2022-11-01
@YITP

DEMON DRIVEN BY GEOMETRICAL PHASE

Based on H Hayakawa and R Yoshii, arXiv:2205.15193 (2022).

(H Hayakawa, VMM Paasonen, R Yoshii, arXiv:2112.12370 (2021).)

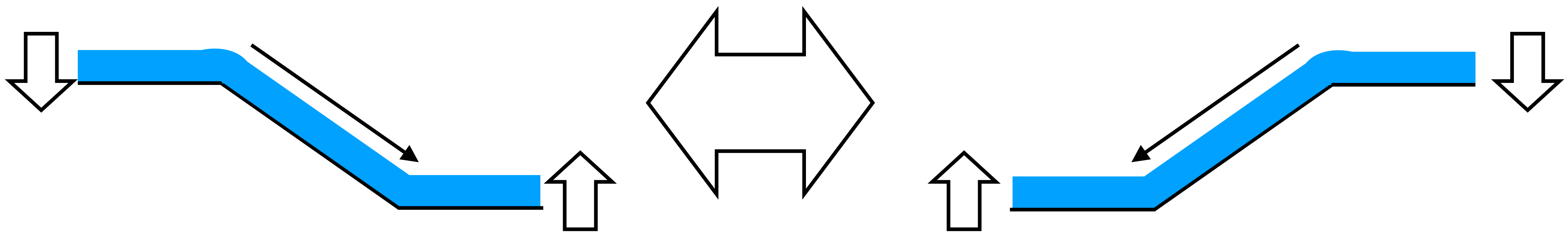
Ryosuke Yoshii
(Sanyo-Onoda City University)

Collaborator
Hisao Hayakawa (YITP)

(Ville M. M. Paasonen)

Introduction: Thouless pumping

Classical pump

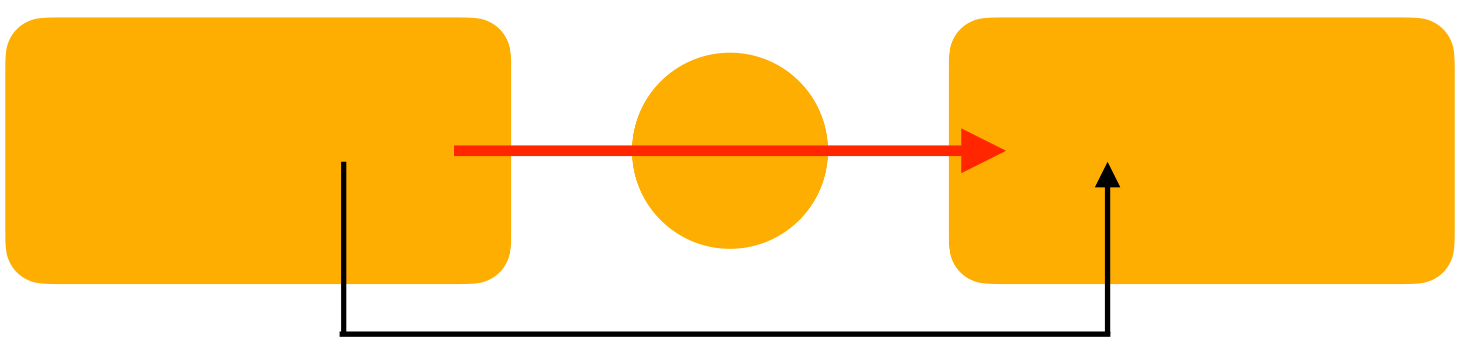


Mean bias = 0 ⇒ mean current = 0

Topological pump

Mean current can flow w/ bias voltages

Mean current!



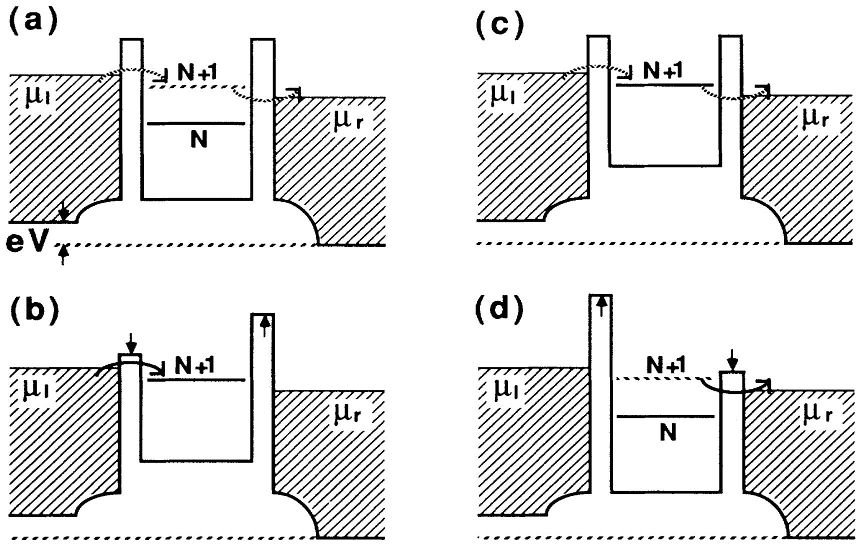
$$\langle V_{\text{bias}} \rangle_{\text{time average}} = 0$$

