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Quantum Mpemba effect: an anomalous thermal relaxation process in quantum matter

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with Amit Kumar Chatterjee (Ramakrishna Mission Vidyamandira)

& Satoshi Takada (Tokyo Univ. Agri. & Tech.),

Seminar at National Central University, Taiwan, February 22nd, 2024.

Refs: PRL**131**, 032901 (2021)=Editors' Suggestion and arXiv:2311.01347.

Self-introduction

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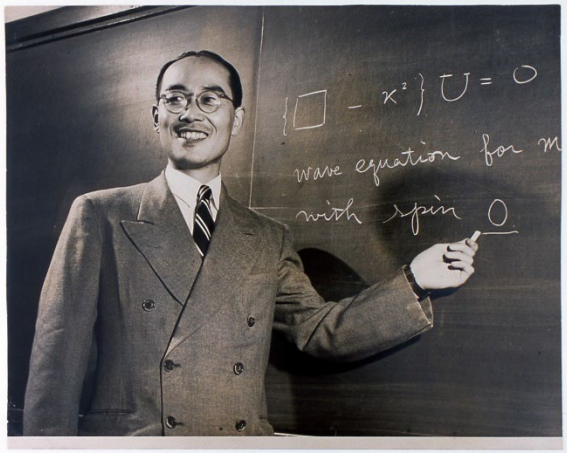
- Thank you for giving me an opportunity to present a seminar in front of you.
- This is the second visit to National Central University, Taiwan(the last one was 1998).



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- Introduction: What is the Mpemba effect?
- Quantum Mpemba effect in Anderson model (PRL **131**, 032901 (2023))
- Quantum Mpemba effect with exceptional points (arXiv:2311.01347)
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Various memory effects

REVIEWS OF MODERN PHYSICS, VOLUME 91, JULY–SEPTEMBER 2019

Memory formation in matter

Kaiser effect, Mullins effect, Kovacs effect…

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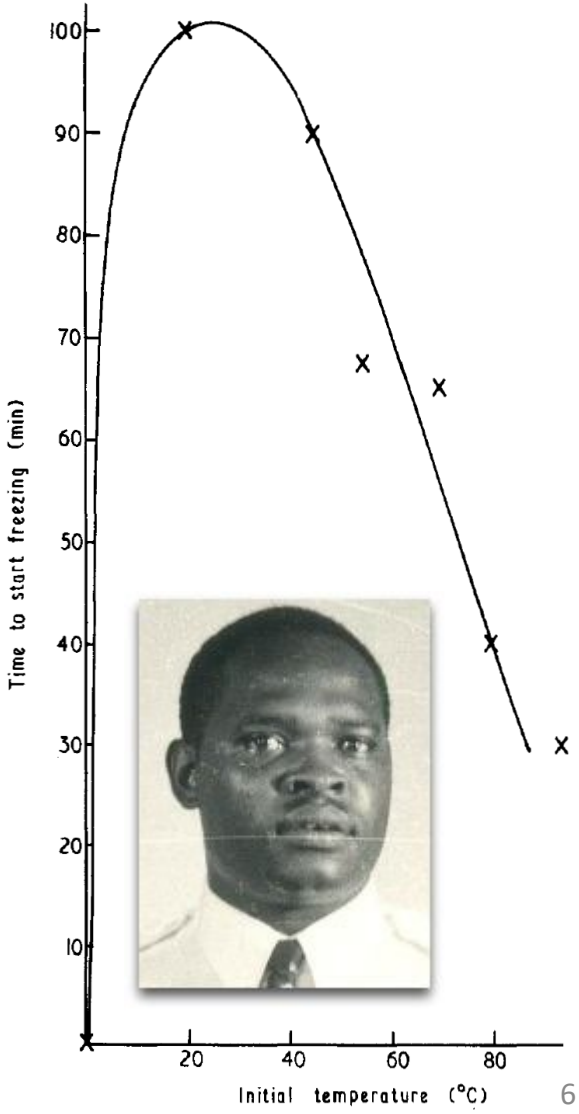
(published 26 July 2019)

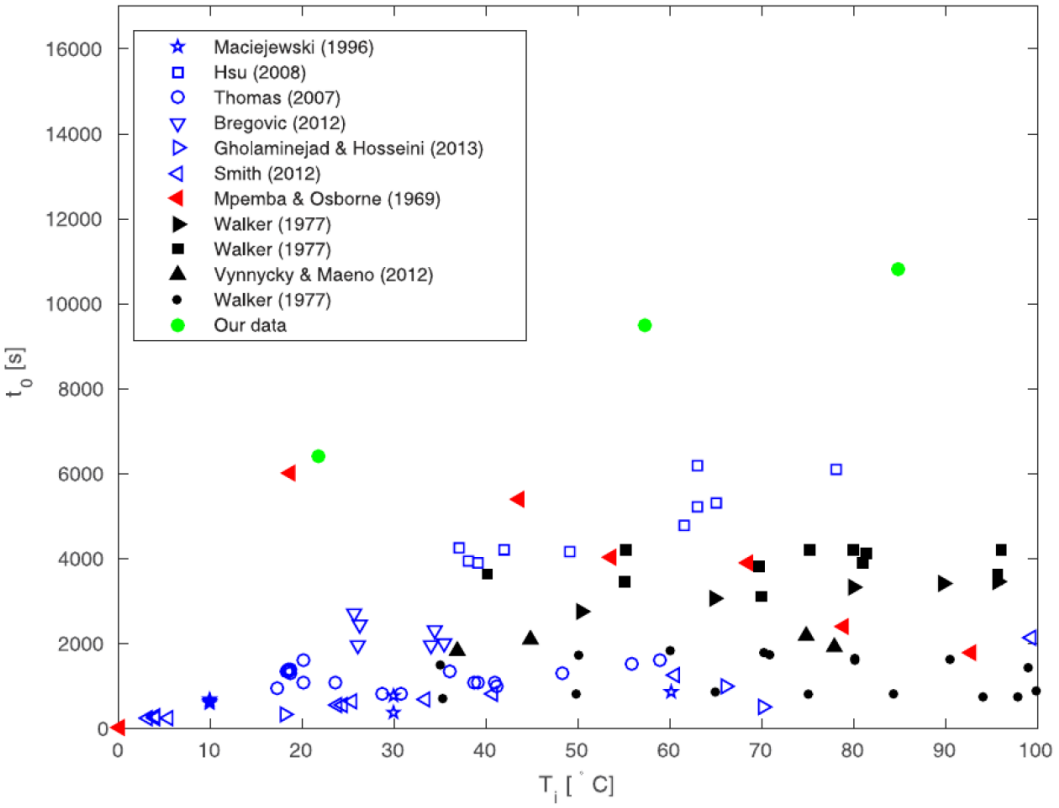
What is the Mpemba effect?

- What is Mpemba effect?
  - Erasto B. Mpemba found that some hot suspensions (*ice cream mix*) can freeze faster than cold (1963).
  - With the help of D. G. Osborne he has published a scientific paper (1969).



Debates

- Poor reproducibility
- The right figure is one counter example of Mpemba effect.
- However, people believe the existence of Mpemba-like phenomena.



Burridge and Linden, Sci. Rep. **6**, 37665 (2016).

