Superconformal defects in tricritical Ising model

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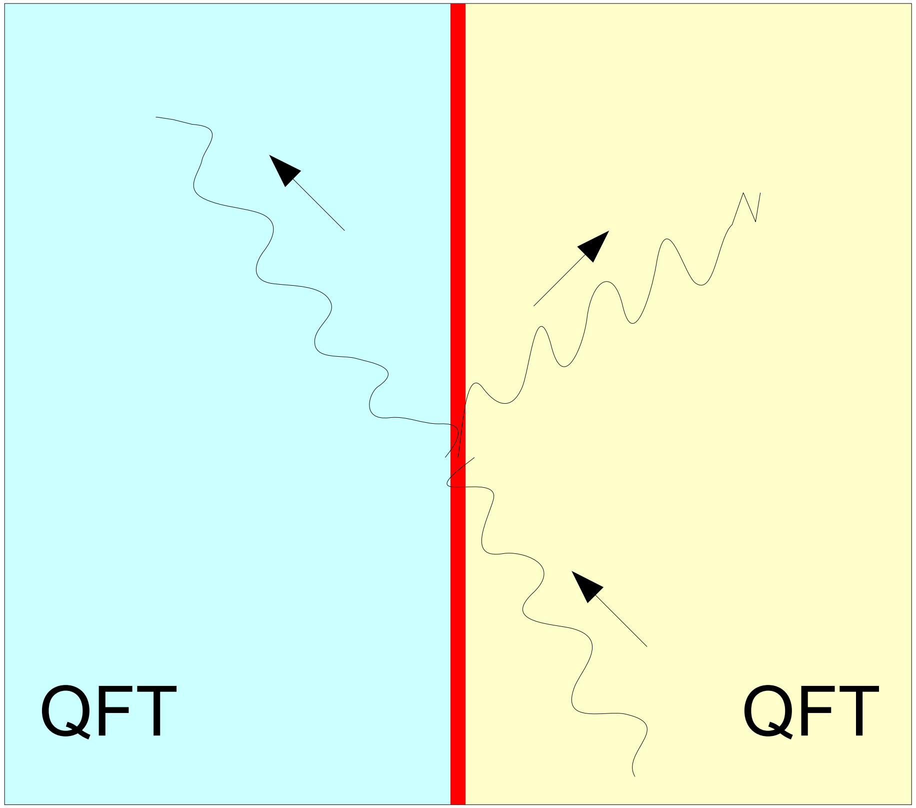


Defect

Large wave length behavior

Conformal field theory

Unitary minimal models

Conformal boundary

Conformal defect

Supersymmetry

Tricritical Ising model

Plan

Conformal field theory (with boundary)