Thouless, 1983

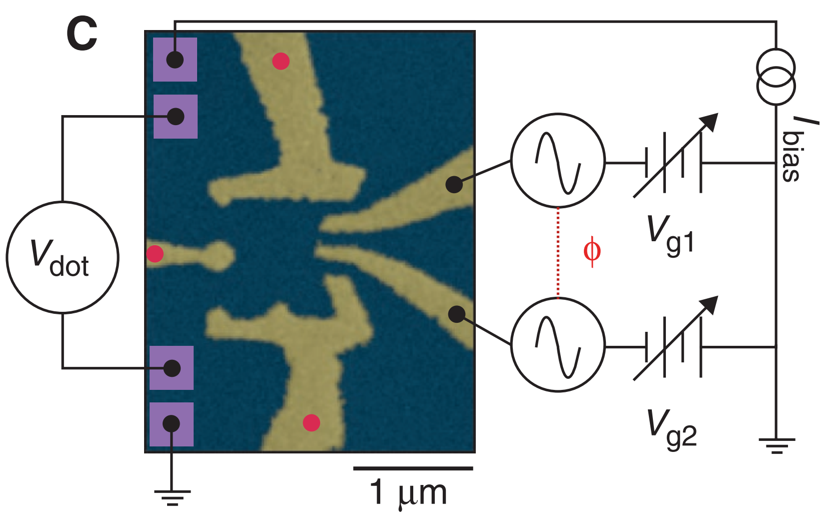
Experimental implementation

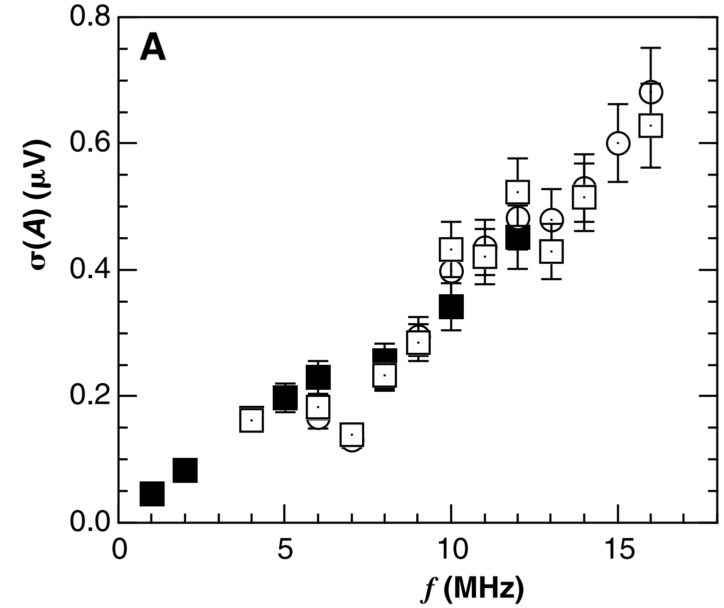
Quantum dot system

Quantum dot turnstile



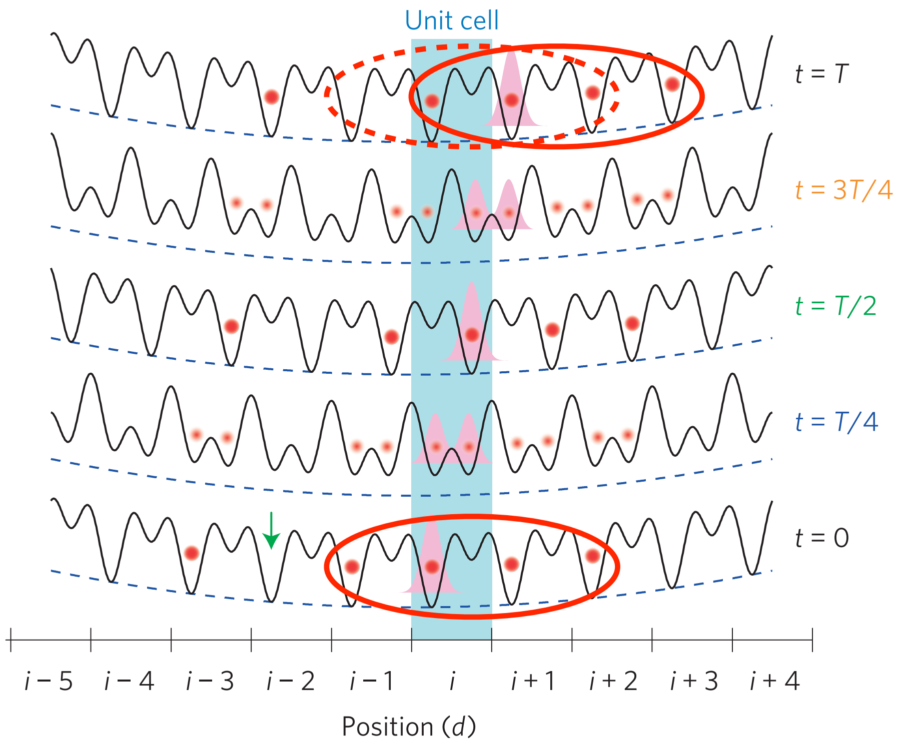
M. Switkes, et.al., Science, **283**, 1905 (1999).

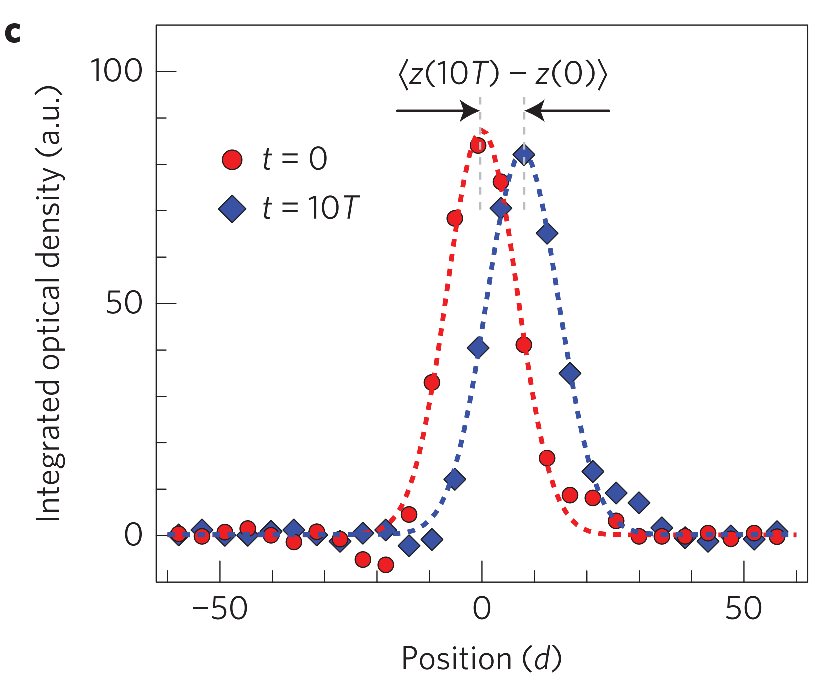




Cold atom system

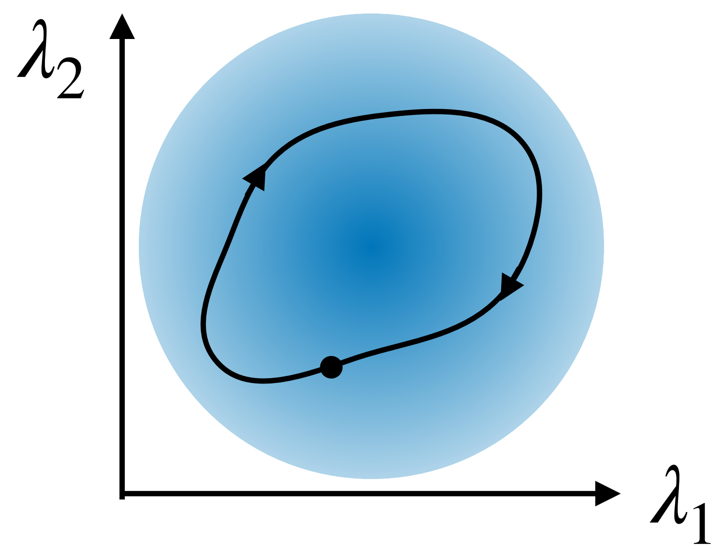
S. Nakajima, et.al., Nat. Phys., **12**, 296 (2016).