Recent trend

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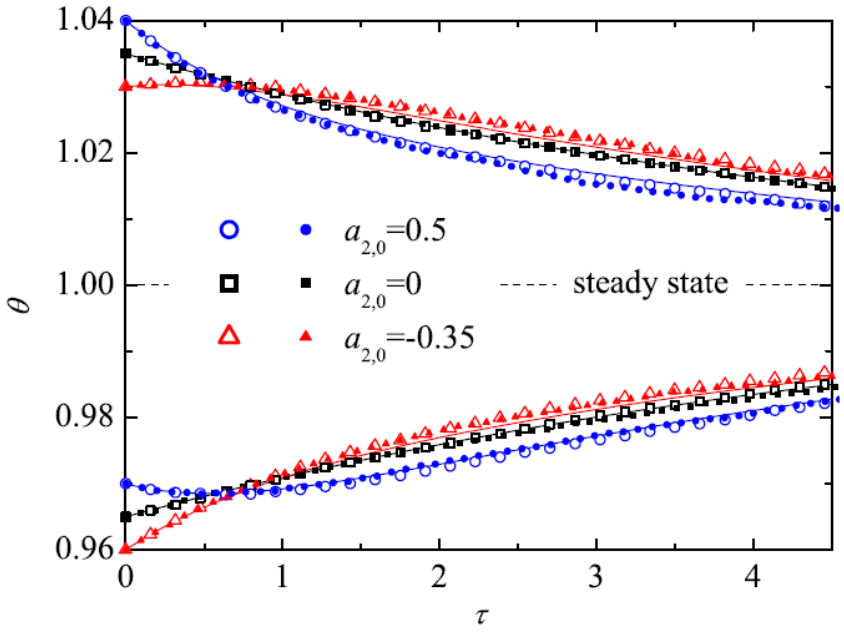
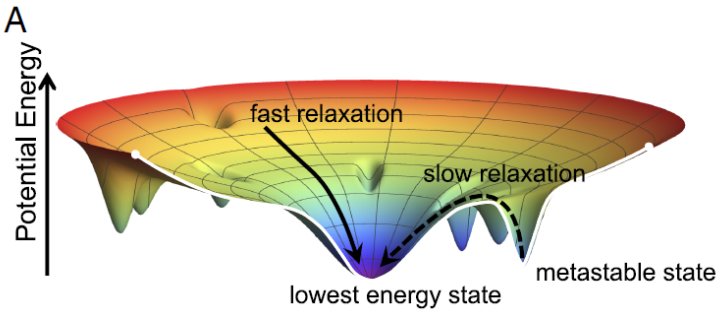
- Now, people are not interested in time to start the freezing, but are interested in the cross point(s) of the relaxation process.
- Namely, if the "temperature" starting from high initial temperature becomes lower temperature than that starting from lower initial temperature, we regard it as Mpemba effect.

Some theoretical studies

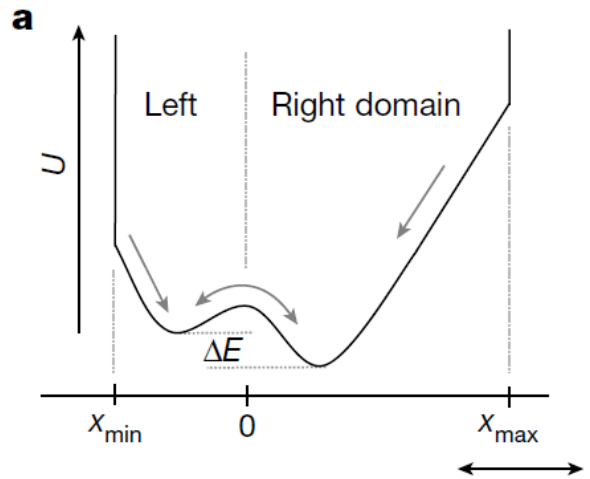
- Lasanta et al. PRL **119**, 148001 (2017) found that a granular gas can have both ME and the inverse ME by controlling *kurtosis*.
- Lu & Raz, PNAS **114**, 5083 (2017) indicated that the slow relaxation can take place by trapping at local minima.
  - But there is no temperature.
- But how can we control?

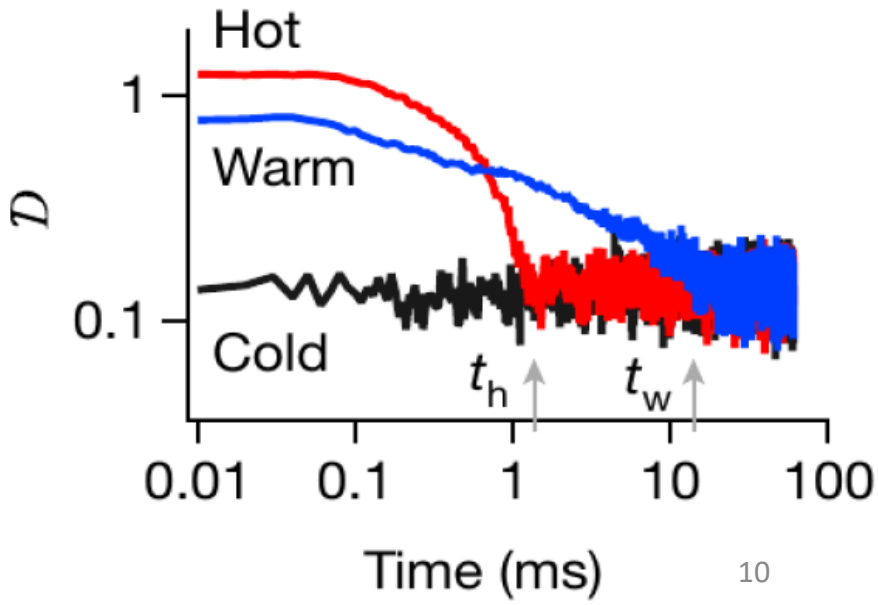




Experimental confirmation

- Kumar & Bechhoeffer, Nature **584**, 64 (2020).
- They have analyzed trapped colloids in a double well potential.
- They observed the distance between the distribution and equilibrium one.





Lu & Raz (PNAS2017)

- They have analyzed the master equation:

$$\frac{\mathrm{d}p_i(t)}{\mathrm{d}t} = \sum_j R_{ij}(T_b)p_j(t) \quad \text{for } i = 1, 2, \cdots, n.$$

- They are interested in the slowest relaxation mode=>approach to the equilibrium state:

$$\vec{p}(t) = \vec{\pi}(T_b) + e^{\lambda_2 t} a_2 \vec{v}_2 + \ldots \qquad \pi_i(T_b) = \frac{e^{-E_i/k_B T_b}}{\sum_i e^{-E_i/k_B T_b}}$$

- The condition for Markovian Mpemba effect:

$$|a_2^c| > |a_2^h|$$

Question to the scenario by Lu-Raz

- Connection with kinetic theory is not clear.
- They are only interested in approaching to the final equilibrium state, but this is not always related to the cross points.
  - Initial relaxation may be important.
- They are only interested in discrete systems.

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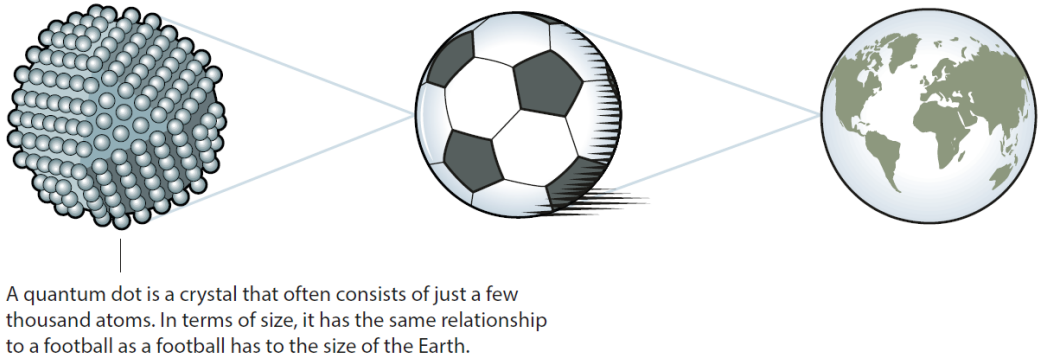
Purpose of this talk

- We analyze quantum Mpemba effect which does not obey the scenario by Lu & Raz.
- In the first part, we analyze the Mpemba effect in the (quasi-classical) Anderson model.
  - a_3 and a_4 are important but a_2 is not.
  - Here a_2 is the coefficient of slowest eigenmode.
- We analyze the Mpemba effect in Hatano's model as an example of fully open quantum systems.

