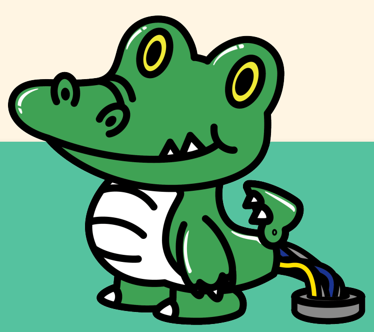
2022/11/1 @YITP

OSAKA UNIVERSITY

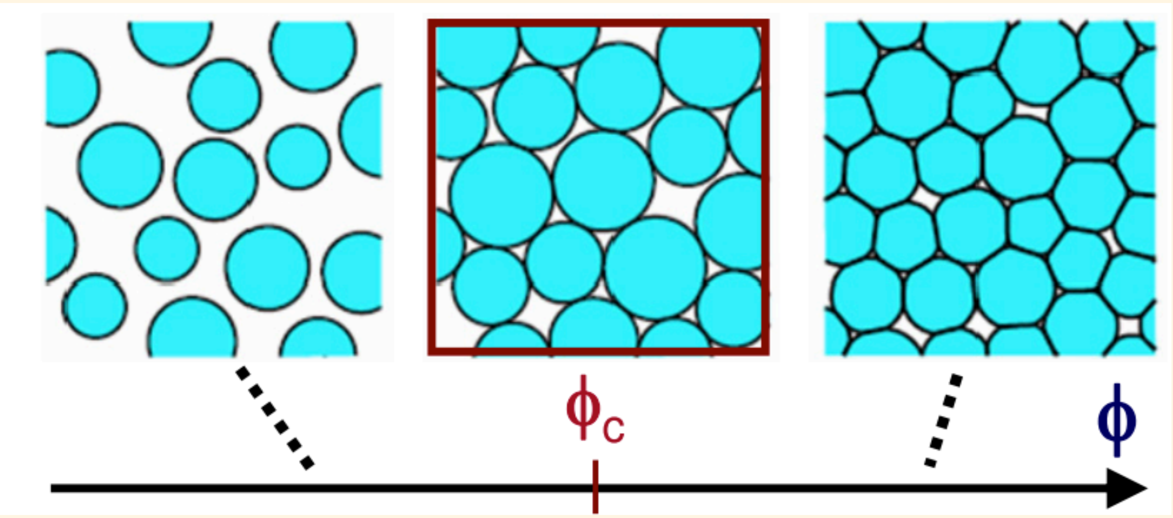
Jamming in channel flow of granular materials



Kiwamu Yoshii, Michio Otsuki
(Osaka Univ.)

Jamming transition of granular material

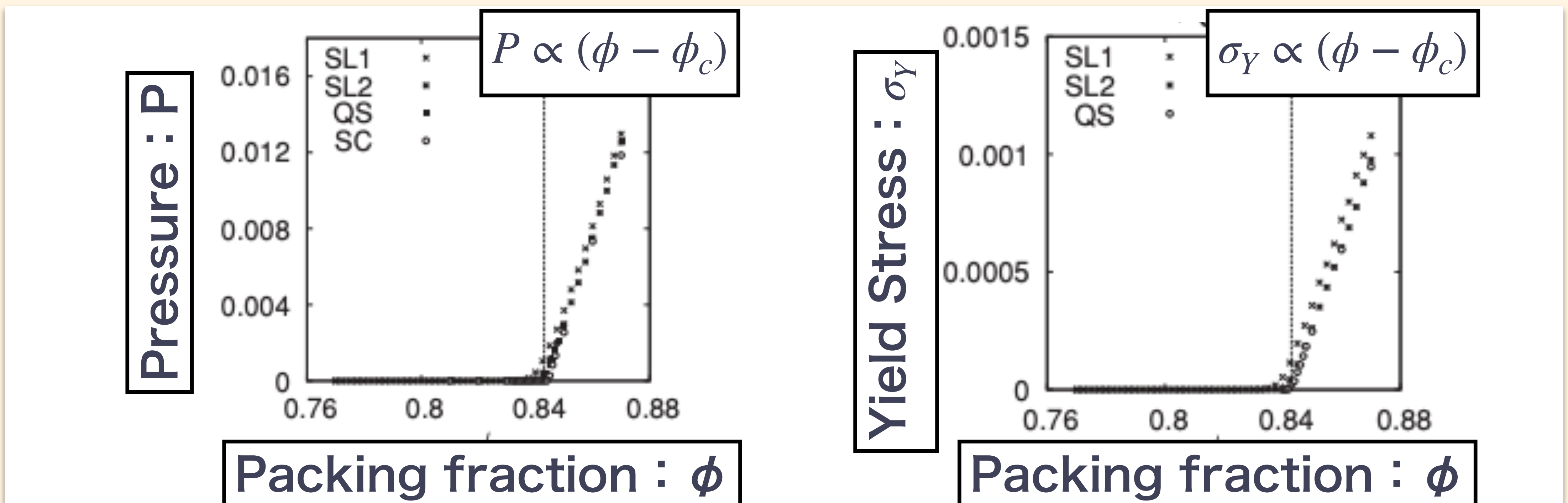
Jamming transition



M. Van Hecke., J. Phys.; Condens Matter (2011)

- Jammed : Solid-like state
  - >> $\phi > \phi_c$
- Unjammed : Fluid-like state
  - >> $\phi < \phi_c$

Critical behavior



M. Otsuki & H. Hayakawa Phys. Rev. E (2011)

Jamming in a realistic situation

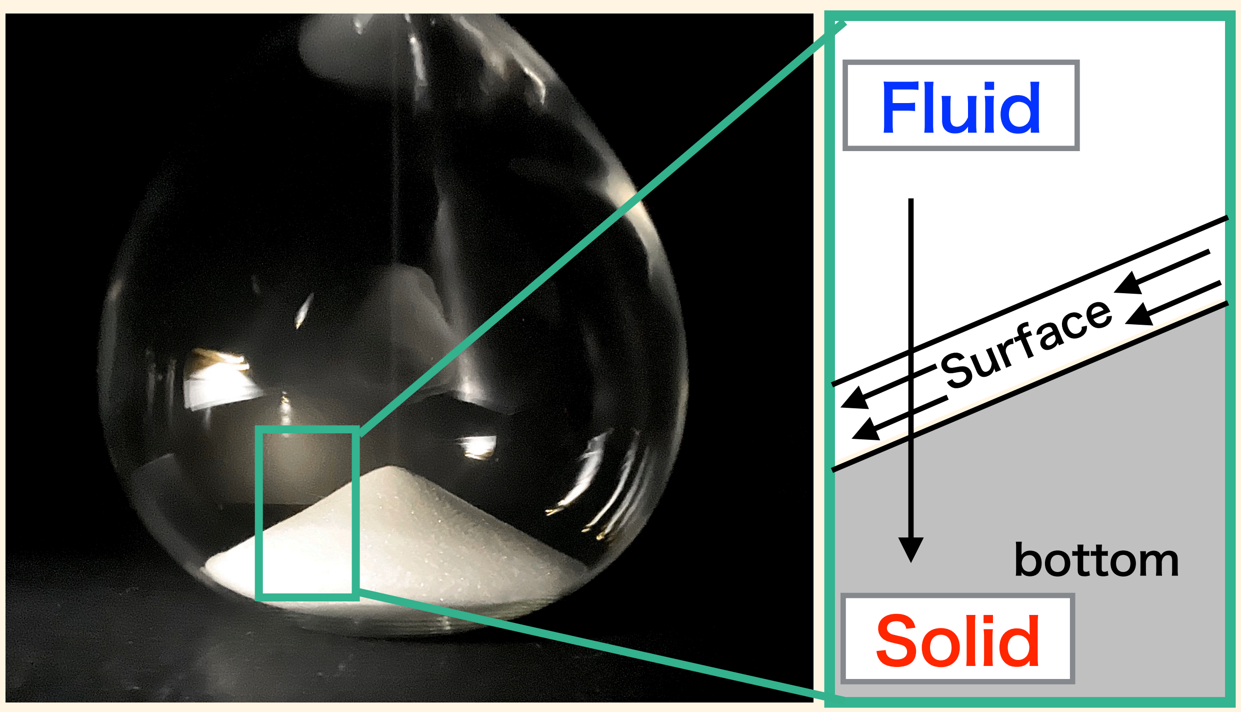
▶ many pervious study of jamming : <u>uniform systems</u>