Geometrical interpretation

Non-trivial curvature in parameter space



Gauss-Bonnet type argument

⇨ Current ~ integration of the curvature inside trajectory

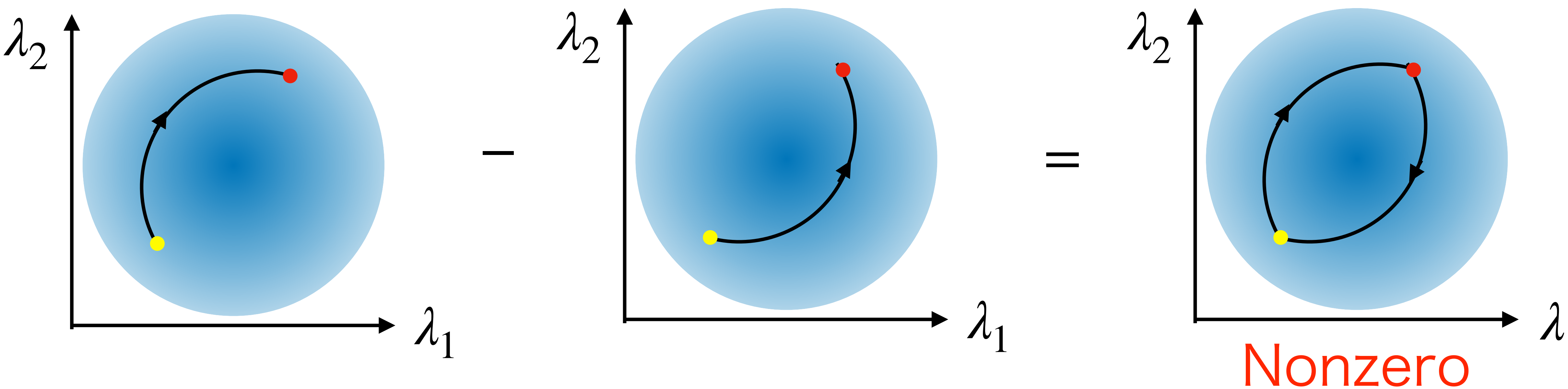
Berry-Sinitsyn-Nemenman(BSN) curvature

N. A. Sinitsyn and I. Nemenman PRL **99**, 220408 (2007).

Connection with thermodynamics

Entropy production depends on the trajectory

T. Sagawa and H. Hayakawa PRE **84**, 051110 (2011).



Vector potential in thermodynamics

K. Tomita and H. Tomita PTP **51**, 6 (1974).

Related topics

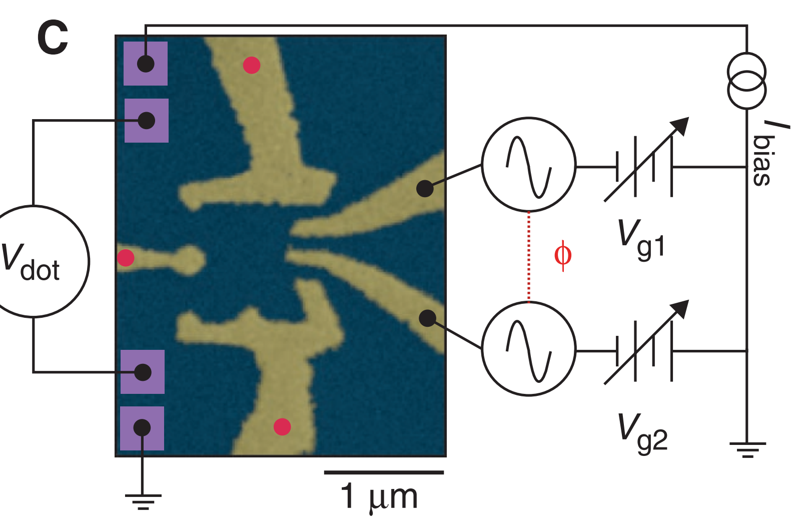
- Brandner-Saito (PRL 2020) : single-bath
- Hino-Hayakawa (PRR2021): 2-baths
- Ito-Dechant (PRX2020)

Purpose

Experimentally realizable system

Quantum dot system

Quantum dot + reservoirs



Fermion with the Coulomb interaction

Quantum effect?

Many body interaction?

Fermi statistics?

Geometrical effect

Work? Efficiency? Entropy production?

Formalism: Master equation

Time evolution of the density matrix ρ

$$\frac{d\rho}{d\theta} = \hat{\mathscr{K}}\rho$$

$\hat{\mathscr{K}}$: linear operator acting on ρ

(CPTP property is satisfied)

θ: time $\hat{\mathscr{K}}(\theta)$: modulated in time

Recasting the density matrix into the vector form

$$\frac{d}{d\theta}|\hat{\rho}\rangle = \hat{\mathscr{K}}|\hat{\rho}\rangle$$

$|r_i(\theta)\rangle\rangle$: right eigenstate of $\hat{\mathscr{K}}$

$\langle\langle \ell_i(\theta)|$: left eigenstate of $\hat{\mathscr{K}}$

ε_i: eigenvalues of $\hat{\mathscr{K}}$

Geometrical state

$$|\rho(\theta)\rangle\rangle = |r_0(\theta)\rangle\rangle - \sum_{i\neq 0}\int_0^\theta d\phi e^{\int_\phi^\theta d\chi \epsilon^{-1}\varepsilon_i(\chi)} \mathscr{A}_\mu \frac{d\Lambda^\mu}{d\phi}|r_i(\theta)\rangle\rangle$$

$$\mathscr{A}_\mu = \langle\langle \ell_i(\phi)|\frac{\partial}{d\Lambda^\mu}|r_0(\phi)\rangle\rangle$$

$\boldsymbol{\Lambda}$: parameters

$$\mathscr{F}^i_{\mu\nu} \equiv \frac{\partial \mathscr{A}^i_\nu}{\partial \Lambda_\mu} - \frac{\partial \mathscr{A}^i_\mu}{\partial \Lambda_\nu}$$

Geometrical state

$$|\rho(\theta)\rangle\rangle = \underline{|r_0(\theta)\rangle\rangle} - \sum_{i\neq 0}\int_0^\theta \underline{d\phi e^{\int_\phi^\theta d\chi \epsilon^{-1}\varepsilon_i(\chi)}} \mathscr{A}_\mu \frac{d\Lambda^\mu}{d\phi}|r_i(\theta)\rangle\rangle$$

$$\mathscr{A}_\mu = \langle\langle \ell_i(\phi)|\frac{\partial}{d\Lambda^\mu}|r_0(\phi)\rangle\rangle$$

