YITP
YUKAWA INSTITUTE FOR THEORETICAL PHYSICS

多重量子Mpemba効果の理論：例外点の効果と振動

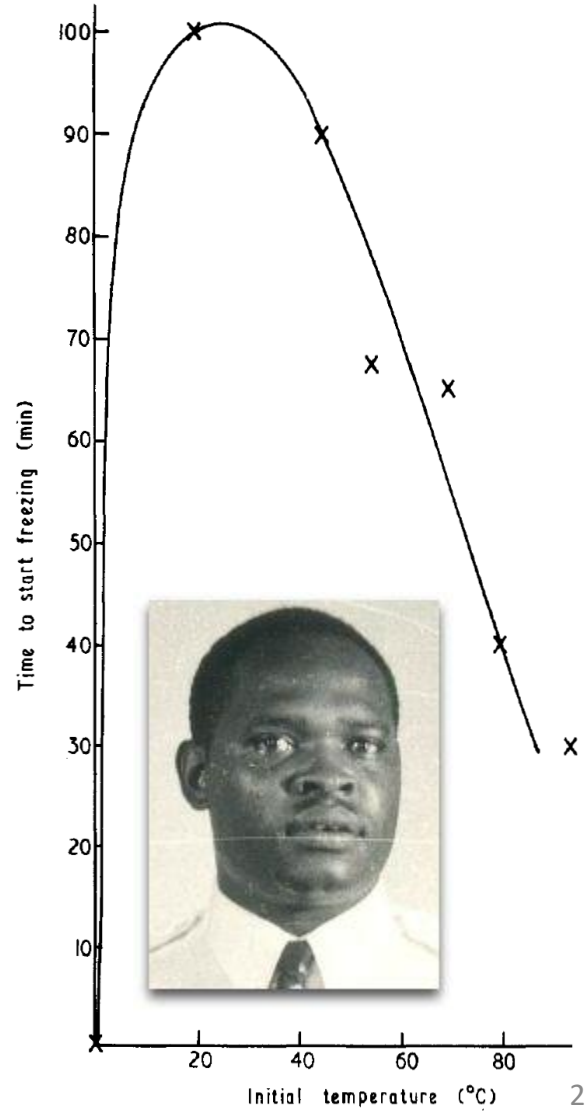
Amit Kumar Chatterjee (Ramakrishna Mission Vidyamandira),高田智史(東京農工大工),
早川尚男(京大基研)

日本物理学会2024年春季大会 18aL3-8

Refs: PRL**131**, 032901 (2021)=Editors' Suggestion and arXiv:2311.01347.

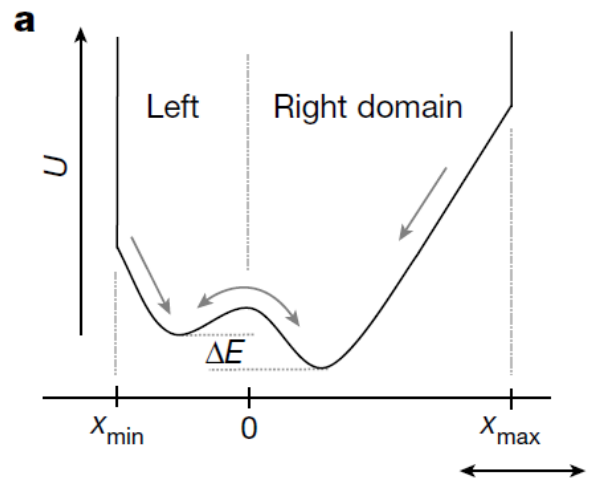
What is the Mpemba effect?