Uniform shear, periodic boundary condition…

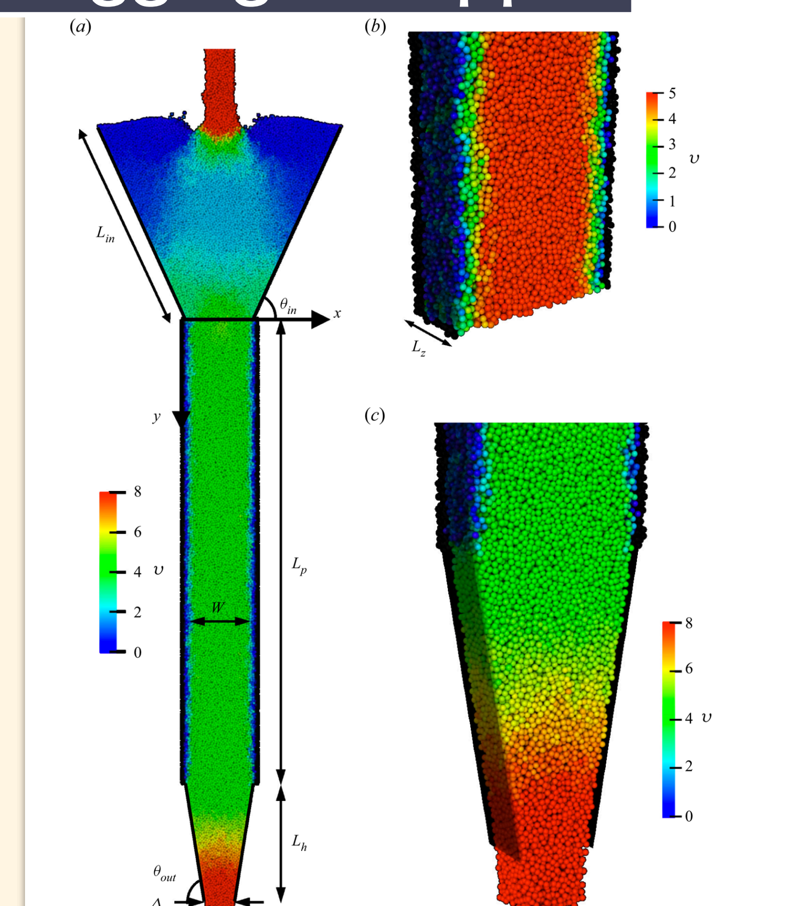
? in <u>non-uniform systems</u>

Phenomena related to jamming

Sand pile



Clogging in hopper



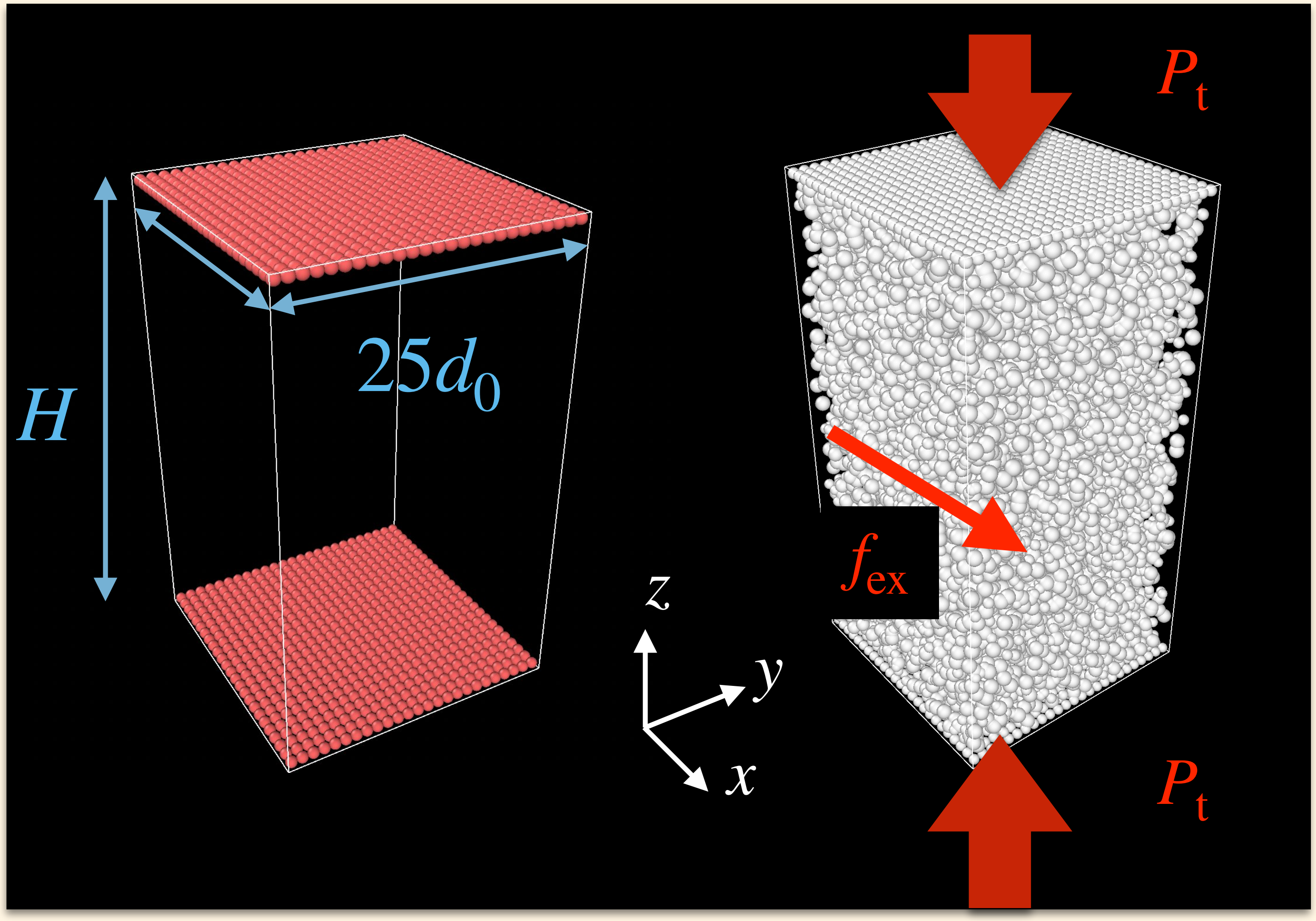
T. Barker, et al. JFM (2021)

Model : Plane Poiseuille flow

- many pervious study of jamming : uniform systems

? in non-uniform systems (Jamming, critical scaling…)

Granular flow between parallel plate



- Particles between parallel plate
  - rough wall consisted of particles
  - $\phi > \phi_c$ or $P > 0$
  - Wall : fixed
  - Bulk : external force f_c

Mass : m, Radius : d_0

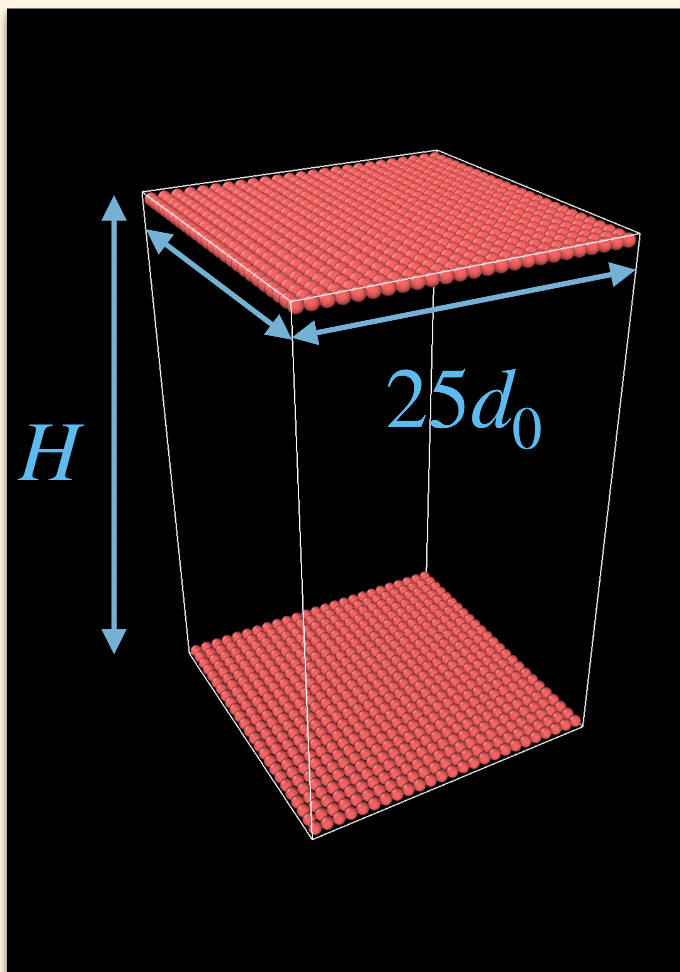
volume : v

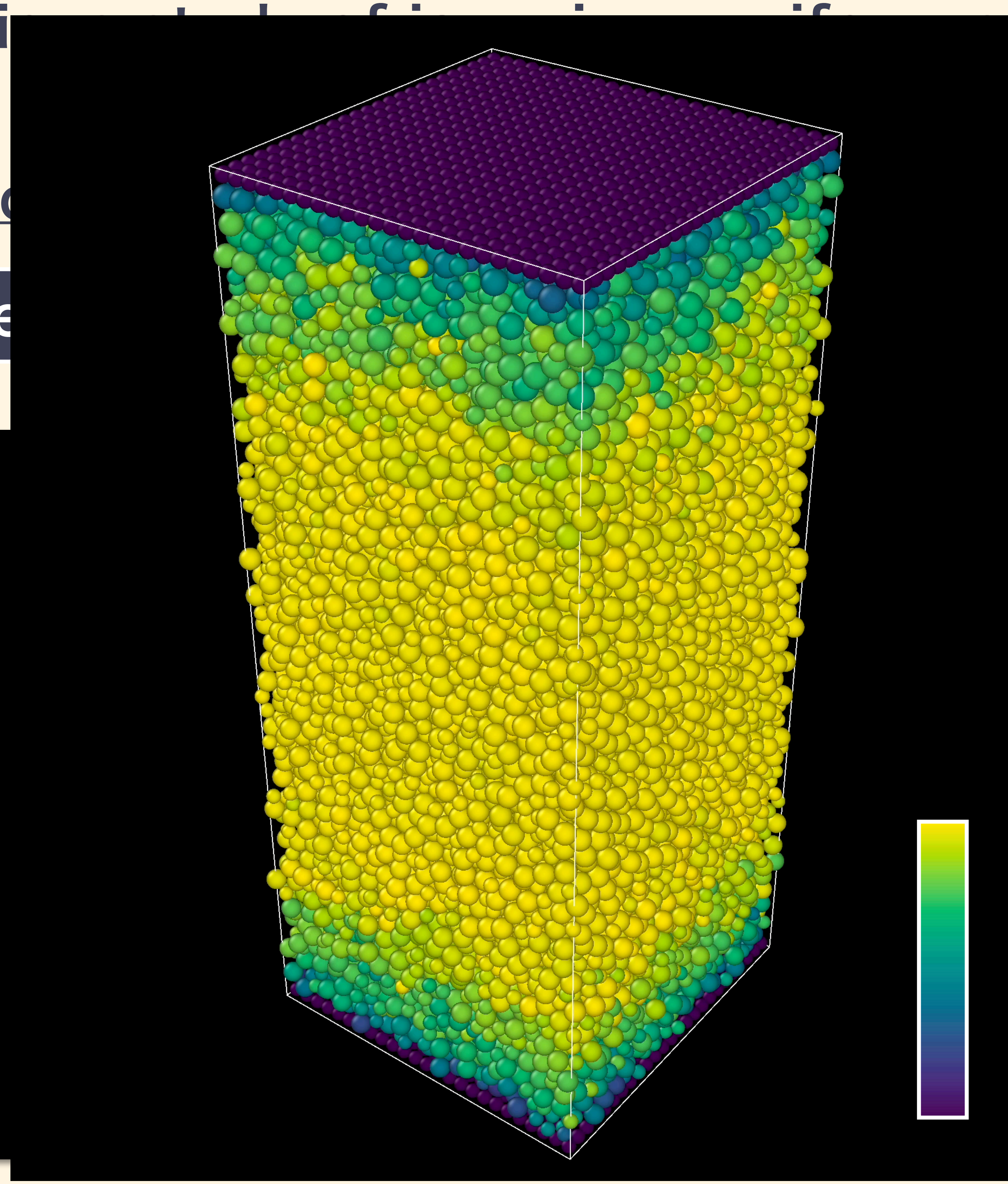
Model : Plane Poiseuille flow

- many pervi… ystems

? in non-unif… aling···)

