Multi species ASEP with impurities: matrix product state, negative mobility and clustering

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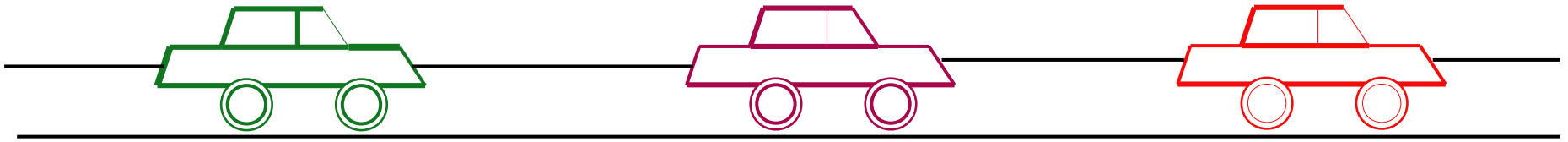
[*References*: *arXiv*:2205.03082 (2022),
arXiv:2208.03297 (2022)]

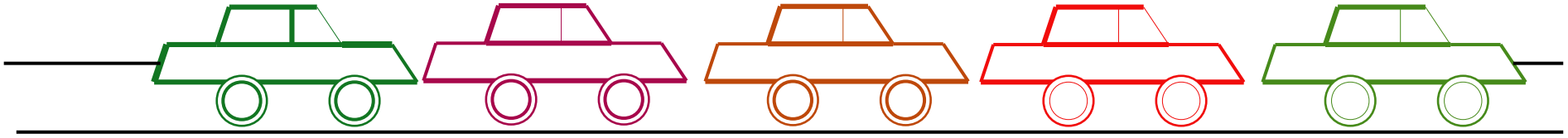
NON-EQUILIBRIUM SYSTEMS

- Transport phenomena

net flow or current [absent in equilibrium]



- Phase transitions *(e.g. traffic jam)*



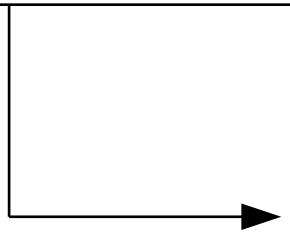
One general aim :

Understanding

COMPLEX NON-EQUILIBRIUM PHENOMENA

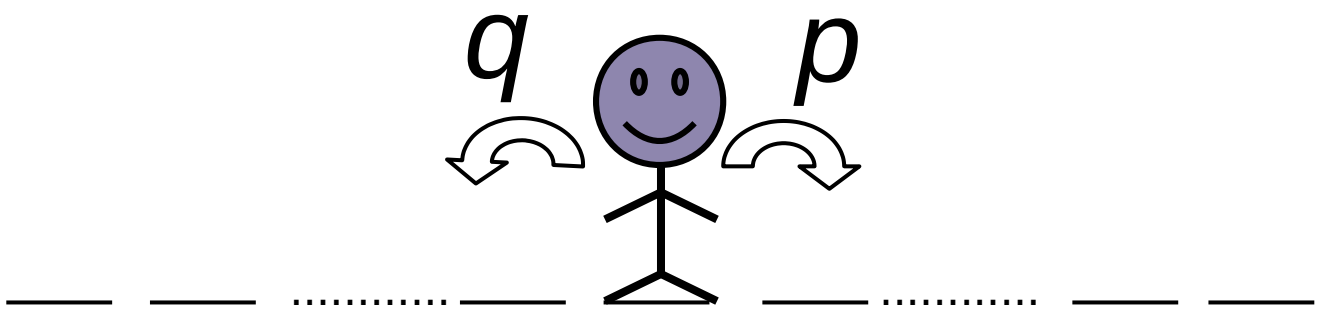
using

EXACTLY SOLVABLE TOY MODELS



Non-equilibrium
steady state
probability distribution

A random walker



periodic lattice

STEADY STATE: all equally likely configurations

Generalization:

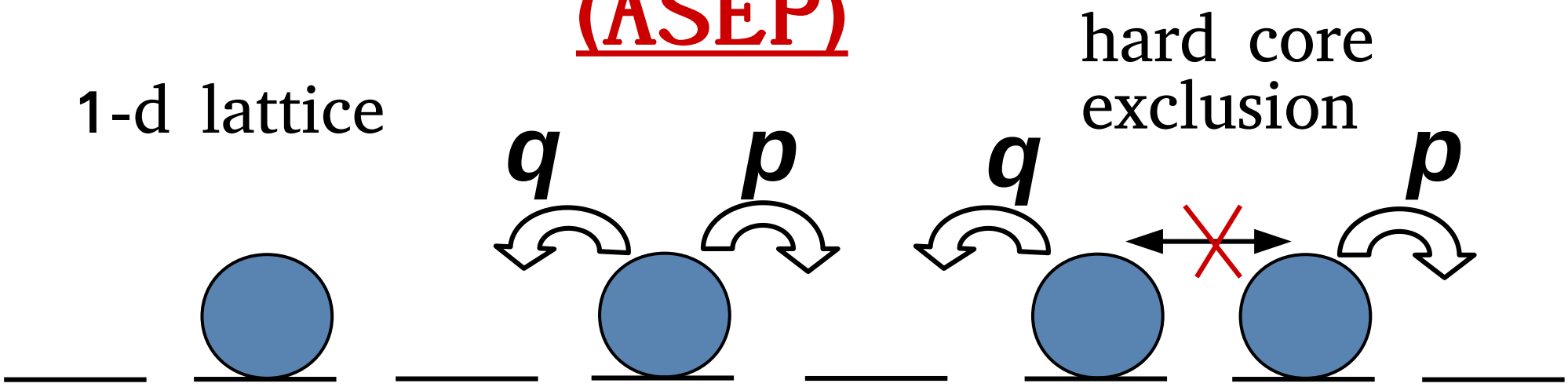
many interacting random walkers

⇩

Model:

