DAEMON DRIVEN BY GEOMETRICAL PHASE

Hisao Hayakawa

ersity)

collaboration with

anyo-Onoda Univ.)

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@YITP, Kyoto Univ.

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Contents

- Introduction
  - Thouless pumping and Berry's phase
  - Trade-off relation of thermodynamic engines
- Geometric Formulation to "thermodynamics" in quantum engines
- Application to Anderson model
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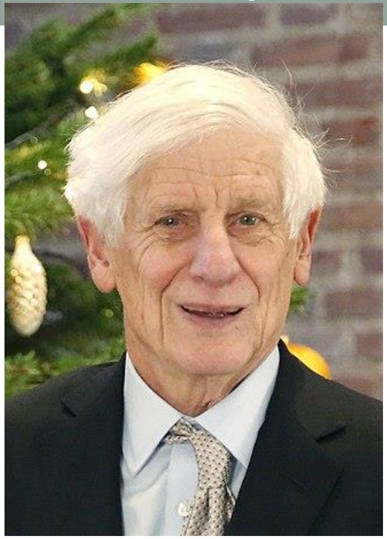
Introduction

- I am talking on geometric pumping proposed by D. Thouless (1983) who got the Nobel prize in 2016.
- The essence of Thouless pumping is Berry's phase proposed by M. Berry (1984).
- The idea by two big shots can be applied to non-equilibrium driven systems.
- Design of (thermodynamic) engines will be important.



Photos taken from wikipedia

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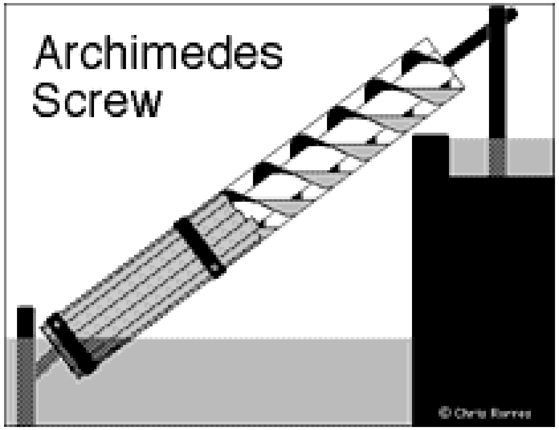
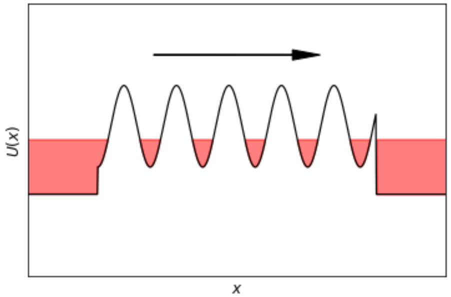
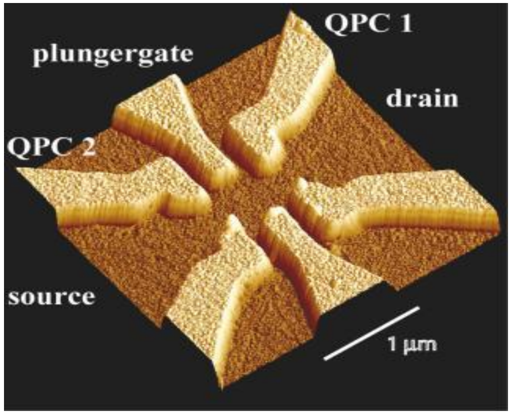
Pumping process

- Pump=>We need a bias.

The current can flow in a mesoscopic system without dc bias =>*Geometric (Thouless) pumping*.