$\mathbf{\Lambda}$: parameters

$$\mathscr{F}^i_{\mu\nu} \equiv \frac{\partial \mathscr{A}^i_\nu}{\partial \Lambda_\mu} - \frac{\partial \mathscr{A}^i_\mu}{\partial \Lambda_\nu}$$

Steady state

Exponential decay

Geometrical state

$$|\rho(\theta)\rangle\rangle = |r_0(\theta)\rangle\rangle - \sum_{i\neq 0}\int_0^\theta d\phi e^{\int_\phi^\theta d\chi \epsilon^{-1}\varepsilon_i(\chi)} \mathscr{A}_\mu \frac{d\Lambda^\mu}{d\phi} |r_i(\theta)\rangle\rangle$$

$$\mathscr{A}_\mu = \langle\langle \ell_i(\phi)| \frac{\partial}{d\Lambda^\mu} |r_0(\phi)\rangle\rangle$$

$$\mathscr{F}^i_{\mu\nu} \equiv \frac{\partial \mathscr{A}^i_\nu}{\partial \Lambda_\mu} - \frac{\partial \mathscr{A}^i_\mu}{\partial \Lambda_\nu}$$

$\mathbf{\Lambda}$: parameters

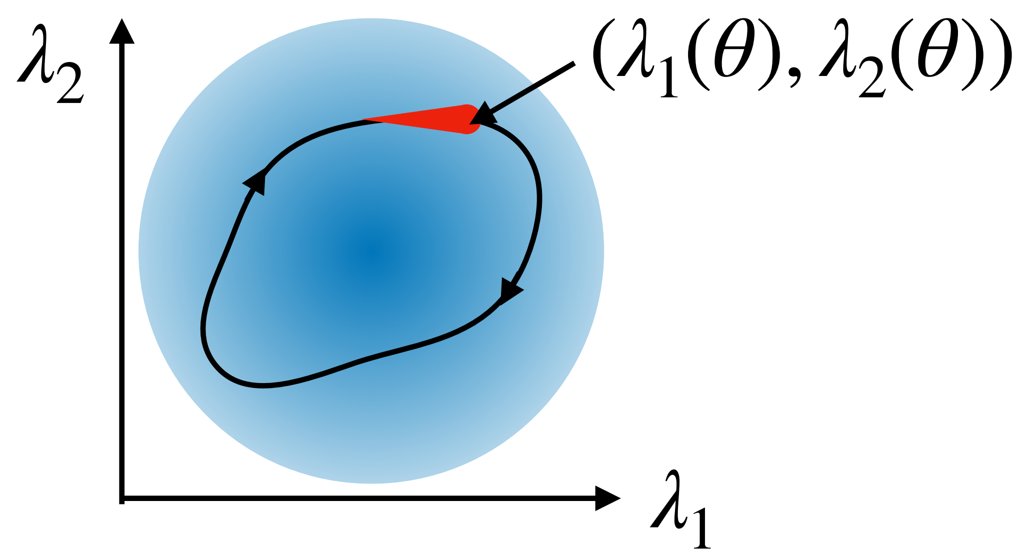
Steady state

Exponential decay

Only $\theta - \epsilon \leq \phi \leq \theta$ contributes

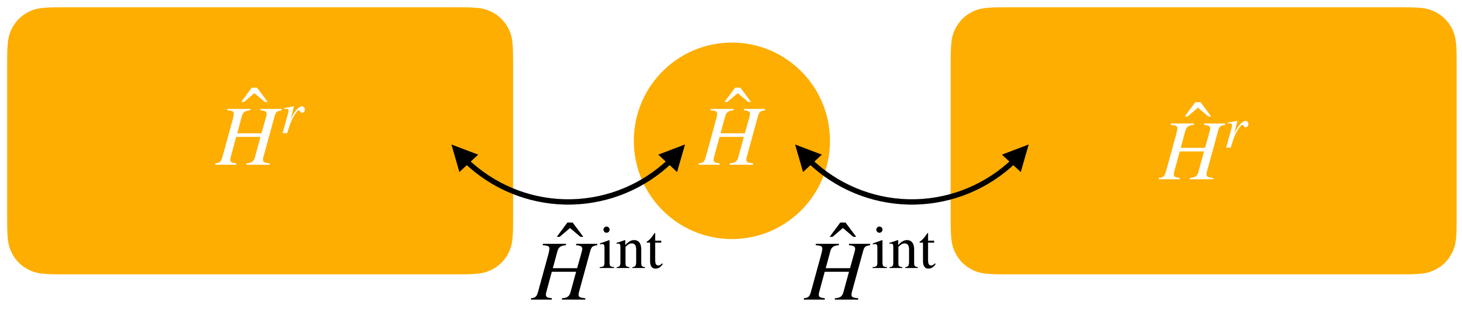
Quasi local in time

"quasi" steady state



Application to Impurity Anderson Model

$$\hat{H}^{\rm tot} = \hat{H} + \hat{H}^r + \hat{H}^{\rm int}$$



$$\hat{H} = \sum_{\sigma} \epsilon_0 \hat{d}^\dagger_\sigma \hat{d}_\sigma + U \hat{n}_\uparrow \hat{n}_\downarrow \quad \left(\hat{n}_{\uparrow/\downarrow} = \hat{d}^\dagger_{\uparrow/\downarrow} \hat{d}_{\uparrow/\downarrow} \right)$$

$\hat{d}^\dagger_\sigma$ ($\hat{d}_\sigma$): creation and annihilation operator in dot (spin σ)

$$\hat{H}^r = \sum_{\alpha,k,\sigma} \epsilon_k \hat{a}^\dagger_{\alpha,k,\sigma} \hat{a}_{\alpha,k,\sigma}$$

