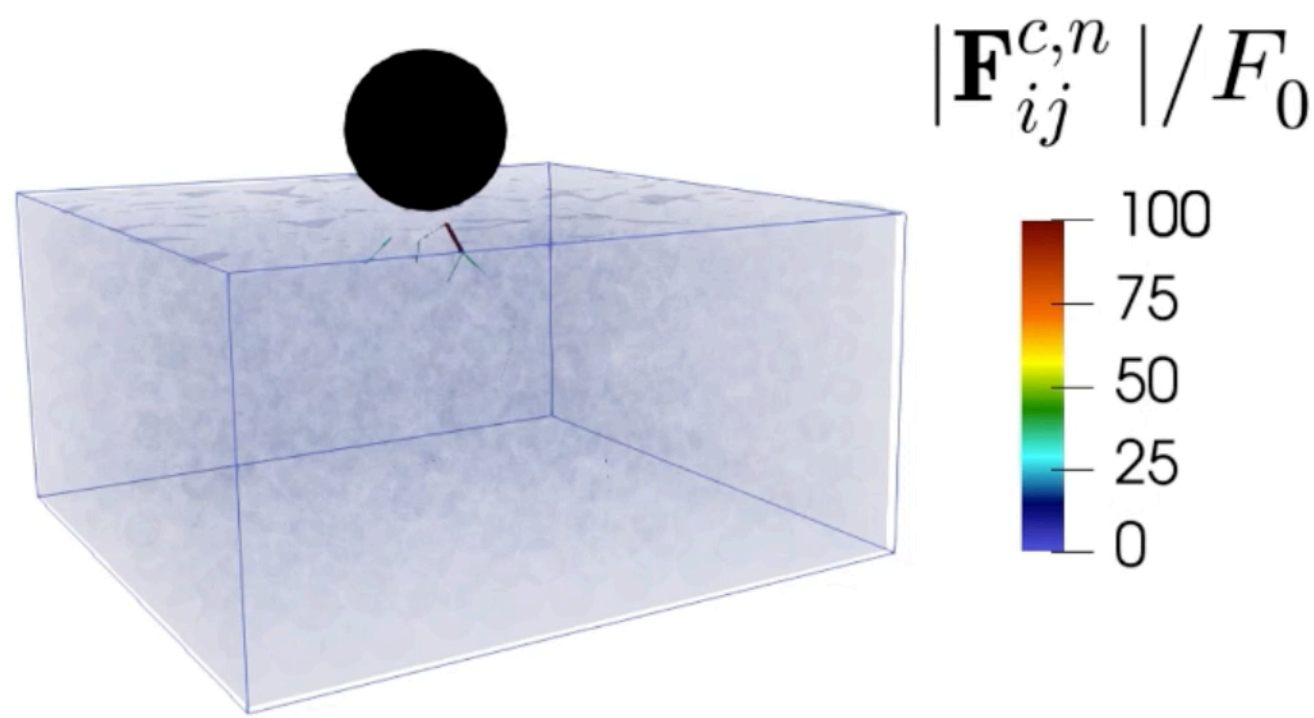
Elastic rebound

Egawa and Katsuragi, Phys. Fluids **31**, 053304 (2019)



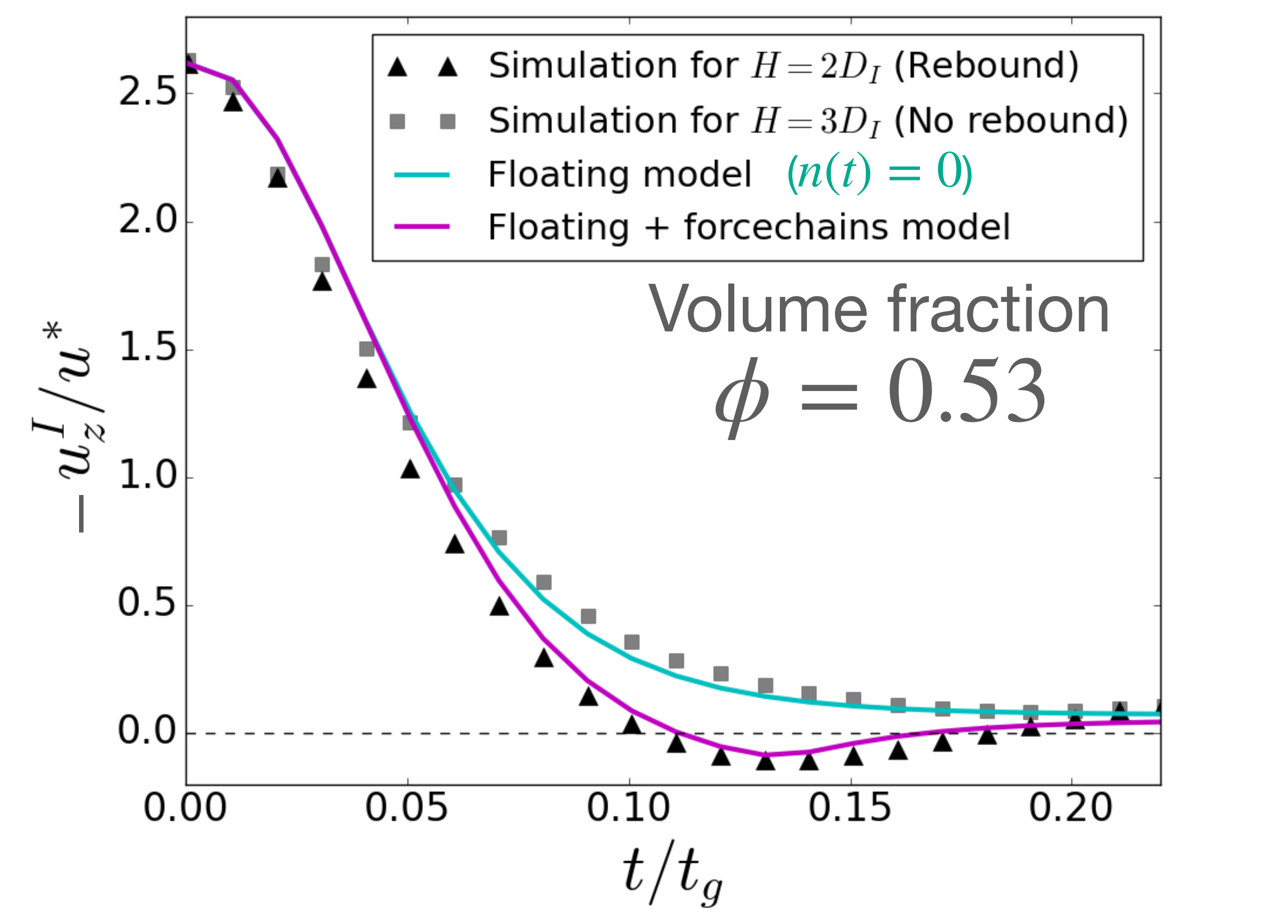
Previous works: Recovered rebound from percolating force chains