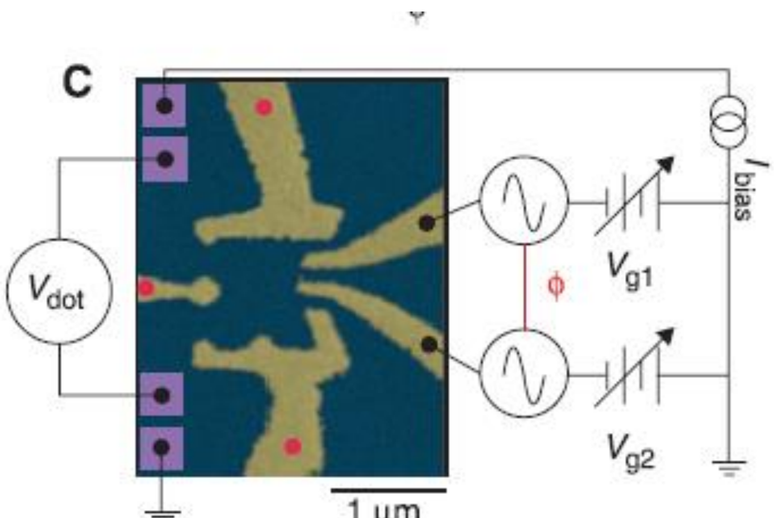


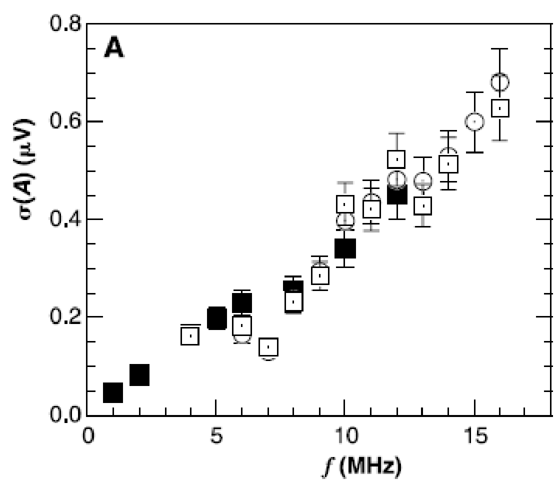


A smart nano-machine

Previous studies

- Experiments
  - Pothier et al. (1992) get a classical pumping for a mesoscopic system.
  - Switkes et al. (1999) control the gate voltage and get 20 electrons current per a cycle in a quantum dot system.





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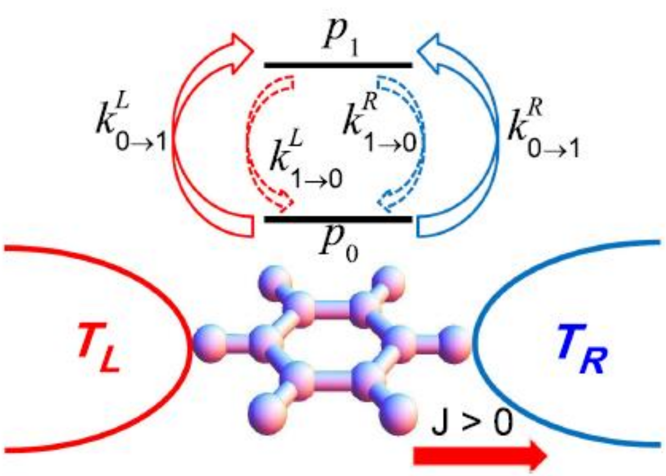
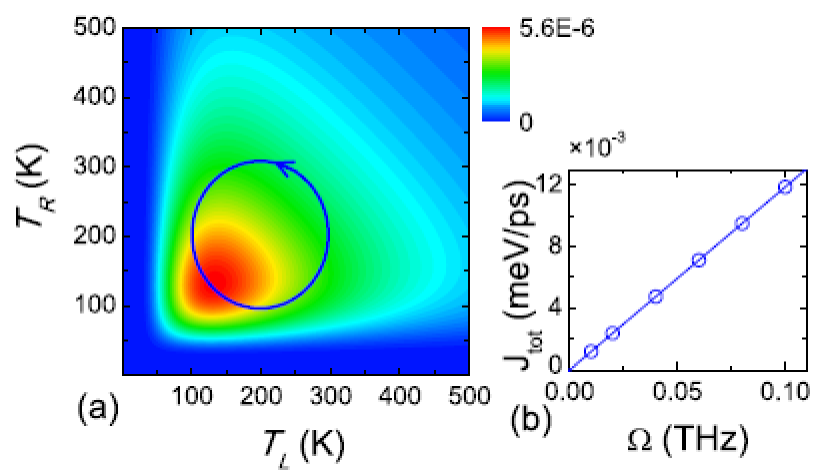
Previous theoretical studies

- Adiabatic geometric pumping (theories)
  - Thouless (1983) for a closed system
  - Open quantum system (P. W. Brower (1998)).
- Sintsyin & Nemenman (2007) indicated that Berry's phase can be used to nonequilibrium stochastic processes.
- Ren-Hänggi-Li (2010) analyzed a spin-boson system and to clarify the role of Berry's phase.