Quantum dot

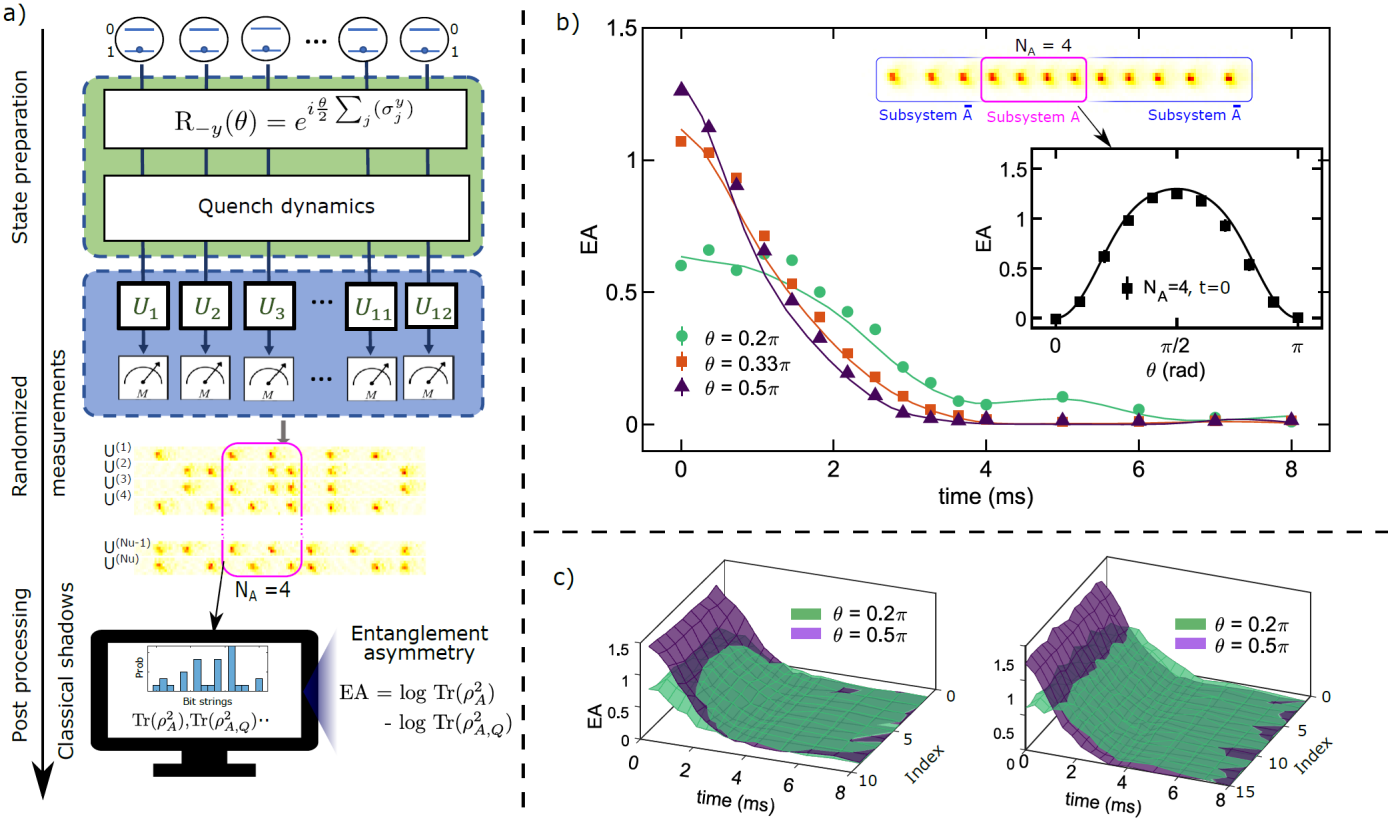
- Nobel prize in chemistry 2023 is awarded for the discovery of quantum dots.
- We use quantum dots to demonstrate the quantum Mpemba effect.



A quantum dot is a crystal that often consists of just a few thousand atoms. In terms of size, it has the same relationship to a football as a football has to the size of the Earth.

Experimental QMPE

- The first experimental report on QMPE exists this year (arXiv:2401.04270).
- This is observed in a trapped quantum simulator.



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Our motivation

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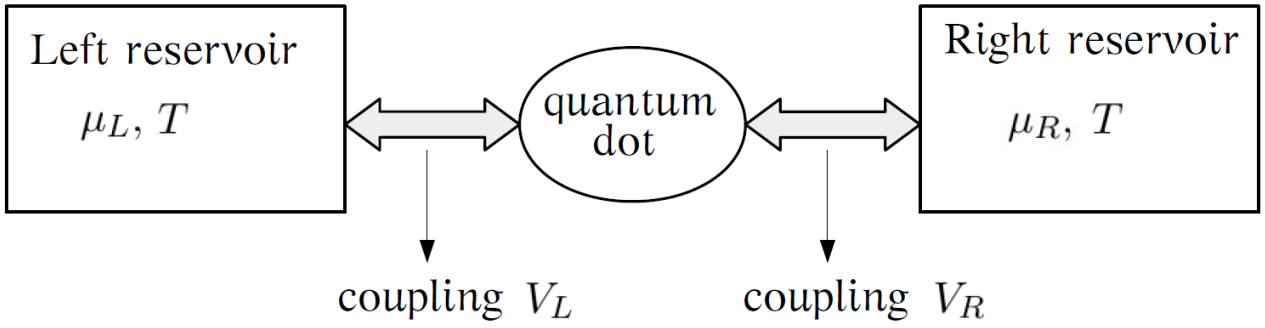
- To clarify the mechanism of quantum Mpemba effect
- To illustrate thermal Mpemba effect (in temperature) for the quantum Mpemba effect
- To explore the role of not-slow modes

Quench dynamics of Anderson model

A single quantum dot connected to two reservoirs



Total Hamiltonian:

$$\hat{H}^{tot} = \hat{H}^{s} + \hat{H}^{r} + \hat{H}^{int}$$

System Hamiltonian

Reservoir Hamiltonian

System-reservoirs interaction Hamiltonian

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$$\hat{H}^s = \sum_{\sigma} \epsilon_0 \hat{d}^\dagger_\sigma \hat{d}_\sigma + U \hat{n}_\uparrow \hat{n}_\downarrow$$

$$\hat{H}^r = \sum_{\gamma,k,\sigma} \epsilon_k \hat{a}^\dagger_{\gamma,k,\sigma} \hat{a}_{\gamma,k,\sigma}$$

$$\hat{H}^{int} = \sum_{\gamma,k,\sigma} V_\gamma \hat{d}^\dagger_\sigma \hat{a}_{\gamma,k,\sigma} + \text{h.c.}$$

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ϵ_0: energy of electron in quantum dot

ϵ_k: energy of electron corresponding to wave number k in reservoirs

U: electron-electron interaction in quantum dot

V_L, V_R: coupling strength between quantum dot and reservoirs

$\hat{d}^\dagger, \hat{d}$: creation and annihilation operators in quantum dot

$\hat{a}^\dagger, \hat{a}$: creation and annihilation operators in reservoirs

$\hat{n}$: number operator $(= \hat{d}^\dagger \hat{d})$

γ: reservoir indices L, R σ: up-spin $(\uparrow)$ or down-spin $(\downarrow)$

Quantum Master equation:

The time evolution of the density matrix (column vector) is given by

$$\frac{d}{dt}|\hat{\rho}(t)\rangle = \hat{K}|\hat{\rho}(t)\rangle$$

with the following Lindbladian (or, rate matrix)

$$\hat{K} = \begin{pmatrix} -2f_-^{(1)} & f_+^{(1)} & f_+^{(1)} & 0 \\ f_-^{(1)} & -f_-^{(0)} - f_+^{(1)} & 0 & f_+^{(0)} \\ f_-^{(1)} & 0 & -f_-^{(0)} - f_+^{(1)} & f_+^{(0)} \\ 0 & f_-^{(0)} & f_-^{(0)} & -2f_+^{(0)} \end{pmatrix}$$

