#### German-Japanese 25<sup>th</sup> Anniversary Symposium

# Multi species asymmetric simple exclusion process with impurity activated flips

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[Reference: arXiv:2205.03082 (2022)]

# **NON-EQUILIBRIUM SYSTEMS**

- Transport phenomena
  - net flow or current [absent in equilibrium]



Phase transitions (e.g. traffic jam)





### Understanding

### COMPLEX NON-EQUILIBRIUM PHENOMENA

### using

#### EXACTLY SOLVABLE TOY MODELS

Non-equilibrium steady state probability distribution

## <u>A random walker</u>



periodic lattice

STEADY STATE: all equally likely configurations

# Generalization:

many interacting random walkers

# Model:

Asymmetric Simple Exclusion Process



- Exactly solvable model: Matrix Product State [\*open boundary condition\*]
- Boundary induced phase transitions
- Applications: protein transport, traffic flow

Multi-lane ASEP: [more realistic]

almost no exact solution with correlation between lanes

(either mean field solution, or, exact solutions factorized over lanes)

Question: Approximate mapping of multi-lane ASEP to 1-d model that allows exact solution ??





## <u>Question:</u> Equivalent 1-d model??











# TASKS: (i) matrix algebra (ii) matrix representations

# **RESULTS**:

Finite dimensional Totally asymmetric motion matrices

Partially asymmetric motion

# Infinite dimensional matrices

[arXiv:2205.03082 (2022)]

# Negative Differential Mobility 11

## **General notion:**



### Drift current of species "K": $J_{K0}$



#### **Counter-flow induced clustering**









Non-ergodic



# [*arXiv*:2205.03082 (2022)]

# **Counter-flow induced clustering**

[arXiv:2208.03297 (2022)]

# THANK YOU

With increasing bias, if current decreases, then we have negative differential mobility

• non-interacting particles



[Baerts et.al., Phys. Rev. E 88, 052109 (2013)]

**Mechanism:** some kind of trapping that decreases dynamical activity

## $0+2011+2 \rightarrow 0+2012+2$ (accessible) $0+2011+2 \rightarrow 0+2021+2$ (not accessible)

only a subspace of the whole configuration space is accessible, for a given initial configuration

# NON-ERGODIC

#### Two point correlations



#### Average species densities

#### Effect of drift on flip dynamics: two species case



$$\begin{array}{lll} \text{drift} \left( \text{species} \right) : & I0 & \overleftarrow{\frac{p_I}{q_I}} & 0I & I = 1, 2, ..., \mu \\ \text{drift} \left( \text{impurity} \right) : & +0 & \xrightarrow{\epsilon} & 0 + \\ \text{flip} : & I + & \overleftarrow{\frac{w_{IK}}{w_{KI}}} & K + I, K = 1, ..., \mu \end{array}$$

$$\mathcal{M}_{i,i+1}\mathbf{X}_i\otimes\mathbf{X}_{i+1} = \tilde{\mathbf{X}}_i\otimes\mathbf{X}_{i+1} - \mathbf{X}_i\otimes\tilde{\mathbf{X}}_{i+1}$$

### Matrix algebra:

$$p_{K}D_{K}E - q_{K}ED_{K} = D_{K}$$

$$\epsilon AE = A$$

$$\sum_{\substack{I=1\\I \neq K}}^{\mu} w_{IK}D_{I}A = D_{K}A\sum_{\substack{I=1\\I \neq K}}^{\mu} w_{KI}$$

Finite dimensional representations: Totally asymmetric motion



Infinite dimensional representations: Partially asymmetric motion