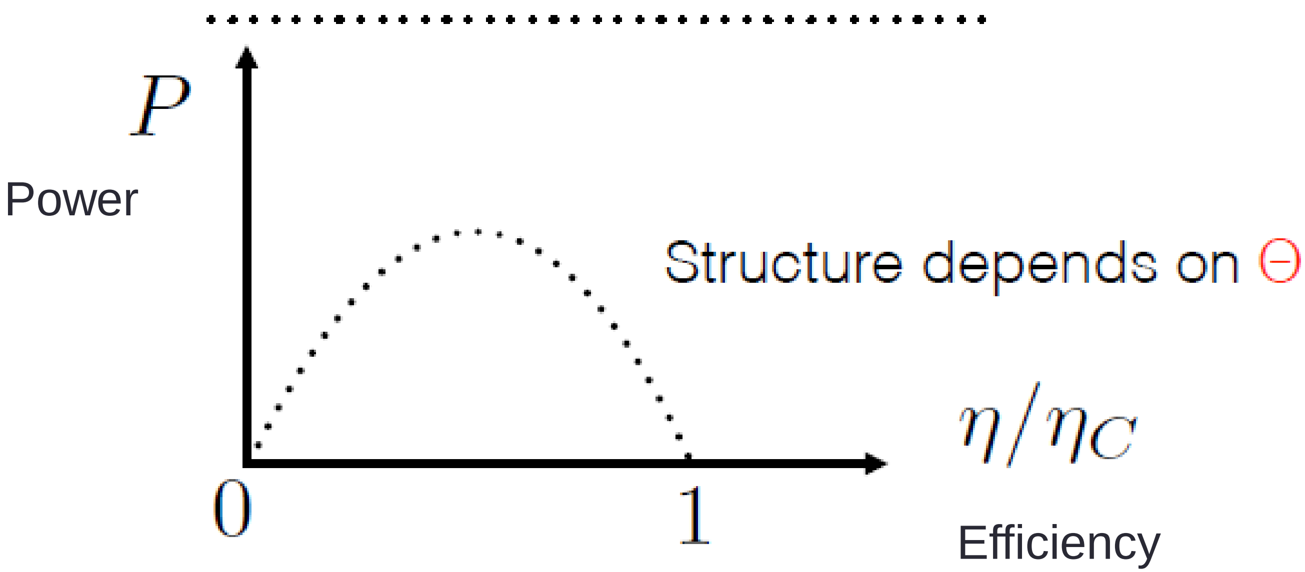




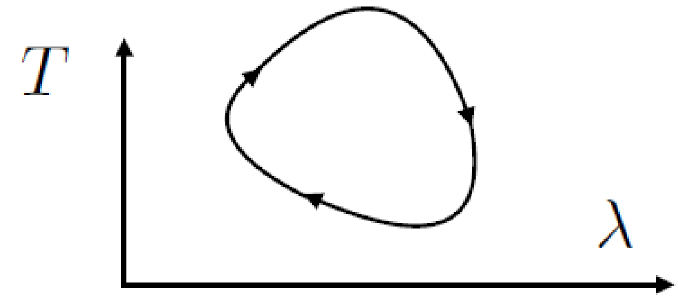
Trade-off relation

- Shiraishi-Saito-Tasaki (2016) gave the trade-off relation for small amplitude and low speed case in Markovian dynamics.

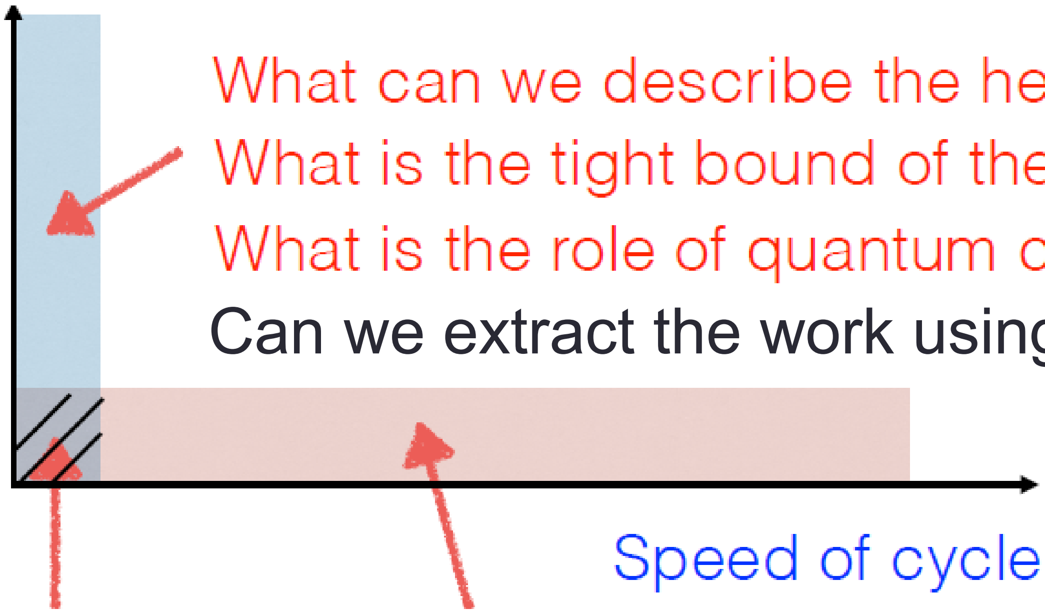
$$\Theta \beta_l (\eta/\eta_C)[1-(\eta/\eta_C)] \geq P$$



Large amplitude modulation



Amplitude of (λ, T)



What can we describe the heat engines ?

What is the tight bound of the power vs. efficiency ?

What is the role of quantum coherence ?

Can we extract the work using Berry's phase?