Asymmetric Simple Exclusion Process

Asymmetric Simple Exclusion Process (ASEP)



- *Exactly solvable model: Matrix Product State*

  [*open boundary condition*]

- *Boundary induced phase transitions*

- *Applications: protein transport, traffic flow*

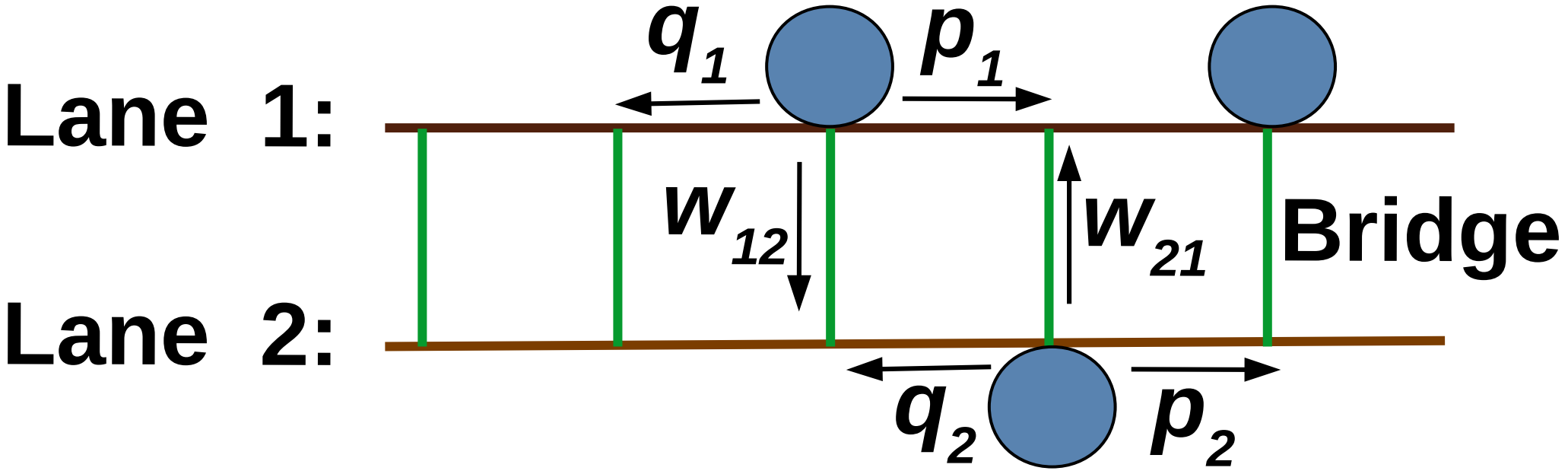
Multi-lane ASEP: [more realistic]

almost no exact solution with correlation between lanes

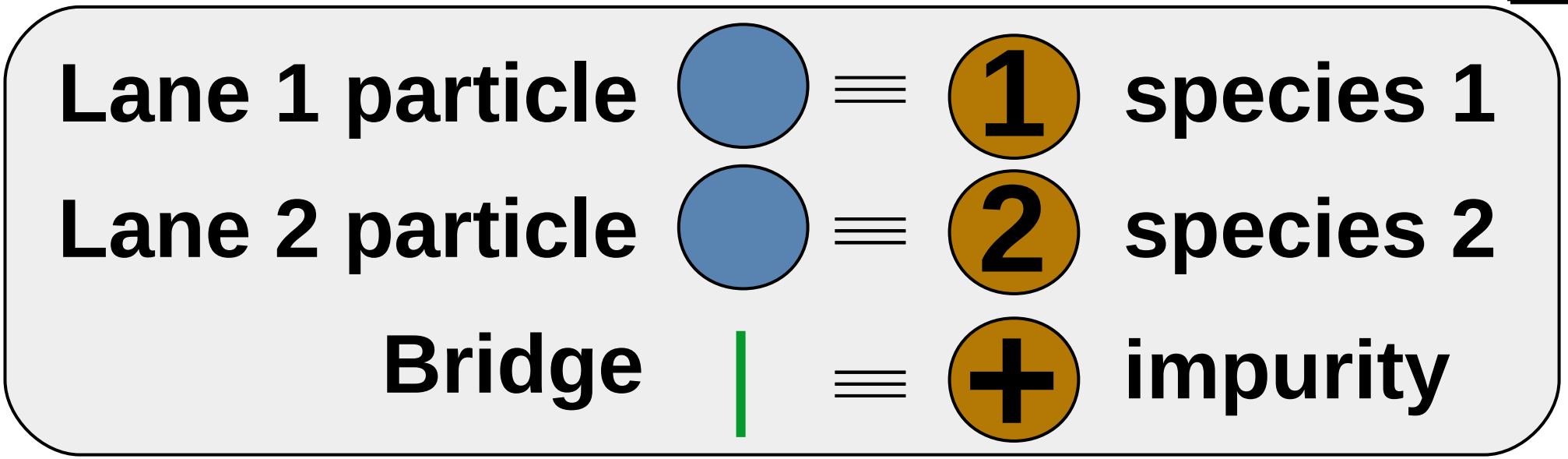
(*either mean field solution, or, exact solutions factorized over lanes*)

Question: Approximate mapping of multi-lane ASEP to 1-d model that allows exact solution ??