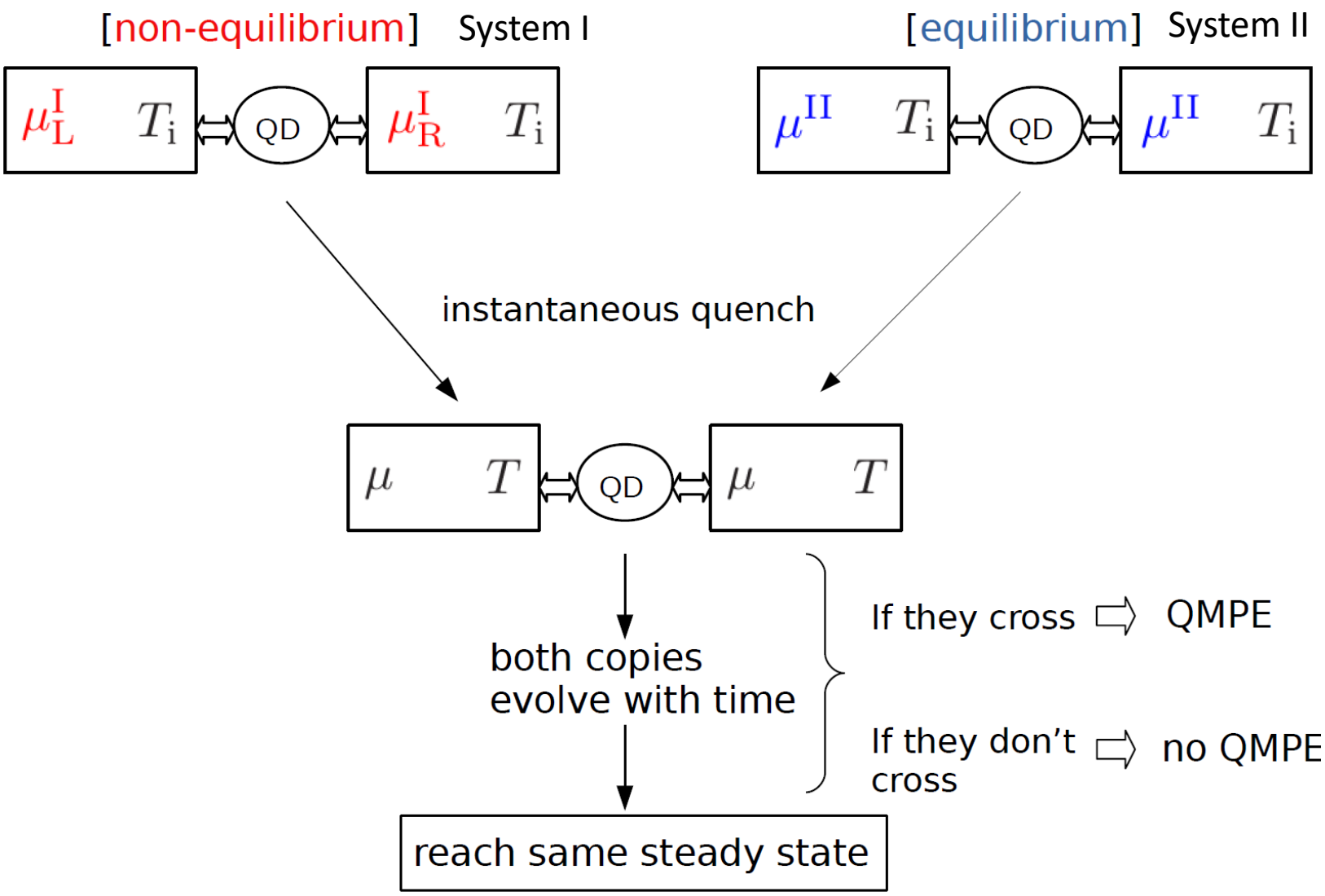
where

$$f_+^{(j)} := f_L^{(j)}(\mu_L, U) + f_R^{(j)}(\mu_R, U) \qquad \text{and} \qquad f_-^{(j)} = 2 - f_+^{(j)}$$

with the Fermi-Dirac distribution:

$$f_\gamma^{(j)}(\mu_\gamma, U) = \frac{1}{1+e^{(\epsilon_0 + jU - \mu_\gamma)/T}}$$

Protocol


[non-equilibrium] System I
[equilibrium] System II
$\mu_{\rm L}^{\rm I}$ $T_{\rm i}$
QD
$\mu_{\rm R}^{\rm I}$ $T_{\rm i}$
$\mu^{\rm II}$ $T_{\rm i}$
QD
$\mu^{\rm II}$ $T_{\rm i}$
instantaneous quench
μ T
QD
μ T
both copies evolve with time
If they cross ⇨ QMPE
If they don't cross ⇨ no QMPE
reach same steady state

QMPE in density matrix

- a_2 is zero $\Longrightarrow$ No contribution from slowest relaxation mode

- To show QMPE in density matrix elements:

$$\Delta\rho_\alpha(\tau) := \rho_\alpha^{\mathrm{I}}(\tau) - \rho_\alpha^{\mathrm{II}}(\tau), \qquad \alpha = 1,2,3,4 \quad (\equiv \uparrow\downarrow, \uparrow, \downarrow, \text{vacant})$$

$$= e^{\lambda_3 \tau} \hat{R}_{\alpha,4} \Delta a_4 \left[S_\alpha + e^{-(\lambda_3 - \lambda_4)\tau} \right]$$

Necessary criterion for QMPE: $S_\alpha < 0 \quad \& \quad |S_\alpha| < 1$

$$S_\alpha := (\hat{R}_{\alpha,3} \Delta a_3)/(\hat{R}_{\alpha,4} \Delta a_4)$$

combined role of the faster relaxation modes on QMPE

Phase diagrams

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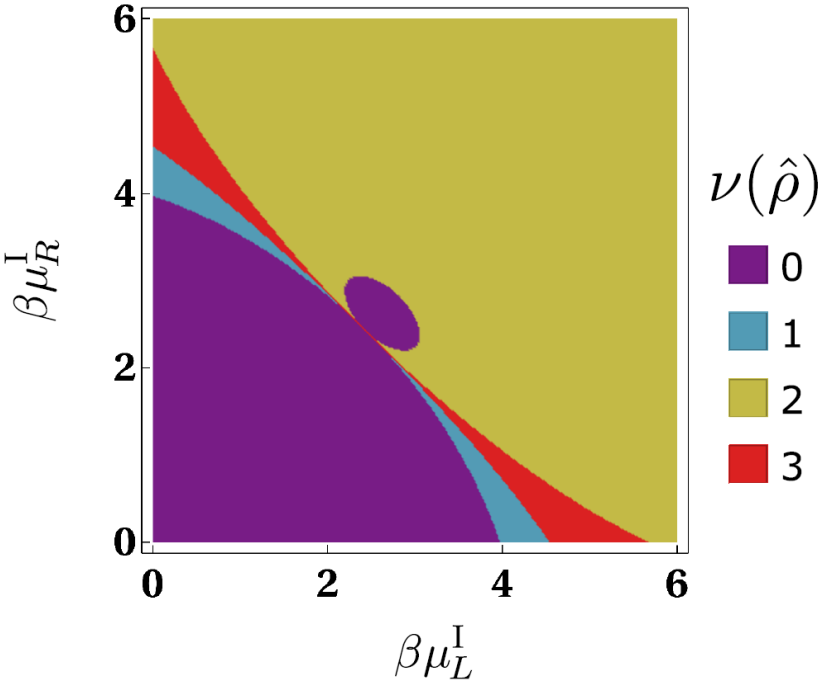
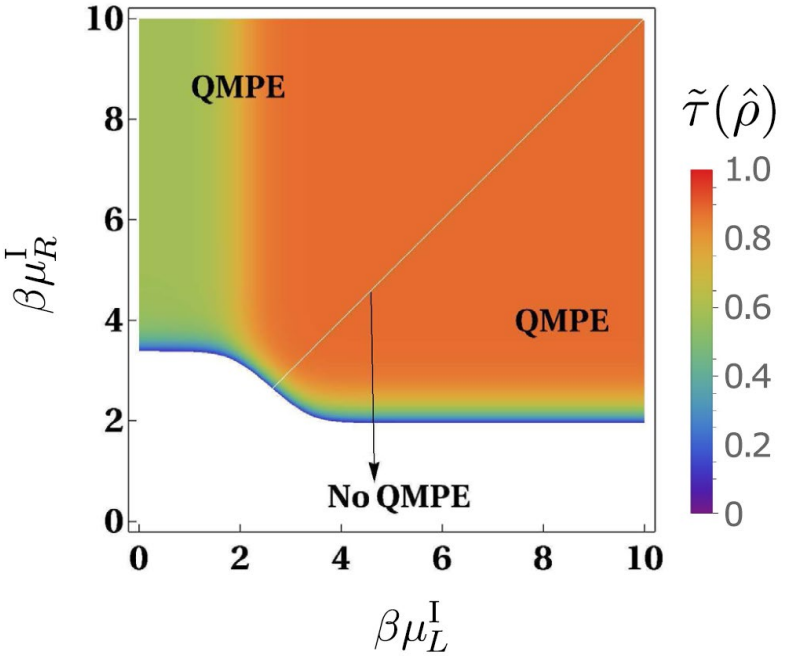
- $\nu(\hat{\rho})$: Number of elements showing QMPE