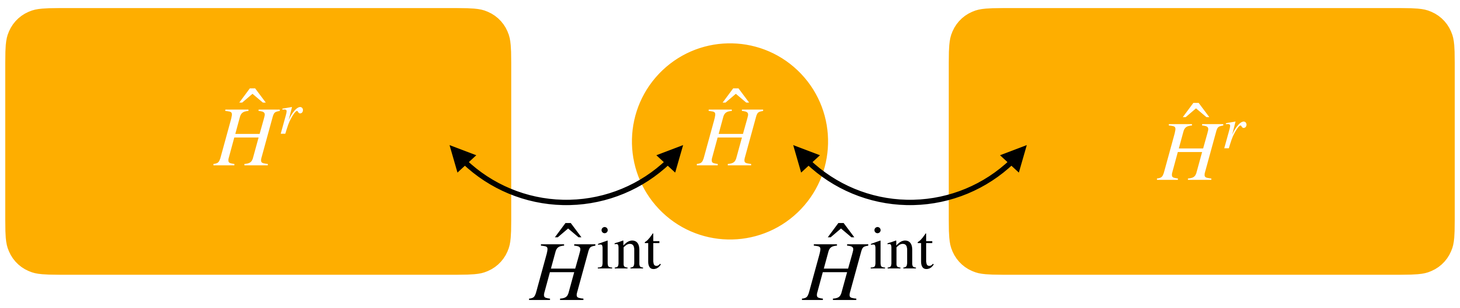
$\hat{a}^\dagger_{\alpha,k,\sigma}$ ($\hat{a}_{\alpha,k,\sigma}$): creation and annihilation operator in leads

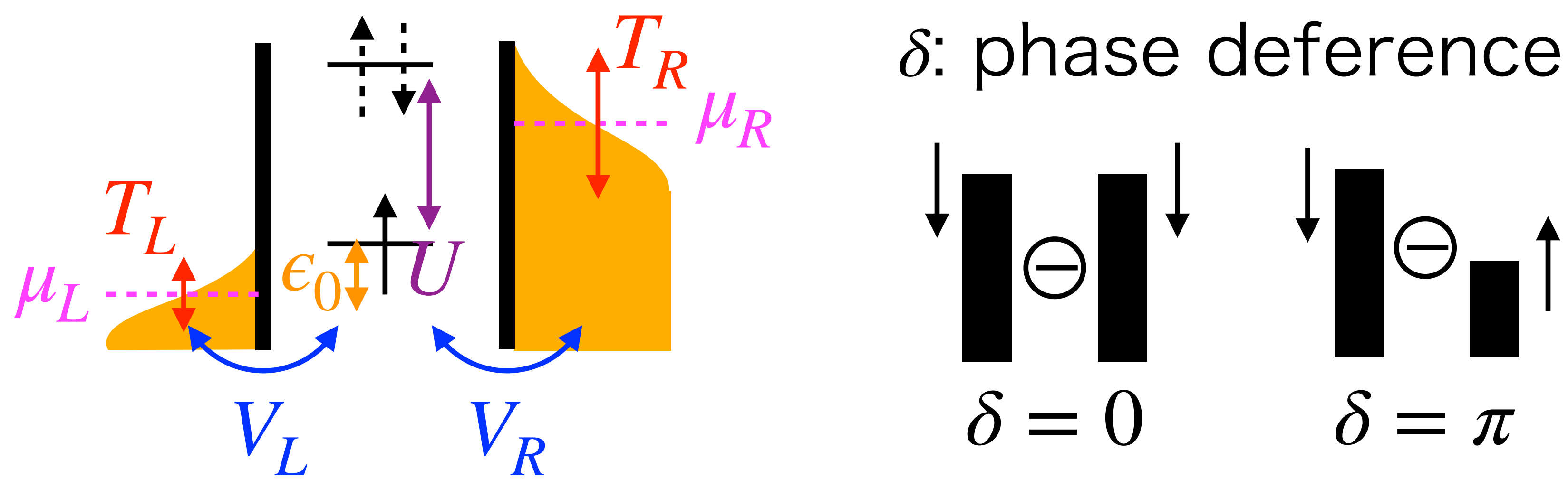
(spin σ, wave number k, $\alpha = \text{left or right}$)

$$\hat{H}^{\rm int} = \sum_{\alpha,k,\sigma} V_\alpha \hat{d}^\dagger_\sigma \hat{a}_{\alpha,k,\sigma} + \mathrm{h.c.},$$

($V_L = V_R$ for simplicity)

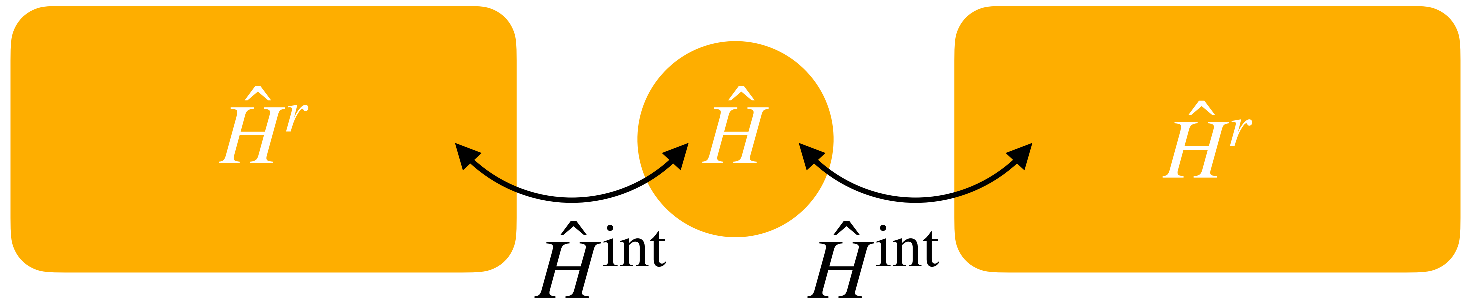
Modulating parameters

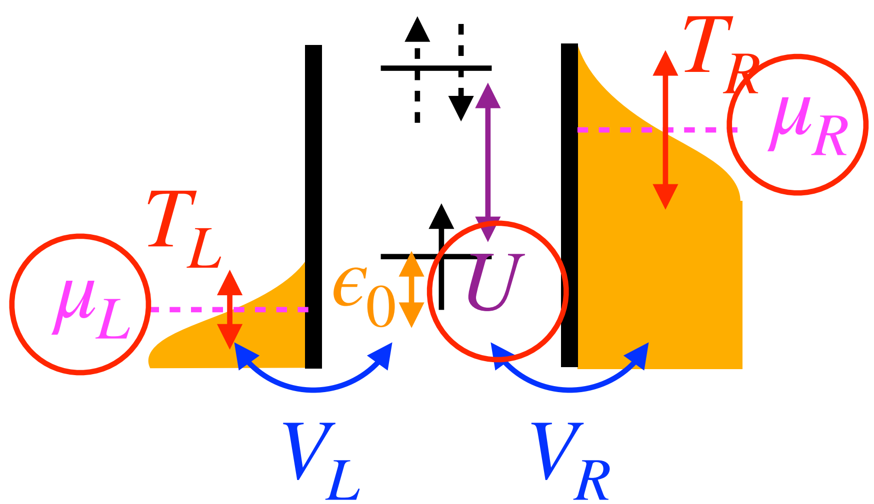




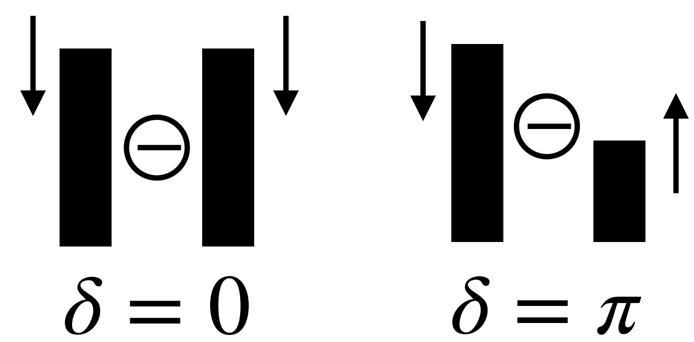
- Temperature is difficult to be controlled
- Parameter in quantum dot must be tuned to extract work