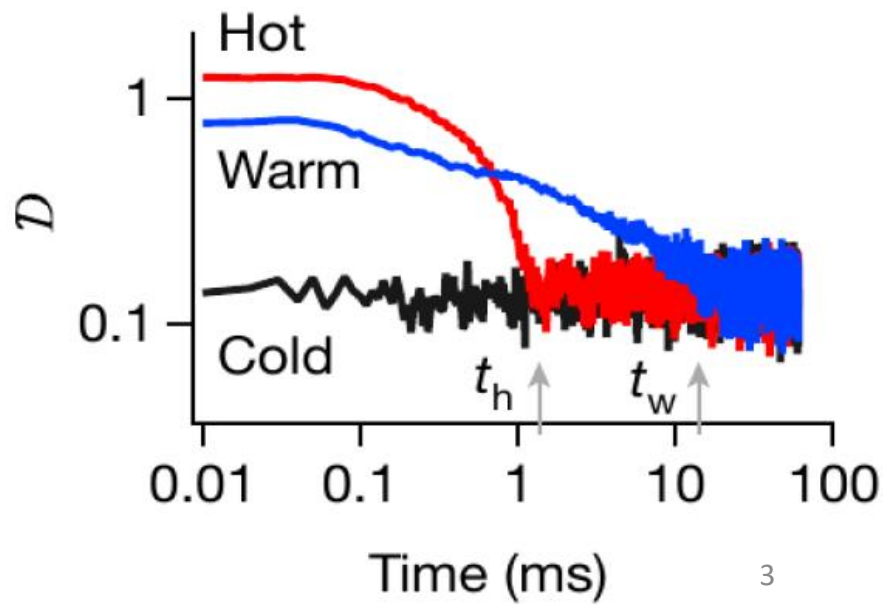
- What is Mpemba effect?
  - Erasto B. Mpemba found that some hot suspensions (*ice cream mix*) can freeze faster than cold (1963).
  - With the help of D. G. Osborne he has published a scientific paper (1969).



