Quantum Mpemba effect: an anomalous thermal relaxation process in quantum matter

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Refs: PRL**131**, 032901 (2021)=Editors' Suggestion and arXiv:2311.01347.

Self-introduction

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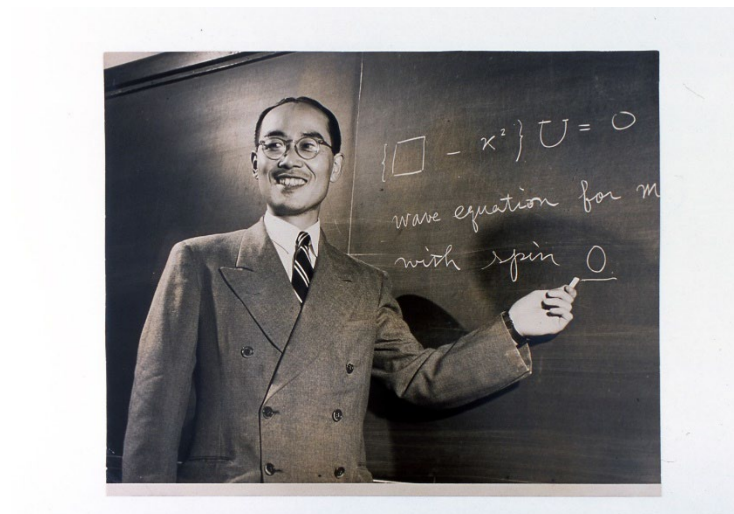
- Thank you for giving me an opportunity to present a seminar in front of you.
- This is the second visit to Academia Sinica (the last one was 1998).



Why quantum?

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- My main subject is granular physics, jamming transition and rheology.
- I also study nonequilibrium statistical mechanics.
- I am a little tired of my old subjects because they are difficult.
- I realize that physics of open quantum systems is much simpler than classical many-body problems.

Contents

Contents

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- Introduction: What is the Mpemba effect?
- Quantum Mpemba effect in Anderson model (PRL 131, 032901 (2023))
- Quantum Mpemba effect with exceptional points
- Discussion
- Concluding remarks

Various memory effects

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REVIEWS OF MODERN PHYSICS, VOLUME 91, JULY–SEPTEMBER 2019

Memory formation in matter

Kaiser effect, Mullins effect, Kovacs effect…

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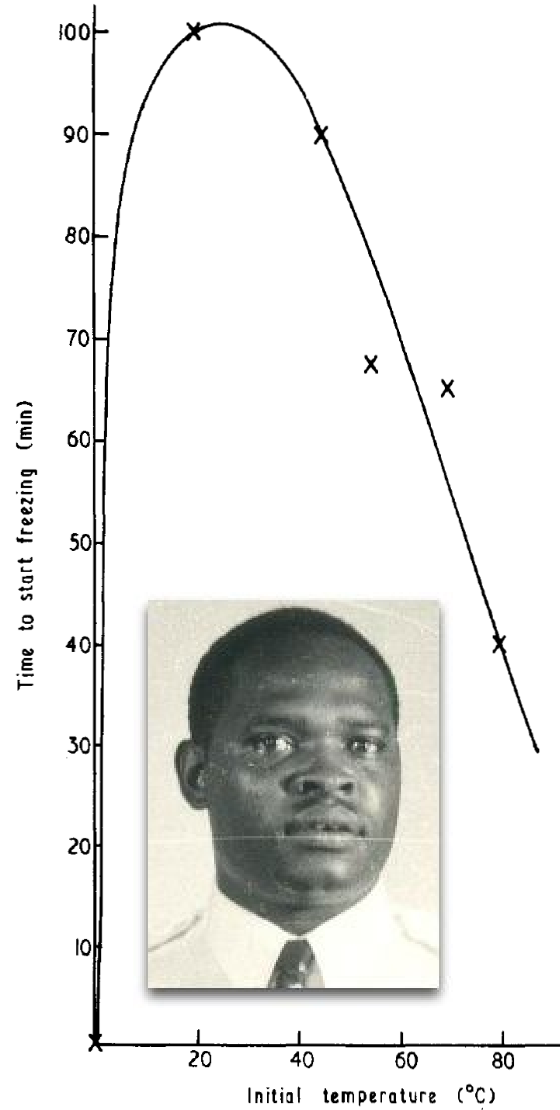
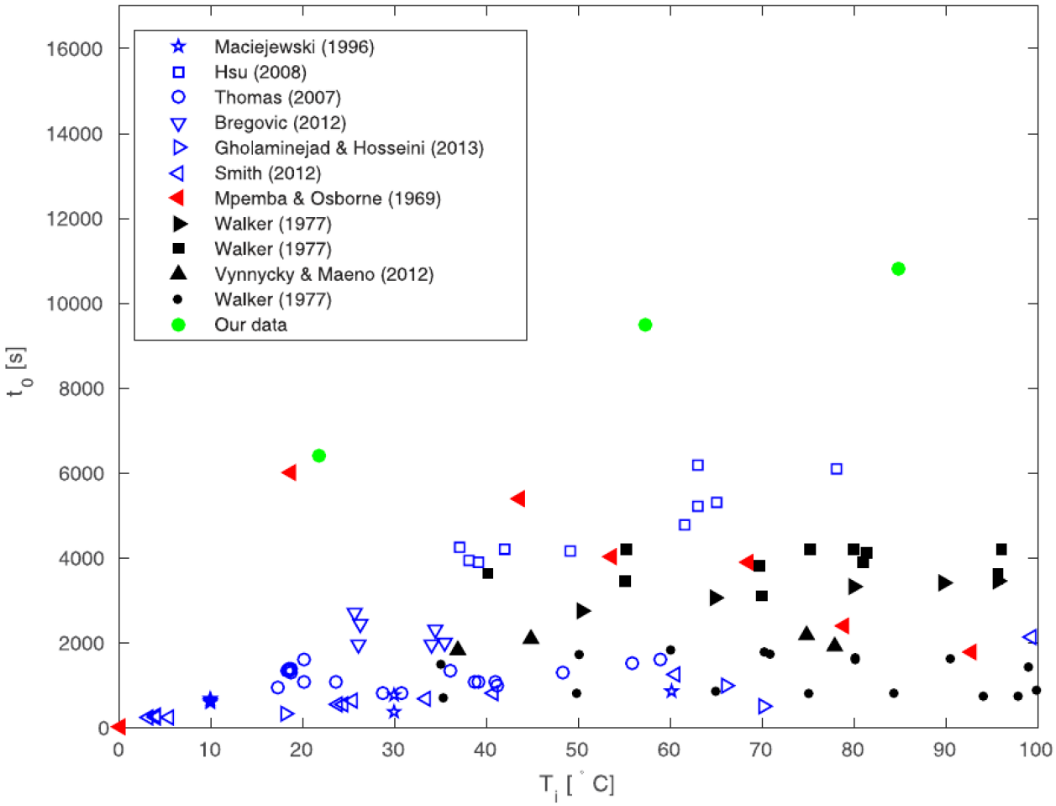
What is the Mpemba effect?

- What is Mpemba effect?
  - Erasto B. Mpemba found that some hot suspensions (*ice cream mix*) can freeze faster than cold (1963).
  - With the help of D. G. Osborne he has published a scientific paper (1969).



Debates

- Poor reproducibility
- The right figure is one counter example of Mpemba effect.
- However, people believe the existence of Mpemba-like phenomena.



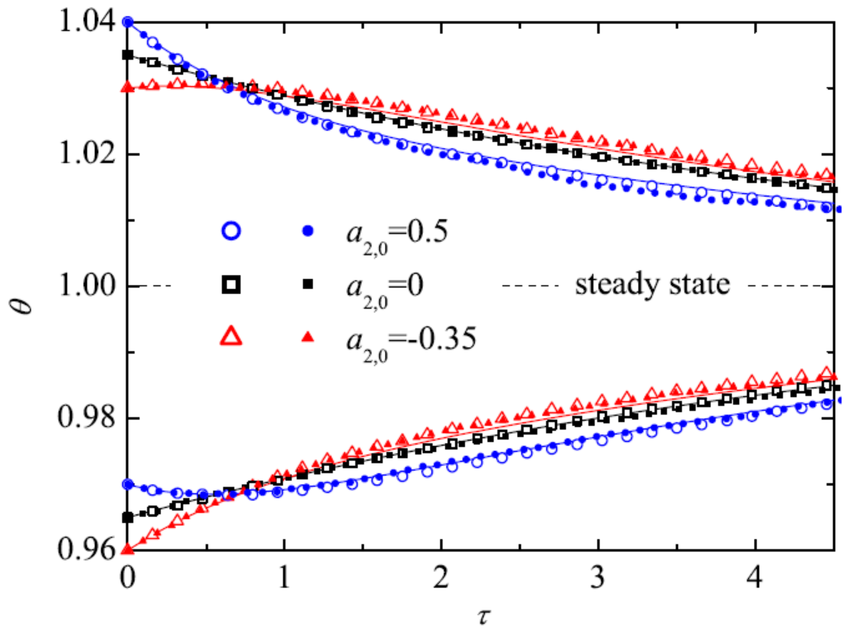
Burridge and Linden, Sci. Rep. **6**, 37665 (2016).

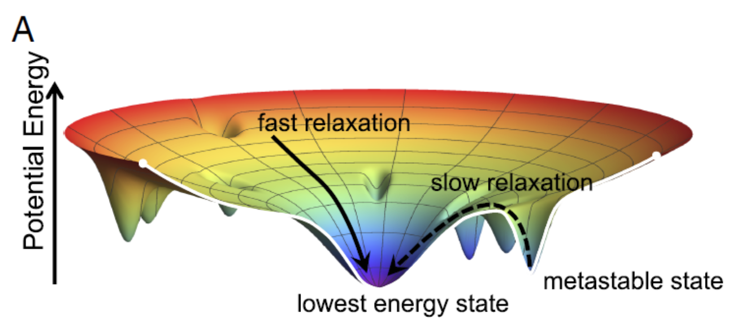
Recent trend

- Now, people are not interested in time to start the freezing, but are interested in the cross point(s) of the relaxation process.
- Namely, if the "temperature" starting from high initial temperature becomes lower temperature than that starting from lower initial temperature, we regard it as Mpemba effect.