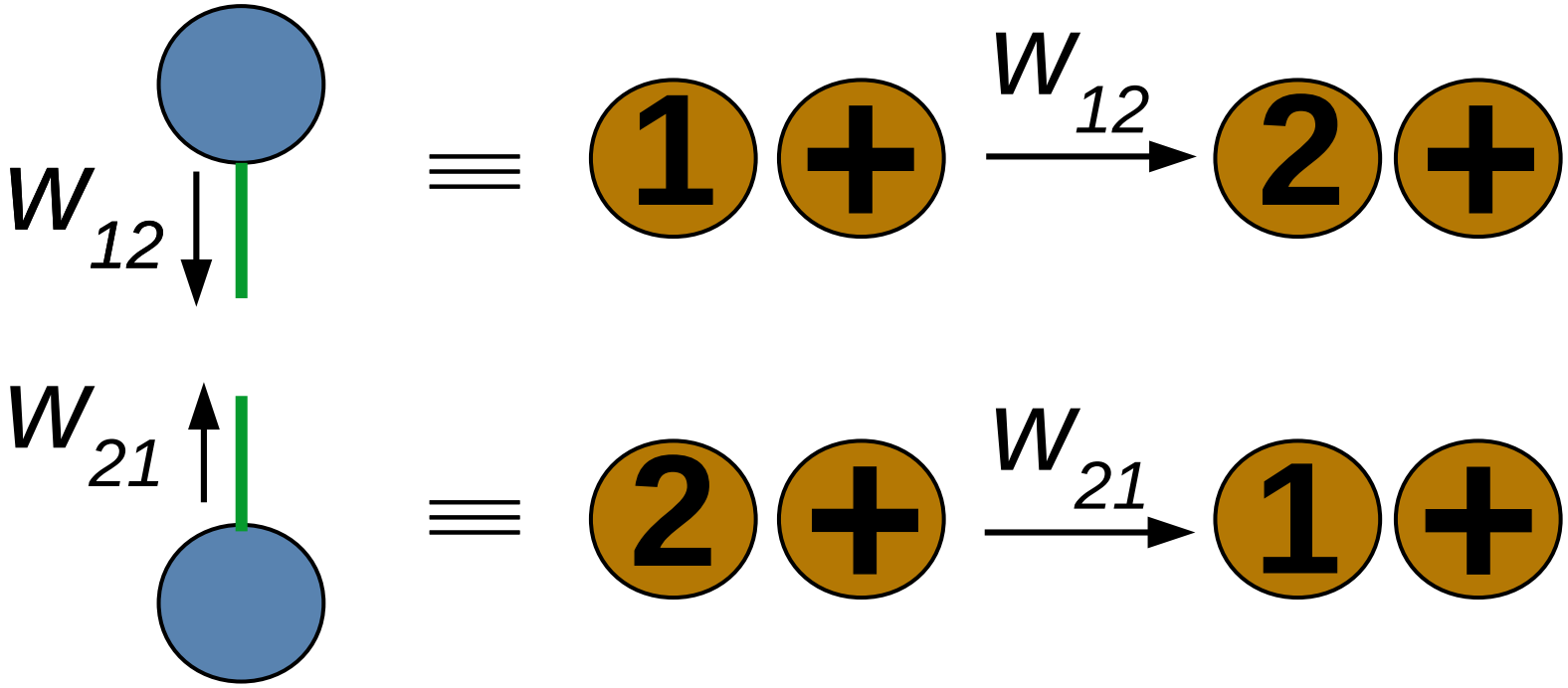
Motivation: Two-lane (multi-lane) to 1-d ASEP with two (multiple) species