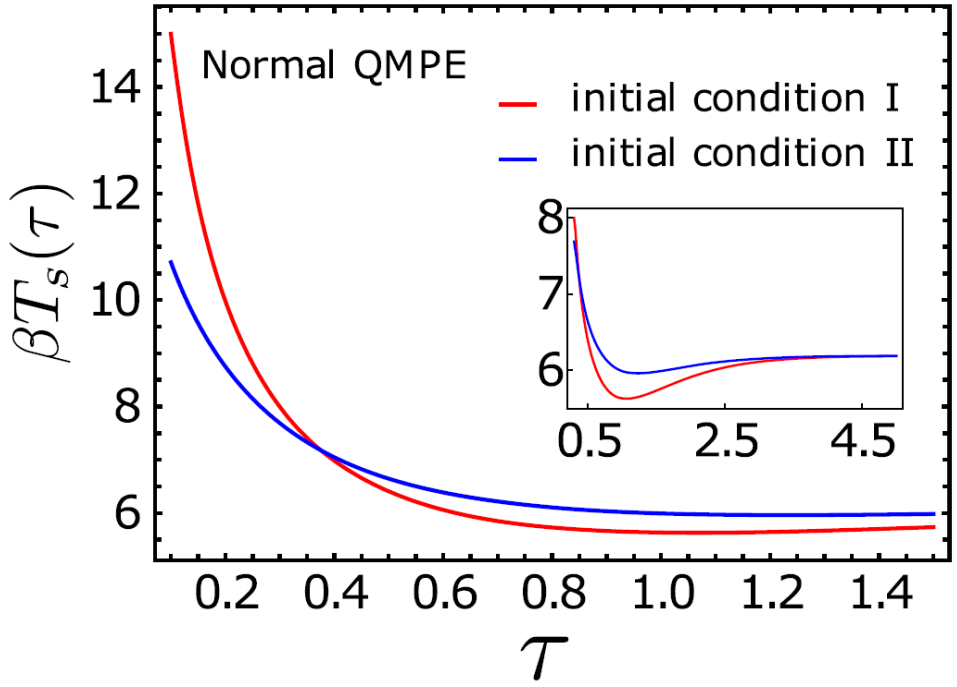


$$\tilde{\tau}(\hat{\rho}) = \max[\tau_1, \tau_2, \tau_3, \tau_4] \quad \text{if } 0 < \tau_\alpha < \infty,$$
$$\tilde{\tau}(\hat{\rho}) \to \infty \quad \text{if no finite } \tau_\alpha \text{ exists} \quad \forall\, \alpha,$$

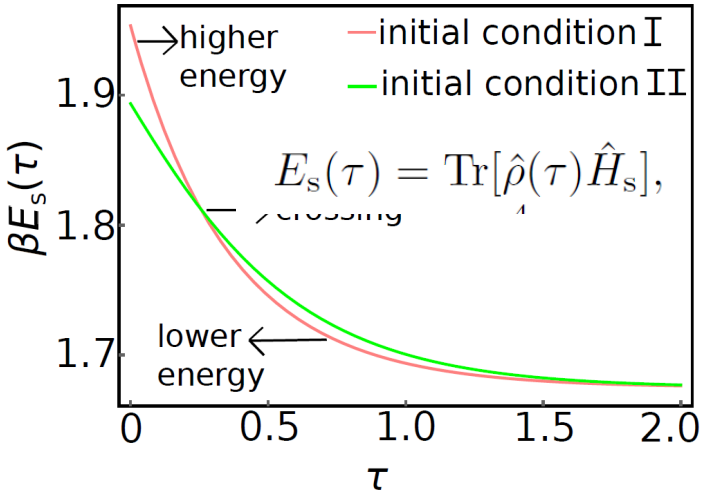


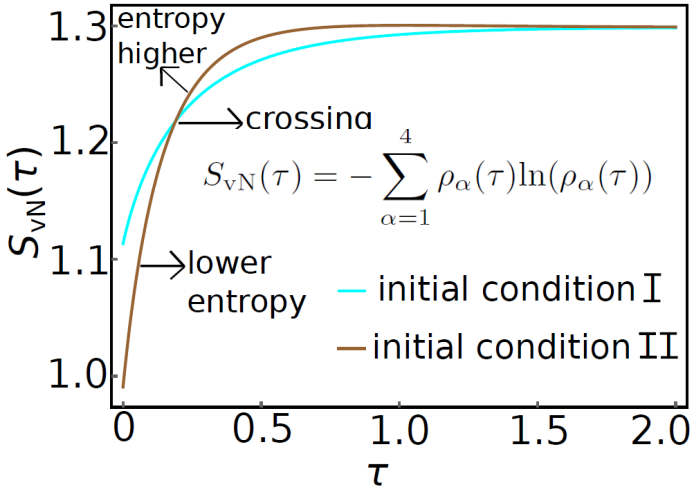
Thermal Mpemba effect

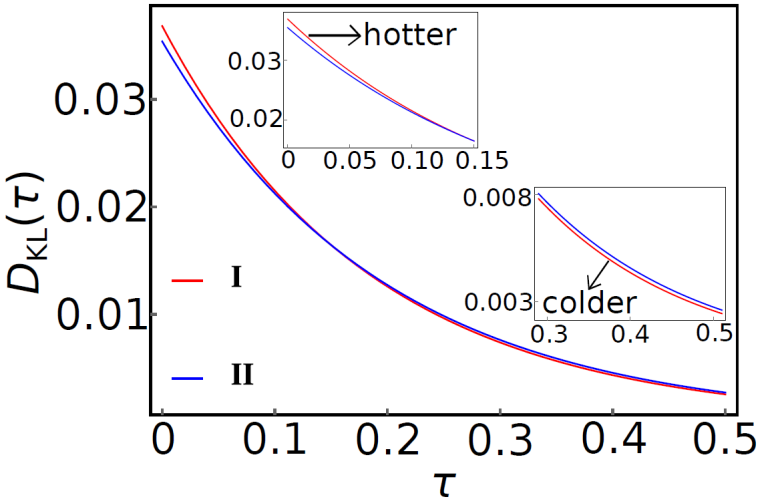
$$T_s(\tau) := \frac{\partial E_s(\tau)}{\partial S_{\mathrm{vN}}(\tau)} = \frac{\partial E_s(\tau)}{\partial \tau} \Big/ \frac{\partial S_{\mathrm{vN}}(\tau)}{\partial \tau} \qquad S_{\mathrm{vN}}(\tau) = -\sum_\alpha \rho_\alpha(\tau) \ln[\rho_\alpha(\tau)]$$



Mpemba effect in the other observables







$$D_{\mathrm{KL}}(\tau) = \sum_{\alpha=1}^{4} \rho_\alpha(\tau) \ln\left(\frac{\rho_\alpha(\tau)}{\rho_{\mathrm{ss},\alpha}}\right)$$

Summary of quantum Mpemba effect

- We have demonstrated the existence of Mpemba-like phenomena after a sudden change of system.
- Such effects can be observed in the density matrix elements, von Neumann , energy and temperature.
- Mpemba effect may be useful to speed-up to get a desired state.

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Why exceptional points?

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- The previous model is quasi-classical because off-diagonal elements of the density matrix do not play any roles.
- We need to know the effect of entanglements.
- The model of open quantum systems may have exceptional points.
- The minimum model to satisfy the above requirement is Hatano's model.