Some theoretical studies

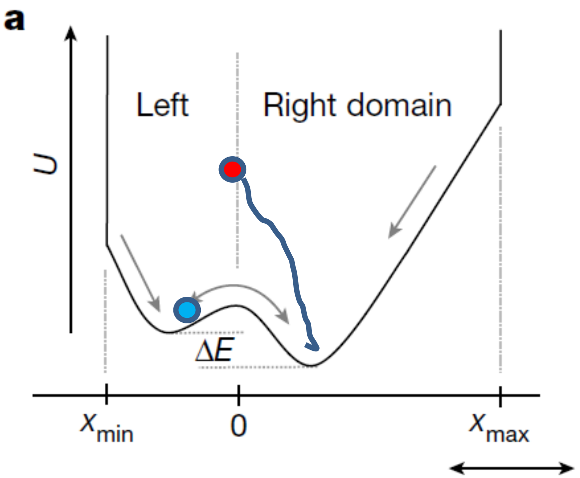
- Lasanta et al. PRL **119**, 148001 (2017) found that a granular gas can have both ME and the inverse ME by controlling *kurtosis*.
- Lu & Raz, PNAS **114**, 5083 (2017) indicated that the slow relaxation can take place by trapping at local minima.
  - But there is no temperature.
- But how can we control?





Physical Mechanism of Mpemba effect

- There are two scenarios to present the Mpemba effect.
- Lu-Raz scenario: A particle at high temperature does not feel the effect of potential, while it at warm temperature is trapped in a potential minimum.
  - The slowest mode is only important.
- Noneq. initial vs eq. initial conditions: Cooling rate is different.



Lu & Raz (PNAS2017)

- They have analyzed the master equation:

$$\frac{\mathrm{d}p_i(t)}{\mathrm{d}t} = \sum_j R_{ij}(T_b)p_j(t) \quad \text{for } i = 1, 2, \cdots, n.$$

- They are interested in the slowest relaxation mode=>approach to the equilibrium state:

$$\vec{p}(t) = \vec{\pi}(T_b) + e^{\lambda_2 t} a_2 \vec{v}_2 + \ldots \qquad \pi_i(T_b) = \frac{e^{-E_i/k_B T_b}}{\sum_i e^{-E_i/k_B T_b}}$$

- The condition for Markovian Mpemba effect:

$$|a_2^c| > |a_2^h|$$

Lu & Raz (2017) no.2

- They have introduced KL divergence:

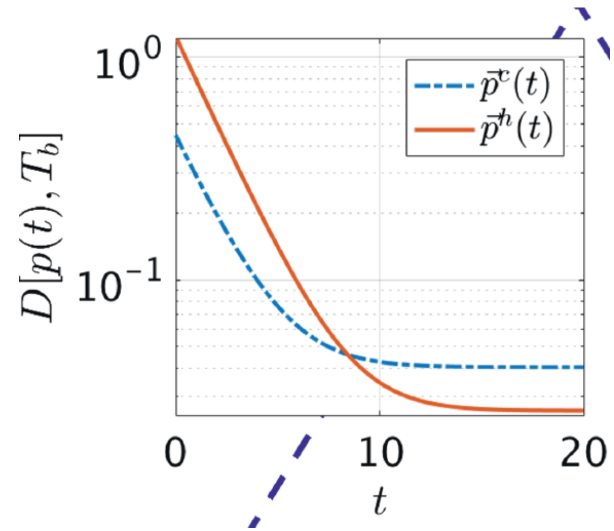
$$D_e[\vec{p};\, T_b] = \sum_i \left(\frac{E_i \Delta p_i}{T_b} + p_i \ln p_i - \pi_i^b \ln \pi_i^b \right),$$

- The Markovian Mpemba effect

$$\left| a_2^c \right| > \left| a_2^h \right|$$

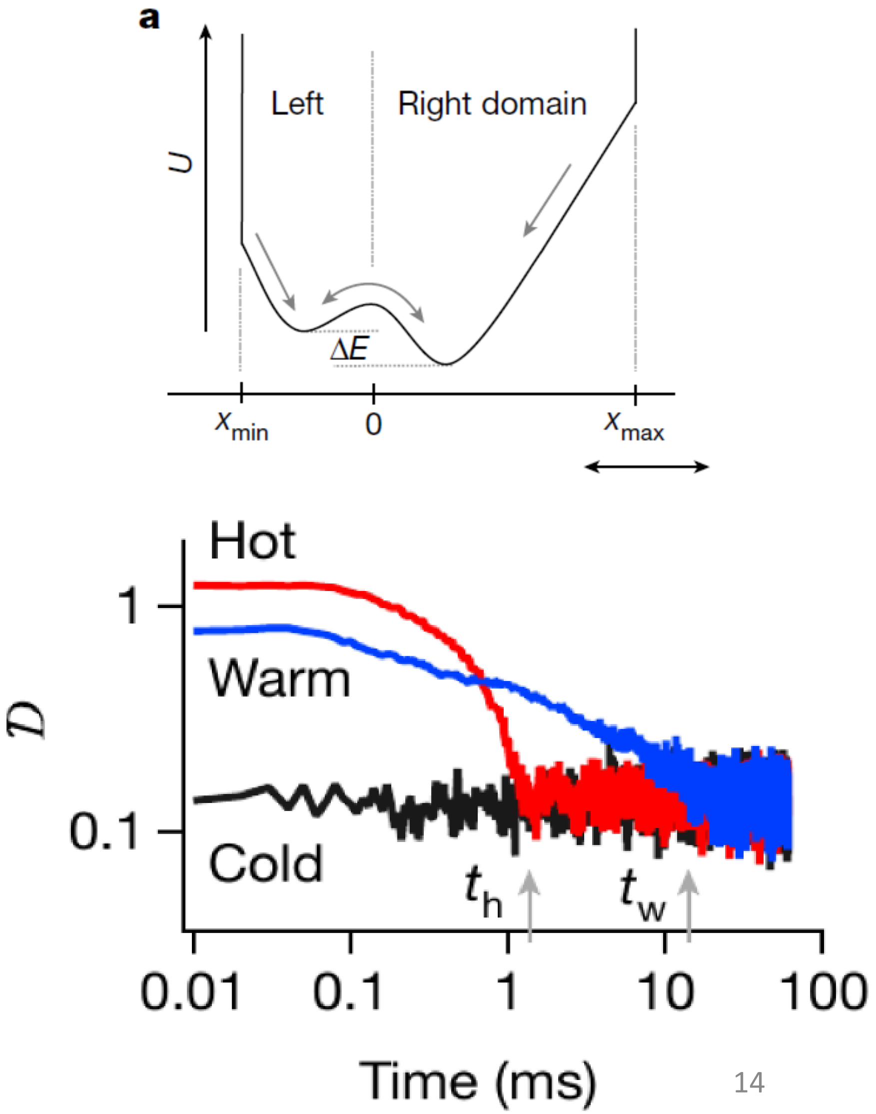
$$\Downarrow$$

$$D_e[\vec{p}^{\,h}(t);\, T_b] < D_e[\vec{p}^{\,c}(t);\, T_b]$$



Experimental confirmation

- Kumar & Bechhoeffer, Nature **584**, 64 (2020).
- They have analyzed trapped colloids in a double well potential.
- They observed the distance between the distribution and equilibrium one.



Question to the scenario by Lu-Raz

- Connection with kinetic theory is not clear.
- They are only interested in approaching to the final equilibrium state, but this is not always related to the cross points.
  - Initial relaxation may be important.
- They are only interested in discrete systems.

The initial cooling depends on how the initial condition is prepared.

- There are many studies which do not follow Lu&Raz scenario.
  - Lasanta et al (PRL2017), Carrolo et al (PRL2021), Takada et al (PRE2021), Ares et al (Nat. Comm. 2023), Chatterjee et al (PRL2023)
- They do not consider the potential landscape.
- Instead, they prepare two systems: one at equilibrium and another at nonequilibrium.
- The difference of the initial cooling rate generates the Mpemba effect.

Purpose of this talk

- We analyze quantum Mpemba effect which does not obey the scenario by Lu & Raz.
- In the first part, we analyze the Mpemba effect in the (quasi-classical) Anderson model.
  - a_3 and a_4 are important but a_2 is not.
- We analyze the Mpemba effect in Hatano's model as an example of fully open quantum systems.

Quantum dot

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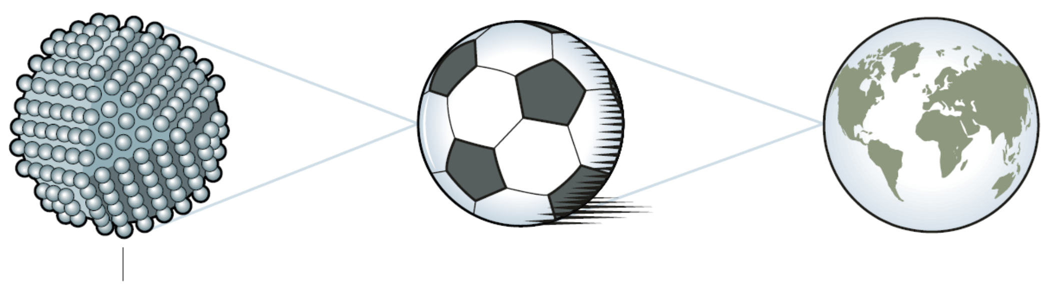
- Nobel prize in chemistry 2023 is awarded for the discovery of quantum dots.
- We use quantum dots to demonstrate the quantum Mpemba effect.



A quantum dot is a crystal that often consists of just a few thousand atoms. In terms of size, it has the same relationship to a football as a football has to the size of the Earth.

Illustration of Mpemba effect in the second scenario

- We can write the energy equation (for an uniform system):

$\dot{\gamma}$: shear rate

$$c_V \dot{T} = -\frac{\dot{\gamma}}{n} P_{xy} \underset{\gtrsim 0}{+} 2 c_V \zeta (T_{\rm env} - T),$$

- If the system is at equilibrium, the viscous heating is absent (P_{xy}=0).
- If the system is in non-equilibrium, the heating term must exist.
- Then, the system at equilibrium must have faster cooling than that at non-equilibrium.