Speed of cycle $1/\tau$

Shiraishi, Saito & Tasaki, PRL (2016)

Brandner & Saito, PRX (2015)

Large amplitude engines

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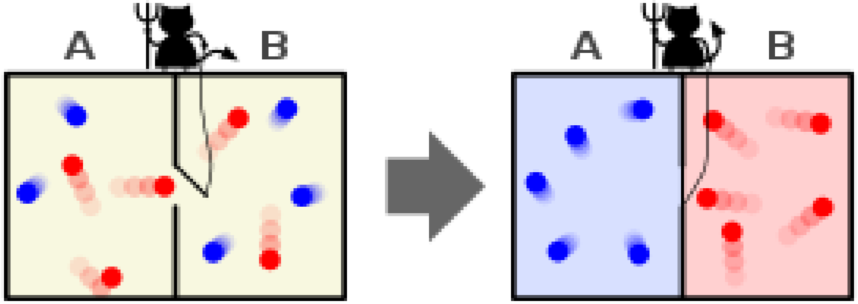
- Brander & Saito, PRL **124**, 040602 (2020) implemented a heat engine with large amplitude.
  - They clarified the role of geometrical metric tensor.
  - They still assumed that there is only one reservoir.
- Heat engines controlled by multiple reservoirs associated with Thouless pumping
  - Hino & Hayakawa, PRR **3**, 013187 (2021).
- Let us discuss a quantum engine by controlling the chemical potentials and Hamiltonian.
  - Extension of the previous works
  - Establishment of quantum engines which are easy to be implemented.

Geometrical Daemon

- Maxwell's daemon utlizes information.



- Geometrical daemon
  - Can we extract the work from the geometrical phase?
  - This is the geometrical daemon!

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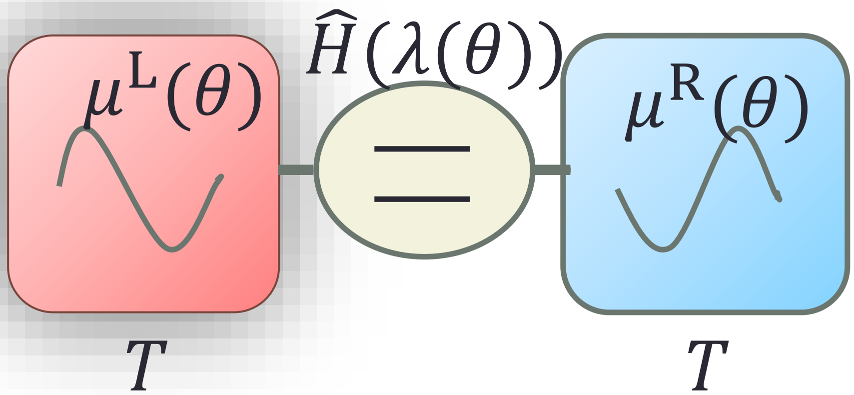
Setup

- System+two reservoirs



- Control parameters:
  - Chemical potentials in heat reservoirs
  - One parameter in the system Hamiltonian to extract the work

$$\mu_L = \overline{\mu}(1 + r_L \sin\theta), \quad \mu_R = \overline{\mu}[1 + r_R \sin(\theta + \delta)],$$

$$\overline{\mu^\alpha} := \frac{1}{\tau_p}\int_0^{\tau_p} dt \mu^\alpha(t) \quad \hat{H}(\lambda(\theta)) = \hat{H}(\lambda(\theta + 2\pi))$$

Strong assumption: we ignore the cost of control the Hamiltonian.

Chemical Engines

- Quantum (Lindblad type) master equation for the density matrix $\hat{\rho}$

$$\boxed{\frac{d}{d\theta}|\hat{\rho}(\theta,\delta)\rangle = \epsilon^{-1}\hat{K}|\hat{\rho}(\theta,\delta)\rangle,}$$

$$\mathbf{\Lambda} := \left(\lambda, \frac{\mu^{\mathrm{L}}}{\overline{\mu^{\mathrm{L}}}}, \frac{\mu^{\mathrm{R}}}{\overline{\mu^{\mathrm{R}}}}\right)$$

$$\epsilon := 1/(\tau_{\mathrm{p}}\Gamma) \qquad \theta := 2\pi(t-t_0)/\tau_{\mathrm{p}}$$

Γ is the coupling strength

KL divergence

- Relative entropy (non-negative)

$$\boxed{S^{\mathrm{HS}}(\hat{\rho}||\hat{\sigma}) := \mathrm{Tr}\left[\hat{\rho}(\ln\hat{\rho} - \ln\hat{\sigma})\right]}$$

- We discuss the entropy production

$$\Delta S(\theta,\delta) := S^{\mathrm{HS}}(\hat{\rho}(\theta,\delta)||\hat{\rho}^{\mathrm{SS}}(\theta,\delta)) - S^{\mathrm{HS}}(\hat{\rho}(2\pi+\theta,\delta)||\hat{\rho}^{\mathrm{SS}}(2\pi+\theta,\delta)),$$

- If we begin with $\hat{\rho}(0,\delta) = \hat{\rho}^{\mathrm{SS}}(0,\delta)$

$$\boxed{\Delta S(\delta) = -S^{\mathrm{HS}}(\hat{\rho}(2\pi,\delta)||\hat{\rho}^{\mathrm{SS}}(2\pi,\delta)) \leq 0,}$$

$$\Delta S(\delta) := \Delta S(\theta = 0,\delta)$$

Entropy production

- Restriction: entropy production $\Delta S(\delta) \leq 0$ due to non-negativity of S^{HS}.
- We expect that $\Delta S(\delta) \geq 0$ because of the monotonicity of the relative entropy $\dot{S}^{HS} \leq 0, if\ the\ dynamics\ satisfies\ CPTP$.
- Is $S^{HS}(\rho(\theta,\delta)||\rho^{SS}(\theta,\delta))$ always zero if we start from $\rho = \rho^{SS}$?
- Answer is No.