Model

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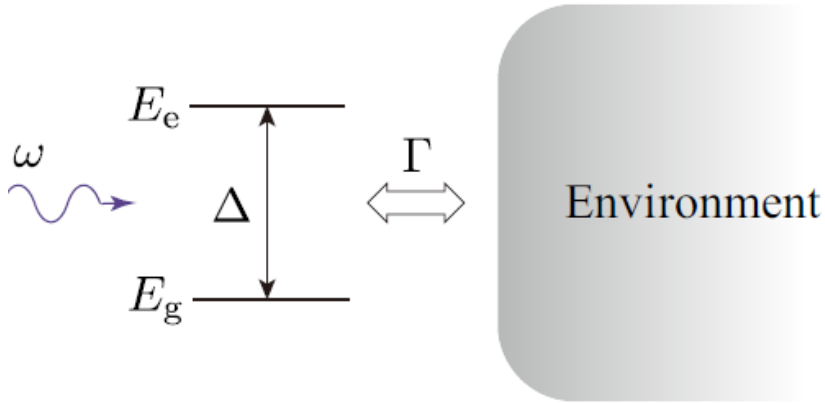
- We consider the Lindblad equation for a two-level open quantum system.
- N. Hatano, Mol. Phys. **117**, 2121 (2019).

$$i\dot{\rho}_{\mathrm{eg}} = \delta\rho_{\mathrm{eg}} - \frac{d}{2}(\rho_{\mathrm{ee}} - \rho_{\mathrm{gg}}) - \frac{i}{2}\Gamma\rho_{\mathrm{eg}},$$

$$i\dot{\rho}_{\mathrm{ge}} = -\delta\rho_{\mathrm{ge}} + \frac{d}{2}(\rho_{\mathrm{ee}} - \rho_{\mathrm{gg}}) - \frac{i}{2}\Gamma\rho_{\mathrm{ge}},$$

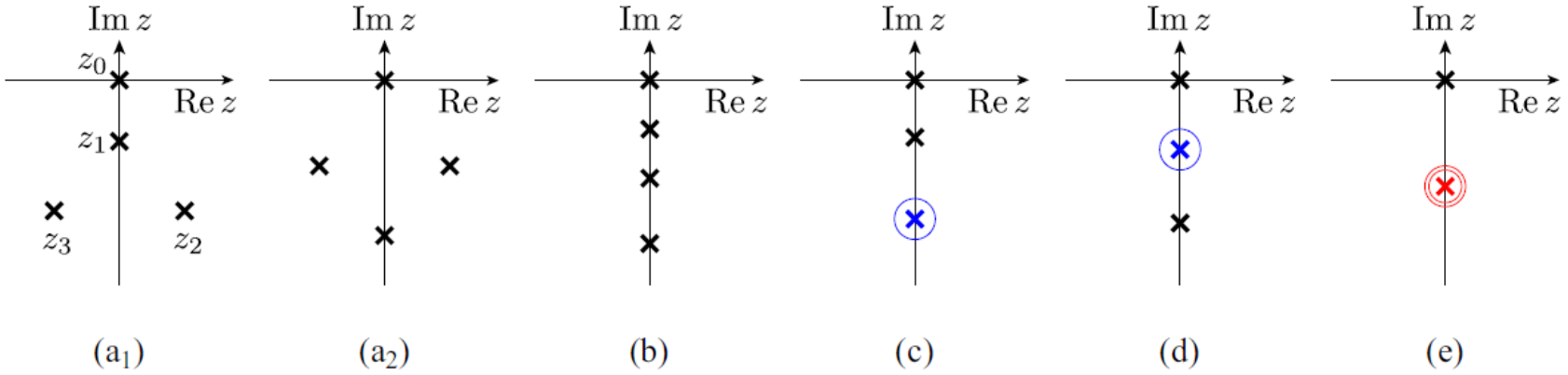
$$i\dot{\rho}_{\mathrm{ee}} = -\frac{d}{2}(\rho_{\mathrm{eg}} - \rho_{\mathrm{ge}}) - i\Gamma\rho_{\mathrm{ee}},$$

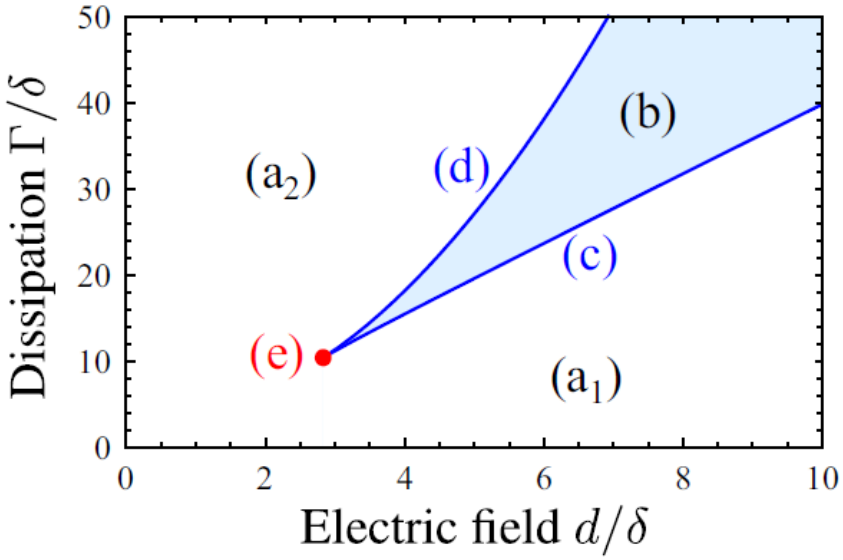
$$i\dot{\rho}_{\mathrm{gg}} = \frac{d}{2}(\rho_{\mathrm{eg}} - \rho_{\mathrm{ge}}) + i\Gamma\rho_{\mathrm{ee}}.$$



d: parameter related to electric field

Eigenvalues & phase diagrams





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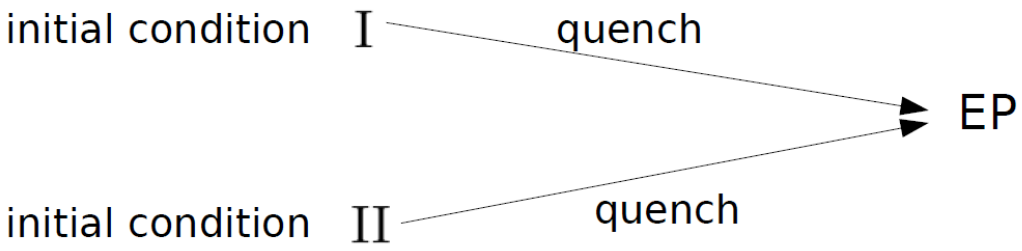
Setup

- To clarify the role of exceptional points, we consider quenches to the exceptional point.



$$i\frac{d}{dt}|\widehat{\rho}(t)\rangle = \widehat{\mathcal{L}}|\widehat{\rho}(t)\rangle.$$

At EP

$$|\hat{\rho}(t)\rangle = \begin{pmatrix} \rho_1(t) \\ \rho_2(t) \\ \rho_3(t) \\ \rho_4(t) \end{pmatrix} \equiv \begin{pmatrix} \rho_{\rm eg}(t) \\ \rho_{\rm ge}(t) \\ \rho_{\rm ee}(t) \\ \rho_{\rm gg}(t) \end{pmatrix} = \begin{pmatrix} \rho_{\rm eg}(t) \\ \rho^*_{\rm eg}(t) \\ 1-\rho_{\rm gg}(t) \\ \rho_{\rm gg}(t) \end{pmatrix} = \begin{pmatrix} \rho_{\rm re}(t)+{\rm i}\,\rho_{\rm im}(t) \\ \rho_{\rm re}(t)-{\rm i}\,\rho_{\rm im}(t) \\ 1-\rho_{\rm gg}(t) \\ \rho_{\rm gg}(t) \end{pmatrix}.$$

contribution contribution

Evolution of density matrix

- The density matrix is given by

$$\rho_j(t) = \sum_{k=1}^{4} e^{-\lambda_k t} r_{k,j} a_k + \mathrm{i}t e^{-\lambda_2 t} r_{2,j} a_3,$$