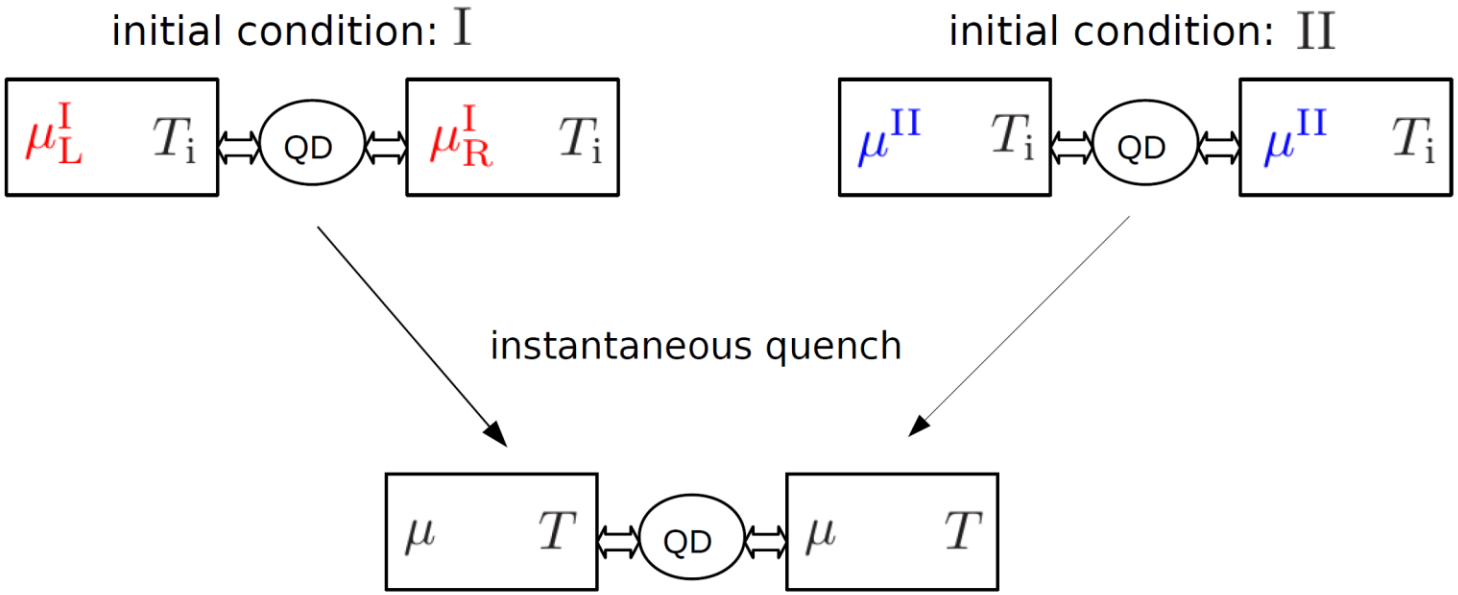
Experimental confirmation

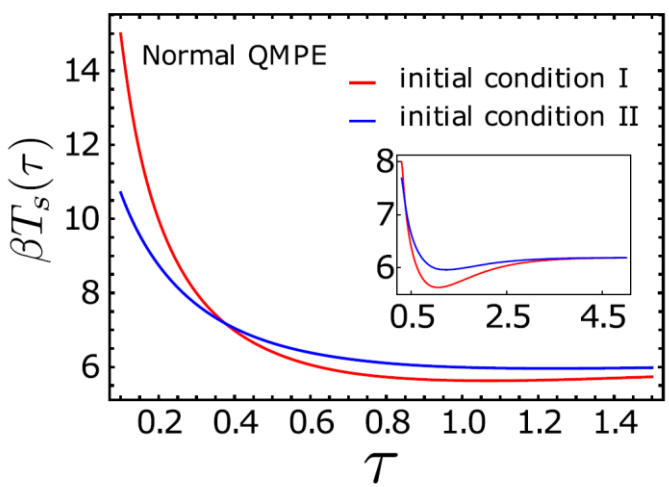
- Kumar & Bechhoeffer, Nature **584**, 64 (2020).
- They have analyzed trapped colloids in a double well potential.
- They observed the distance between the distribution and equilibrium one.





Chatterjee, Takada & Hayakawa, PRL **131**, 032901 (2023)





$$T_s(\tau) := \frac{\partial E_s(\tau)}{\partial S_{\mathrm{vN}}(\tau)} = \frac{\partial E_s(\tau)}{\partial \tau} \Big/ \frac{\partial S_{\mathrm{vN}}(\tau)}{\partial \tau}$$

$$S_{\mathrm{vN}}(\tau) = -\sum_\alpha \rho_\alpha(\tau) \ln[\rho_\alpha(\tau)]$$

YITP YUKAWA INSTITUTE FOR THEORETICAL PHYSICS

Why exceptional points?

- Our previous analysis was quasi-classical because off-diagonal elements of the density matrix do not play any roles.
- We need to know the effect of entanglements.
- The model of open quantum systems may have exceptional points.
- The minimum model to satisfy the above requirement is Hatano's model.

Model

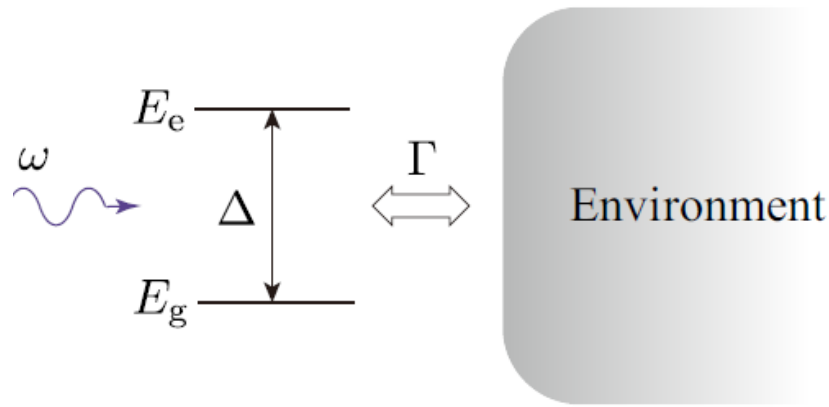
- We consider the Lindblad equation for a two-level open quantum system.
- N. Hatano, Mol. Phys. **117**, 2121 (2019).

$$i\dot{\rho}_{\mathrm{eg}} = \delta\rho_{\mathrm{eg}} - \frac{d}{2}(\rho_{\mathrm{ee}} - \rho_{\mathrm{gg}}) - \frac{i}{2}\Gamma\rho_{\mathrm{eg}},$$

$$i\dot{\rho}_{\mathrm{ge}} = -\delta\rho_{\mathrm{ge}} + \frac{d}{2}(\rho_{\mathrm{ee}} - \rho_{\mathrm{gg}}) - \frac{i}{2}\Gamma\rho_{\mathrm{ge}},$$

$$i\dot{\rho}_{\mathrm{ee}} = -\frac{d}{2}(\rho_{\mathrm{eg}} - \rho_{\mathrm{ge}}) - i\Gamma\rho_{\mathrm{ee}},$$