The evolution of the density matrix

- One cycle evolution of the density matrix given by

$$\boxed{\Delta|\hat{\rho}\rangle := |\hat{\rho}(2\pi,\delta)\rangle - |\hat{\rho}(0,\delta)\rangle = \sum_{i\neq 0} \mathcal{C}_i(\delta)|r_i(0,\delta)\rangle,}$$

- where

$$\mathcal{C}_i(\delta) := \int_0^{2\pi} d\phi e^{\epsilon^{-1}\int_\phi^{2\pi} dz \varepsilon_i(z,\delta)} \mathcal{A}_i^\mu \frac{\partial \Lambda_\mu}{\partial \phi}.$$

- Here, the BSN connection $\mathcal{A}_i^\mu$ is defined as

$$\mathcal{A}_i^\mu(\phi,\delta) := -\langle \ell_i(\Lambda(\phi,\delta))|\frac{\partial}{\partial \Lambda_\mu}|r_0(\Lambda(\phi,\delta))\rangle.$$

Work relations

- We can introduce the "work" (see Jarzynski 1997)

$$W(\delta) := \int_0^{2\pi} d\theta \mathscr{P}(\theta, \delta), \quad \boxed{\mathscr{P}(\theta, \delta) := \mathrm{Tr}\left[\hat{\rho}(\theta, \delta) \frac{\partial \hat{H}(\lambda(\theta))}{\partial \lambda(\theta)}\right] \dot{\lambda}(\theta).}$$

- Absorbing and release heat

$$Q_{A/R}(\delta) := \int_0^{2\pi} d\theta \mathscr{P}_{A/R}(\theta, \delta). \quad \mathscr{P}_{A/R}(\theta, \delta) := \frac{\mathscr{P}(\theta, \delta) \pm |\mathscr{P}(\theta, \delta)|}{2},$$

- The efficiency

$$\boxed{\eta(\delta) := \frac{|W(\delta)|}{Q_A(\delta)}.}$$

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Application to Anderson model

- The total Hamiltonian is given by

$$\hat{H}^{\mathrm{tot}} := \hat{H} + \hat{H}^{\mathrm{r}} + \hat{H}^{\mathrm{int}},$$

- where

$$\hat{H} = \sum_{\sigma} \epsilon_0 \hat{d}_\sigma^\dagger \hat{d}_\sigma + U(\theta)\hat{n}_\uparrow \hat{n}_\downarrow,$$

$$\hat{H}^{\mathrm{r}} = \sum_{\alpha,k,\sigma} \epsilon_k \hat{a}_{\alpha,k,\sigma}^\dagger \hat{a}_{\alpha,k,\sigma},$$

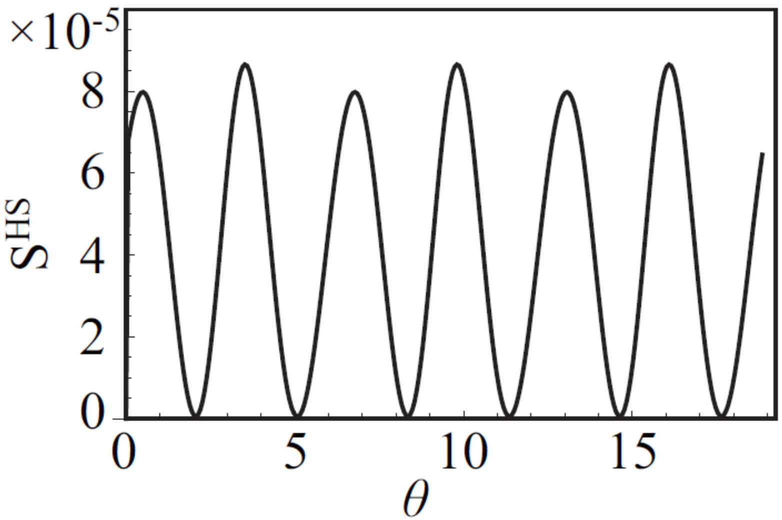
$$\hat{H}^{\mathrm{int}} = \sum_{\alpha,k,\sigma} V_\alpha \hat{d}_\sigma^\dagger \hat{a}_{\alpha,k,\sigma} + \mathrm{h.c.},$$