Granular flow be





een parallel plate

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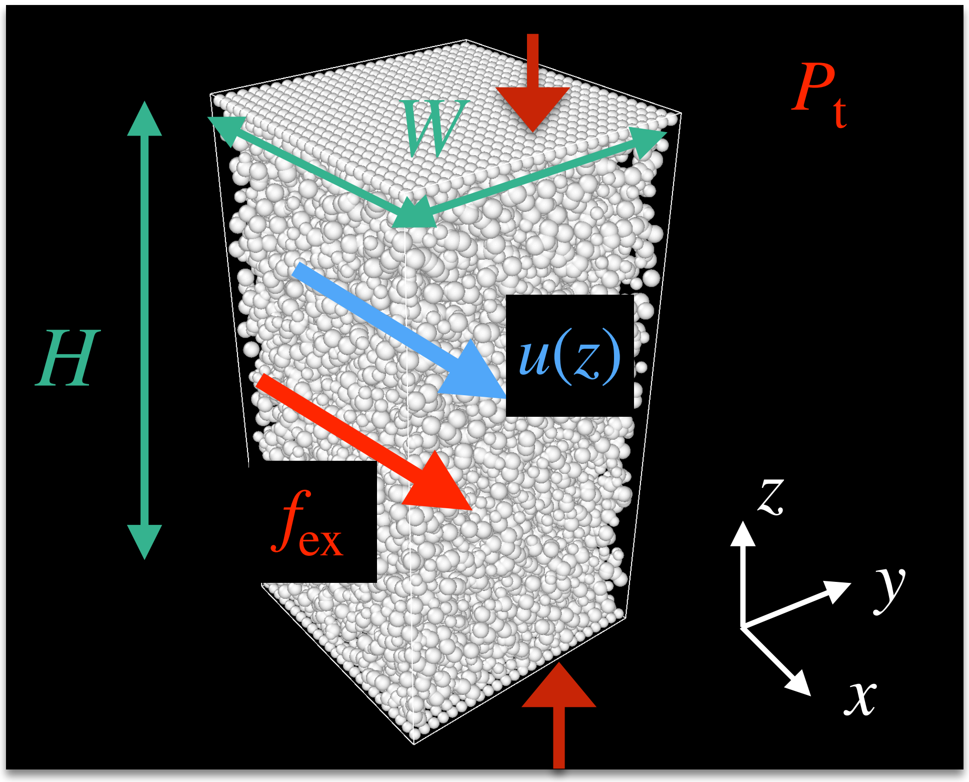
0

al force f_c

lius : d_0

Continuum analysis

Setup



- Velocity along the x-direction : u(z)
- Uniform in the x,y-directions

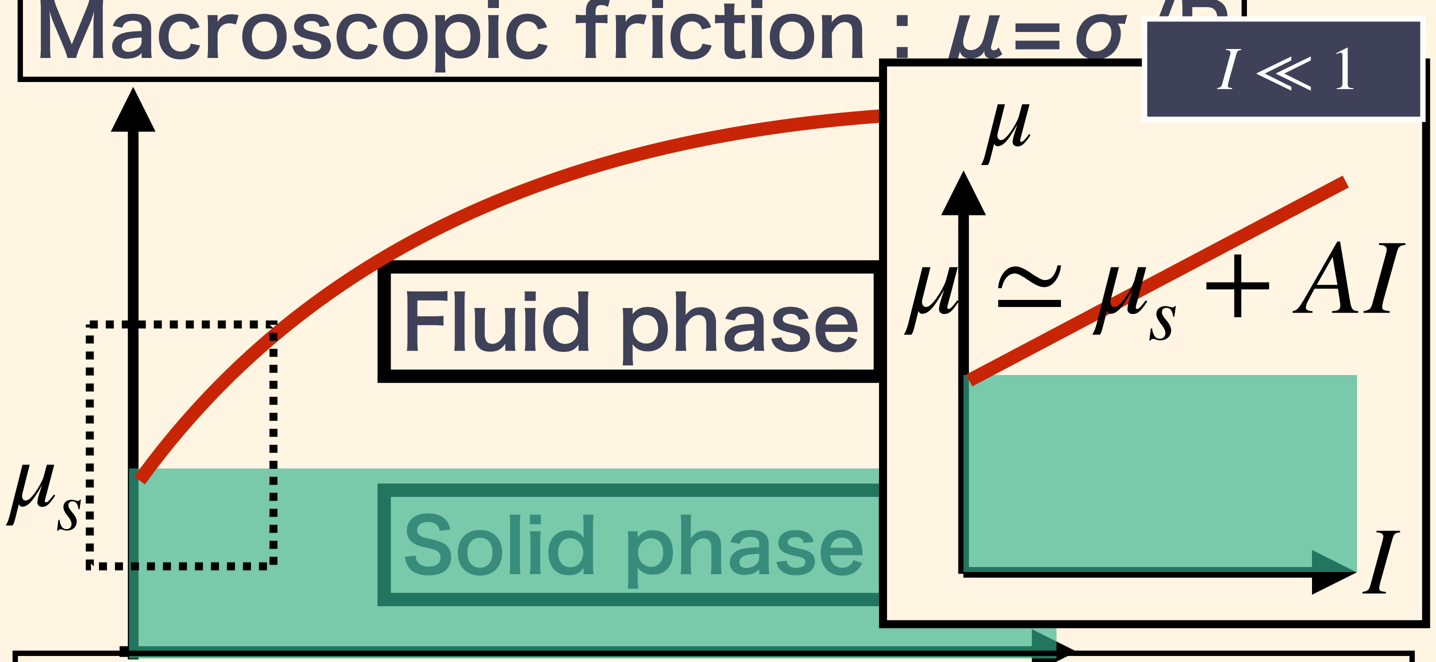
Shear rate: $\dot{\gamma}(z) =$	$\frac{du(z)}{dz}$	, Stress: $\sigma(z)$

Momentom Balance: $\partial_z \sigma(z) = \phi f / v$

Pressure: $P(z) = P_t$

Volume of particle : v

Macroscopic friction : $\mu = \sigma / P$



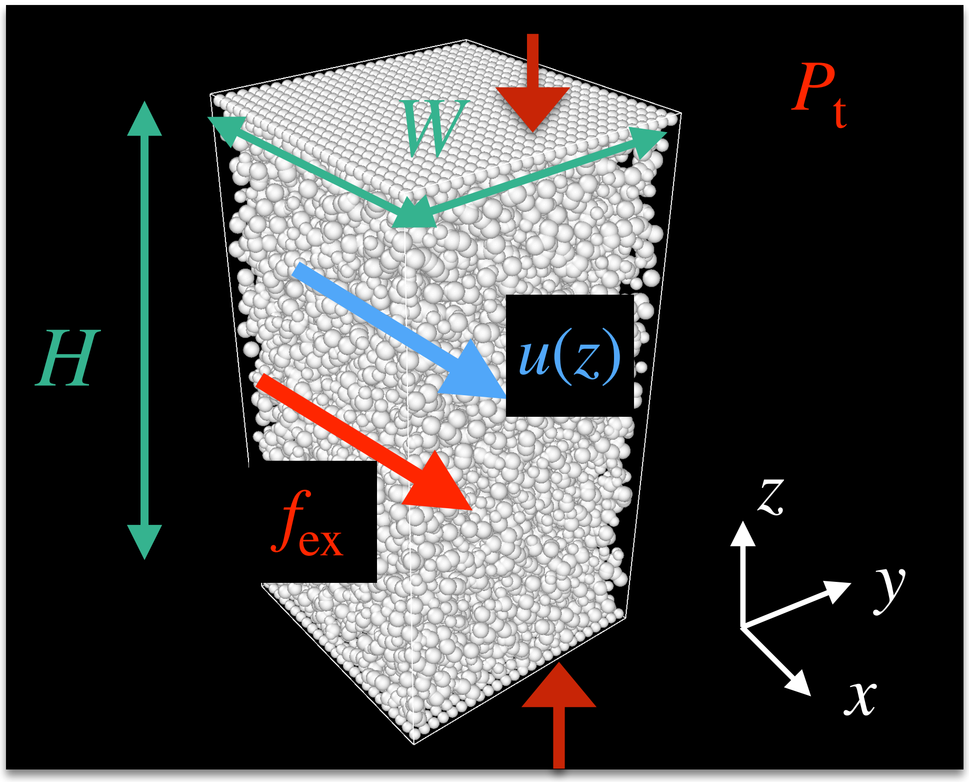
Dimensionless shear rate : $I = \dot{\gamma}\sqrt{Pd/m}$

- $\mu > \mu_s$ 👉 $\dot{\gamma} > 0$ (Fluid phase)
- $\mu < \mu_s$ 👉 $\dot{\gamma} = 0$ (Solid phase)
- $I \ll 1$ 👉 $\mu \propto I$