Defect conformal field theory

Review

Result of our work

Folded theory

Superconformal defects in tricritical Ising

technical detail

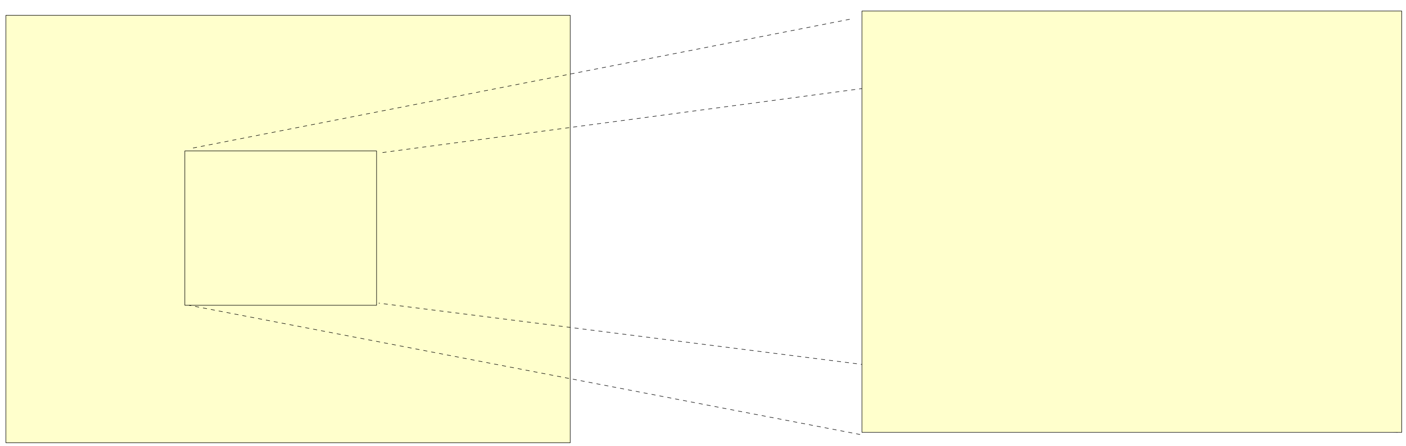
Summary

Conformal field theory (with boundary)

Conformal Field Theory (CFT)

=Quantum field theory invariant under conformal transformation

⟺ Scale invariance

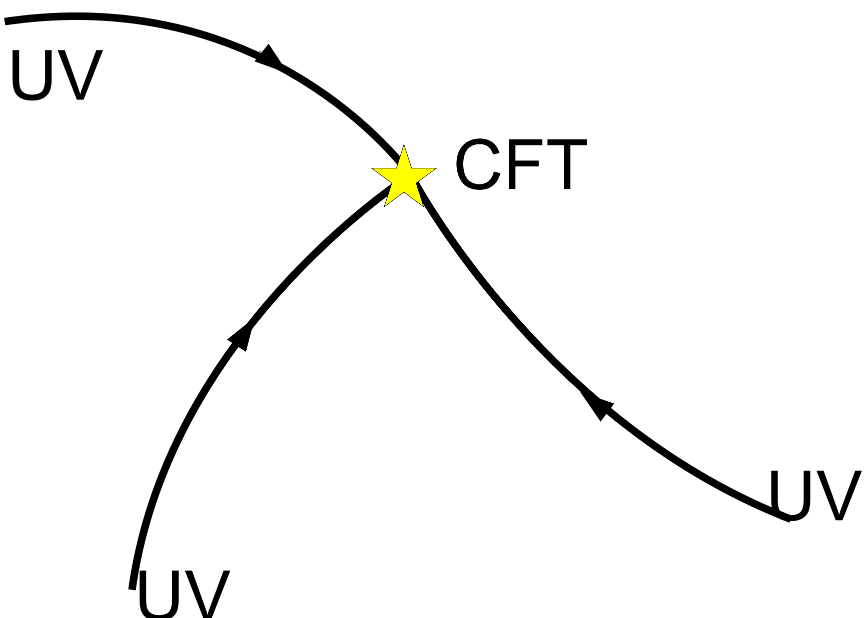


Example:

- Maxwell theory
- Free massless scalar theory
- 4-dim N=4 Super Yang-Mills

IR limit of ANY local quantum field theory is a CFT (could be empty)

Universal IR physical quantities independent of the detail of the microscopic theory



RG flow

Space of all QFTs

2 dim conformal field theory

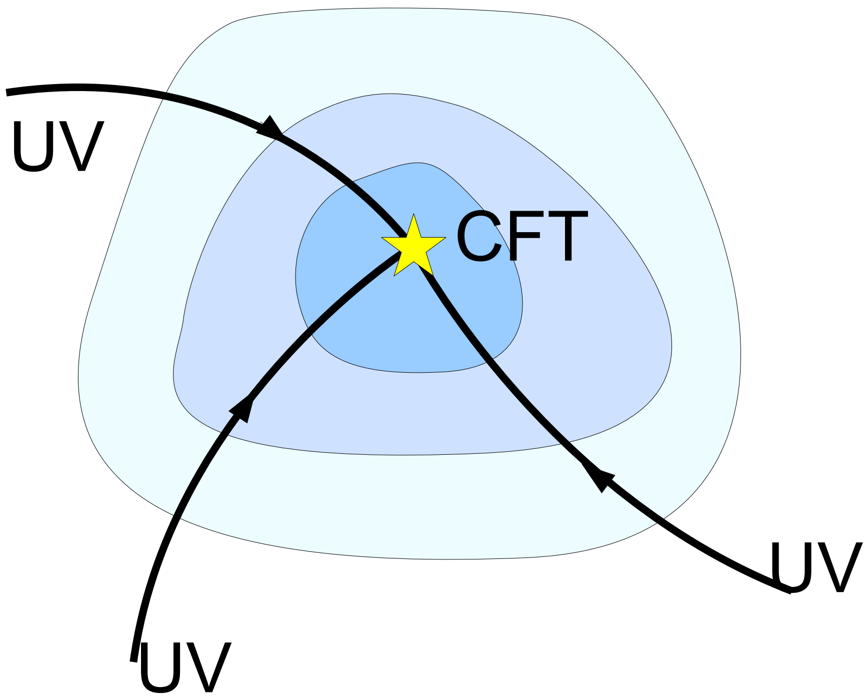
- Condensed matter: 2dim classical statistical system.
- Condensed matter: 1dim quantum statistical system.
- String theory: worldsheet theory