Pradipto and H. Hayakawa, Phys. Fluids **33**, 093110 (2021)
Pradipto and H. Hayakawa, Phys. Rev. Fluids **6**, 033301 (2021)



LBM-DEM Simulation

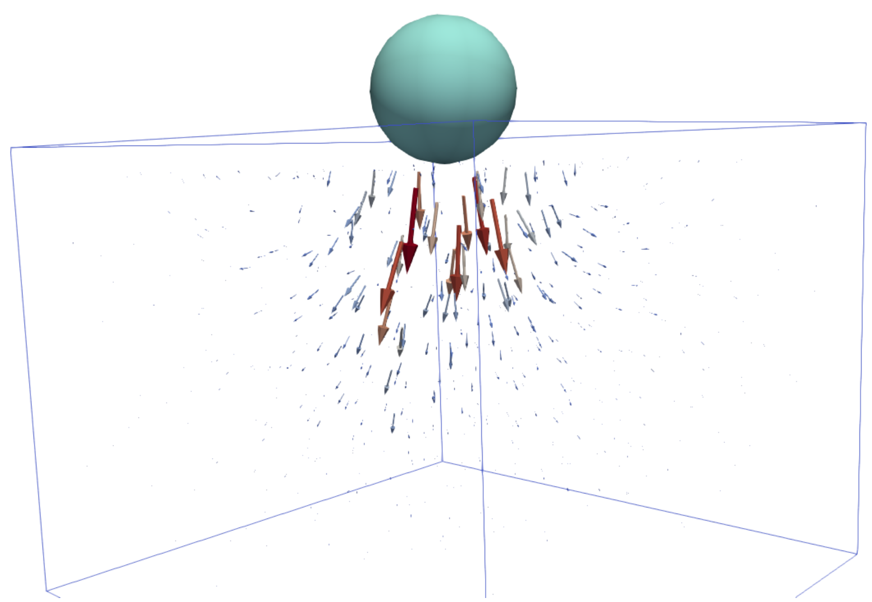
Contacting suspended particles form networks → force chains



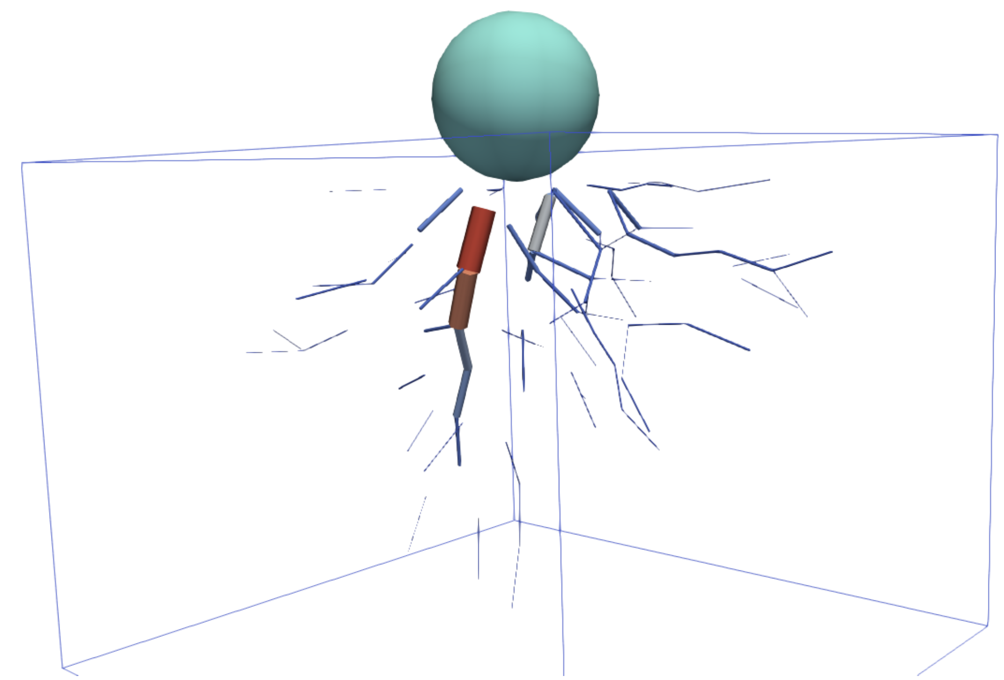
- Calculate number of percolating force chains $n(t)$
  - Incorporate elastic response from $n(t)$

- Do elastic response exists when **no** force chains percolate?
- What about actual running motion?