μ_s : static friction coefficient

Continuum analysis

Setup



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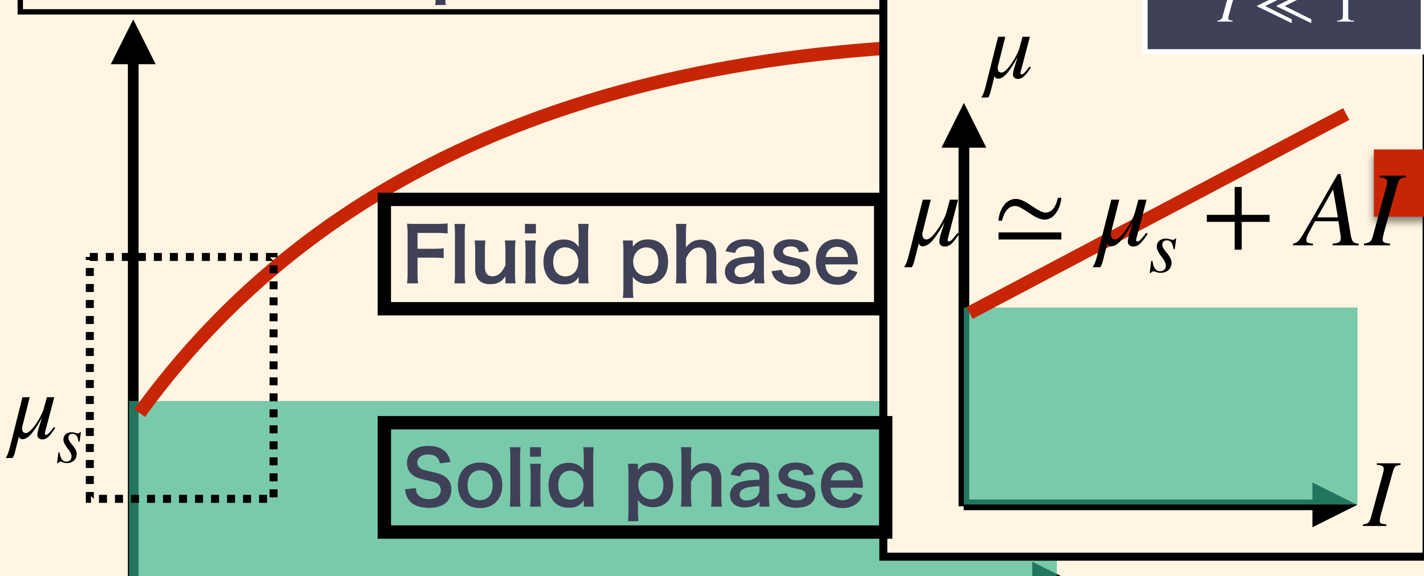
Pressure: $P(z) = P_t$

Volume of particle : v

Macroscopic friction : $\mu=\sigma/P$



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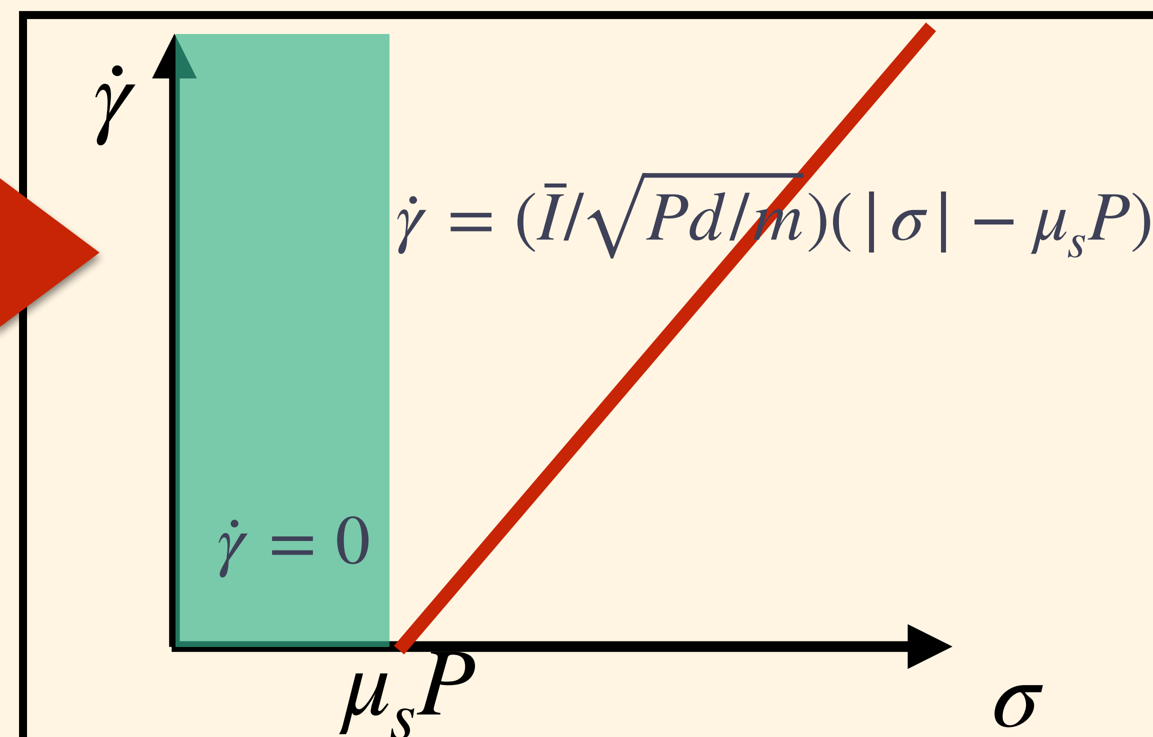


Stress distribution

Momentom Balance: $\partial_z \sigma(z) = \phi f_{\text{ex}}/v$

Pressure: $P(z) = P_{\text{t}}$

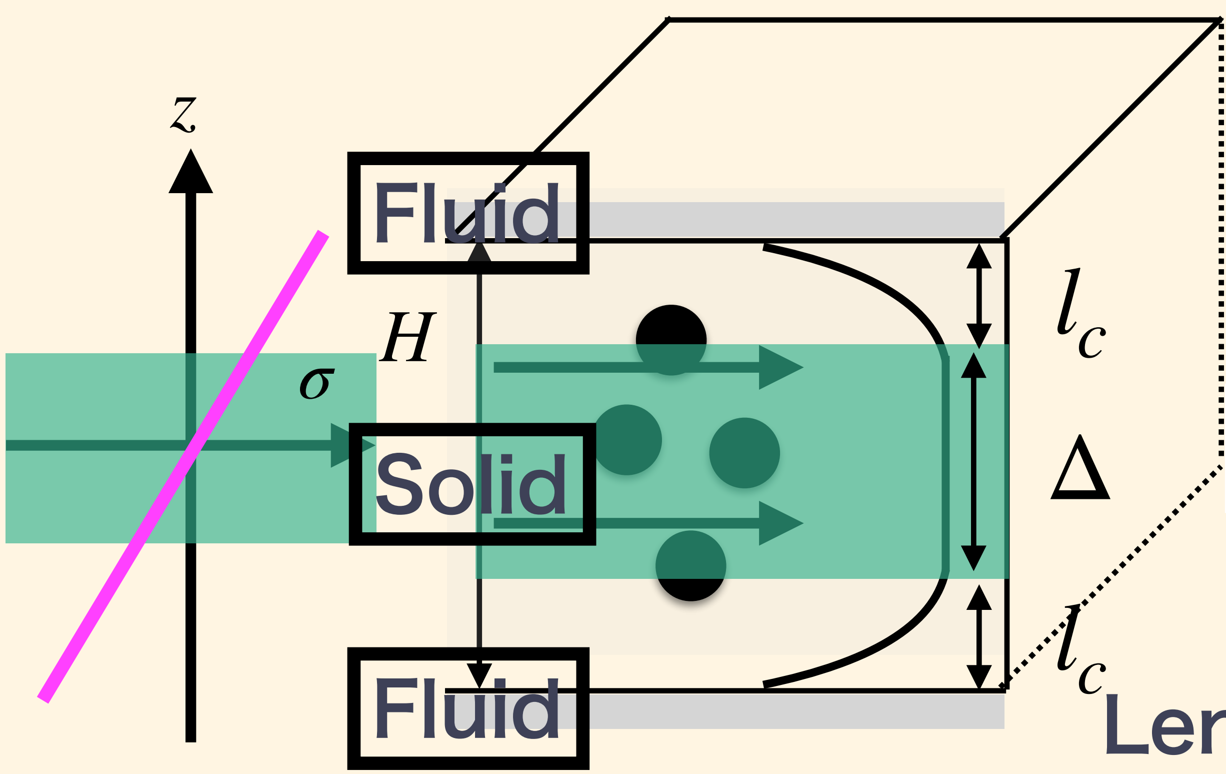
- Fluid phase : $|\sigma(z)| > \mu_s P_{\text{t}}$
- Solid phase : $|\sigma(z)| < \mu_s P_{\text{t}}$

Stress distribution: $\sigma(z) = \frac{\phi f_{\text{ex}}}{v} z$

Fluid phase :

$$|\sigma(z)| = \frac{\phi f_{\text{ex}} |z|}{v} > \mu_s P_{\text{t}}$$

$$\sigma_{\text{Max}} = |\sigma(z)|_{z=\pm\frac{H}{2}} = \frac{\phi f_{\text{ex}} H}{2v}$$