Modulating parameters





δ: phase deference



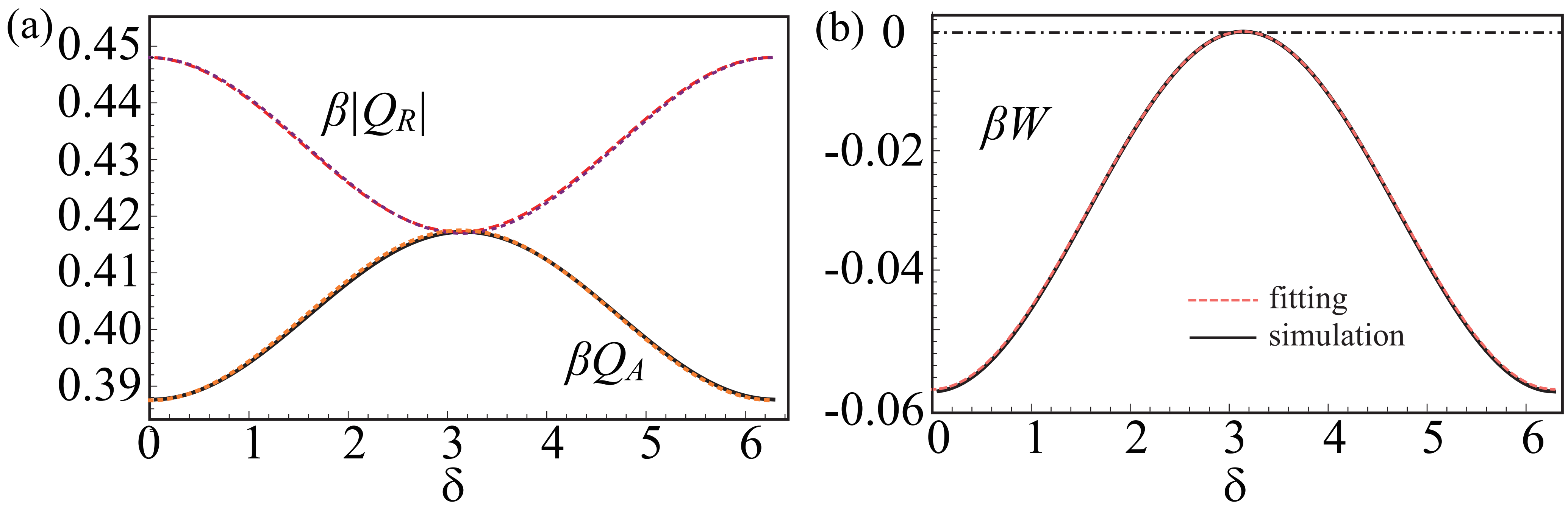
- Temperature is difficult to be controlled
- Parameter in quantum dot must be tuned to subtract work

⇨ Modulating parameters U, μ_L, μ_R

$$U = U_0(1+\lambda),\ \lambda = \cos\theta, \quad \mu_L = \mu\sin\theta, \quad \mu_R = \mu\sin(\theta+\delta)$$

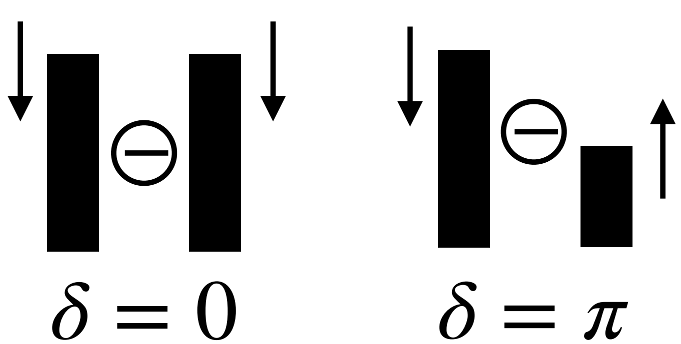
Absorb and release of heat & work

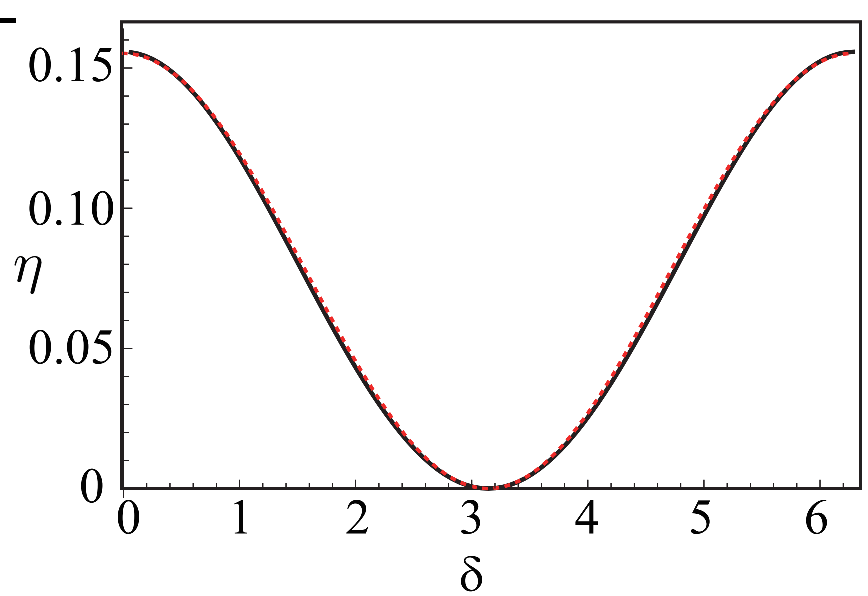
$$Q_A = \oint \frac{\mathscr{P}_\mu + \left|\mathscr{P}_\mu\right|}{2} d\Lambda^\mu, \quad Q_R = \oint \frac{\mathscr{P}_\mu - \left|\mathscr{P}_\mu\right|}{2} d\Lambda^\mu, \quad W = \oint \mathscr{P}_\mu d\Lambda^\mu, \quad \mathscr{P}_\mu = \mathrm{Tr}\left(\rho \frac{\partial H}{\partial \Lambda^\mu}\right)$$