2 dim conformal field theory

Special because of

- Infinite dimensional symmetry “Virasoro”
- “Central charge” as degrees of freedom

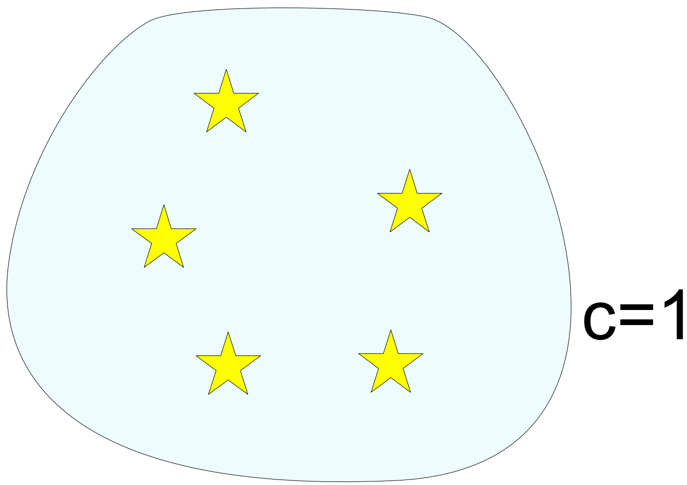
“Height” for RG flow



Fact: All unitary CFTs c<1 are classified

[Friedan, Qiu, Shenker]

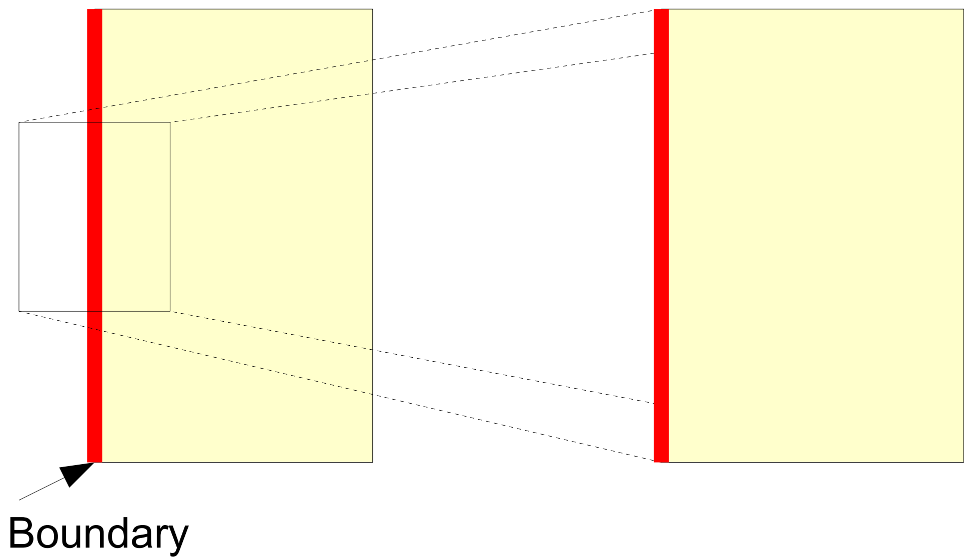
"Unitary minimal series"



$$c = 1 - \frac{6}{m(m+1)}, \quad (m = 3,4,5,6,\cdots)$$

These CFTs are "solved"