Fluid phase disappear :

$$\sigma = \frac{\phi f_{\text{ex}} H}{2v} < \mu_s P_{\text{t}}$$

Critical force : $f_c(\phi, H) = \frac{2v\mu_s P_{\text{t}}}{\phi H}$

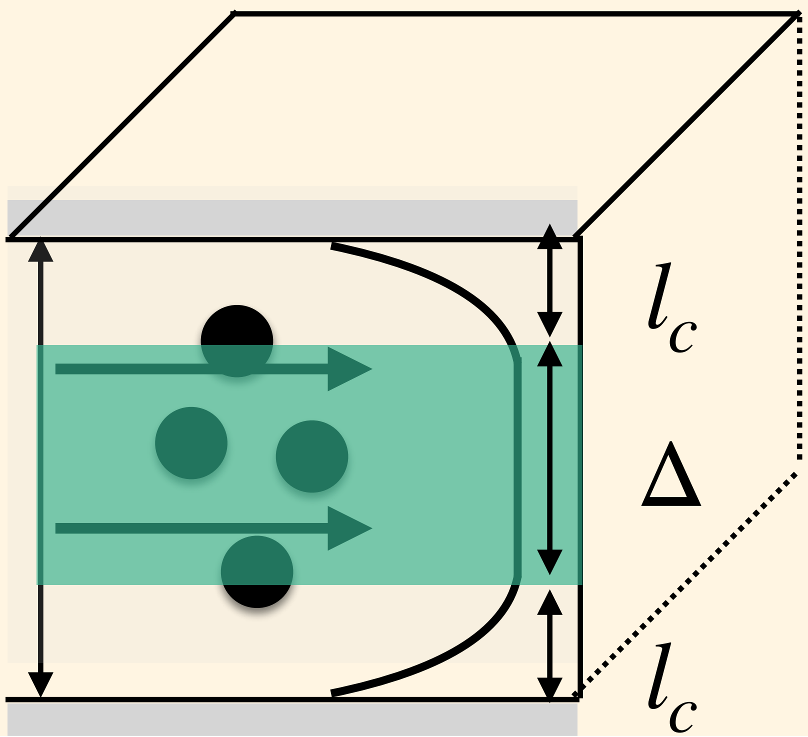
Length of fluid phase : $l_c \simeq \frac{H}{2} \frac{f_{\text{ex}} - f_c}{f_c}$

Velocity profile

Stress distribution: $\sigma(z) = \frac{\phi f_{\mathrm{ex}}}{v} z$

$$\dot\gamma = \begin{cases} \tilde I_0 \sqrt{\frac{d}{mP_\mathrm{t}}}\left(|\sigma| - \mu_s P_t\right), & |\sigma| > \mu_s P_\mathrm{t} \\ 0, & |\sigma| < \mu_s P_\mathrm{t} \end{cases}$$

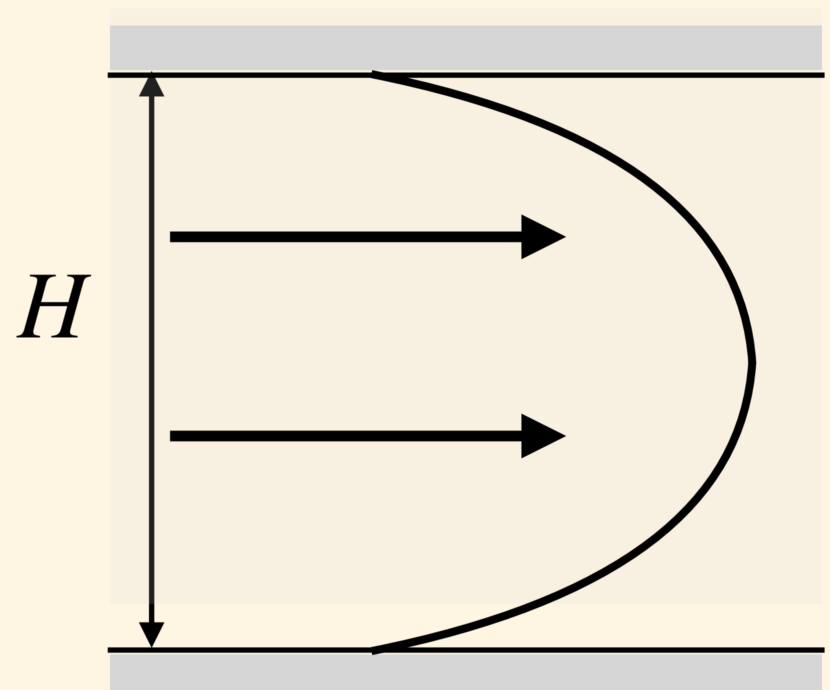
$$\dot\gamma = \frac{\tilde I_0 \sqrt{d/m}\,\phi f}{v\sqrt{P_\mathrm{t}}}\left(|z| - \Delta/2\right)$$



Poiseuille flow of fluid

$$\dot\gamma = \frac{\rho K}{\eta} z \quad , u_{\max} = \frac{\rho K H^2}{8\eta}$$

K : body force, ρ:density, η:viscosity



$$2l_c \rightleftarrows H \qquad \frac{\tilde I_0 \sqrt{d/m}\,\phi f_{\mathrm{ex}}}{v\sqrt{P_\mathrm{t}}} \rightleftarrows \frac{\rho K}{\eta}$$

Maximum velocity in granular case :

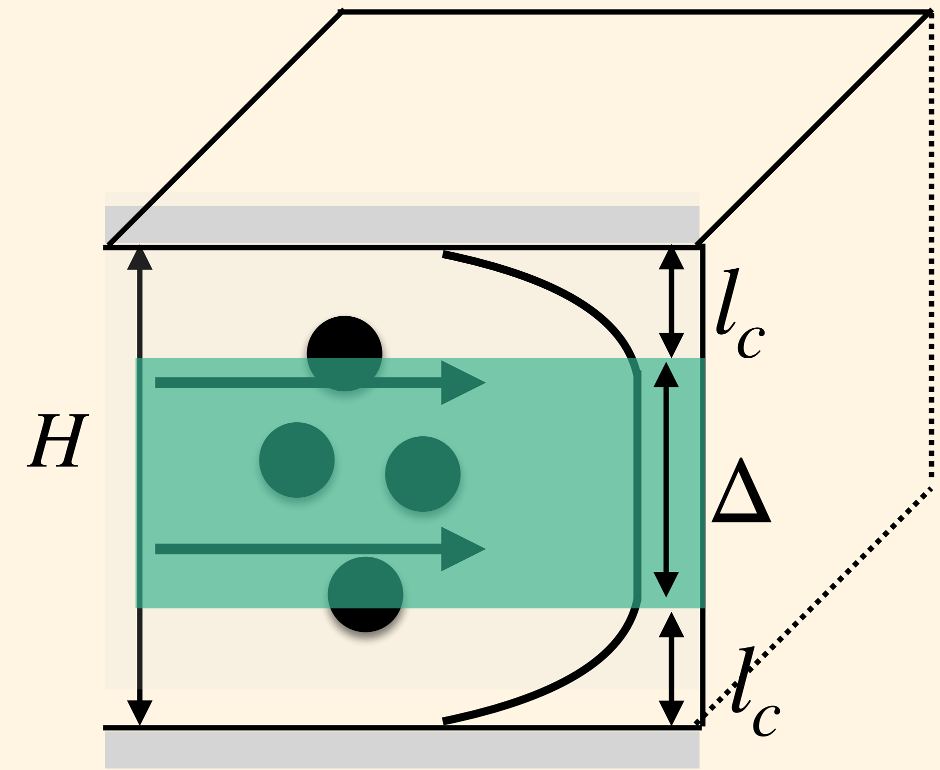
$$u_{\max} = \frac{I_0 \sqrt{d/m}\,\phi f}{2\Delta \mu v \sqrt{P_\mathrm{t}}} l_c^2$$

Mass flux

Maximum velocity : $u_{\rm max} = \dfrac{I_0\sqrt{d/m}\phi f}{2\Delta\mu v\sqrt{P_{\rm t}}} l_c^2$

Length of fluid phase : $l_c \simeq \dfrac{H}{2}\dfrac{f_{\rm ex}-f_c}{f_c}$

Critical force : $f_c(\phi, H) = \dfrac{2v\mu_s P_{\rm t}}{\phi H}$



Mass flux Q

$$Q = \frac{m\phi}{v} W \int_0^H dz u(z) \simeq \frac{m\phi}{v} W H u_{\rm max}$$

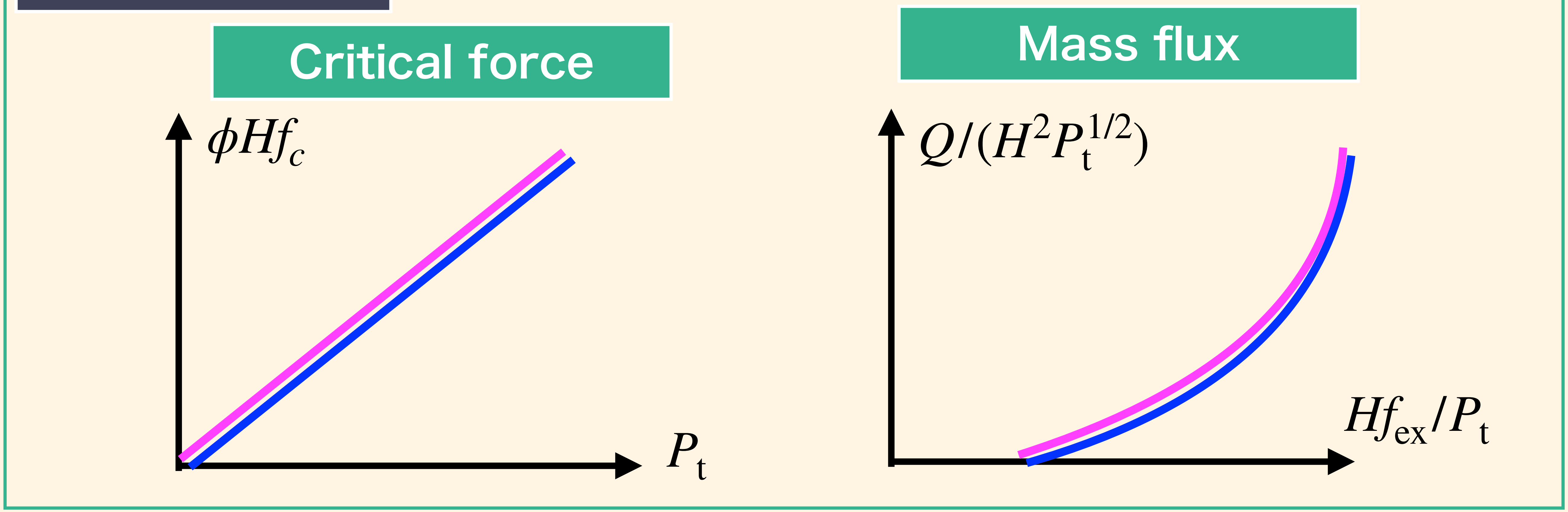
$$Q = \frac{I_0 v\sqrt{md}W\phi^2}{8\Delta\mu v\sqrt{P_{\rm t}} f_c(\phi, H)} H^3 \left(f_{\rm ex} - f_c(\phi, H)\right)^2$$