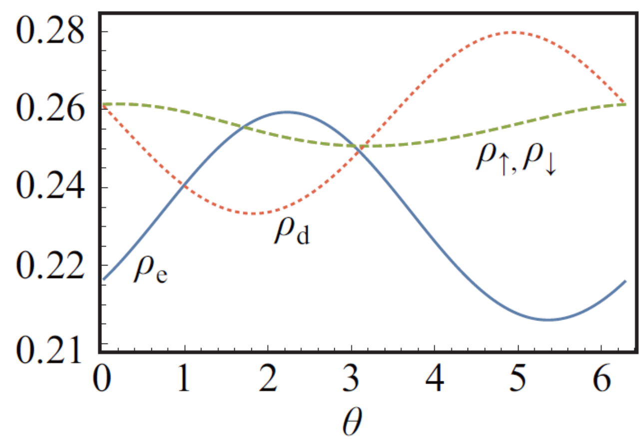
$$U(\theta) := \check{U}_0 \lambda(\theta)$$

$$\hat{n}_\sigma = \hat{d}_\sigma^\dagger \hat{d}_\sigma.$$

Time evolution of relative entropy

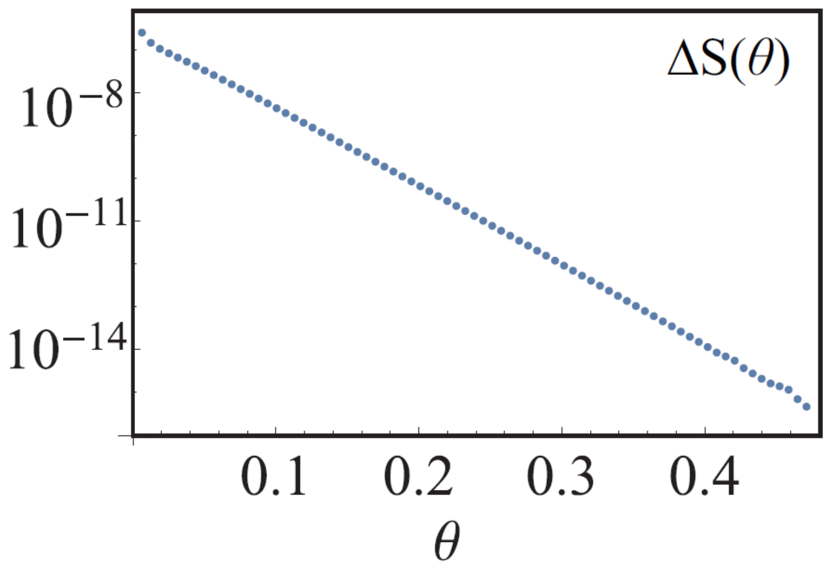
- The relative entropy is non-monotonic, though the dynamic satisfies CPTP!

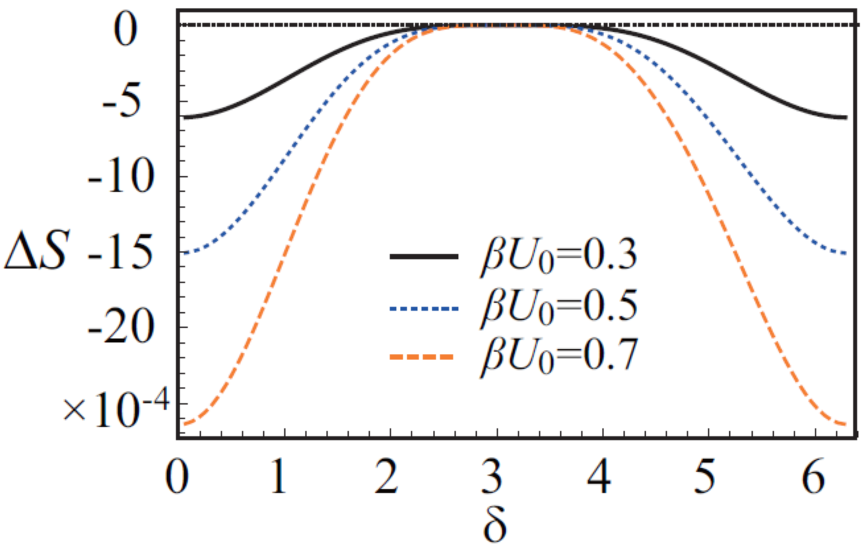




Time evolution of relative entropy (2)

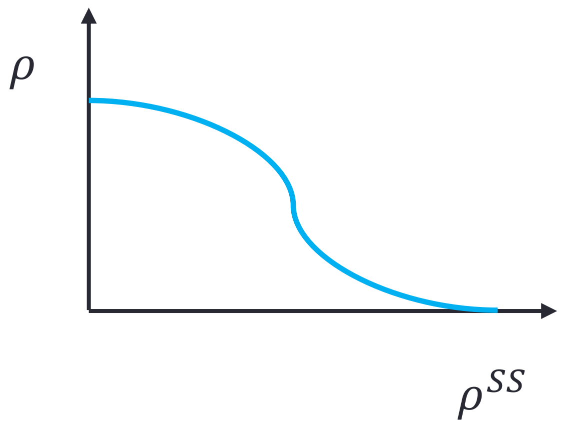
- Initial exponential damping=> decrease the entropy