Conformal boundary



Quantum field theory
with boundary
scale invariant

Statistical system with boundary

Open string theory = D-brane

- Fix the bulk CFT ⇒ Boundary RG flow
- Any IR limit is a conformal boundary
- "Height" = boundary entropy (conjecture)

- Conformal boundary is classified when bulk CFT is a unitary minimal model

  [Cardy], [Behrend, Pearce, Petkova, Zuber]

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$\Big\}$ technical detail

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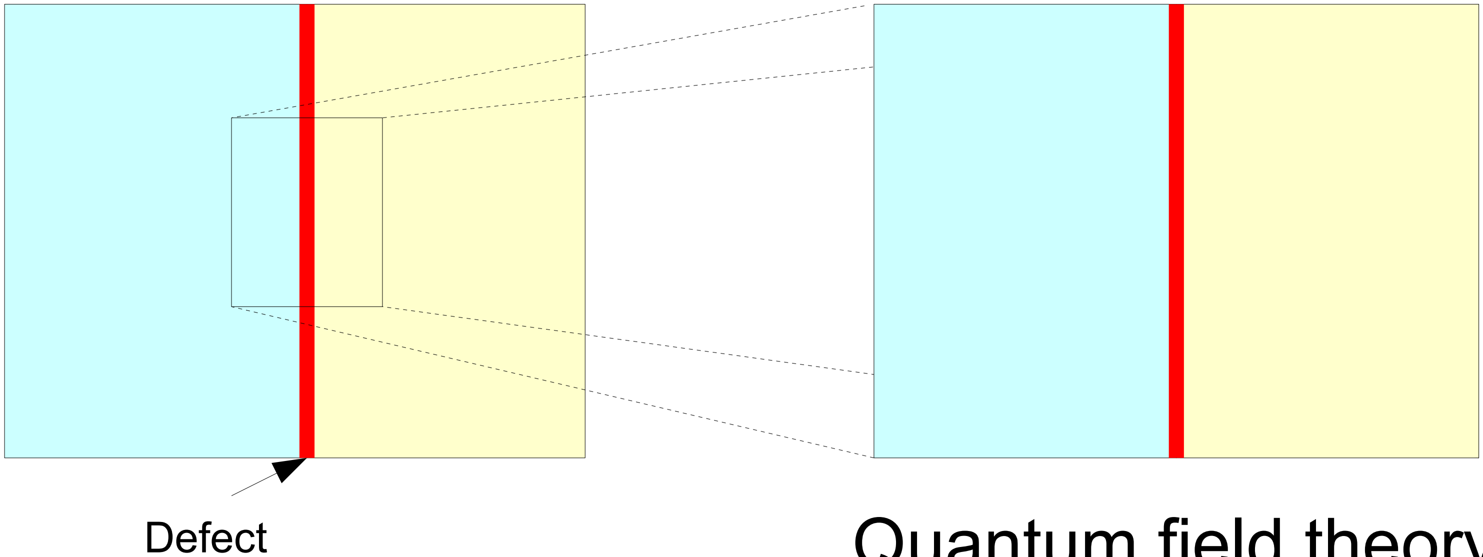
Defect CFT

Conformal defect



Quantum field theory
with defect
scale invariant

1 dim quantum statistical system with impurity

Two different system connected by a line

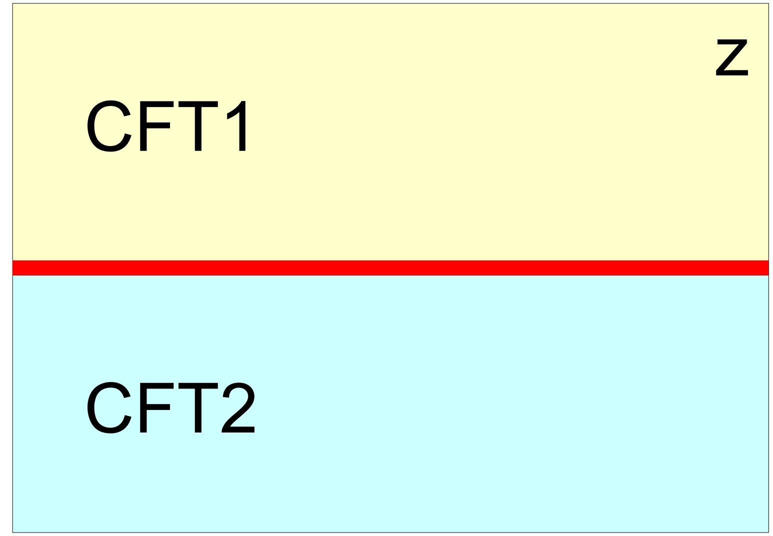
String worldsheet theory!? (some difficulty)