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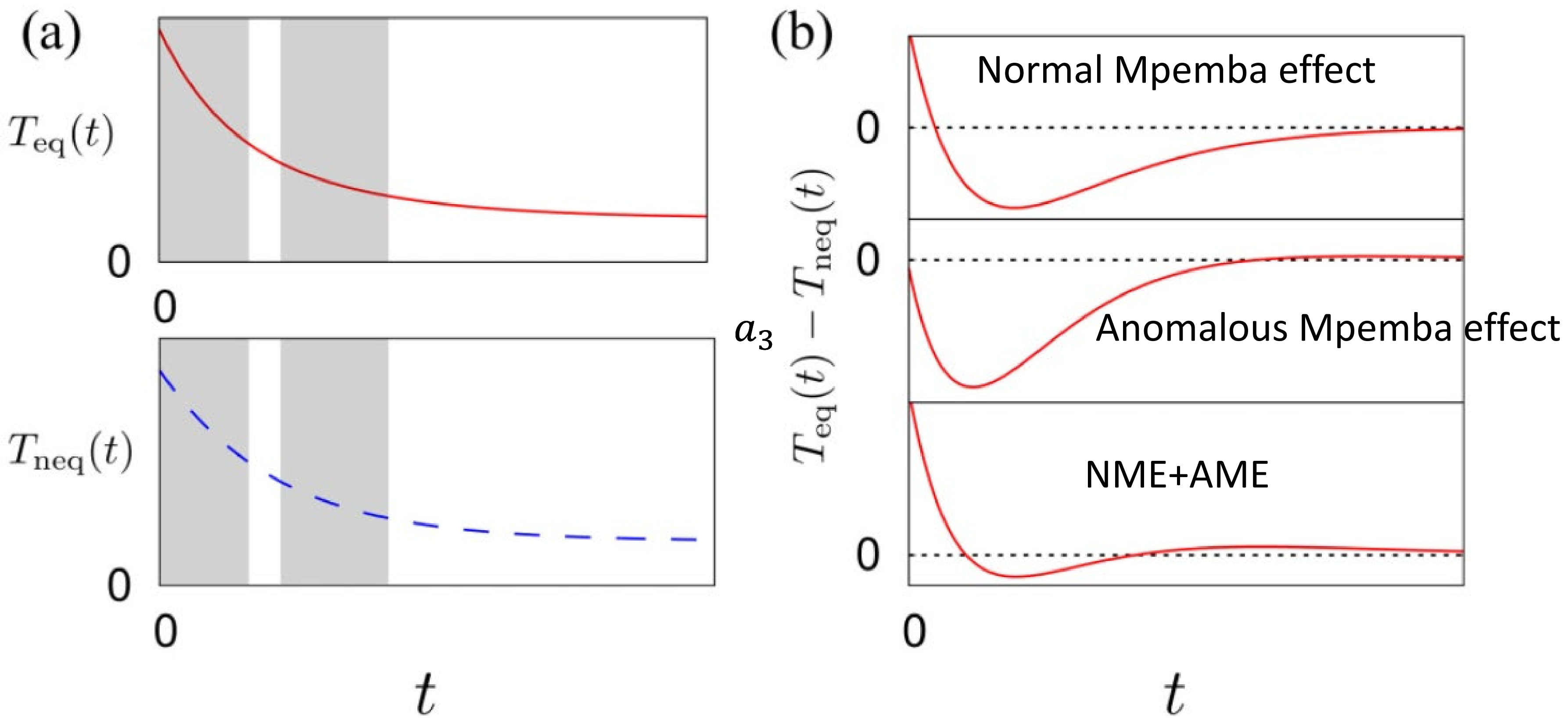
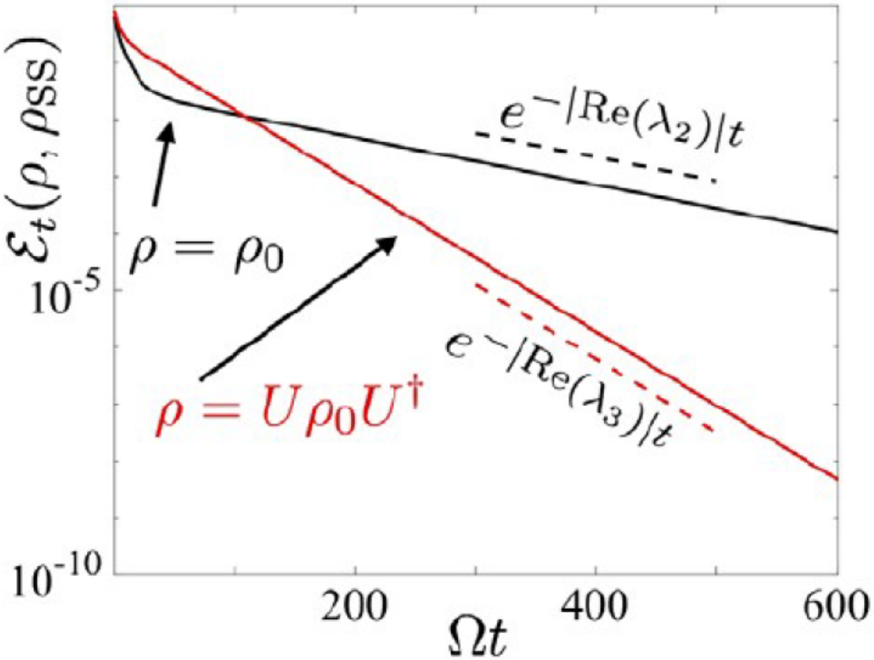
Essence of Mpemba effect

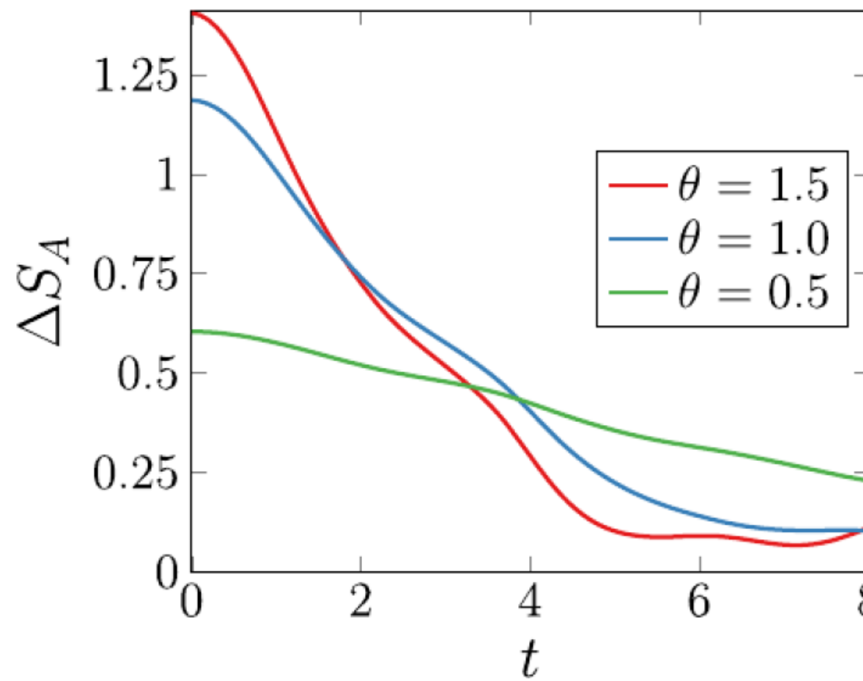


Quantitative argument following this scenario can be found in Takada, Hayakawa & Santos, PRE**103**, 032901 (2021)

Contents

Quantum Mpemba effect





Distance from equilibrium in dissipative Dicke model

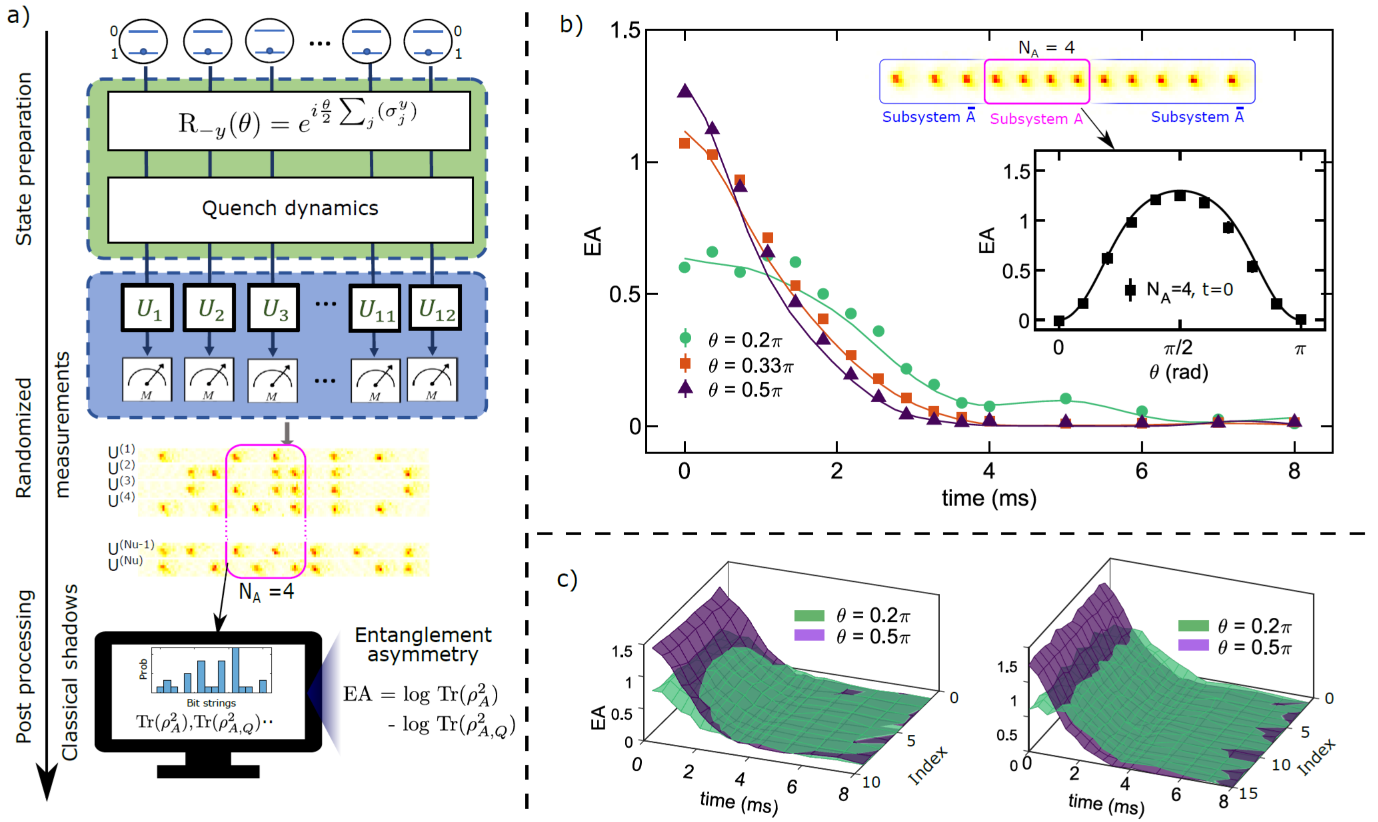
Entangle asymmetry in XXZ spin chain

[Carollo, Lasanta and Lesanovsky, PRL 127, 060401 (2021)]