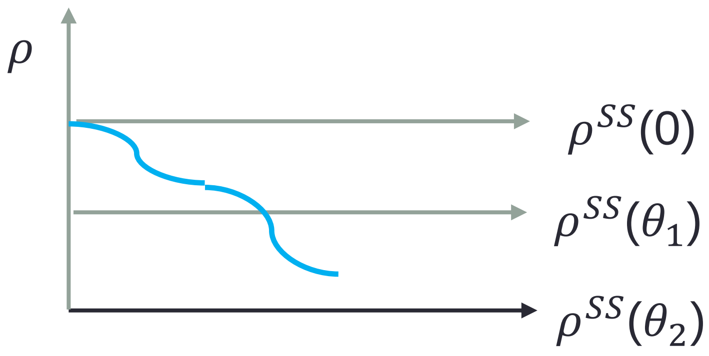


Why is the monotonicity of entropy violated?

- If the dynamics keeps CPTP, the relative entropy cannot increase with time.
- Nevertheless, it increases with time.
- Why?



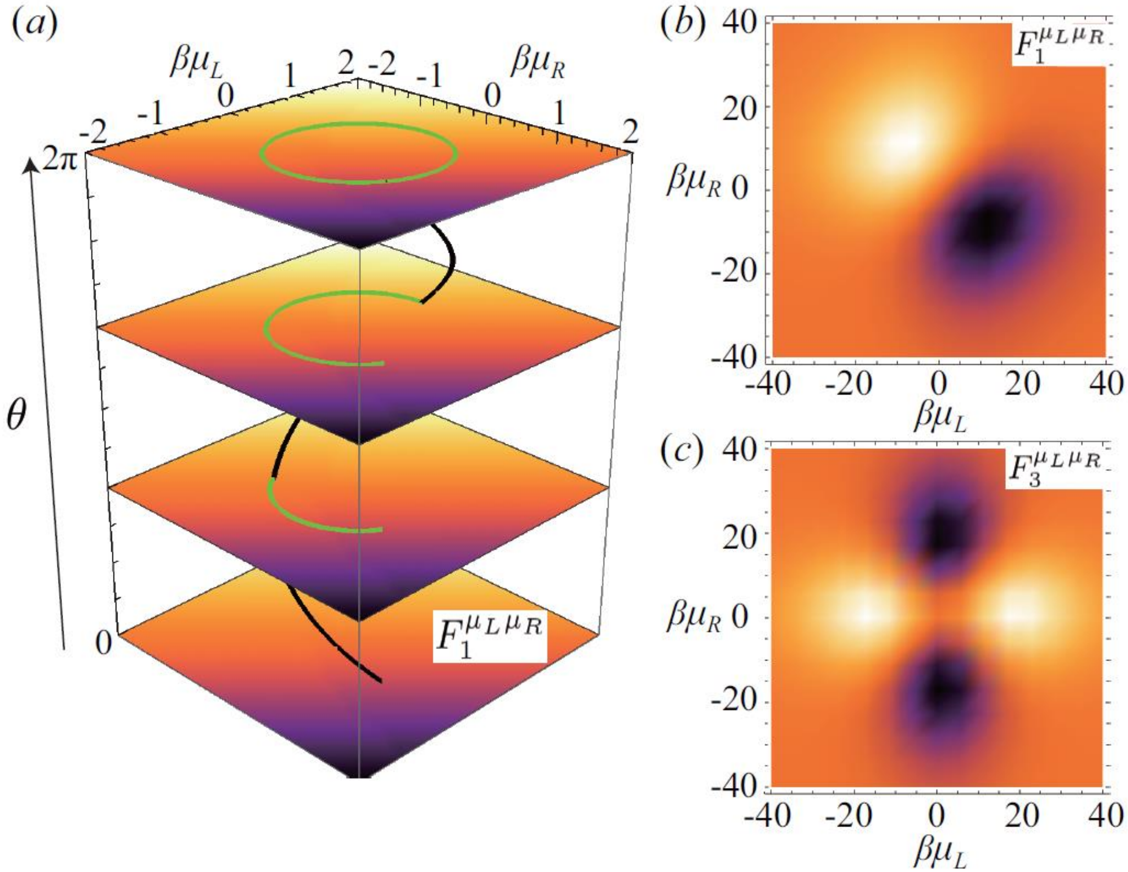
No modulation



With modulation

The state cannot be relaxed to a simple “steady state” under a modulation=> Non-adiabatic effect.