Obtaining fields inside the hardening suspensions



Particles velocity (discrete)



Force chains (contact between particles)

Coarse graining J. Zhang, R. P. Behringer, I. Goldhirsch, Prog. Theor. Phys. Supp **184**, 16 (2010)

Tensorial basis adapted to local flow conditions Giusteri and Seto, J. Rheol. **62**, 713 (2018)

Strain rate tensor

$$\overleftrightarrow{D} = \frac{1}{2}(\nabla \mathbf{u} + \nabla \mathbf{u}^T)$$

Scalar strain rate

$$\dot{\varepsilon} = \sqrt{2\overleftrightarrow{D} : \overleftrightarrow{D}}$$

Strain tensor (from particle overlap)

$$\overleftrightarrow{L} = \frac{1}{2}(\nabla \delta_n + \nabla \delta_n^T)$$

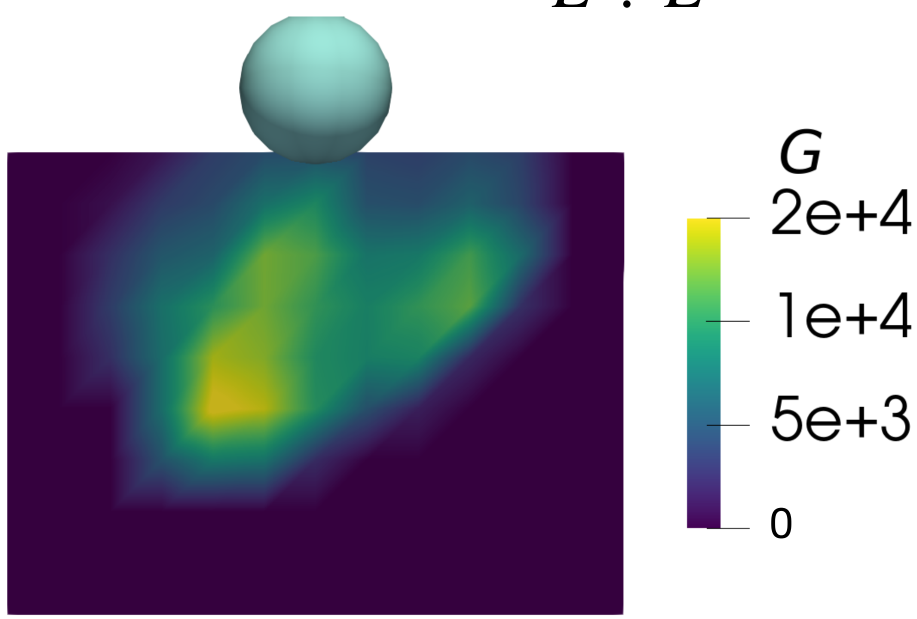
Scalar strain

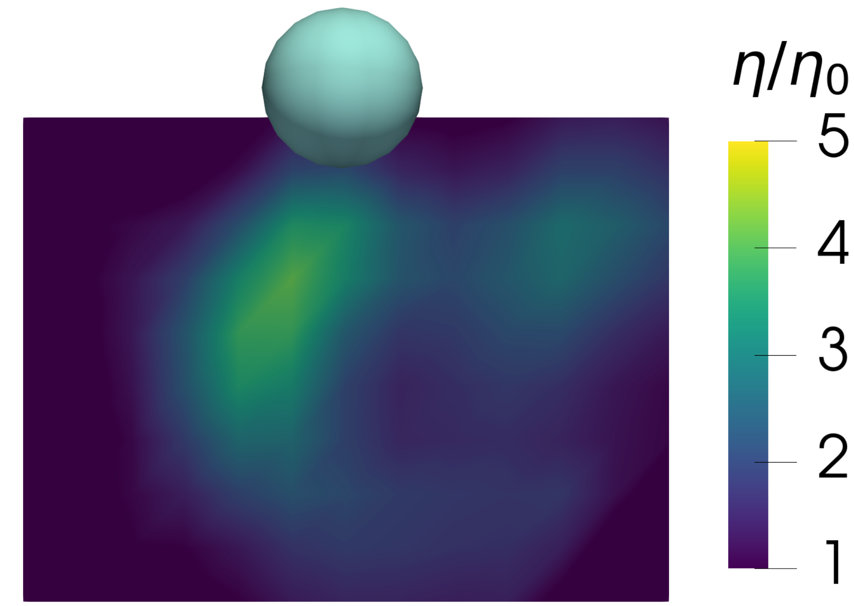
$$\varepsilon = \sqrt{2\overleftrightarrow{L} : \overleftrightarrow{L}}$$

Dynamically jammed region → region with finite rigidity

Rigidity $G = \frac{1}{2}\frac{\overleftrightarrow{\sigma} : \overleftrightarrow{L}}{\overleftrightarrow{L} : \overleftrightarrow{L}}$

Viscosity $\eta = \frac{1}{2}\frac{\overleftrightarrow{\sigma} : \overleftrightarrow{D}}{\overleftrightarrow{D} : \overleftrightarrow{D}}$



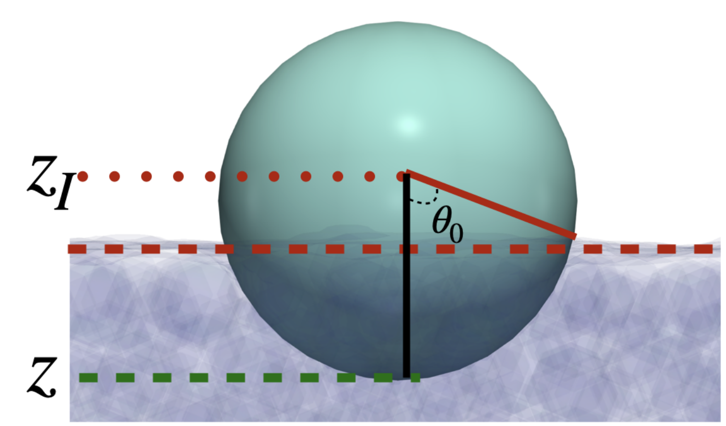


Time-dependent effective viscosity and elasticity from the DJR

$$\eta_{\text{eff}} = \eta_0 + \frac{1}{V_{\text{djr}}}\int_d \eta dV \qquad G_{\text{eff}} = \frac{1}{V_{\text{djr}}}\int_d G dV$$