AdS/CFT correspondence

- Fix the bulk CFT $\Rightarrow$ Defect RG flow
- Any IR limit is a conformal defect
- “Height” = defect entropy (conjecture)

- Conformal defects are NOT Classified even if bulk CFTs are unitary minimal models.

Conformal defect



Preserve one Virasoro symmetry

Gluing condition

$$T^1(z) - \bar{T}^1(\bar{z}) = T^2(z) - \bar{T}^2(\bar{z})$$

at the defect

Energy momentum tensor

$$T^{tot}(z) := \begin{cases} T^1(z) + \bar{T}^2(\bar{z}) & \Im z \geq 0 \\ T^2(z) + \bar{T}^1(\bar{z}) & \Im z \leq 0 \end{cases}$$

is preserved

Special cases: two virasoros are preserved

Gluing condition $$T^1(z)-\bar{T}^1(\bar{z})=T^2(z)-\bar{T}^2(\bar{z})$$

Purely reflective defect

- $T^1(z)=\bar{T}^1(\bar{z}), \quad T^2(z)=\bar{T}^2(\bar{z})$

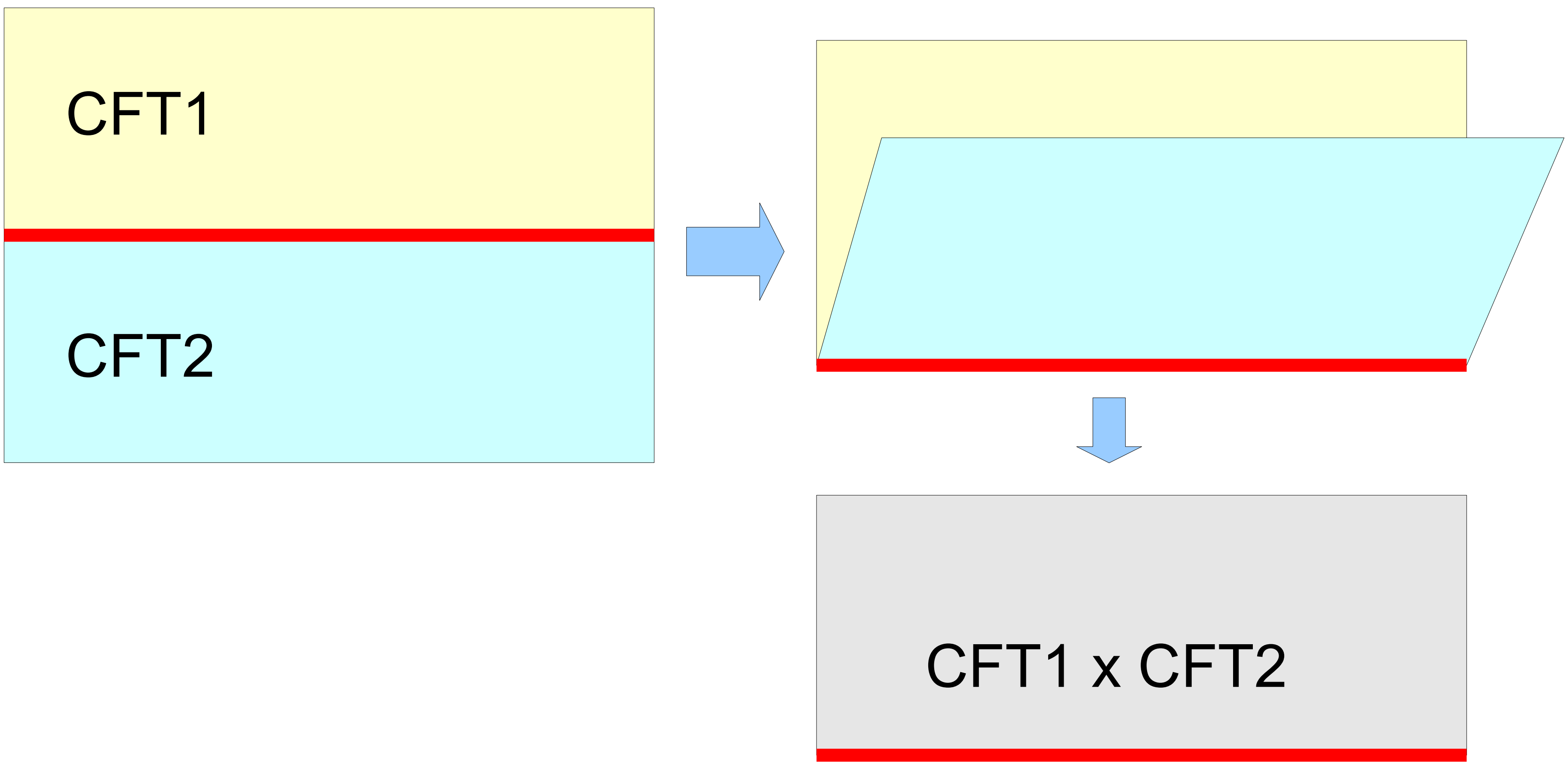
$\Rightarrow$ CFT1 and CFT2 are decoupled