Schematics of contour and BSN curvature



$$F_i^{\mu\nu}(\theta,\delta) := \left(\frac{\partial \mathcal{A}_i^{\nu}}{\partial \Lambda_\mu}\right)_\theta - \left(\frac{\partial \mathcal{A}_i^{\mu}}{\partial \Lambda_\nu}\right)_\theta$$

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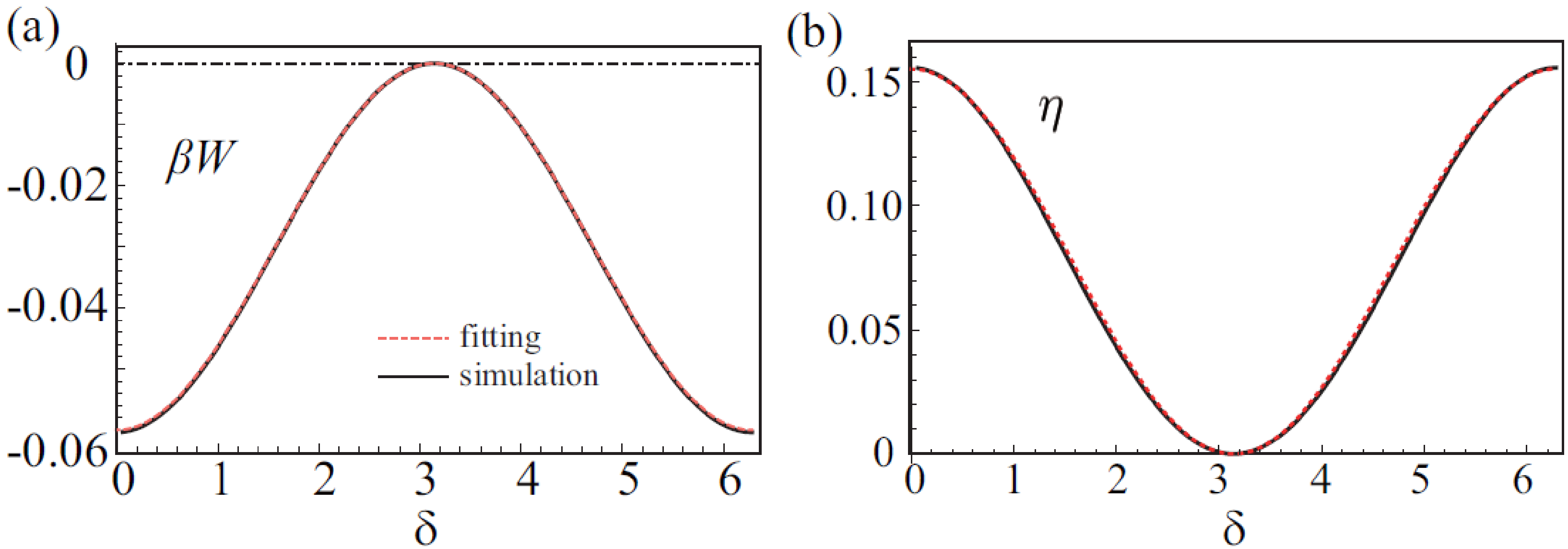
Work and efficiency

- The efficiency is maximum at $\delta = 0$ without phase modulation of the chemical potential.
- The efficiency is zero at $\delta = \pi$.



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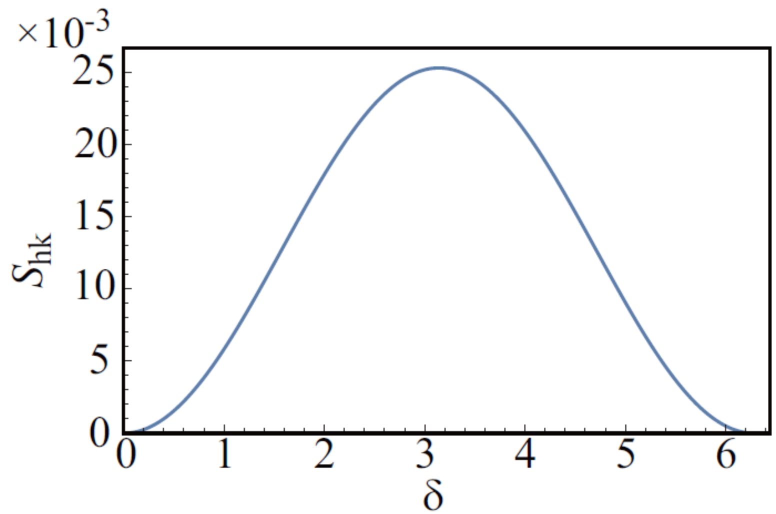
Discussion

- The cost caused by the current ⇔ house keeping entropy to make ρ^{ss}



- The house keeping entropy is zero at $\delta = 0$ because of the absence of the chemical potential gradient.
- We need to take into account the cost of Hamiltonian control.

Summary

- We demonstrate that we can extract work if we control the Hamiltonian and chemical potential simultaneously.
  - Geometrical Daemon
- This causes the negative entropy production
  - This is obtained by the initial relaxation process.
- The initial steady state<= we wait for its achievement.
- See R. Yoshii and H. Hayakawa, arXIv:2205.15193.
- See also H. Hayakawa et al. arXiv:2112.12370.