DJR model (data-assisted)

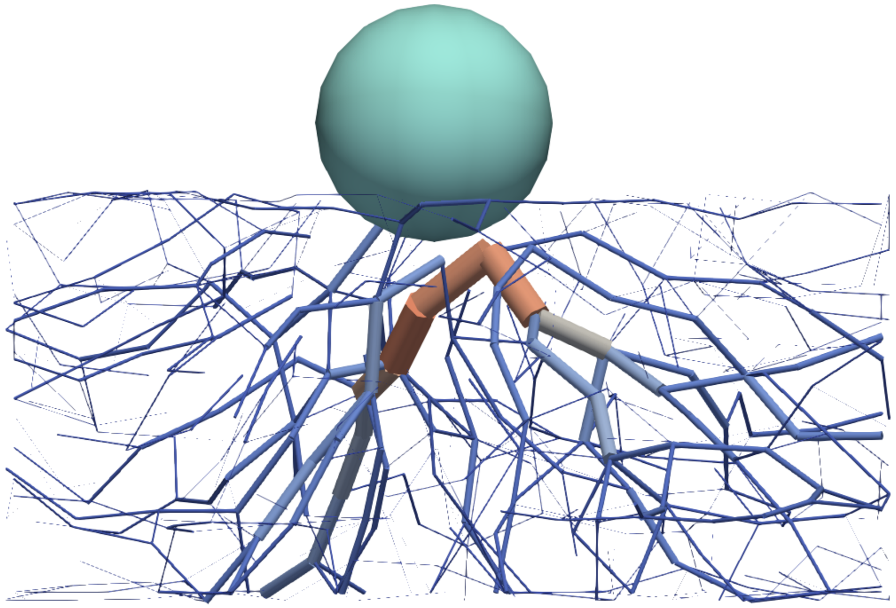
$$m_I \ddot{z}_I = -m_I g - 3\pi\eta_{\text{eff}} z \dot{z}_I + C\frac{A_{DJR}}{H_{DJR}} G_{\text{eff}} z$$