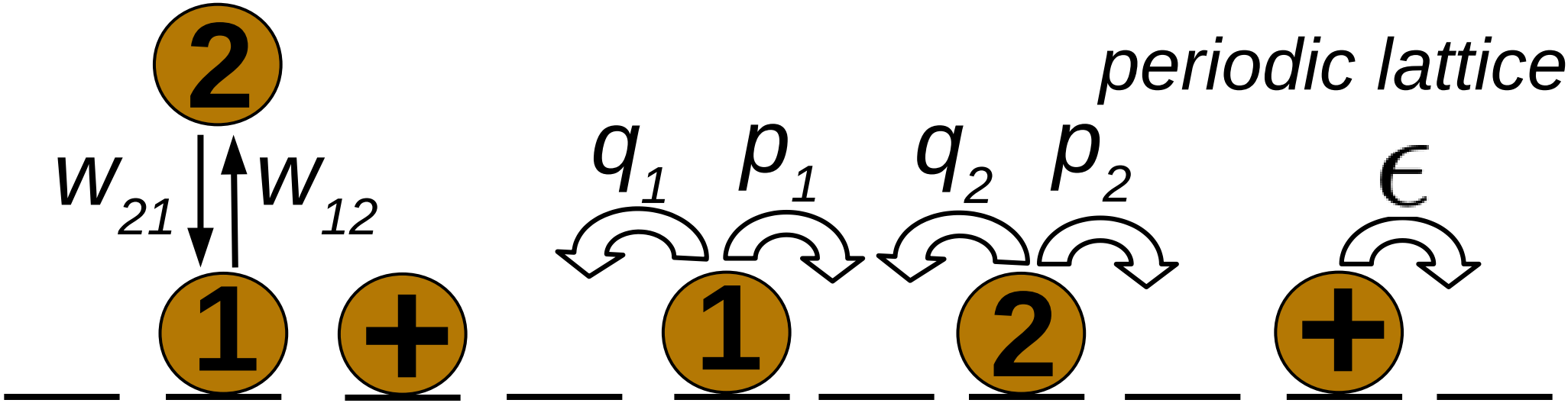
Question: Equivalent 1-d model ??


Lane 1 particle
≡
1
species 1
Lane 2 particle
≡
2
species 2
Bridge
≡
+
impurity

Lane change:


W12
≡
1
+
W12
2
+
W21
≡
2
+
W21
1
+

Multi species ASEP with impurity activated flips



(i) disordered

(ii) non-conserved

(iii) non-ergodic

Exact steady state and Observables ??

Steady state: Matrix Product Ansatz

component represented by matrix :

$$\text{species "K"} \longrightarrow D_K \qquad \text{impurity} \longrightarrow A$$

$$\text{vacancy} \longrightarrow E$$

Probability of any configuration:

$$P(\{s_i\}) \propto \mathrm{Tr}\left[\prod_{i=1}^{L} X_i\right],$$

$$X_i = E\,\delta_{s_i,0} + A\,\delta_{s_i,+} + \sum_{K=1}^{\mu} D_K\,\delta_{s_i,K}.$$

TASKS: (i) matrix algebra
(ii) matrix representations

Matrix algebra: (2-species ASEP with impurities)

$$p_1 D_1 E - q_1 E D_1 = D_1$$

$$p_2 D_2 E - q_2 E D_2 = D_2$$

$$\epsilon A E = A$$

$$w_{12} D_1 A = w_{21} D_2 A$$

⇨ Solution ?

RESULTS:

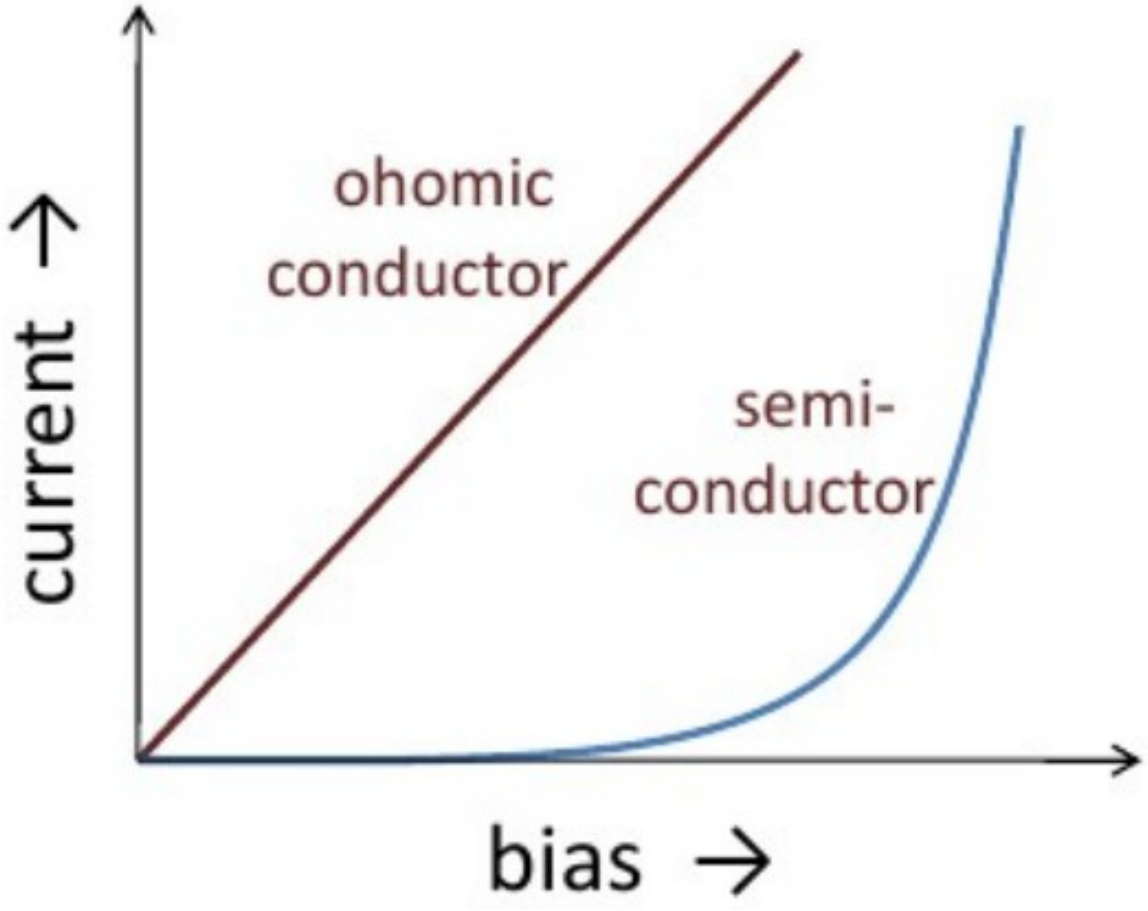
Totally asymmetric motion $(q_1=0=q_2)$ → Finite dimensional matrices