[Ares, Murciano and Calabrese, Nature Communications 14, 2036 (2023)]

Experimental observation

- The first experimental report on QMPE exists this year (arXiv:2401.04270).
- This is observed in a trapped quantum simulator.



Our motivation

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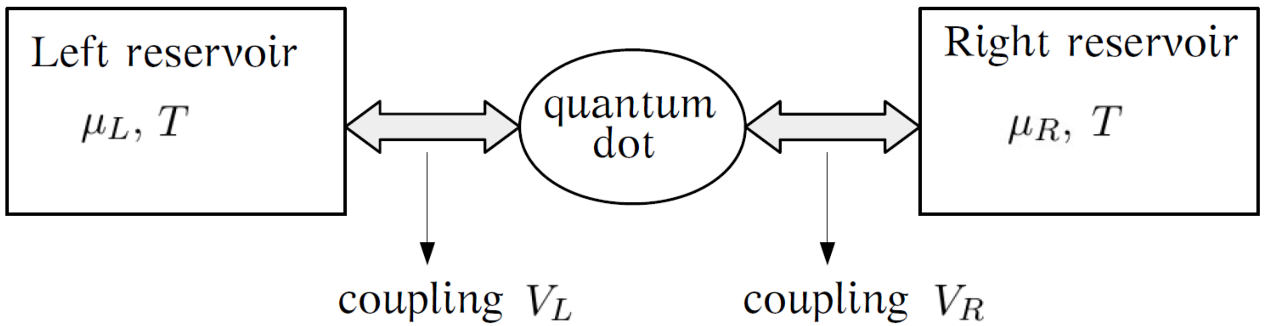
- To clarify the mechanism of quantum Mpemba effect
- To illustrate thermal Mpemba effect (in temperature) for the quantum Mpemba effect
- To explore the role of not-slow modes

Quench dynamics of Anderson model

A single quantum dot connected to two reservoirs



Total Hamiltonian:

$$\hat{H}^{tot} = \hat{H}^{s} + \hat{H}^{r} + \hat{H}^{int}$$

System Hamiltonian

Reservoir Hamiltonian

System-reservoirs interaction Hamiltonian

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$$\hat{H}^s = \sum_\sigma \epsilon_0 \hat{d}_\sigma^\dagger \hat{d}_\sigma + U\hat{n}_\uparrow \hat{n}_\downarrow$$

$$\hat{H}^r = \sum_{\gamma,k,\sigma} \epsilon_k \hat{a}_{\gamma,k,\sigma}^\dagger \hat{a}_{\gamma,k,\sigma}$$

$$\hat{H}^{int} = \sum_{\gamma,k,\sigma} V_\gamma \hat{d}_\sigma^\dagger \hat{a}_{\gamma,k,\sigma} + \text{h.c.}$$

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ϵ_0: energy of electron in quantum dot

ϵ_k: energy of electron corresponding to wave number k in reservoirs

U: electron-electron interaction in quantum dot

V_L, V_R: coupling strength between quantum dot and reservoirs

$\hat{d}^\dagger, \hat{d}$: creation and annihilation operators in quantum dot

$\hat{a}^\dagger, \hat{a}$: creation and annihilation operators in reservoirs

$\hat{n}$: number operator $(= \hat{d}^\dagger \hat{d})$

γ: reservoir indices L, R σ: up-spin $(\uparrow)$ or down-spin $(\downarrow)$

Quantum Master equation:

The time evolution of the density matrix (column vector) is given by

$$\boxed{\frac{d}{dt}|\hat{\rho}(t)\rangle = \hat{K}|\hat{\rho}(t)\rangle}$$

with the following Lindbladian (or, rate matrix)

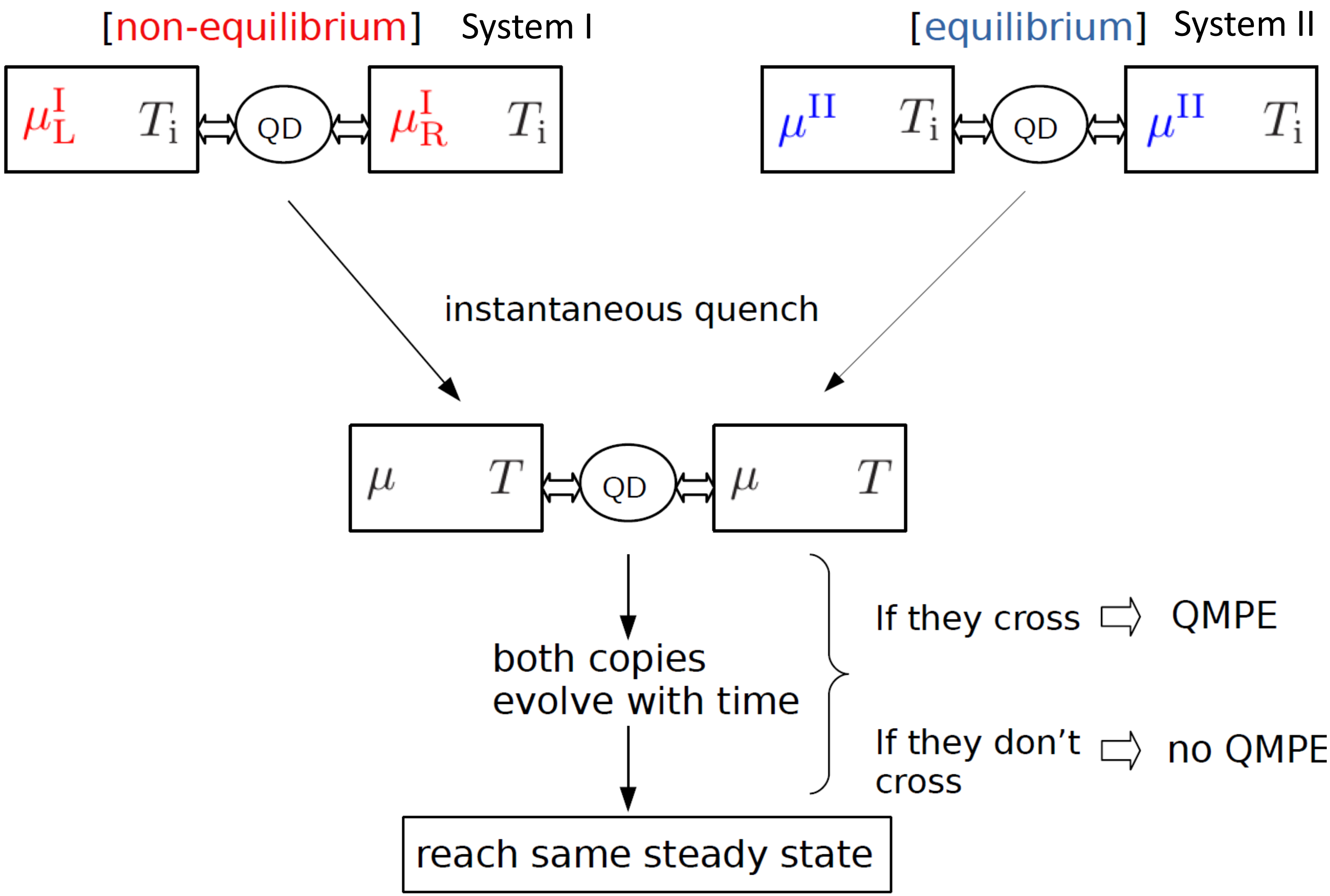
$$\hat{K} = \begin{pmatrix} -2f_-^{(1)} & f_+^{(1)} & f_+^{(1)} & 0 \\ f_-^{(1)} & -f_-^{(0)} - f_+^{(1)} & 0 & f_+^{(0)} \\ f_-^{(1)} & 0 & -f_-^{(0)} - f_+^{(1)} & f_+^{(0)} \\ 0 & f_-^{(0)} & f_-^{(0)} & -2f_+^{(0)} \end{pmatrix}$$

where

$$f_+^{(j)} := f_L^{(j)}(\mu_L, U) + f_R^{(j)}(\mu_R, U) \qquad \text{and} \qquad f_-^{(j)} = 2 - f_+^{(j)}$$

with the Fermi-Dirac distribution: $\boxed{f_\gamma^{(j)}(\mu_\gamma, U) = \frac{1}{1+e^{(\epsilon_0 + jU - \mu_\gamma)/T}}}$

Protocol


[non-equilibrium] System I
[equilibrium] System II
μ_L^I T_i QD μ_R^I T_i
μ^{II} T_i QD μ^{II} T_i
instantaneous quench
μ T QD μ T
both copies
evolve with time
If they cross ⇨ QMPE
If they don't cross ⇨ no QMPE
reach same steady state