Prediction

- **Critical force:** $$f_c(\phi, H) = \frac{2\nu\mu_s P_\mathrm{t}}{\phi H}$$

- **Mass flux:** $$Q(f, \phi, H) \propto \frac{\phi^2 H^3}{\sqrt{P_\mathrm{t} f_c(\phi, H)}} \left(f_\mathrm{ex} - f_c(\phi, H)\right)^2$$

$f_\mathrm{ex} \simeq f_c,\ \phi \simeq \phi_c$



DEM Simulation

Bulk

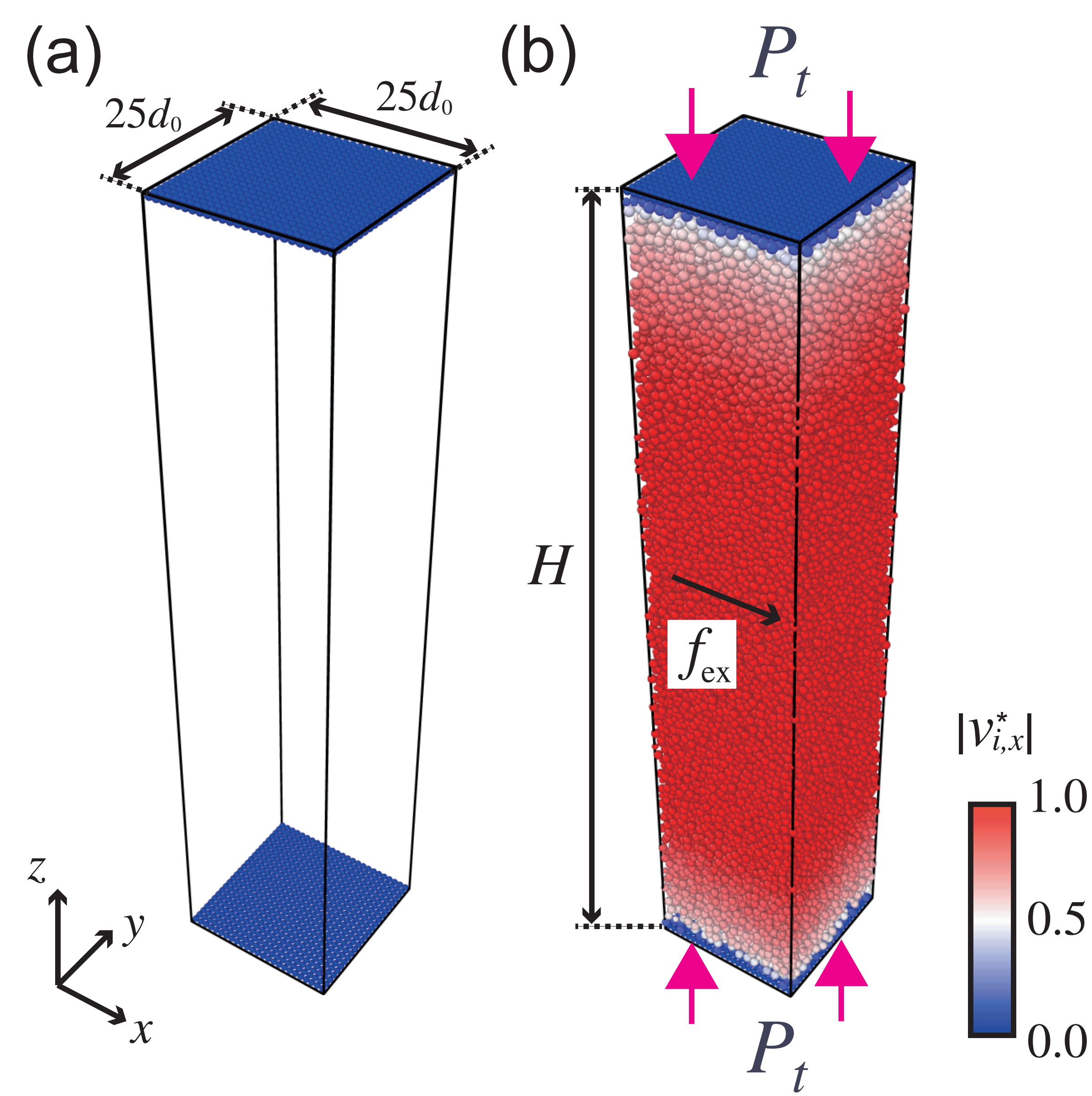
$$m_i \frac{d^2 \boldsymbol{r}_i}{dt^2} = \sum_{j \neq i} F_{ij}^{\mathrm{rep}} \boldsymbol{n}_{ij} + F_{\mathrm{ex}} \boldsymbol{e}_x$$

Wall

$$m_i \frac{d^2 r_{i,w}}{dt^2} = - \sum_{j \neq i (\in \mathrm{bulk})} F_{ij}^{\mathrm{rep}} - P_{\mathrm{t}} L^2$$

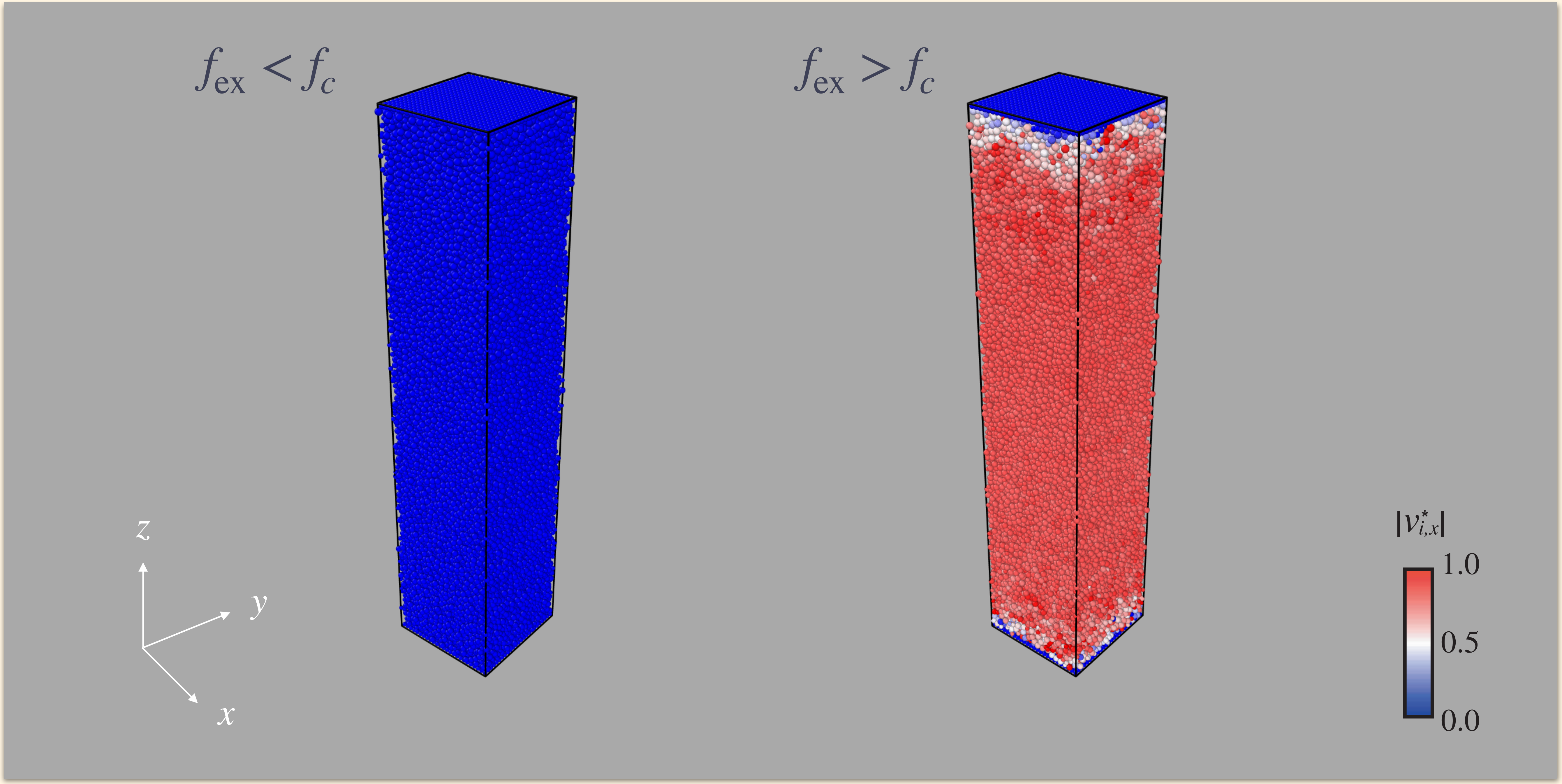
- frictionless particle
- # of particles
  bulk : 25000 or 50000
  wall : 25×25×2
- particle size : d_0, $1.4d_0$
- b.c. :
  x,y : p.b.c., z : wall of particle