Negative work

Efficiency: $\eta \equiv \frac{|W|}{Q_A} = \frac{|Q_A + Q_R|}{Q_A}$



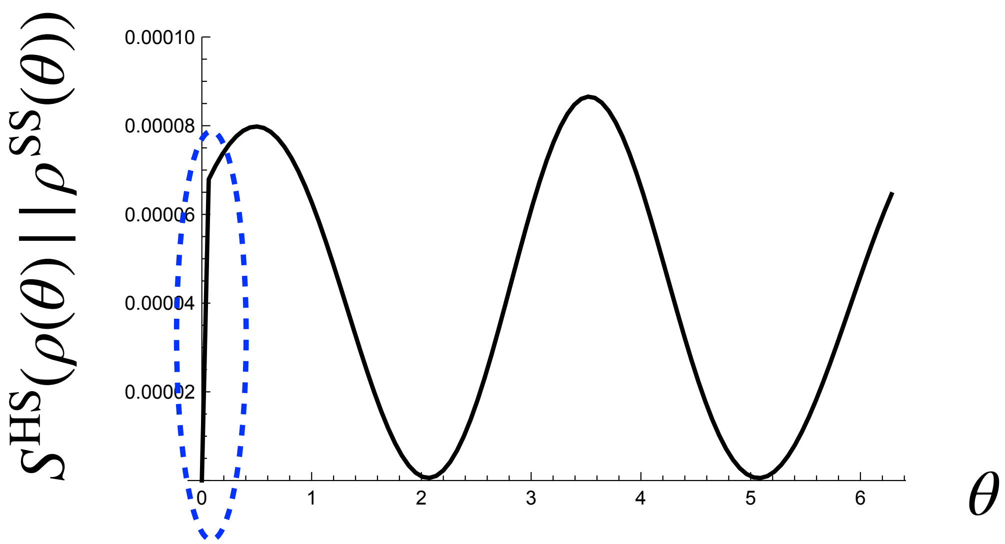


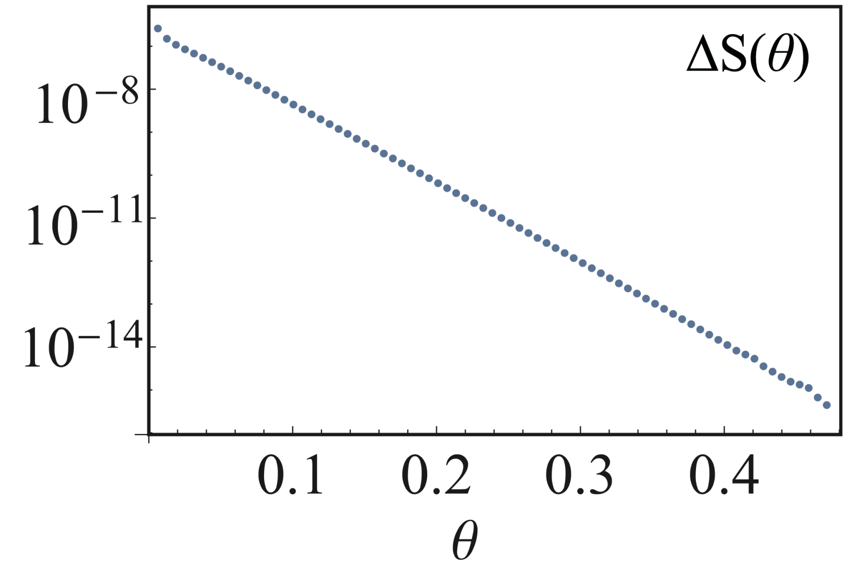
Time evolution of relative entropy

Relative entropy: $S^{\mathrm{HS}}(\rho(\theta)\,||\,\rho^{\mathrm{SS}}(\theta)) = \mathrm{Tr}\rho(\theta)\left[\ln\rho(\theta) - \ln\rho^{\mathrm{SS}}(\theta)\right]$

θ : time

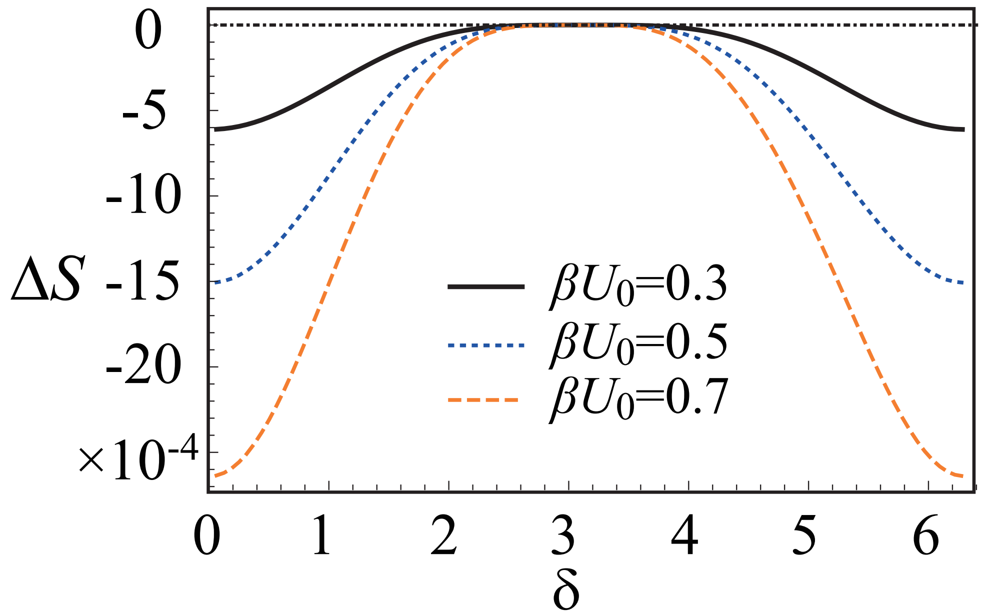
Initial decay toward quasi steady state





Exp. Decay

Initial relaxation → negative ΔS



$$\Delta S = -S^{\mathrm{HS}}(\rho(2\pi)\,||\,\rho^{\mathrm{SS}}(2\pi)) + S^{\mathrm{HS}}(\rho^{\mathrm{SS}}(0)\,||\,\rho^{\mathrm{SS}}(0))$$