QMPE in density matrix

- a_2 is zero $\Longrightarrow$ No contribution from slowest relaxation mode

- To show QMPE in density matrix elements:

$$\Delta\rho_\alpha(\tau) := \rho^{\mathrm{I}}_\alpha(\tau) - \rho^{\mathrm{II}}_\alpha(\tau), \qquad \alpha = 1,2,3,4 \quad (\equiv \uparrow\downarrow, \uparrow, \downarrow, \text{vacant})$$

$$= e^{\lambda_3\tau}\hat{R}_{\alpha,4}\Delta a_4\left[S_\alpha + e^{-(\lambda_3-\lambda_4)\tau}\right]$$

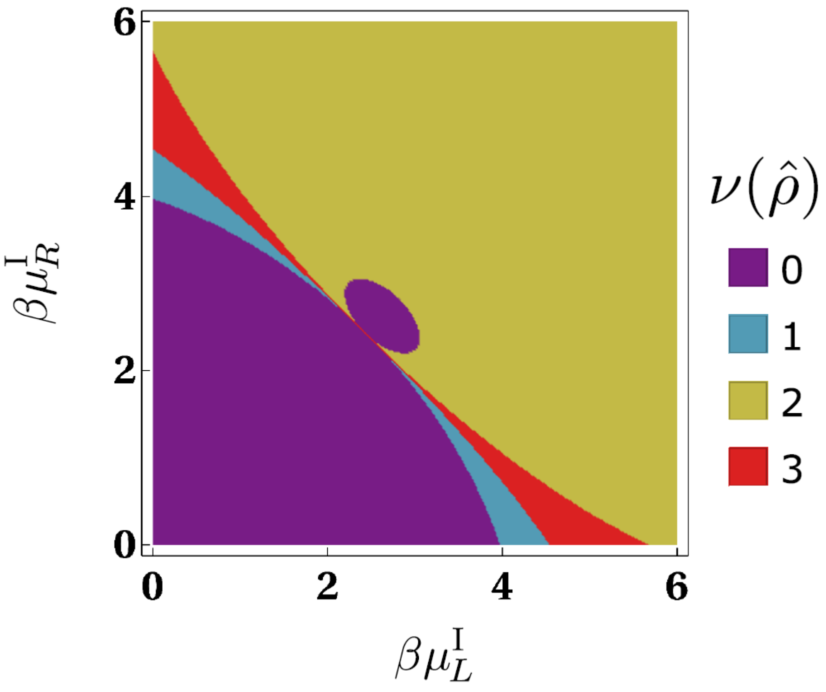
Necessary criterion for QMPE: $S_\alpha < 0 \quad \& \quad |S_\alpha| < 1$

$$\boxed{S_\alpha := (\hat{R}_{\alpha,3}\Delta a_3)/(\hat{R}_{\alpha,4}\Delta a_4)}$$

combined role of the faster relaxation modes on QMPE

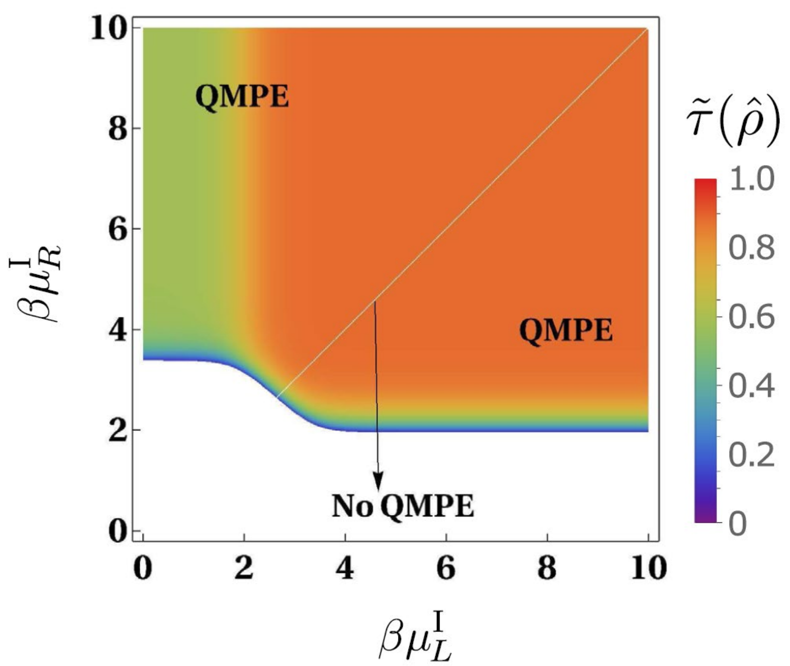
Phase diagrams

- $\nu(\hat{\rho})$: Number of elements showing QMPE



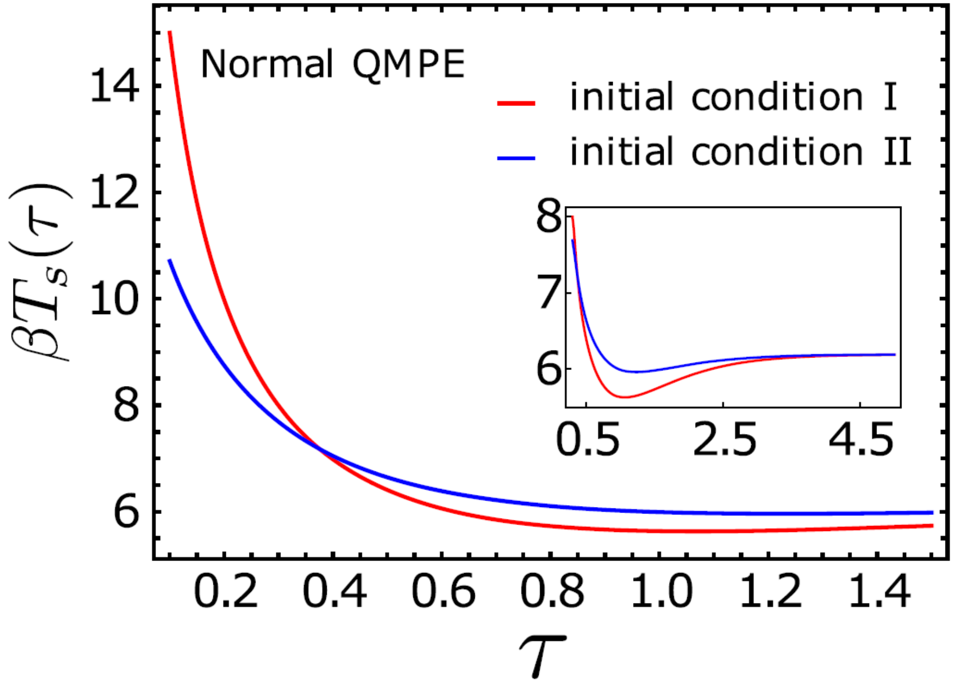
$$\tilde{\tau}(\hat{\rho}) = \max[\tau_1, \tau_2, \tau_3, \tau_4] \quad \text{if } 0 < \tau_\alpha < \infty,$$

$$\tilde{\tau}(\hat{\rho}) \to \infty \quad \text{if no finite } \tau_\alpha \text{ exists} \quad \forall\, \alpha,$$



Thermal Mpemba effect

$$T_s(\tau) := \frac{\partial E_s(\tau)}{\partial S_{\mathrm{vN}}(\tau)} = \frac{\partial E_s(\tau)}{\partial \tau} \Big/ \frac{\partial S_{\mathrm{vN}}(\tau)}{\partial \tau} \qquad S_{\mathrm{vN}}(\tau) = -\sum_\alpha \rho_\alpha(\tau) \ln[\rho_\alpha(\tau)]$$



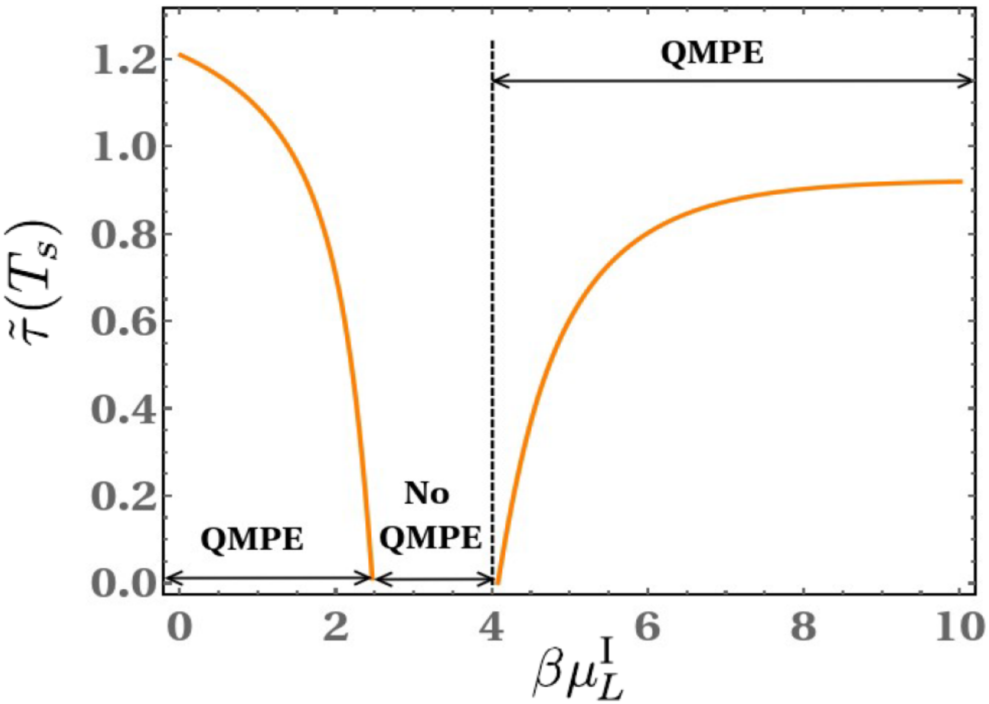
Intersection time

$\Delta T_{\mathrm{s}} := T_{\mathrm{s}}^{\mathrm{I}} - T_{\mathrm{s}}^{\mathrm{II}}$

crossing time: $\tilde{\tau}(T_{\mathrm{s}})$: solution of $\Delta T_{\mathrm{S}} = 0$