Constant pressure



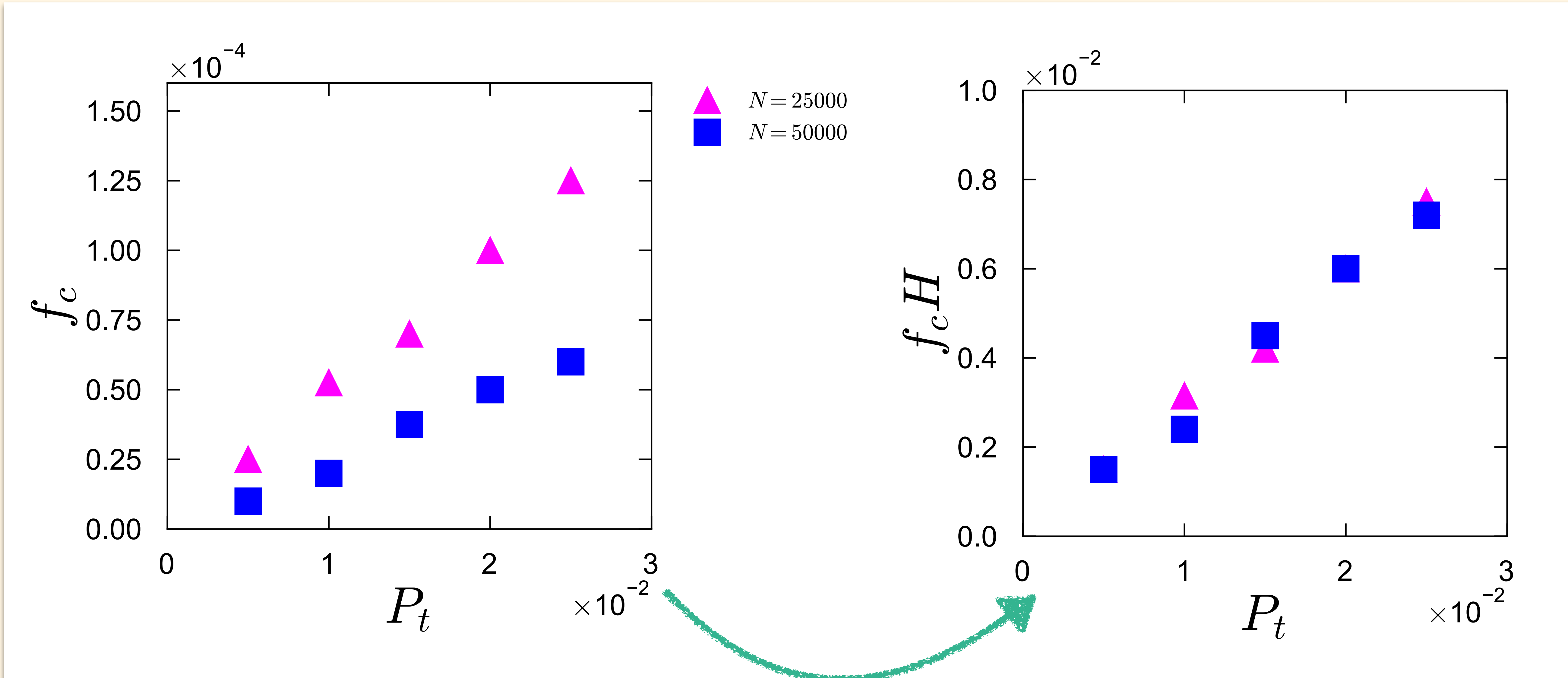
Granular Poiseuille flow

# of particles : 50,000 (Hight $H \simeq 120 d_0$) , $P_t = 0.01$



Solid-like (zero mass flux) : $f_{\rm ex} < f_c$ /Flow (finite mass flux) : $f_{\rm ex} > f_c$

Critical force f_c



Scaling relation

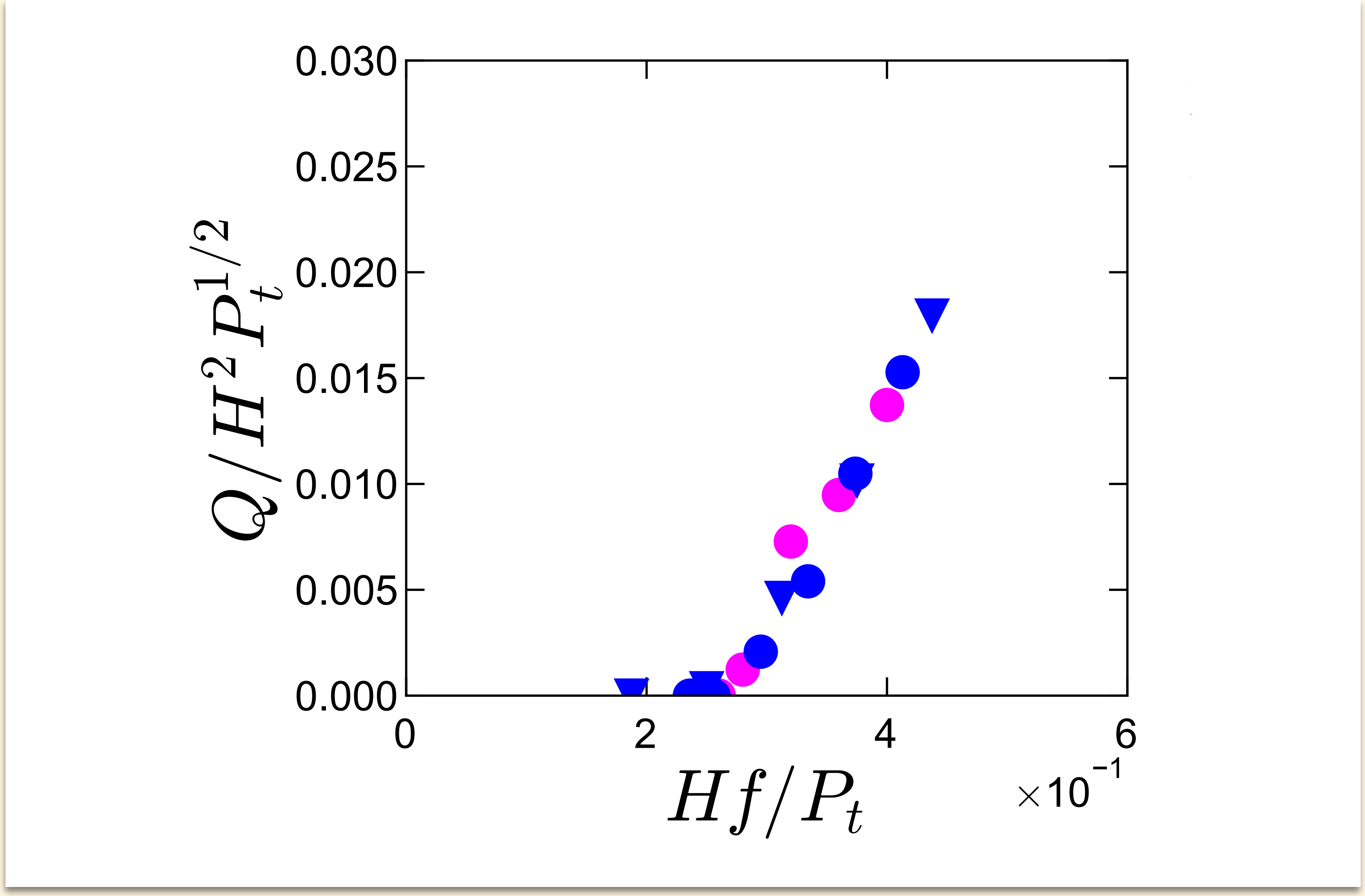
$$f_c(\phi, H) \propto \frac{P_t}{\phi H}$$