Purely transmissive defect

- $T^1(z)=T^2(z), \quad \bar{T}^1(\bar{z})=\bar{T}^2(\bar{z})$

$\Rightarrow$ left-mover and right-mover are decoupled

Folding trick



Conformal defect between CFT1 and CFT2

= Conformal boundary of CFT1 x CFT2

Conformal defect $$T^1(z) - \bar{T}^1(\bar{z}) = T^2(z) - \bar{T}^2(\bar{z})$$

Folded theory

$$T(z) := T^1(z) + \bar{T}^2(\bar{z})$$
$$\bar{T}(\bar{z}) := \bar{T}^1(\bar{z}) + T^2(z)$$

$$\Downarrow$$

Conformal boundary $$T(z) - \bar{T}(\bar{z}) = 0$$

CFT1 x CFT2 theory $$c = c_1 + c_2$$

Even if both CFT1 and CFT2 are minimal models,
CFT1 x CFT2 is NOT a minimal model

central charge of minimal model $$c = \frac{1}{2}, \frac{7}{10}, \frac{4}{5}, \cdots$$

Classification seems difficult
other than the case with $c_1 = c_2 = 1/2$

(defect in Ising model [Oshikawa, Affleck])

Possible systematic approach to conformal boundary of direct product theory

- Factorized boundary state

$$T^1(z) - \bar{T}^1(\bar{z}) = 0, \quad T^2(z) - \bar{T}^2(\bar{z}) = 0$$

$\Rightarrow$ purely reflective defect

- Permutation brane when CFT1=CFT2

$$T^1(z) - \bar{T}^2(\bar{z}) = 0, \quad T^2(z) - \bar{T}^1(\bar{z}) = 0$$

$\Rightarrow$ purely transmissive defect

It is not easy to obtain a conformal defect
not purely transmissive nor purely reflective.

Our idea

Go to superconformal field theory

Superconformal field theories with c<3/2 are classified.

$$c = \frac{3}{2}\left(1 - \frac{8}{m(m+2)}\right), \qquad m = 3,4,5,\cdots$$

"N=1 superconformal unitary minimal series"

A 2-dimensional unitary supersymmetric conformal field theory c<3/2 must be one of them.