Partially asymmetric motion $(q_1\neq 0, q_2\neq 0)$ → Infinite dimensional matrices

Explicit representation of the matrices ⟹ [*arXiv*:2205.03082 (2022)]

Negative Differential Mobility

General notion:



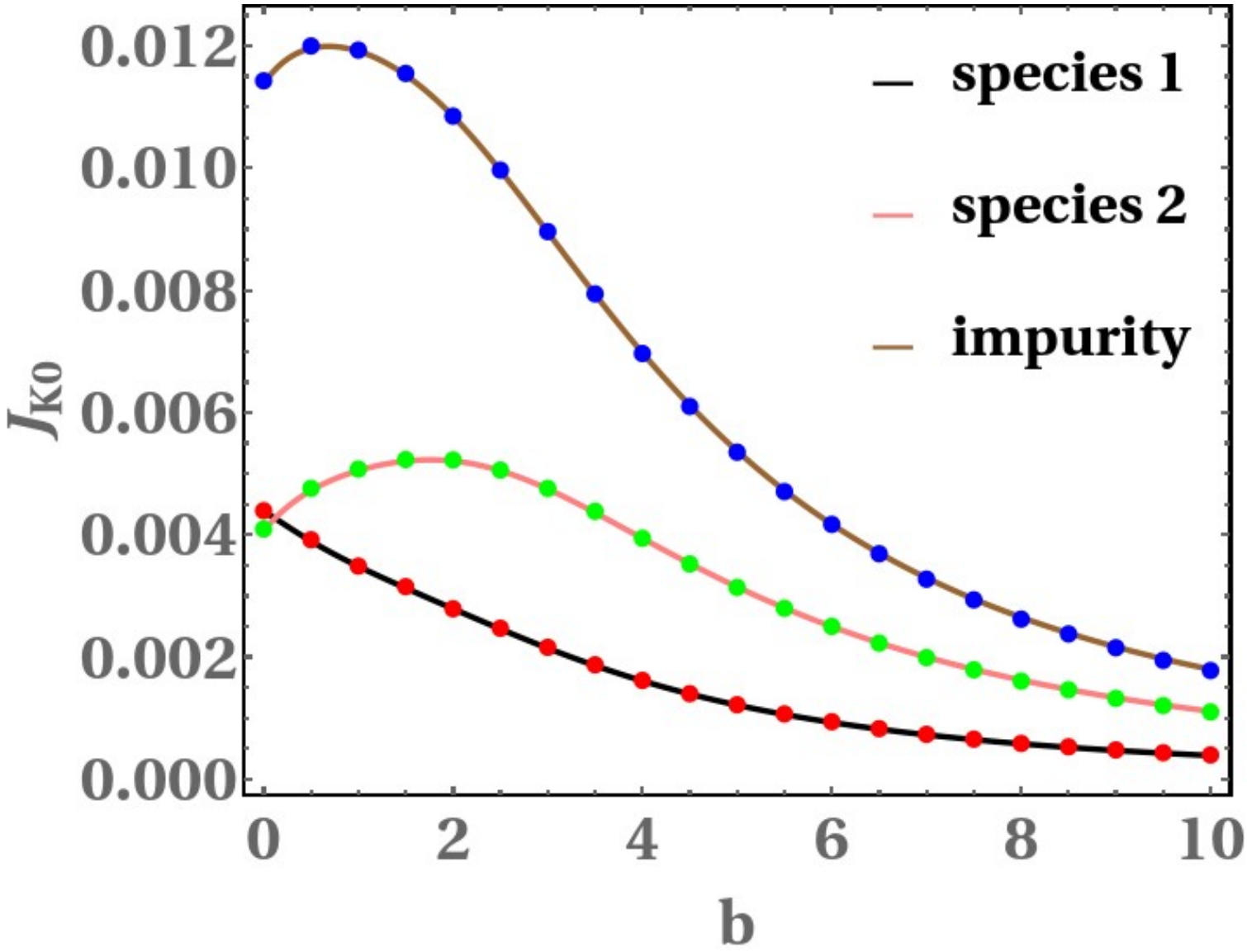
current increases with increasing bias

Our observation:

Decreasing current with increasing bias

Drift current of species "*K*": J_{K0}

Bias: $\ln(p_1/q_1) = b$ $p_1 = 1, q_1 = e^{-b}$ $p_2 = \frac{1}{1+b^2} = q_2$



Decreasing current with increasing bias

Counter-flow induced clustering

Cluster formation

- *Jamming in granular materials*
- *Traffic jam in vehicular/pedestrian traffic flow*
- *Active matter, cellular jamming in cancer*