$0 < \tilde{\tau}(T_{\mathrm{s}}) < \infty$: thermal QMPE

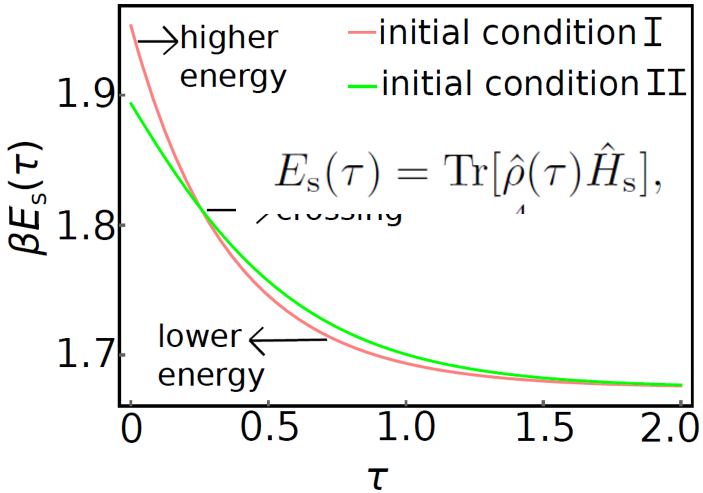
$\tilde{\tau}(T_{\mathrm{s}}) \to \infty$: no QMPE

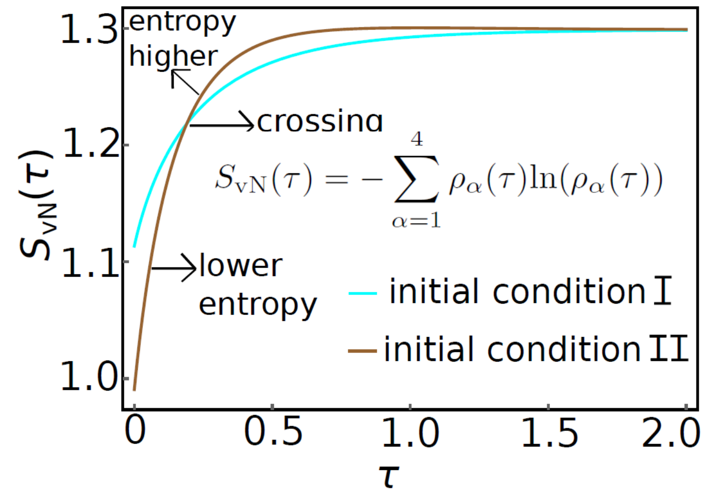


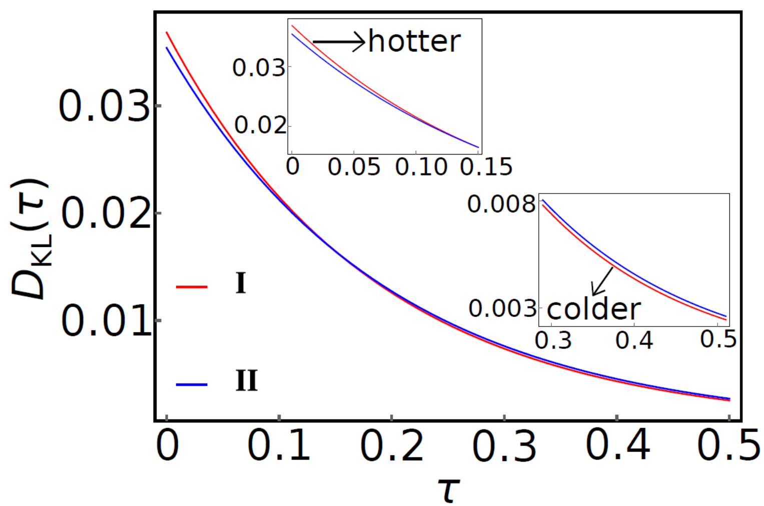
thermal QMPE is rather generic than occasional

$\beta\epsilon_0 = 2.0, \beta U = 1.25, \beta\mu_{\mathrm{R}}^{\mathrm{I}} = 1.0, \beta\mu^{\mathrm{II}} = 2.43, \beta T_{\mathrm{i}} = 1.15, \beta\mu = 2.0.$

Mpemba effect in the other observables







$$D_{\rm KL}(\tau) = \sum_{\alpha=1}^{4} \rho_\alpha(\tau) \ln\left(\frac{\rho_\alpha(\tau)}{\rho_{{\rm ss},\alpha}}\right)$$

Summary of quantum Mpemba effect

- We have demonstrated the existence of Mpemba-like phenomena after a sudden change of system.
- Such effects can be observed in the density matrix elements, von Neumann , energy and temperature.
- Mpemba effect may be useful to speed-up to get a desired state.

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Contents

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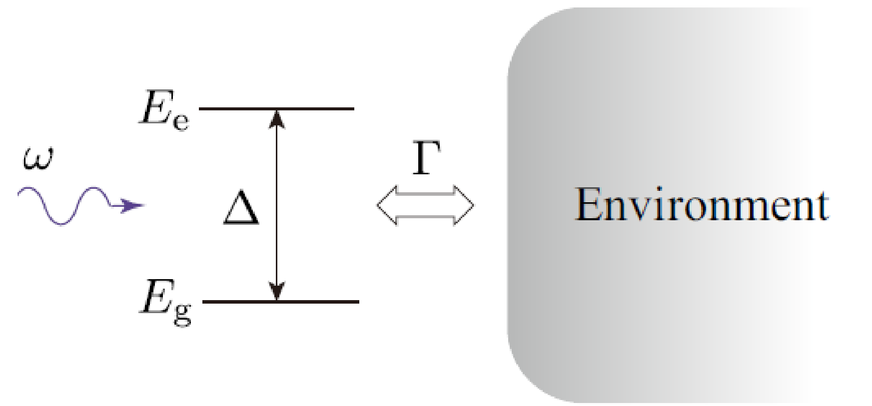
Why exceptional points?

- The previous model is quasi-classical because off-diagonal elements of the density matrix do not play any roles.
- We need to know the effect of entanglements.
- The model of open quantum systems may have exceptional points.
- The minimum model to satisfy the above requirement is Hatano's model.

Model

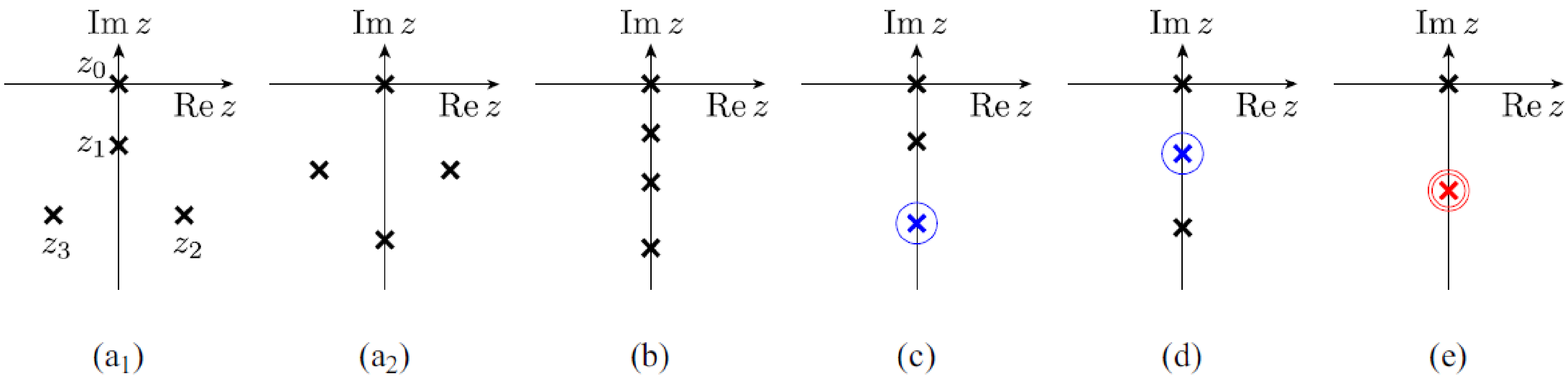
- We consider the Lindblad equation for a two-level open quantum system.
- N. Hatano, Mol. Phys. **117**, 2121 (2019).

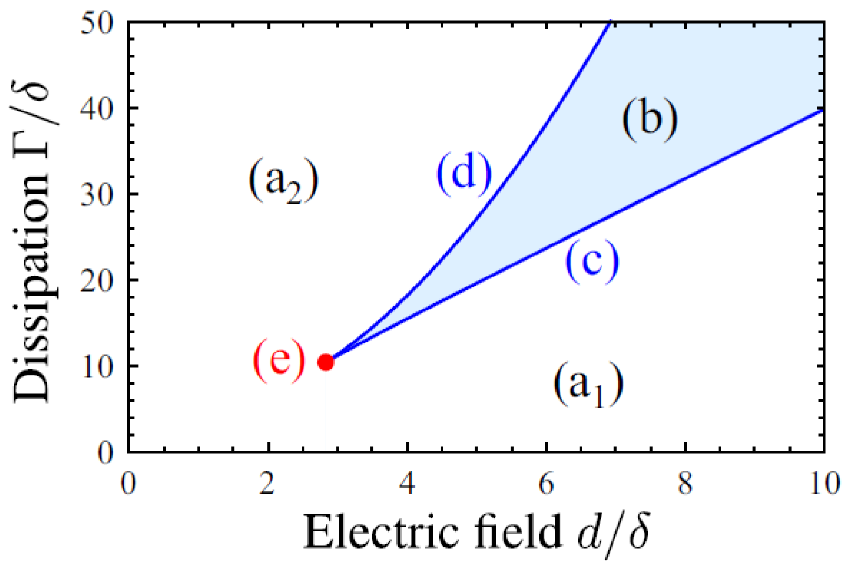
$$
\begin{aligned}
i\dot{\rho}_{\mathrm{eg}} &= \delta\rho_{\mathrm{eg}} - \frac{d}{2}(\rho_{\mathrm{ee}} - \rho_{\mathrm{gg}}) - \frac{i}{2}\Gamma\rho_{\mathrm{eg}}, \\
i\dot{\rho}_{\mathrm{ge}} &= -\delta\rho_{\mathrm{ge}} + \frac{d}{2}(\rho_{\mathrm{ee}} - \rho_{\mathrm{gg}}) - \frac{i}{2}\Gamma\rho_{\mathrm{ge}}, \\
i\dot{\rho}_{\mathrm{ee}} &= -\frac{d}{2}(\rho_{\mathrm{eg}} - \rho_{\mathrm{ge}}) - i\Gamma\rho_{\mathrm{ee}}, \\
i\dot{\rho}_{\mathrm{gg}} &= \frac{d}{2}(\rho_{\mathrm{eg}} - \rho_{\mathrm{ge}}) + i\Gamma\rho_{\mathrm{ee}}.
\end{aligned}
$$



d: parameter related to electric field

Eigenvalues & phase diagrams





Setup

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- To clarify the role of exceptional points, we consider quenches to the exceptional point.