$$a_k = \sum_{n=1}^{4} \ell_{k,n} \rho_n(0),$$

- The difference of density element in two copies

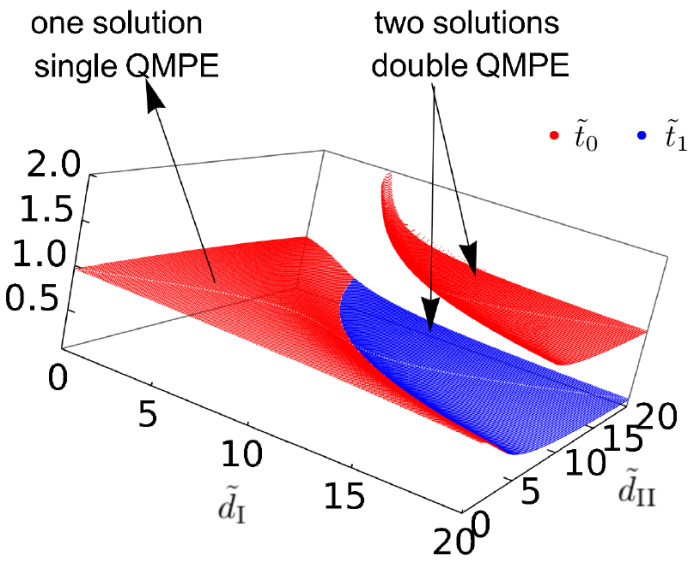
$$\Delta\rho_{\mathrm{gg}}(t) = -e^{-\lambda_2 t}\left[\alpha_1 e^{-(\lambda_4-\lambda_2)t} + t\,\alpha_2 + \alpha_3\right],$$

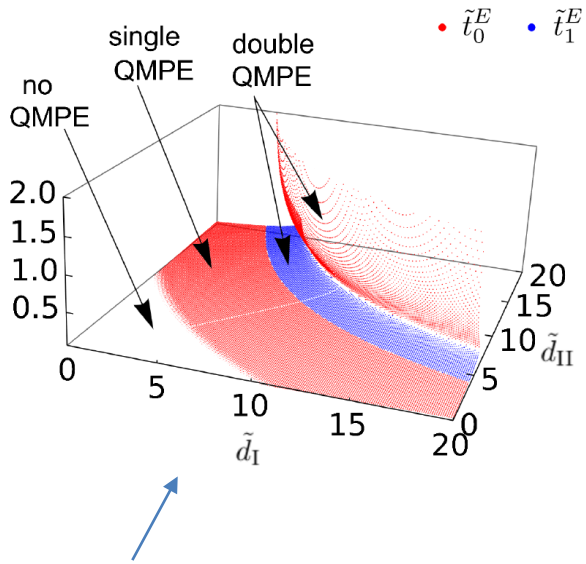
$$\alpha_1 = a_4^{\mathrm{I}} - a_4^{\mathrm{II}},\ \alpha_2 = -\mathrm{i}(a_3^{\mathrm{I}} - a_3^{\mathrm{II}}),\ \alpha_3 = a_2^{\mathrm{I}} - a_2^{\mathrm{II}}.$$

It is not difficult to get the condition of $\Delta\rho_{gg}$=0.

Time of intersections

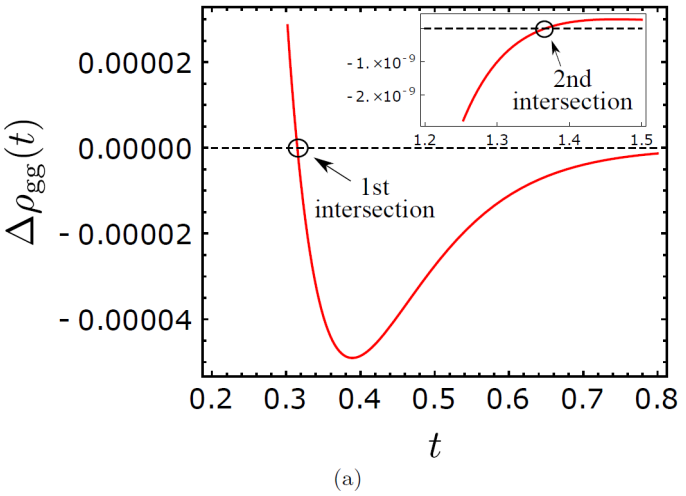
- We obtain the exact time for the intersection:



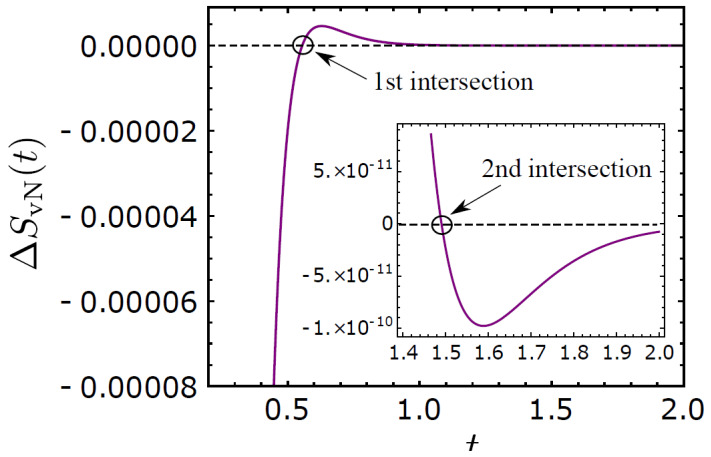


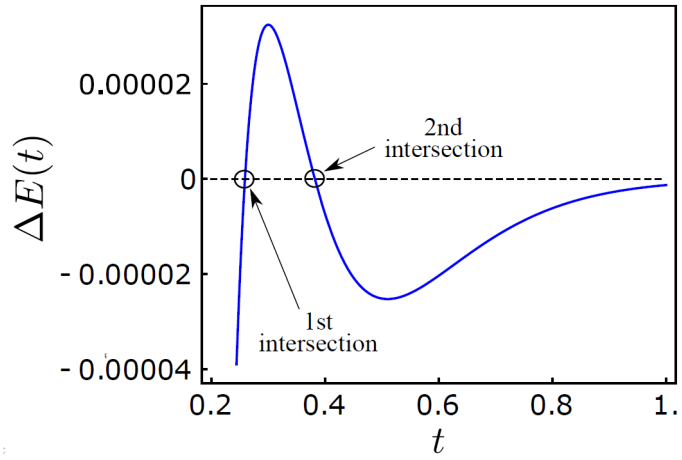
Intersection time for the energy

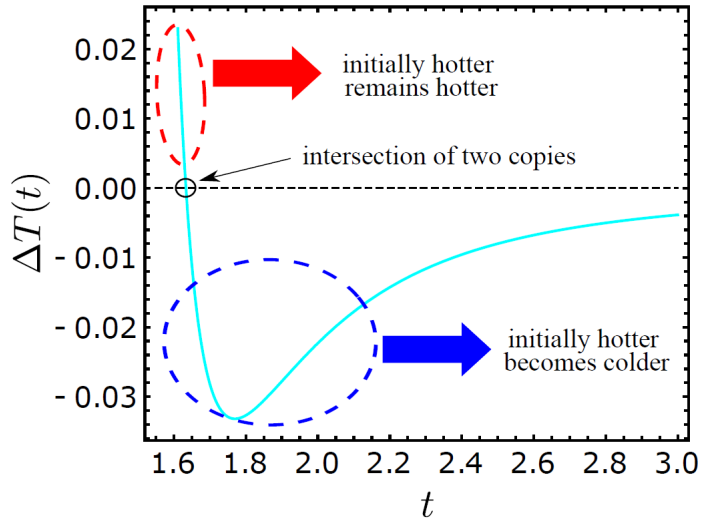
QMPE for various variables



(a)