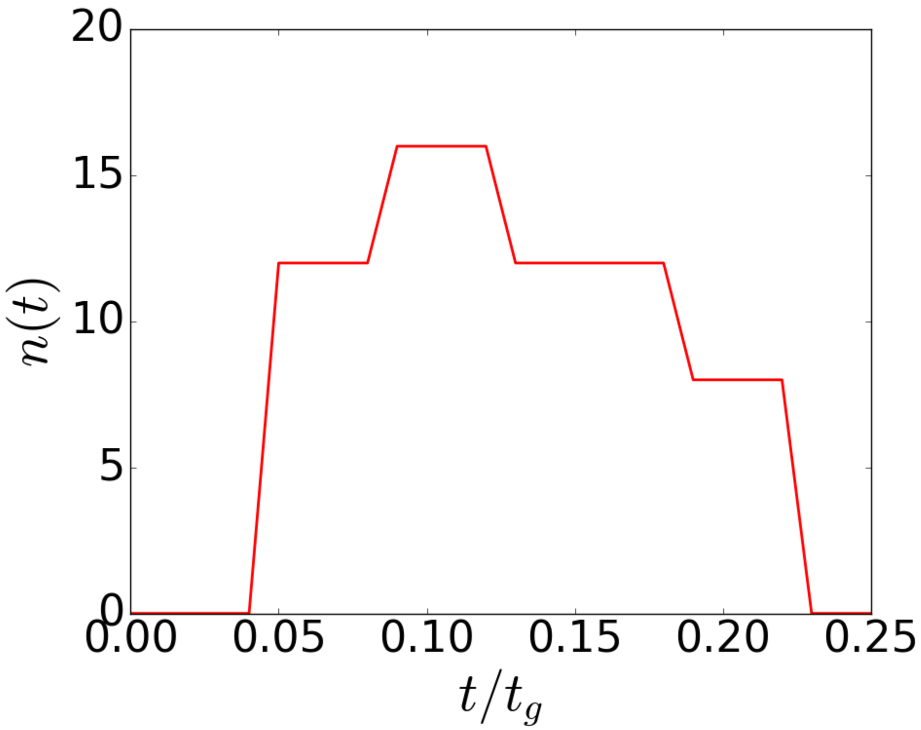
$z = z_I - a_I$

Force chains vs DJR model

Floating + force chains model

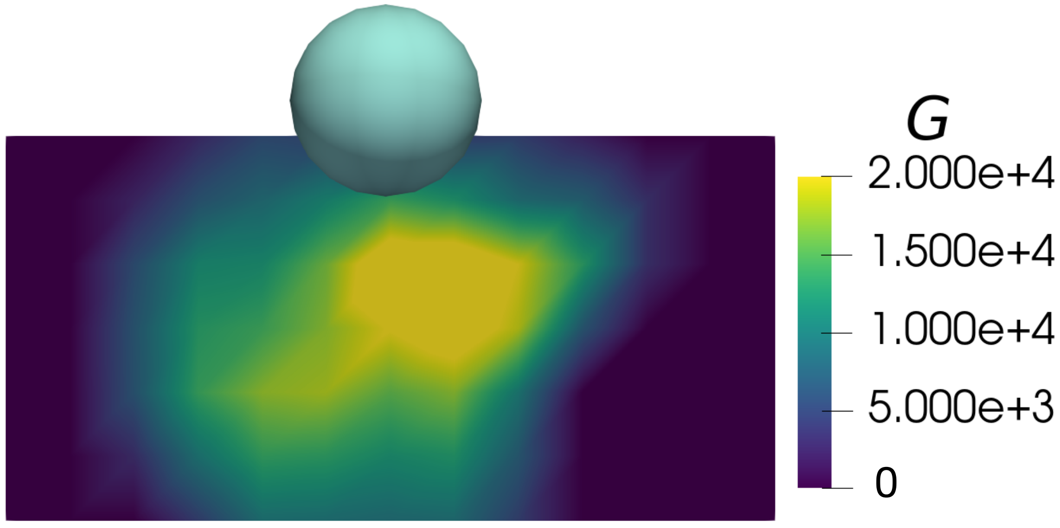


Count number of percolating force chains $n(t)$

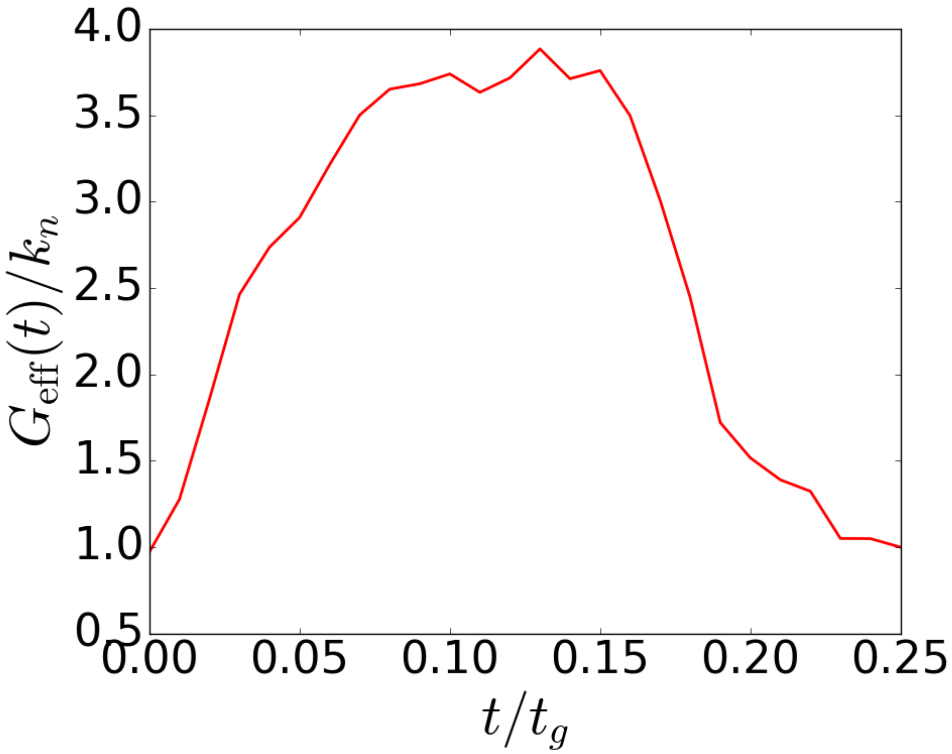


Finite rigidity even when no percolating force chains