initial condition I $\xrightarrow{\text{quench}}$ EP

initial condition II $\xrightarrow{\text{quench}}$ EP

$$i\frac{d}{dt}|\widehat{\rho}(t)\rangle = \widehat{\mathcal{L}}|\widehat{\rho}(t)\rangle.$$

<u>At EP</u>

$$|\hat{\rho}(t)\rangle = \begin{pmatrix} \rho_1(t) \\ \rho_2(t) \\ \rho_3(t) \\ \rho_4(t) \end{pmatrix} \equiv \begin{pmatrix} \rho_{\mathrm{eg}}(t) \\ \rho_{\mathrm{ge}}(t) \\ \rho_{\mathrm{ee}}(t) \\ \rho_{\mathrm{gg}}(t) \end{pmatrix} = \begin{pmatrix} \rho_{\mathrm{eg}}(t) \\ \rho^*_{\mathrm{eg}}(t) \\ 1-\rho_{\mathrm{gg}}(t) \\ \rho_{\mathrm{gg}}(t) \end{pmatrix} = \begin{pmatrix} \rho_{\mathrm{re}}(t) + \mathrm{i}\,\rho_{\mathrm{im}}(t) \\ \rho_{\mathrm{re}}(t) - \mathrm{i}\,\rho_{\mathrm{im}}(t) \\ 1-\rho_{\mathrm{gg}}(t) \\ \rho_{\mathrm{gg}}(t) \end{pmatrix}.$$

contribution contribution

Regin (d): 2nd order exceptional point

- We cannot diagonalize if there is an exceptional point such as $\widehat{\mathcal{L}}_{\mathrm{J}} = \widehat{L}\widehat{\mathcal{L}}\widehat{R}$,

$$\widehat{R} = \left(\ |r_1\rangle, \ \ |r_2\rangle, \ \ |r_3\rangle, \ \ |r_4\rangle \ \right), \qquad \widehat{\mathcal{L}}_{\mathrm{J}} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -\mathrm{i}\lambda_2 & 1 & 0 \\ 0 & 0 & -\mathrm{i}\lambda_2 & 0 \\ 0 & 0 & 0 & -\mathrm{i}\lambda_4 \end{pmatrix}.$$

$$\widehat{L} = \left(\ \langle \ell_1|, \ \ \langle \ell_2|, \ \ \langle \ell_3|, \ \ \langle \ell_4| \ \right)^T.$$

- The eigenvalues are given by

$$\lambda_2 = \frac{2\tilde{\Gamma}}{3} + 2\cos\left(\frac{2\pi}{3}\right)\left(\frac{\tilde{\Gamma}}{6}\left(1 - \frac{\tilde{d}^2}{2} + \frac{\tilde{\Gamma}^2}{36}\right)\right)^{1/3}$$

$$\lambda_4 = \frac{2\tilde{\Gamma}}{3} + 2\left(\frac{\tilde{\Gamma}}{6}\left(1 - \frac{\tilde{d}^2}{2} + \frac{\tilde{\Gamma}^2}{36}\right)\right)^{1/3}.$$

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Evolution of density matrix

- The density matrix is given by

$$\rho_j(t) = \sum_{k=1}^{4} e^{-\lambda_k t} r_{k,j} a_k - \mathrm{i} t e^{-\lambda_2 t} r_{2,j} a_3,$$

$$a_k = \sum_{n=1}^{4} \ell_{k,n} \rho_n(0),$$

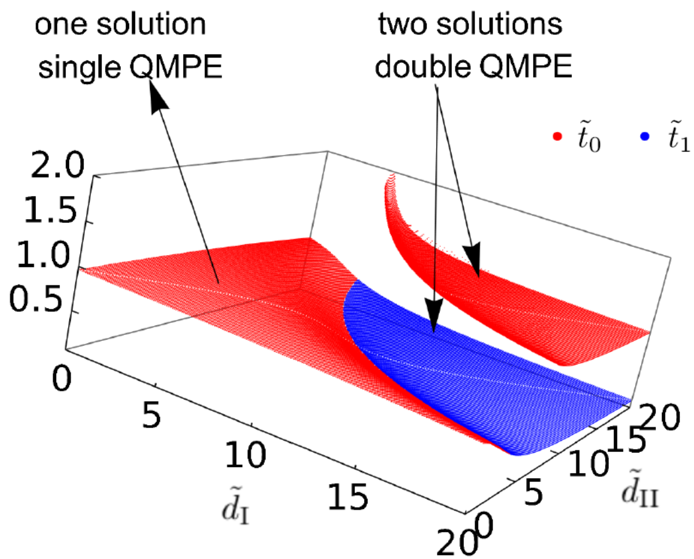
- The difference of density element in two copies

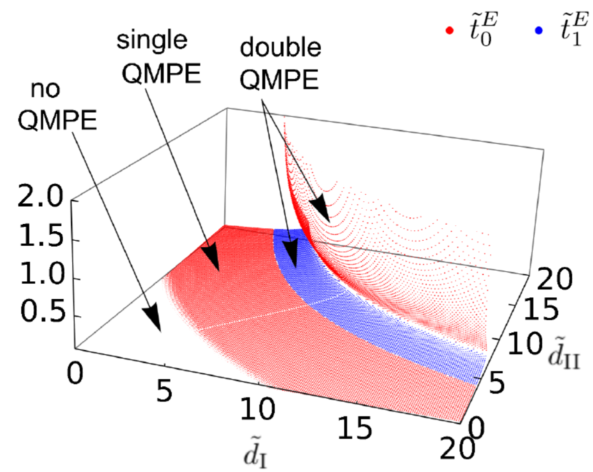
$$\Delta\rho_{\mathrm{gg}}(t) = -e^{-\lambda_2 t}\left[\alpha_1 e^{-(\lambda_4-\lambda_2)t} + t\,\alpha_2 + \alpha_3\right],$$

$$\alpha_1 = a_4^{\mathrm{I}} - a_4^{\mathrm{II}},\ \alpha_2 = -\mathrm{i}(a_3^{\mathrm{I}} - a_3^{\mathrm{II}}),\ \alpha_3 = a_2^{\mathrm{I}} - a_2^{\mathrm{II}}.$$

Time of intersections

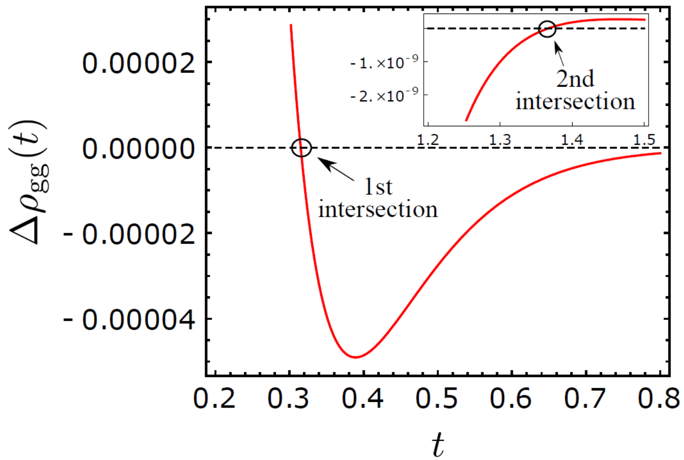
- We obtain the exact time for the intersection:



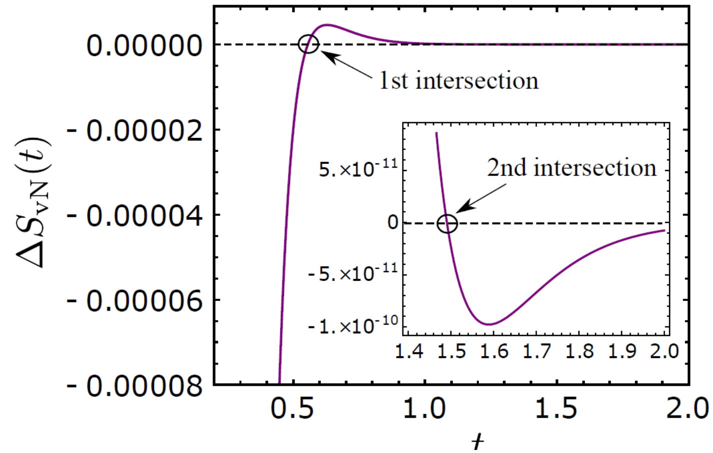


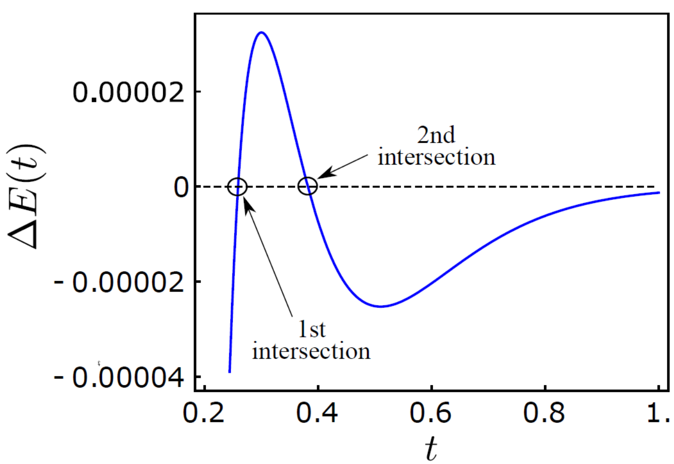
Intersection time for the energy

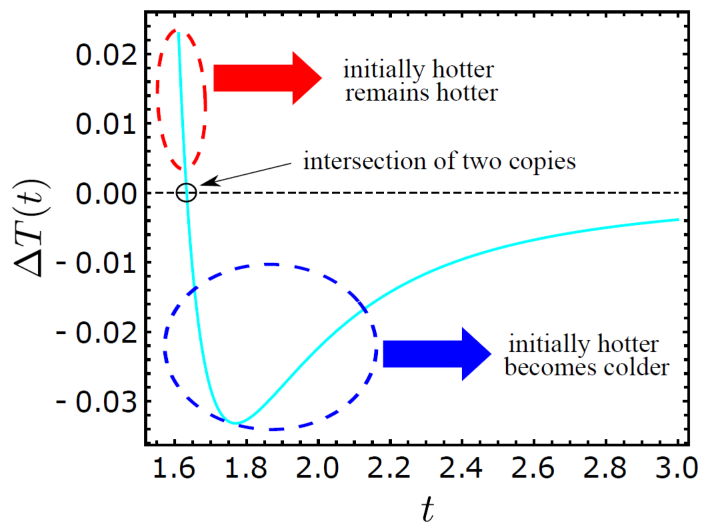
QMPE for various variables



(a)