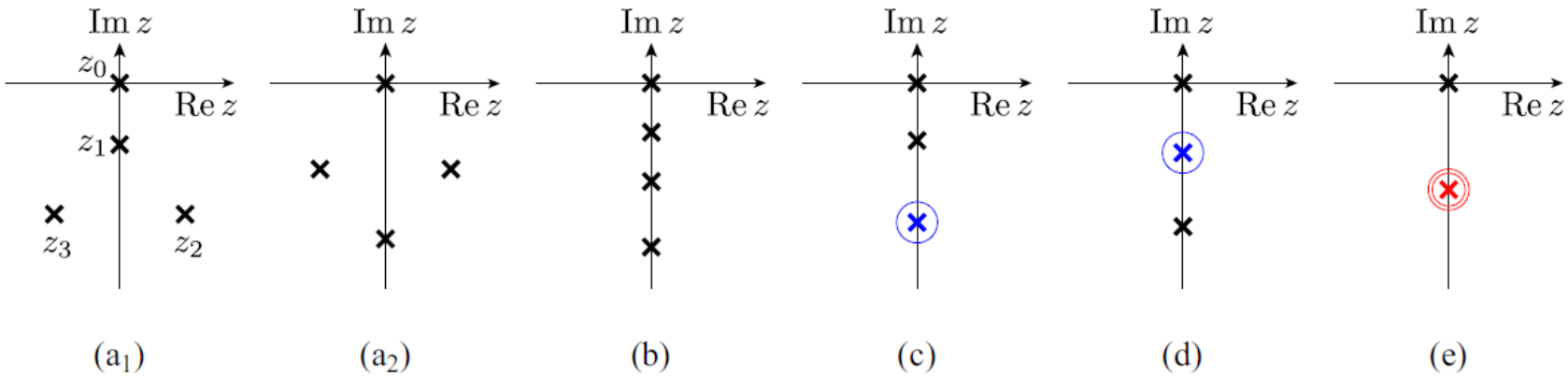
$$i\dot{\rho}_{\mathrm{gg}} = \frac{d}{2}(\rho_{\mathrm{eg}} - \rho_{\mathrm{ge}}) + i\Gamma\rho_{\mathrm{ee}}.$$

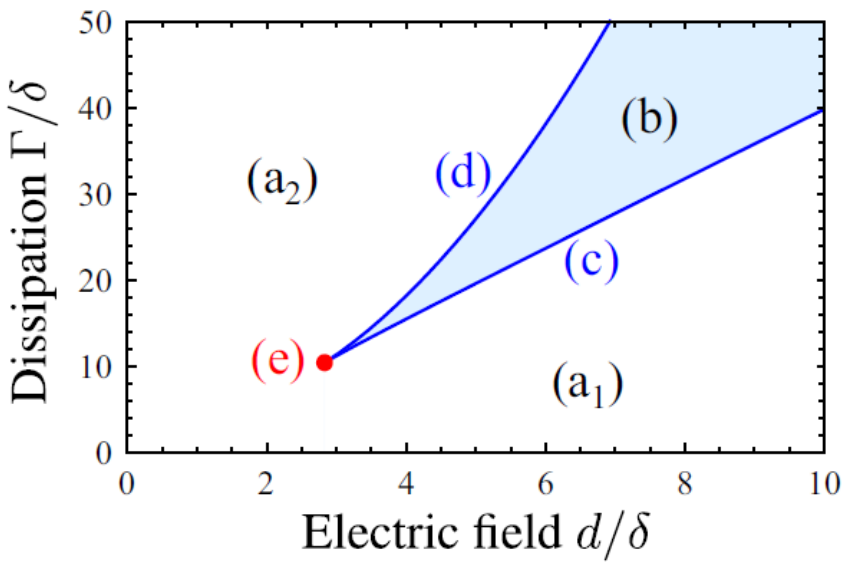


d: parameter related to electric field

$\delta := \Delta - \omega$

Eigenvalues & phase diagrams





Preparing two systems and quench into the same point, we compare the dynamics.

YUKAWA INSTITUTE FOR THEORETICAL PHYSICS

Evolution of density matrix

- The density matrix at an EP is given by

$$\rho_j(t) = \sum_{k=1}^{4} e^{-\lambda_k t} r_{k,j} a_k - \boxed{\mathrm{i}te^{-\lambda_2 t} r_{2,j} a_3},$$