DJR model

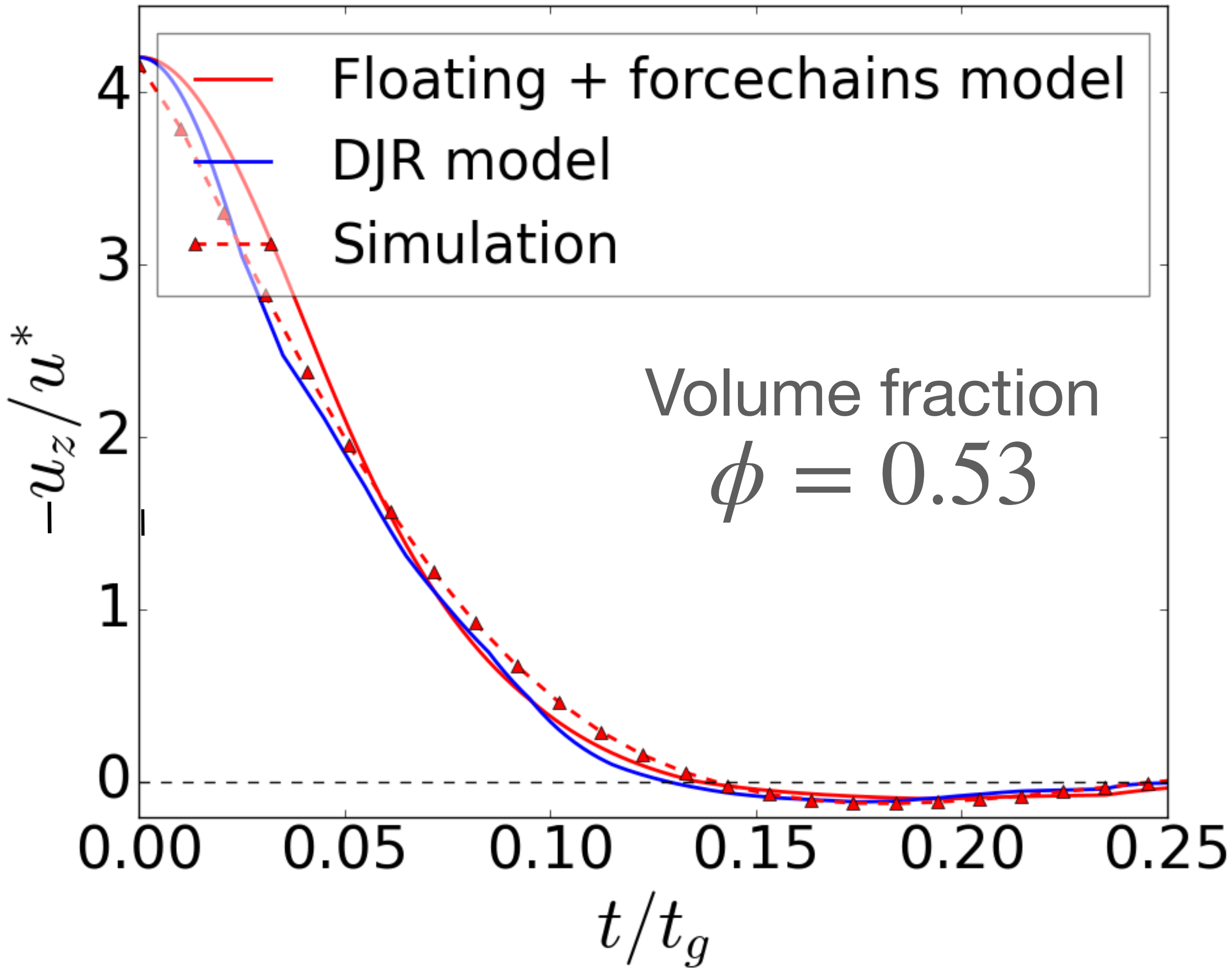


$$G_{\text{eff}} = \frac{1}{V_{\text{djr}}} \int_d G dV$$

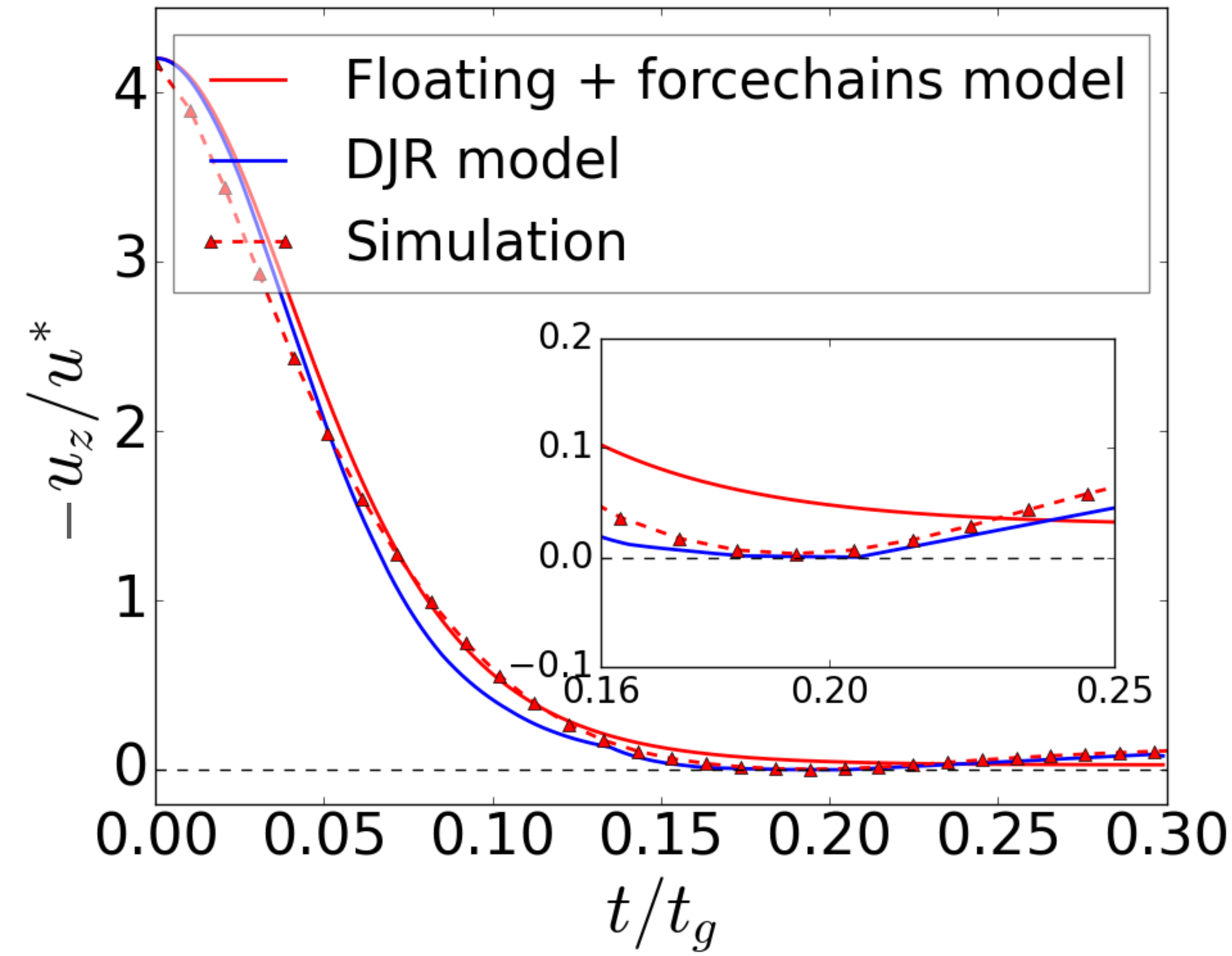


Force chains vs DJR model

Shallow container



Deep container



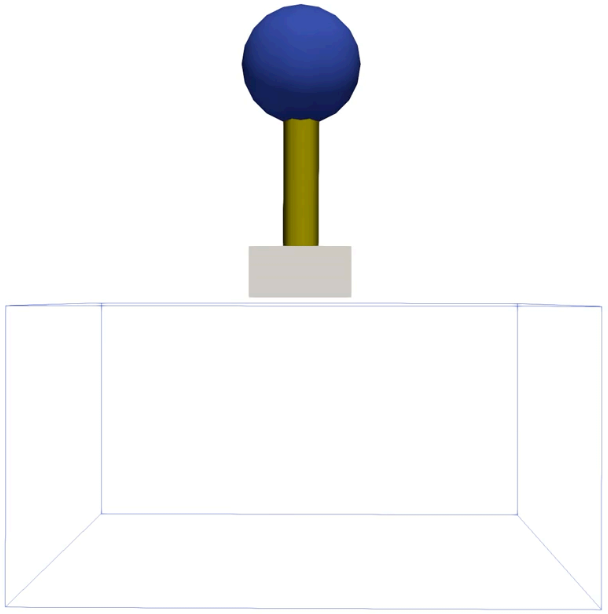
Both model work well