Clustering from counter-flow

pedestrians moving in opposite directions in a narrow lane

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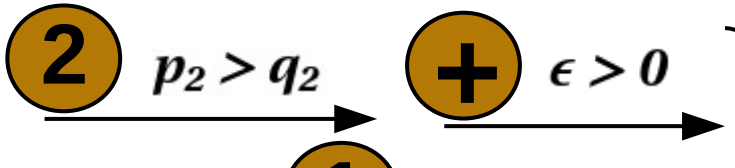
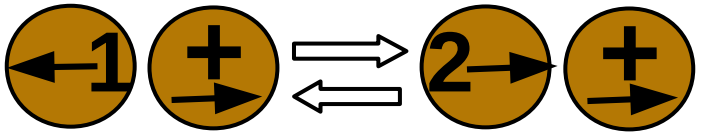
traffic jam at high density

Counter-flow in our model

net bias:



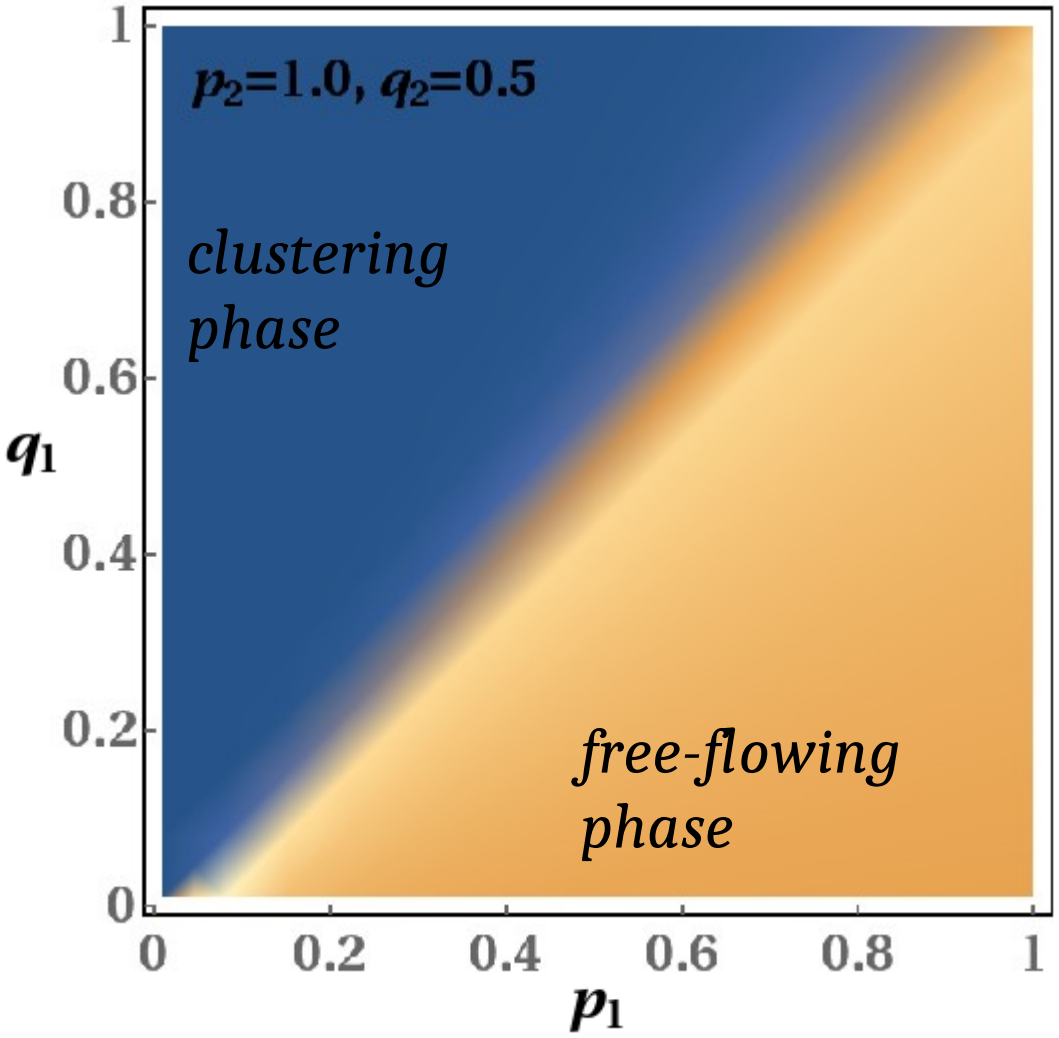
flip:

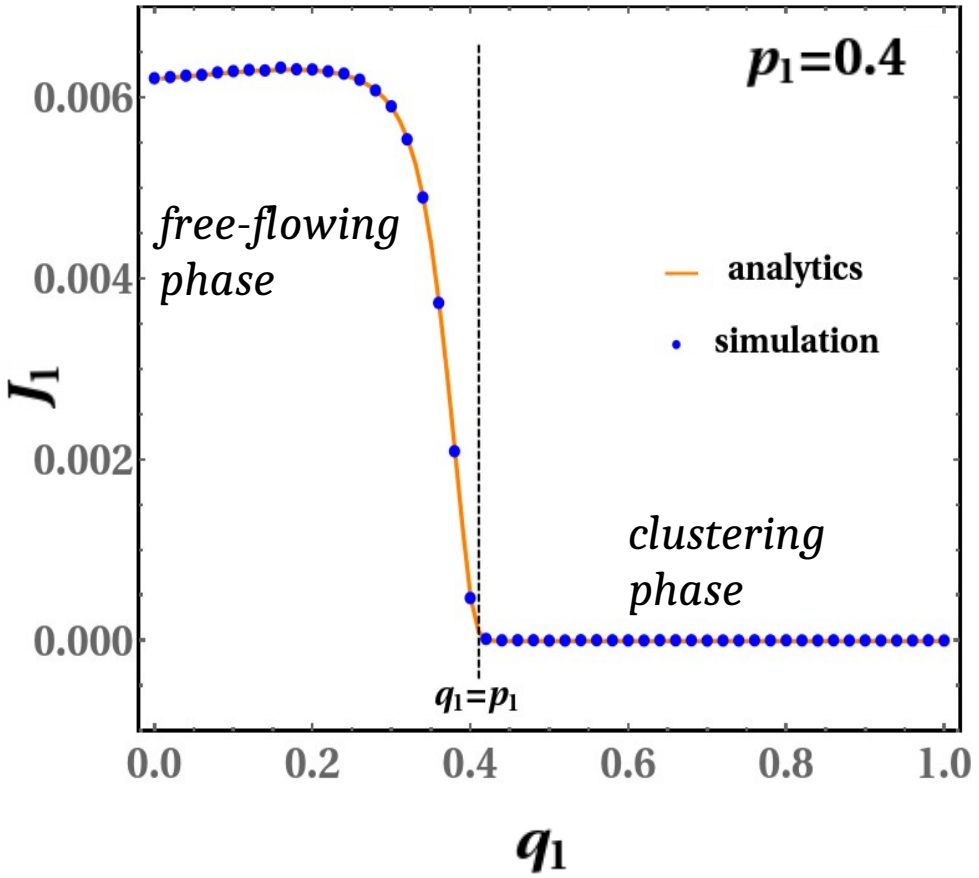


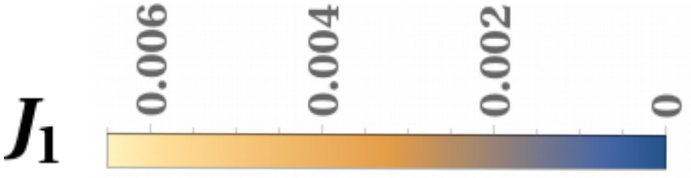
⇨ CLUSTER formation ? ?