- Critical force : good agreement between continuum analysis with simulation

Mass flux Q

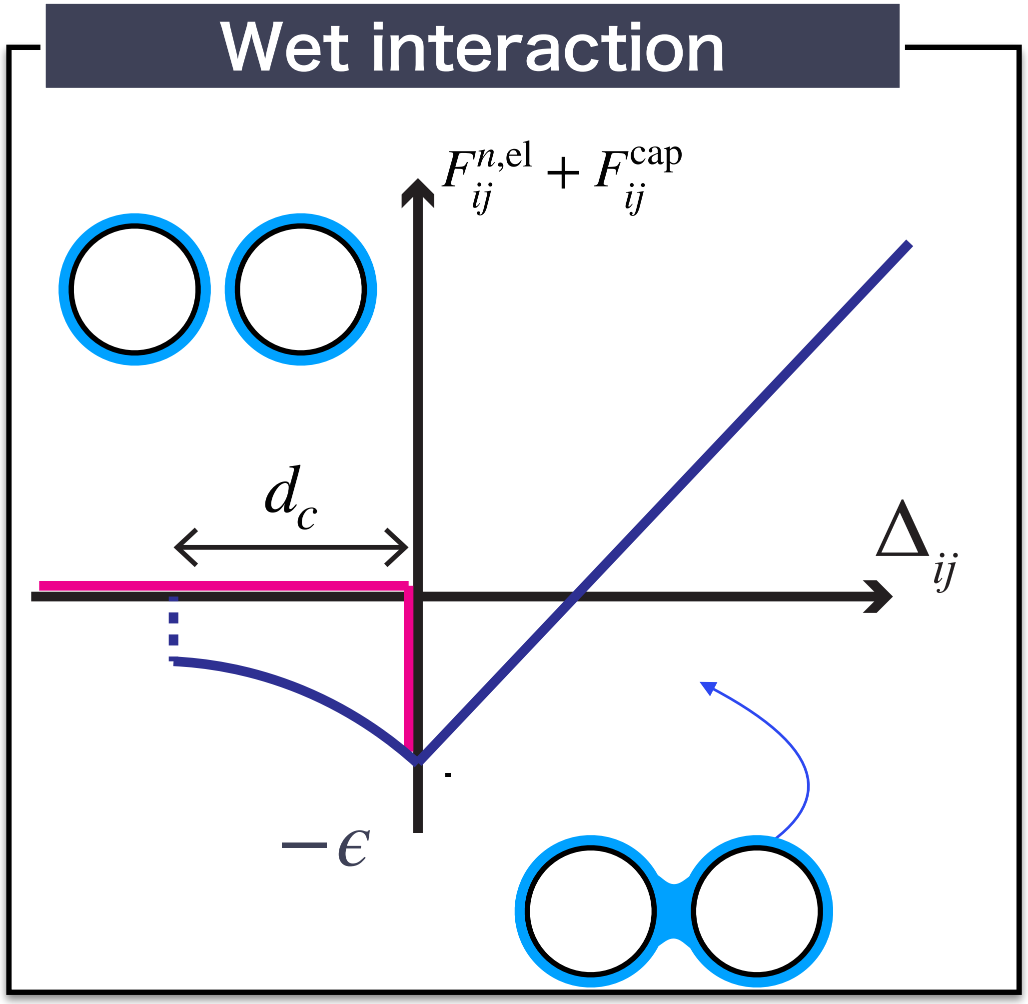
Scaling relation

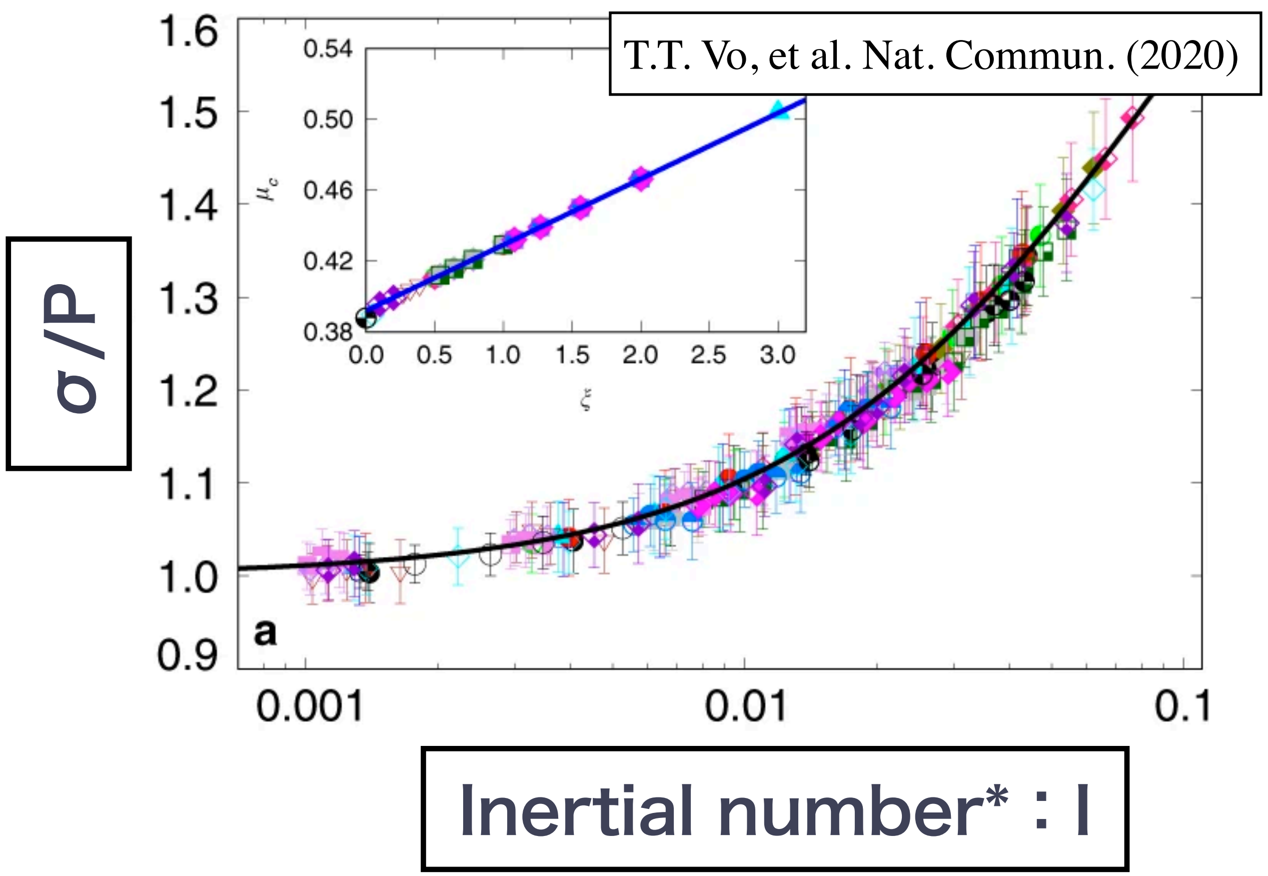
$$\frac{Q(f,\phi,H)}{H^2 P_t^{1/2}} \propto \mathcal{Q}\left(\frac{Hf}{P_t}\right)$$



- Mass flux : good agreement between continuum analysis with simulation

Cohesive case





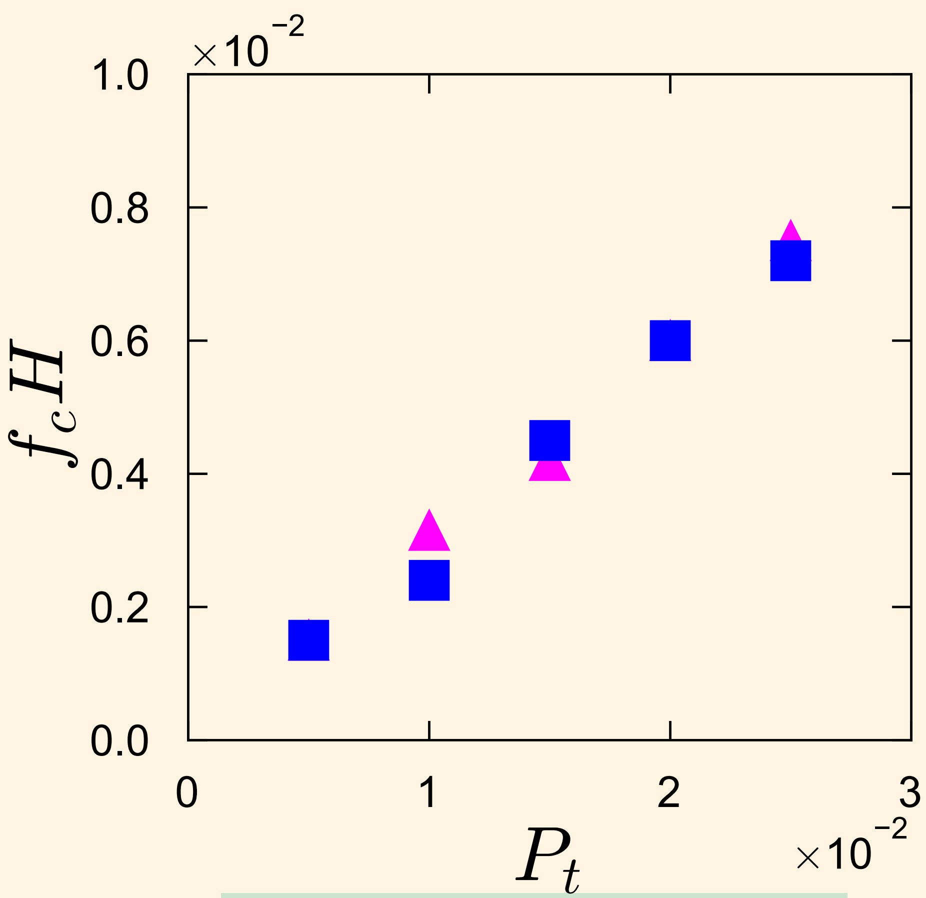
- Cohesive system →μ- I rheology
- The effect of cohesiveness & friction :work in progress

Summary

- Based on continuum analysis with μ- I rheology, we predict scaling laws.

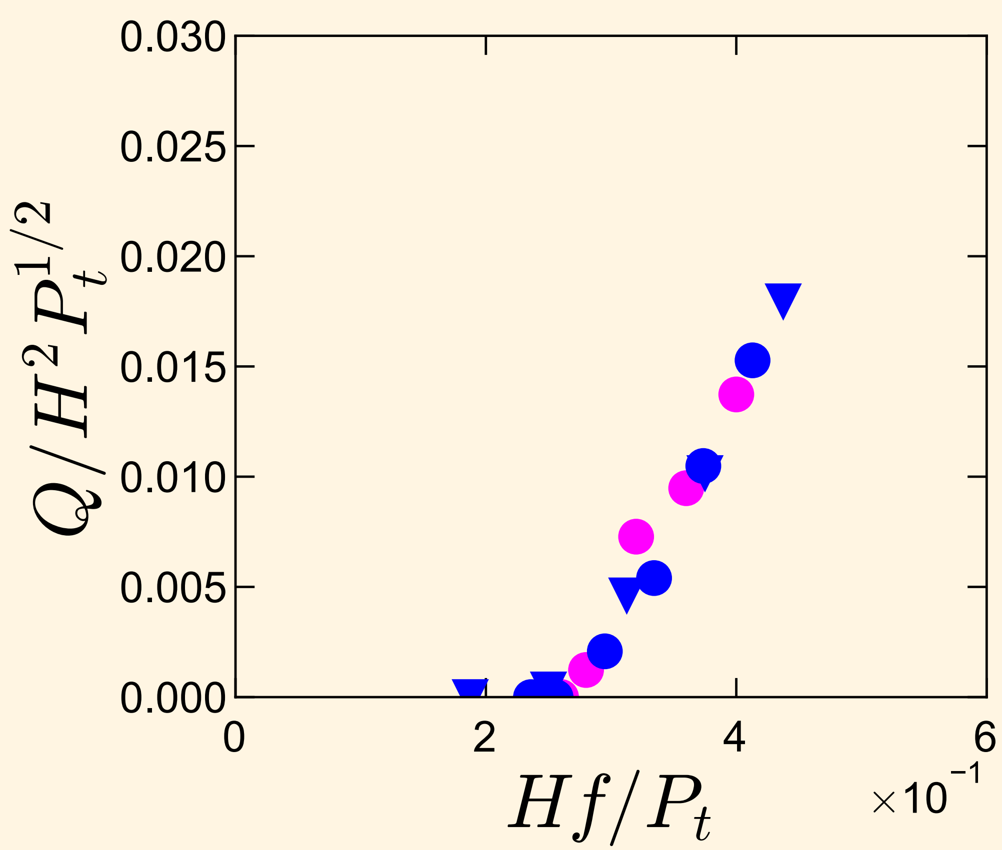
- Scaling laws are verified by DEM simulations in $f_{\rm ex} \simeq f_c$.

Critical force



$$f_c(\phi, H) \propto \frac{P_t}{\phi H}$$

Mass flux



$$\frac{Q(f, \phi, H)}{H^2 P_T^{1/2}} \propto \mathcal{Q}\left(\frac{Hf}{P_t}\right)$$