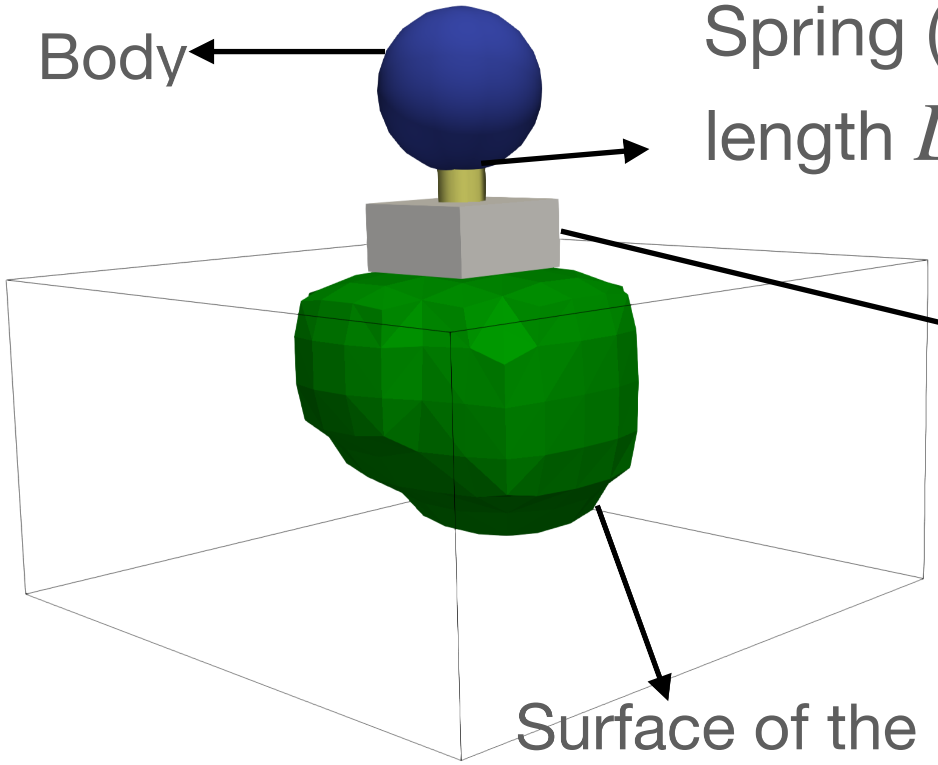
- DJR model works better
- Mild elastic effect captured

Hopping robot on top of dense suspensions

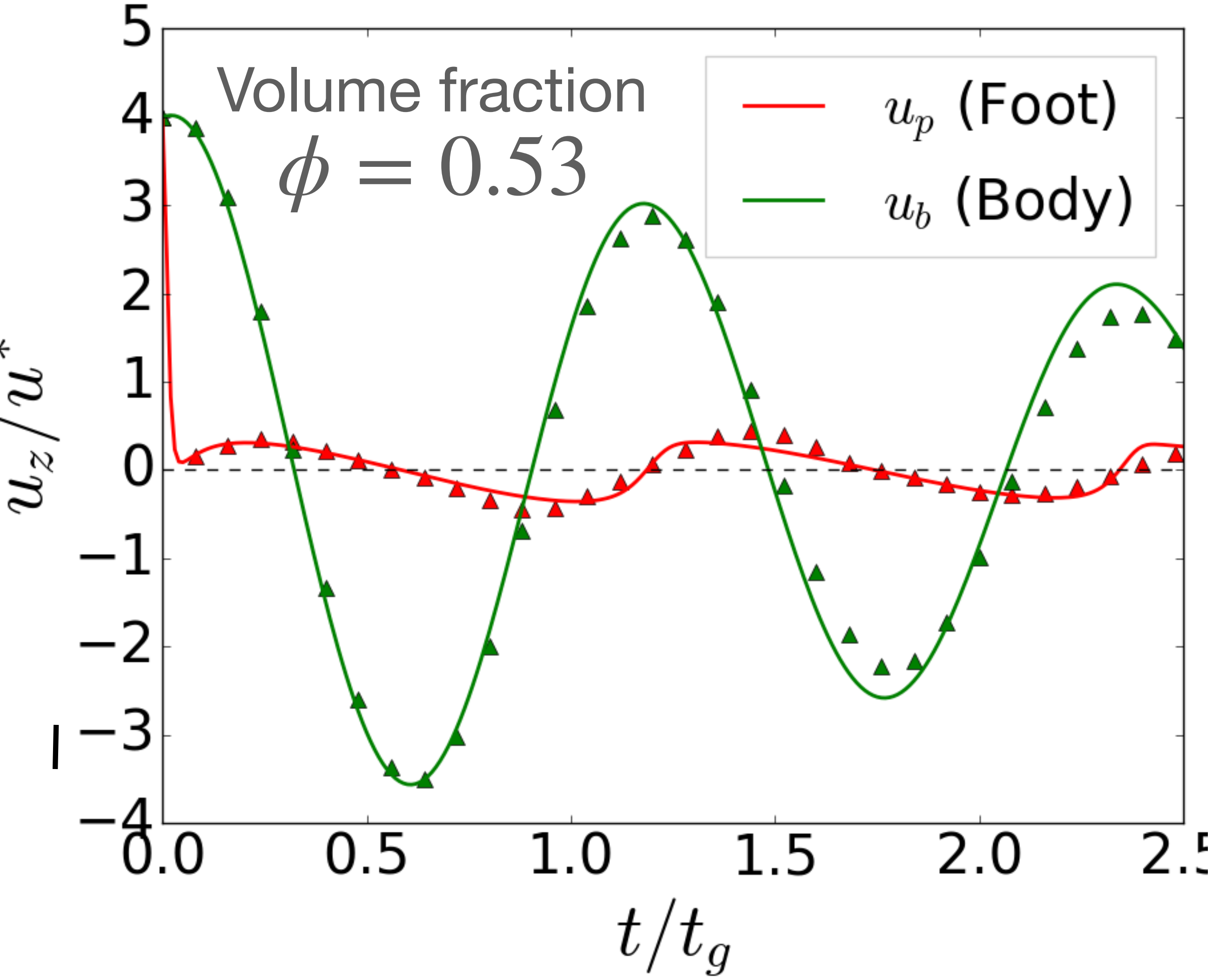
Spring-mass model for one legged hopping robot Raibert and Tello, IEEE Expert **1**, 89 (1986)
(Raibert's hopper)