Drift current : two different phases

$$J_1 = p_1\langle 10\rangle - q_1\langle 01\rangle$$

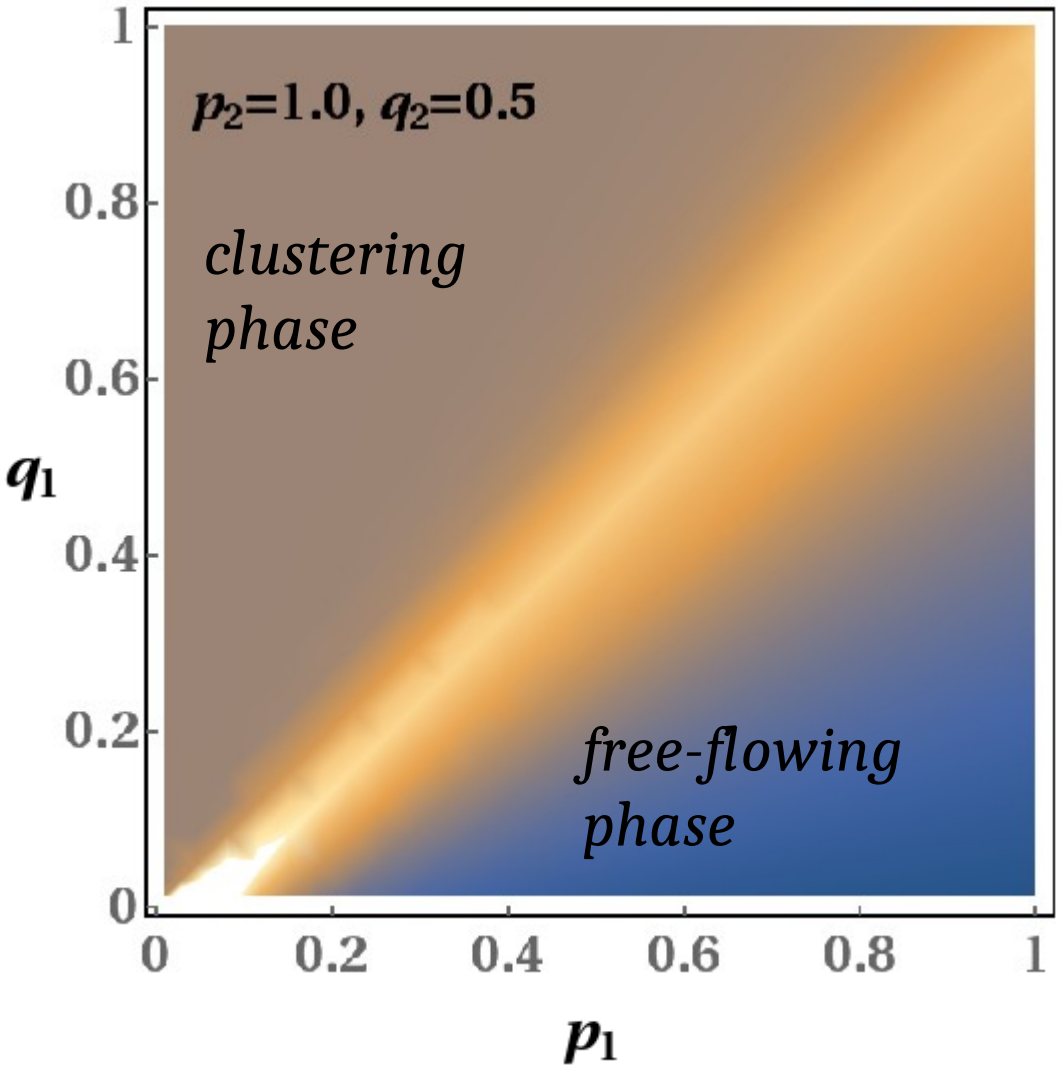


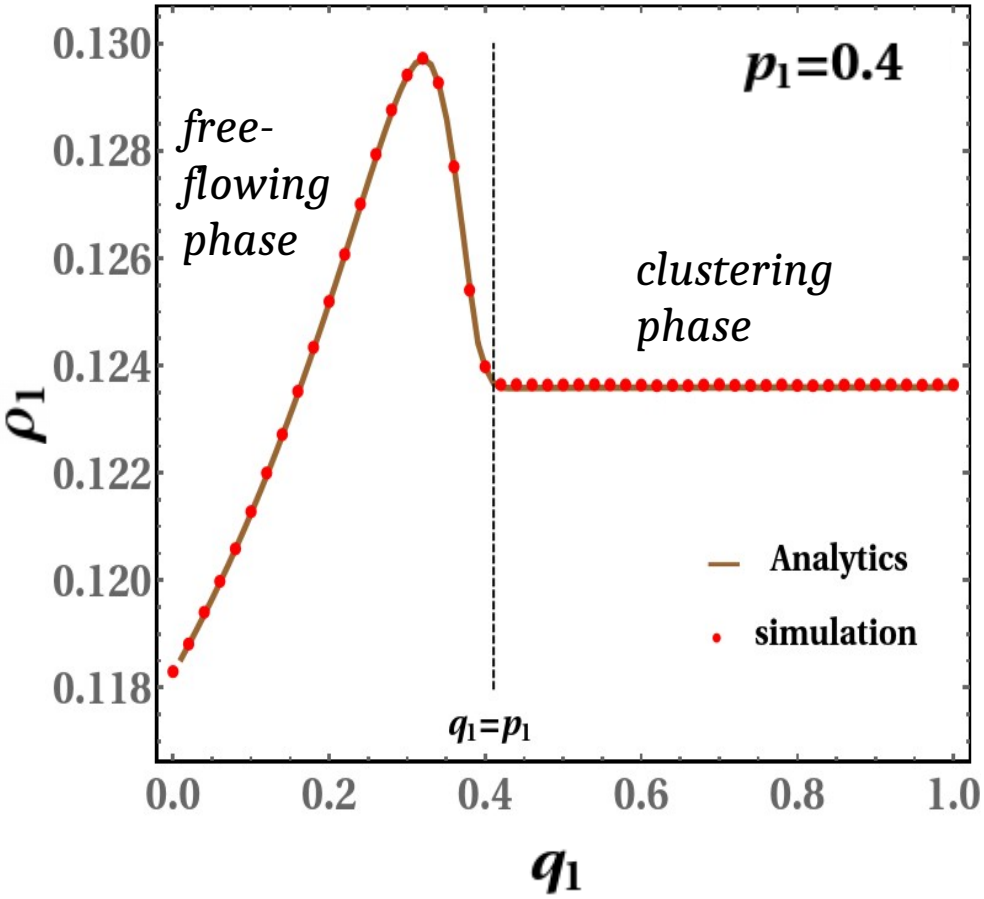




Density : two different phases

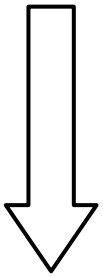
$\rho_1 = \langle 1 \rangle$





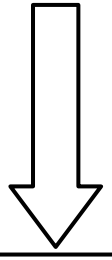
SUMMARY

- Multi-lane traffic flow



1-d approximation

Multi-species ASEP with impurity activated flips



Exact matrix product steady state

- Motion and matrix dimension

Totally asymmetric → *Finite dimensional*

Partially asymmetric → *Infinite dimensional*

[*arXiv*:2205.03082 (2022)]

- *Disordered* *Non-conserved*

Non-ergodic

- *Negative differential mobility*

★ *Counter-flow induced clustering*

free flowing phase (*non-monotonic density and non-zero current*)

clustering phase (*constant density and vanishing current*)

Future directions

1) Generalization of the model with open boundaries, allowing entry/exit of particles, closer to more realistic transport situations.

2) Exact solution of the full dynamical problem (Bethe Ansatz).

3) Rigorous study of multi-lane models to establish the usefulness of the present 1-d approximation.

4) Analytical study of geometric phase in ASEP and related models.

5) Exploring possible quantum version of the present model.

THANK YOU