First one

$$m = 3 \quad \Rightarrow \quad c = \frac{7}{10}$$

"tricritical Ising model"

Consider defect with CFT1=CFT2=tricritical Ising model

Tensor product theory

$$c=\frac{7}{10}+\frac{7}{10}=\frac{7}{5}<\frac{3}{2}$$

This tensor product theory must be a minimal model!

$$m=10 \Rightarrow c=\frac{7}{5}$$

We can use this theory to study superconformal defects in tricritical Ising model.

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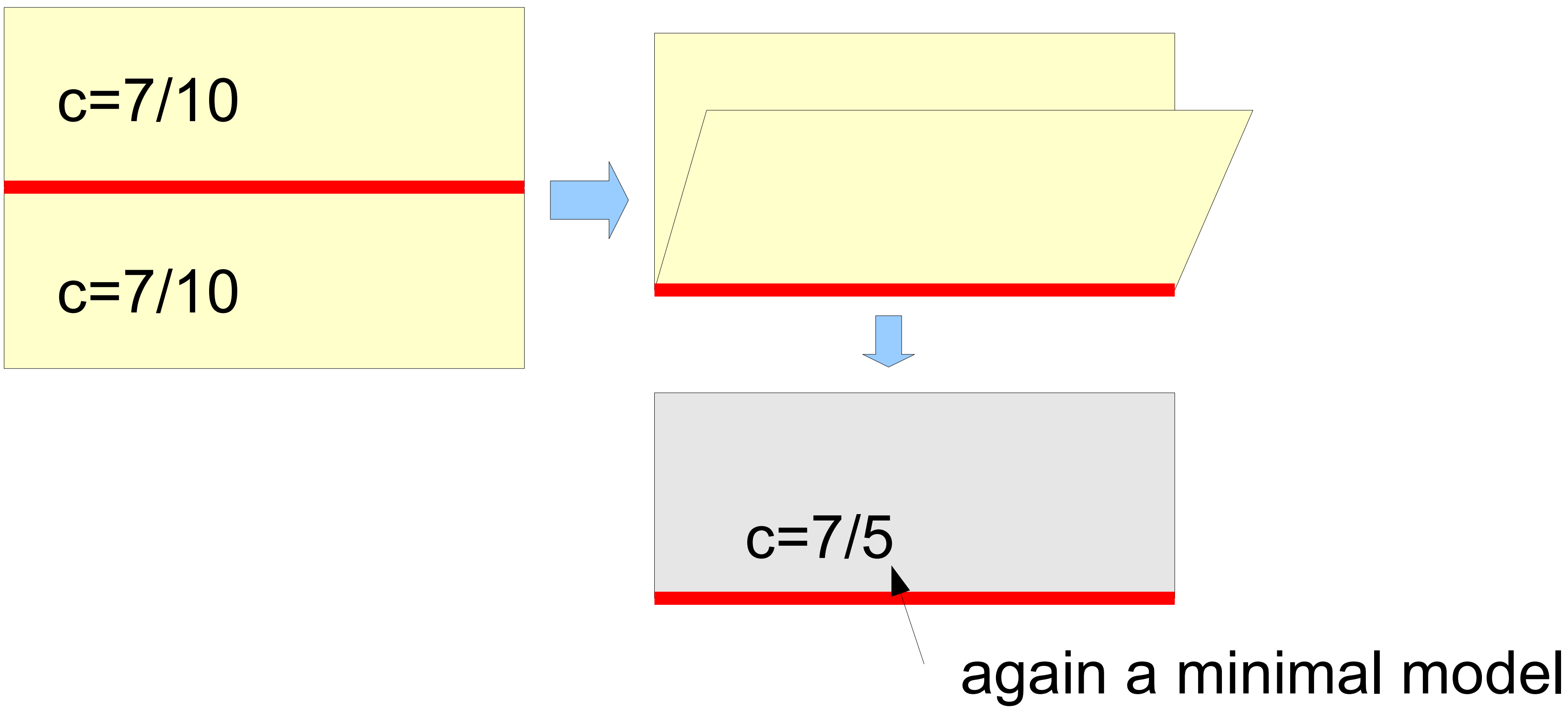
Result of our work