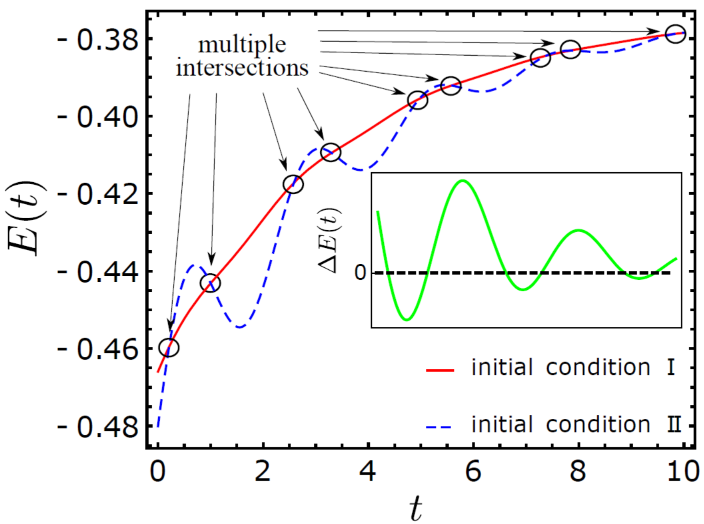


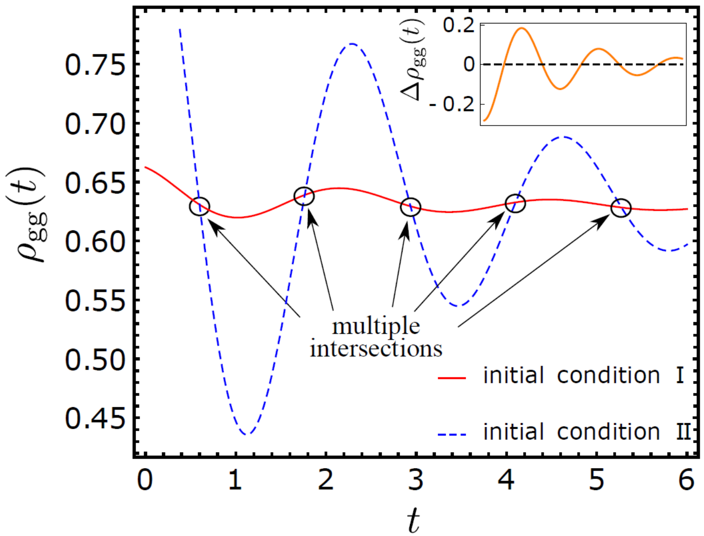


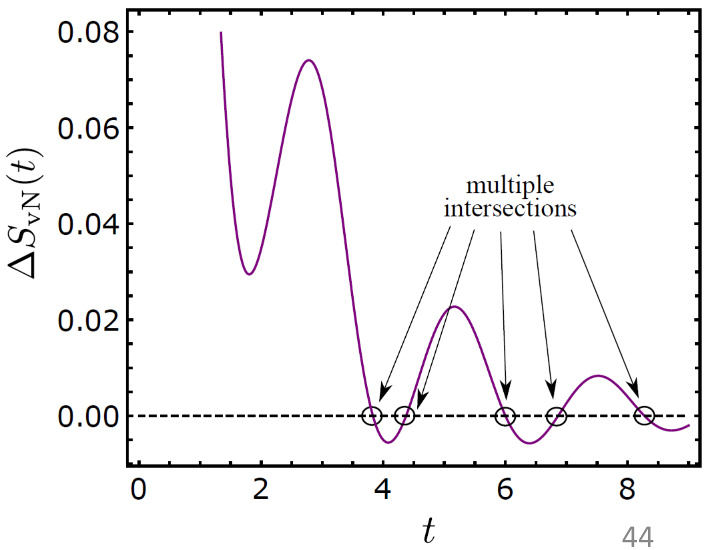


Multiple Mpemba effect in (a_1)

- The region (a_1) has complex eigenvalues.=>Oscillations

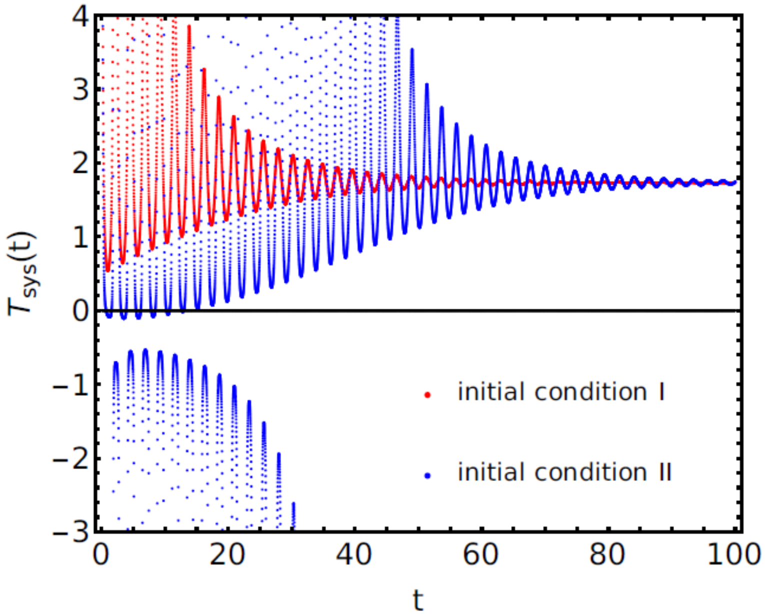


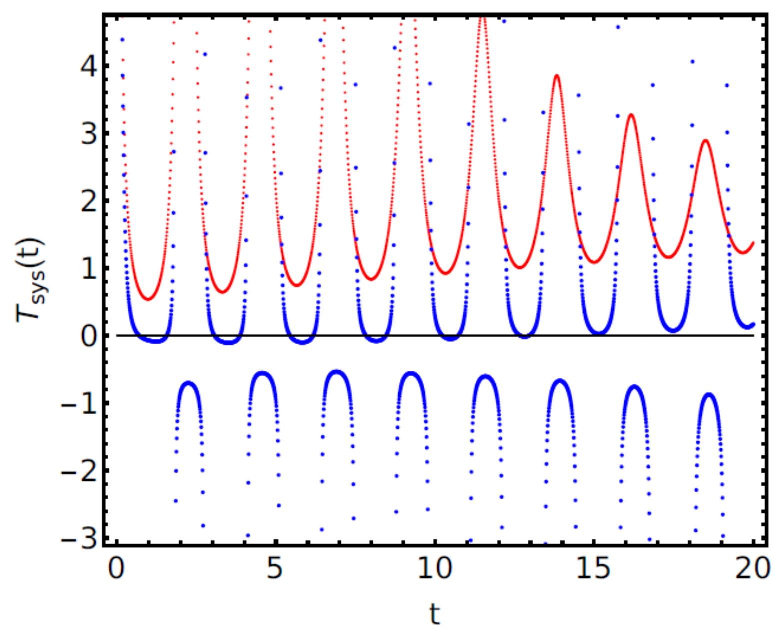




Multiple Thermal Mpemba effect

- If the system has complex eigenvalues, the behavior can be oscillate.
- Then, multiple Mpemba effect in region (a_1) can be observed.





Brief summary of QMPE in Hatano's model

- If we are interested in the exceptional points, we understand that Mpemba effect is generated by the algebraic part of the exceptional point.
- If we are interested in the region with complex eigenvalues, there are multiple interesections.
- => Multiple QMPE

Contents

- Introduction: What is the Mpemba effect?
- Quantum Mpemba effect in Anderson model (PRL 131, 032901 (2023))
- Quantum Mpemba effect in Hatano's model
- Discussion
- Concluding remarks

Discussion

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- It is not difficult to generate MPE by the control of initial condition.
  - Nonequilibrium initial conditions have lower symmetries than that in equilibrium (Ares et al. 2023).
  - We can eliminate the slowest eigenmode by the unitary transformation of the initial condition (Carollo et al, 2021).
- What is the best protocol to get the fastest relaxation?
  - Connection to the speed-limit problem?

Future directions

- The analyzed model to emphasize the initial condition is oversimplified one.
  - We should combine potential landscape effect.
  - If we stress the role of potential, we may discuss quantum tunneling Mpemba effect.
  - Of course, it is possible to discuss quantum thermal Mpemba effect in a double well potential.
- We need to analyze quantum Mpemba effect in many-body systems.
  - Integrable or non-integrable systems

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Summary

- We demonstrate the occurrence of quantum Mpemba effect (QMPE) in Anderson model and Hatano's model.
  - Thermal QMPE can be observed easily.
  - The slow modes are not always important.
  - Difference of the relaxation rate between equilibrium and nonequilibrium initial conditions is important.
- QMPE is generic.
- If there exist exceptional points, the observation of QMPE is easier than that in the absence of EP.
- Multiple QMPE can be observed easily.