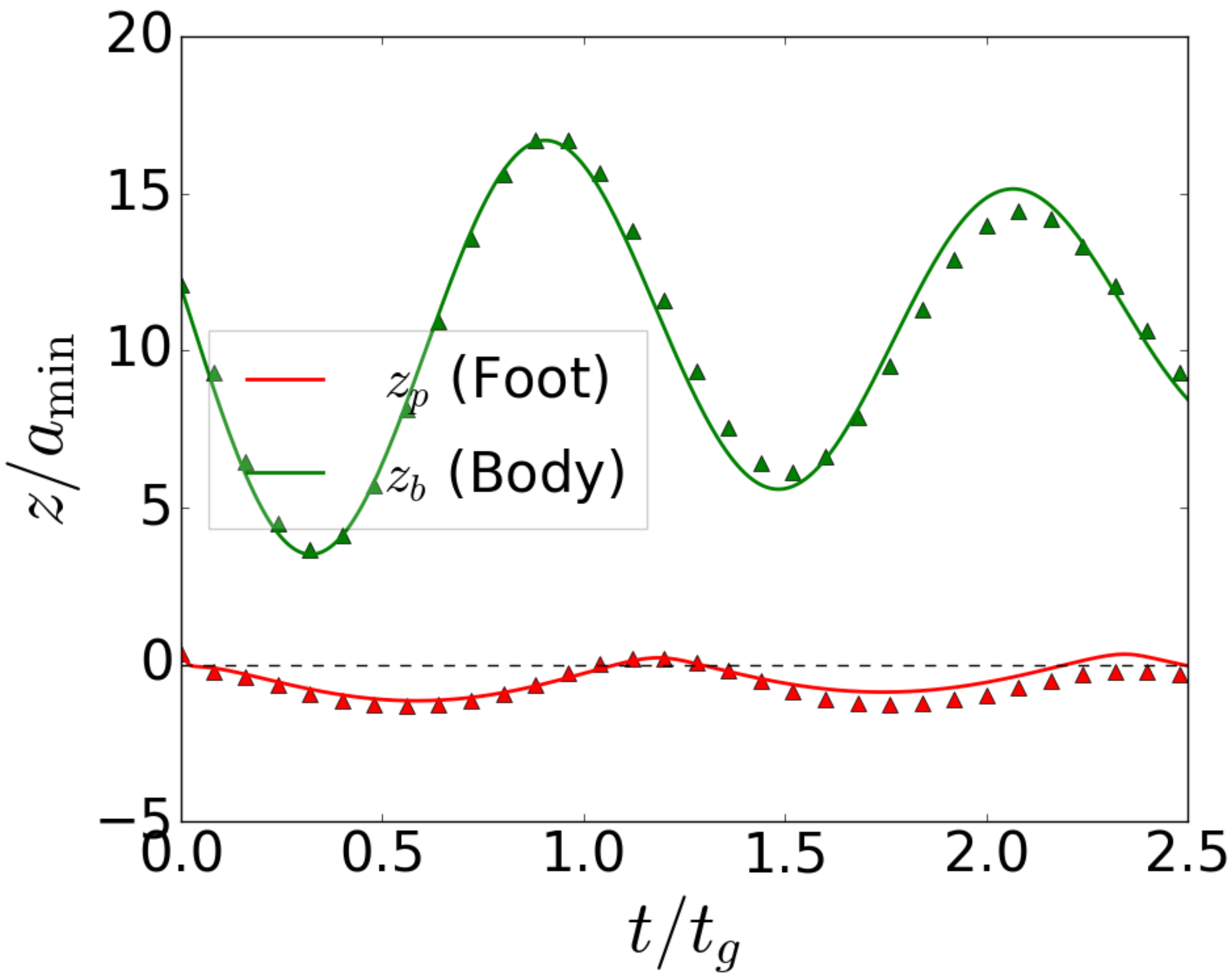
Source: https://youtu.be/XFXj81mvInc





Motion of spring-mass-body system





Foot undergoes multiple rebounds ($u_p > 0$) and also hops ($z_p > 0$)

Summary

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- Dynamically jammed region (DJR) is formed inside the suspensions after the impact → Region with finite rigidity inside the suspensions

- Elastic response exists even without percolating force chains → We proposed model that contains effective viscosity and elasticity of the DJR

- Simulation of foot-spring-body system to mimic hopping →observed multiple rebounds and hops