$$a_k = \sum_{n=1}^{4} \ell_{k,n} \rho_n(0),$$

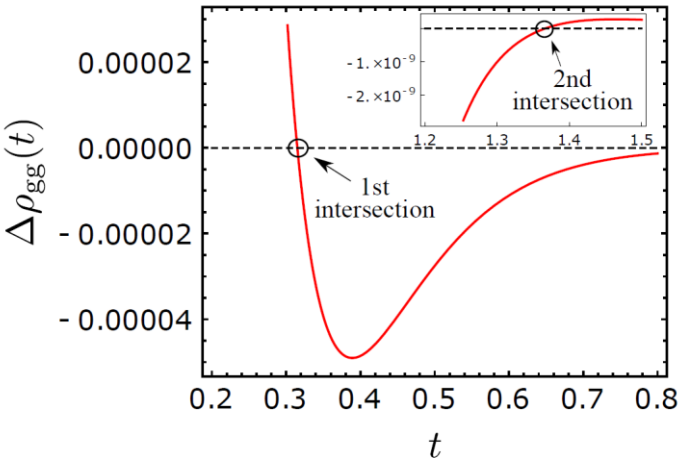
- The difference of density element in two copies

$$\Delta\rho_{\mathrm{gg}}(t) = -e^{-\lambda_2 t}\left[\boxed{\alpha_1 e^{-(\lambda_4-\lambda_2)t} + t\,\alpha_2} + \alpha_3\right],$$

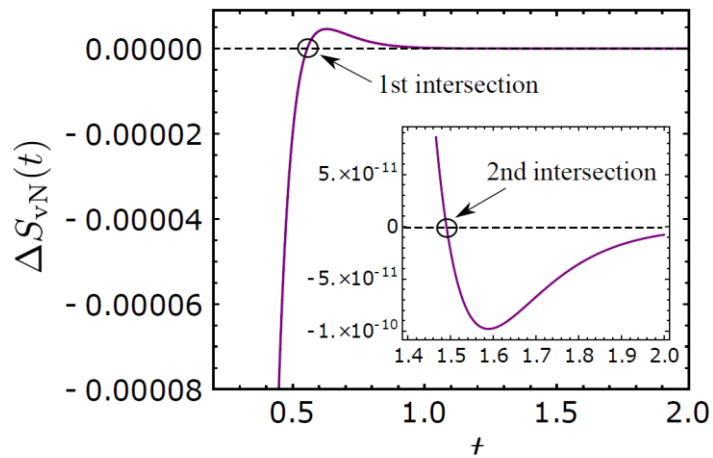
$$\alpha_1 = a_4^{\mathrm{I}} - a_4^{\mathrm{II}},\ \alpha_2 = -\mathrm{i}(a_3^{\mathrm{I}} - a_3^{\mathrm{II}}),\ \alpha_3 = a_2^{\mathrm{I}} - a_2^{\mathrm{II}}.$$

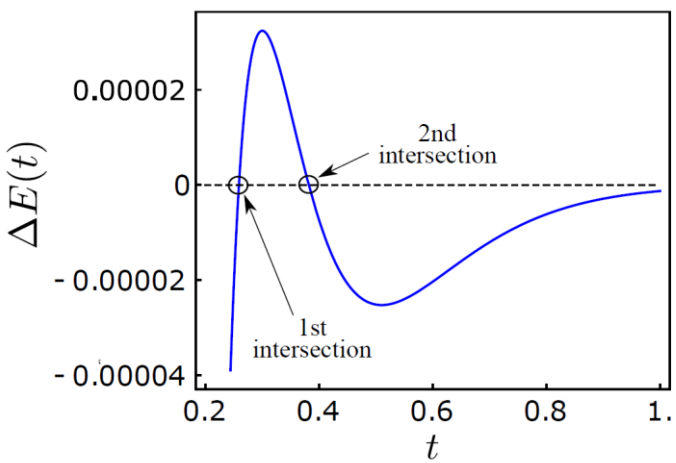
It is not difficult to get the condition of $\Delta\rho_{gg}$=0.

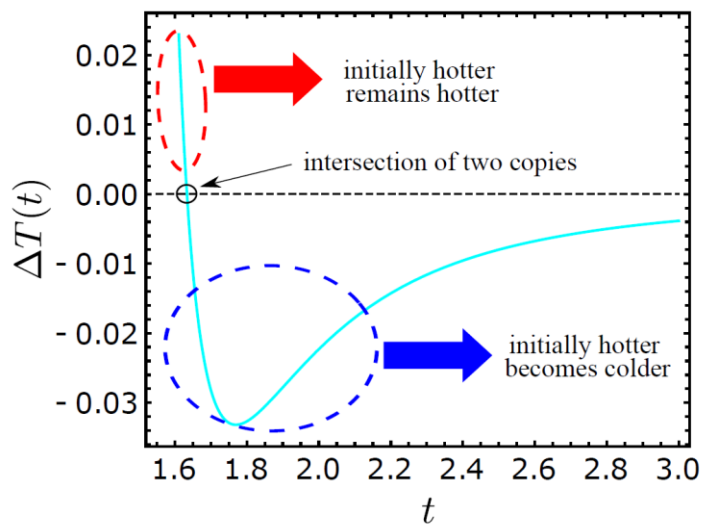
QMPE for various variables



(a)