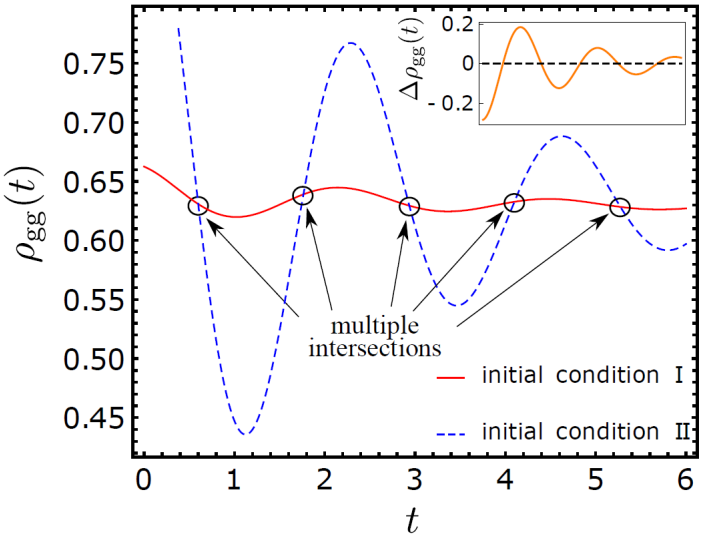


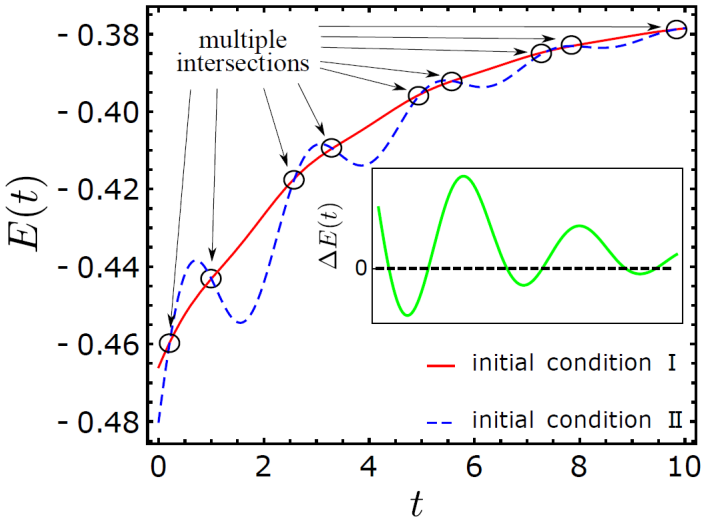


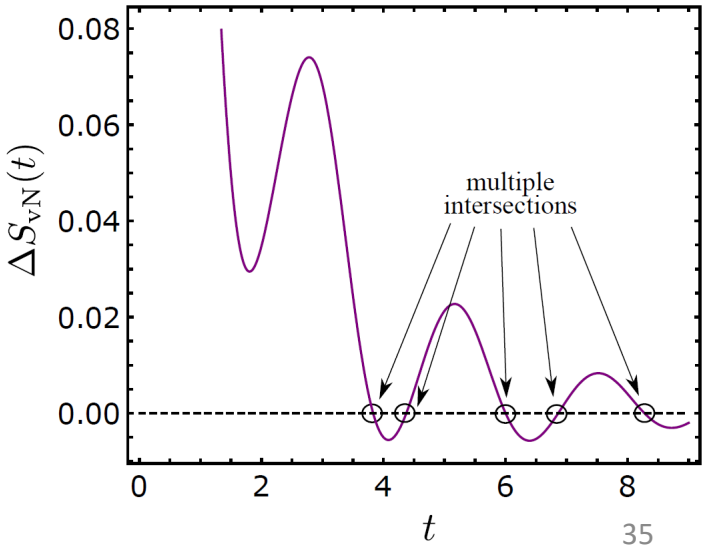


Multiple Mpemba effect in (a_1)

- The region (a_1) has complex eigenvalues.=>Oscillations

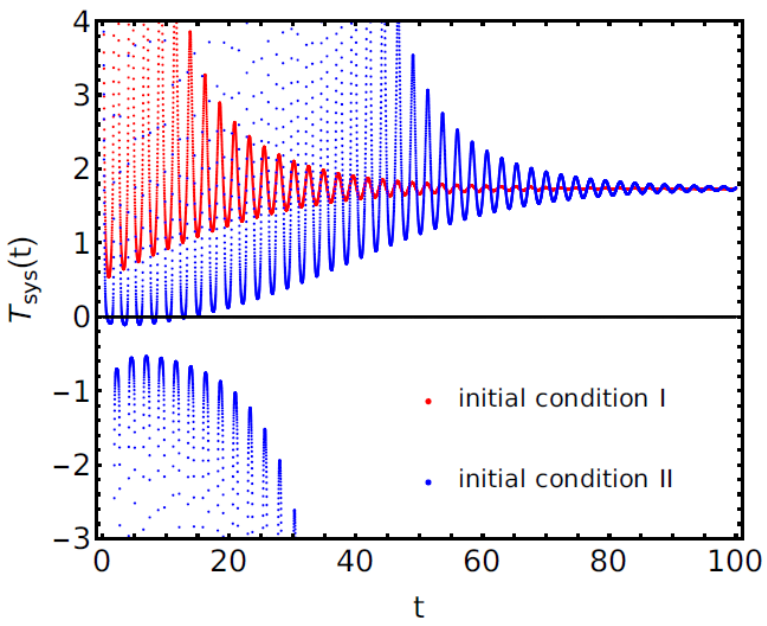


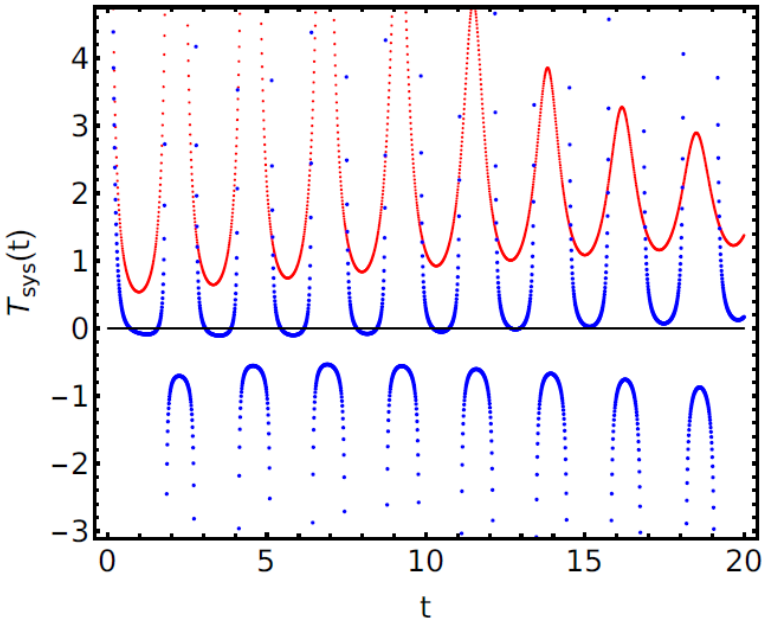




Multiple Thermal Mpemba effect

- If the system has complex eigenvalues, the behavior can be oscillate.
- Then, multiple Mpemba effect in region (a_1) can be observed.





Brief summary of QMPE in Hatano's model

- If we are interested in the exceptional points, we understand that Mpemba effect is generated by the algebraic part of the exceptional point.
- If we are interested in the region with complex eigenvalues, there are multiple interesections.
- => Multiple QMPE

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- **Discussion**
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Discussion

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- It is not difficult to generate MPE by the control of initial condition.
  - Nonequilibrium initial conditions have lower symmetries than that in equilibrium (Ares et al. 2023).
  - We can eliminate the slowest eigenmode by the unitary transformation of the initial condition (Carollo et al, 2021).
- What is the best protocol to get the fastest relaxation?
  - Connection to the speed-limit problem?

Future directions

- The analyzed model to emphasize the initial condition is oversimplified one.
  - We should combine potential landscape effect.
  - If we stress the role of potential, we may discuss quantum tunneling Mpemba effect.
  - Of course, it is possible to discuss quantum thermal Mpemba effect in a double well potential.
- We need to analyze quantum Mpemba effect in many-body system.
  - Integrable or non-integrable systems

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Summary

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- We demonstrate the occurrence of quantum Mpemba effect (QMPE) in Anderson model and Hatano's model.
  - Thermal QMPE can be observed easily.
  - The slow modes are not always important.
  - Difference of the relaxation rate between equilibrium and nonequilibrium initial conditions is important.
- QMPE is generic.
- If there exist exceptional points, the observation of QMPE is easier than that in the absence of EP.
- Multiple QMPE can be observed easily.