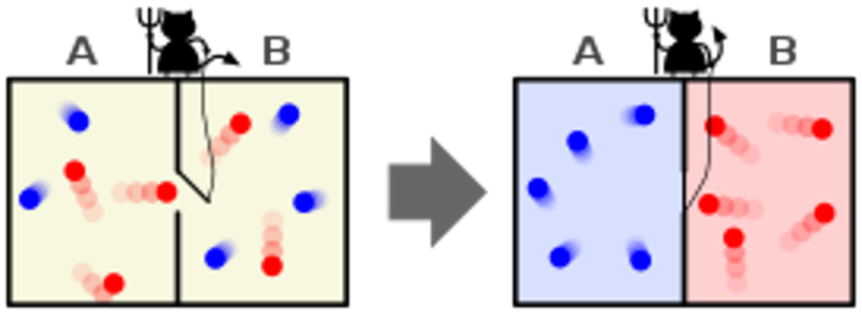
2π : one-cycle

Interpretation of the results

Negative work

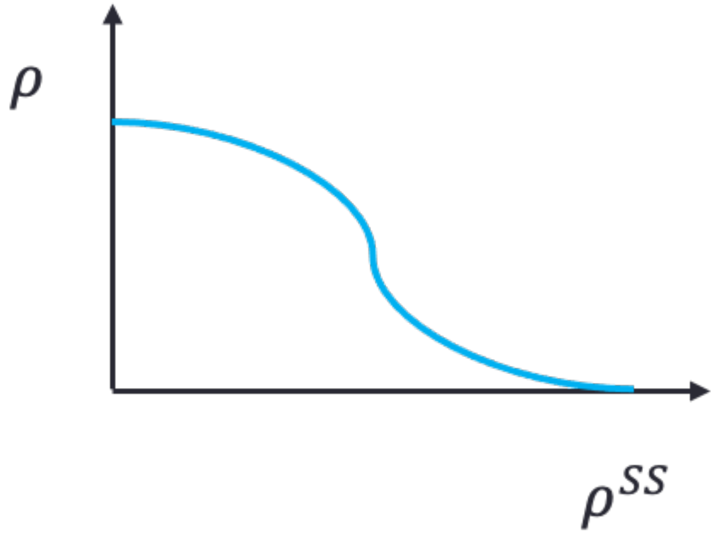
Extracted from the geometrical phase

"Geometrical demon"

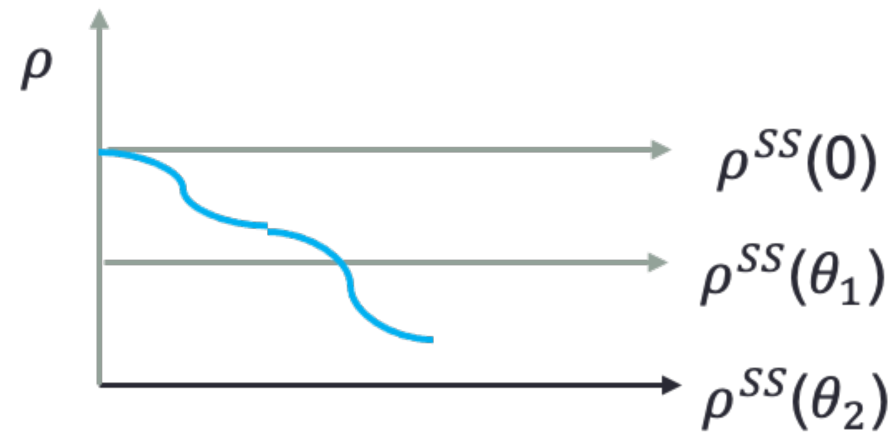


Cf. Maxwell demon
utilizes information

Relative entropy decrease in the CPTP process



No modulation



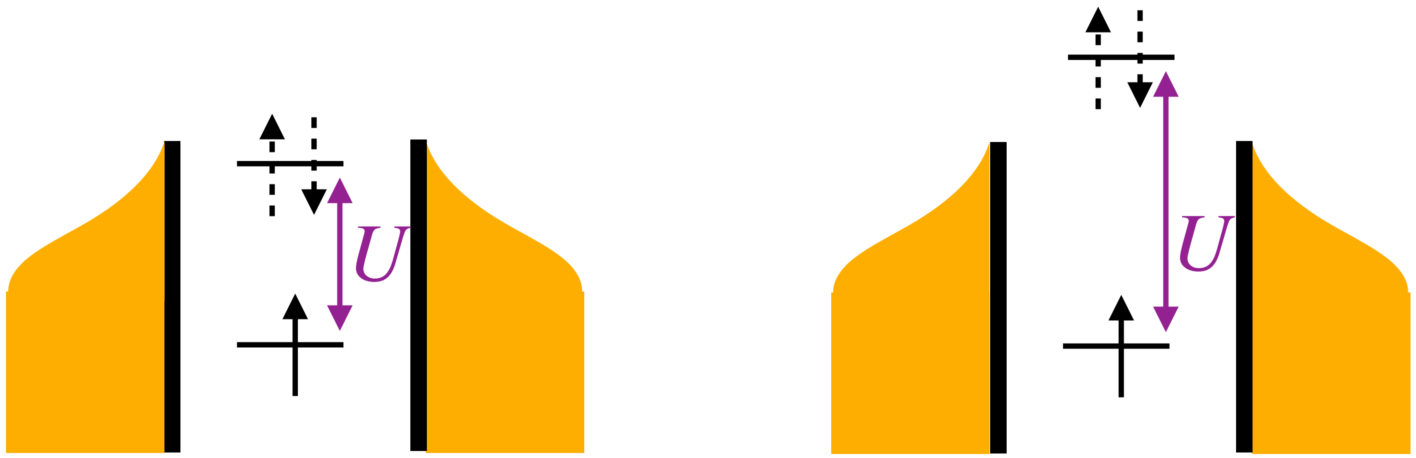
With modulation

State cannot relax to a simple "steady state" under a modulation

=> Non-adiabatic effect.

Remaining issues (on going)

First and Second laws of thermodynamics?



The cost is not considered

(Interaction inside the system is modulated)

Joule heating is generated

(Second law would be maintained)

SUMMARY

- Quasi steady state is realized under parameter modulation

  $|\rho(\theta)\rangle\rangle$: quasi local in time

- Work can be extracted by utilizing the geometrical phase

  "Geometrical Demon"

- Relative entropy can be decreased by geometrical effect

  Initial relaxation process

- Extracted Work exponentially decays

  One needs to wait to initialize the state

- For details, please see

H. Hayakawa and R. Yoshii, arXiv:2205.15193 (2022).

H. Hayakawa, VMM Paasonen, R. Yoshii, arXiv:2112.12370 (2021).