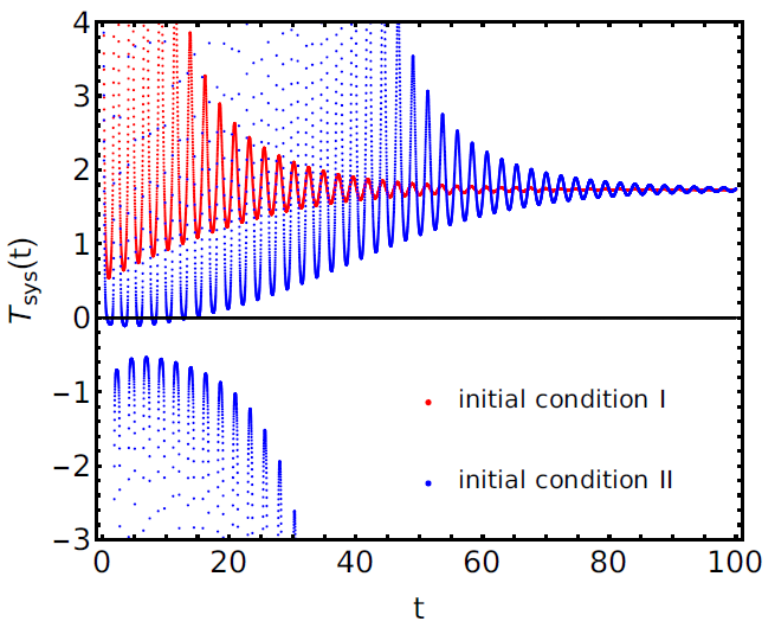


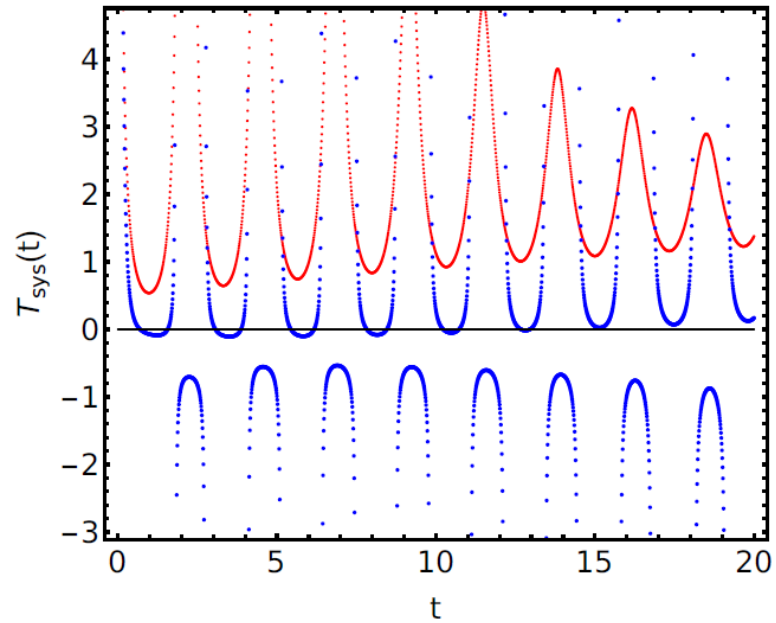




Multiple Thermal Mpemba effect

- If the system has complex eigenvalues, the behavior can be oscillate.
- Then, multiple Mpemba effect in region (a_1) can be observed.





Summary

- If we are interested in the exceptional points, we understand that Mpemba effect is generated by the algebraic part of the exceptional point.
- If we are interested in the region with complex eigenvalues, there are multiple interesections.
- => Multiple QMPE
- See arXiv:2311. 01347 for details.

Experimental QMPE

- The first experimental report on QMPE exists this year (arXiv:2401.04270).
- This is observed in a trapped quantum simulator.