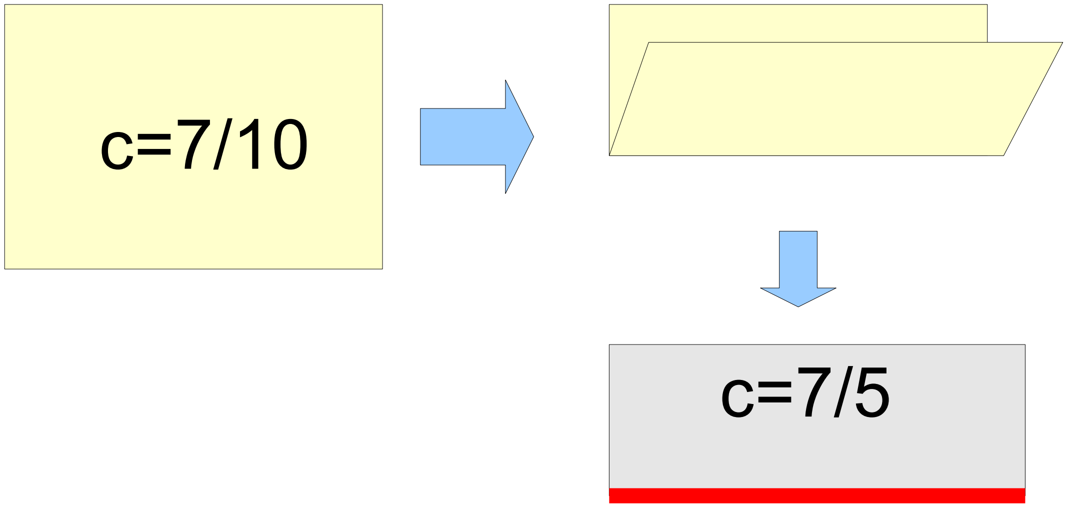
We consider super conformal defects
in tricritical Ising model using folding trick,
and tried to classify them.



Consistent set of defects

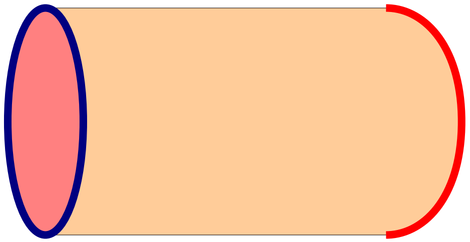
Criterion

- Includes “no defect”



- Satisfies Cardy condition

(Consistency of annulus amplitudes)



We found 18 such defects

Transmitting ratio

$T=0$	4 defects	Purely reflective
$T=1$	4 defects	Purely transmissive
$T=\dfrac{3}{3+\sqrt{3}}$	8 defects	
$T=\dfrac{\sqrt{3}}{3+\sqrt{3}}$	2 defects	

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Folded Theory

Direct product of two tricritical Ising model
(with spin structure aligned)

$c=7/5$

D_6-E_6 modular invariant

N=1 superconformal minimal model

= coset model $\frac{\widehat{SU}(2)_{m-2} \otimes \widehat{SU}(2)_2}{\widehat{SU}(2)_m}$ $c = \frac{3}{2}\left(1 - \frac{8}{m(m+2)}\right)$

Representations are labeled by (r,t,s)

$$r = 1, 2, \ldots, m-1, \qquad t = 1, 2, 3, \qquad s = 1, 2, \ldots, m+1,$$

$$r + t + s = (\text{odd integer}),$$

Identification

$$(r, t, s) \sim (m - r, 4 - t, m + 2 - s).$$

NS sector: t=1,3

R sector: t=2

Characters

$$\chi^{(m)}_{r,t,s}(\tau) = Tr_{(r,t,s)}\, q^{L_0 - c/24}$$

$$q := e^{2\pi i \tau}$$

NS sector

$$\mathrm{ch}^{(m)}_{r,s}(\tau) := \chi^{(m)}_{r,1,s}(\tau) + \chi^{(m)}_{r,3,s}(\tau) = K^{(m)}_{r,s}(\tau)\; q^{-\frac{1}{16}} \prod_{n=1}^{\infty} \frac{1+q^{n-\frac{1}{2}}}{1-q^n},$$

$$\widetilde{\mathrm{ch}}^{(m)}_{r,s}(\tau) := \chi^{(m)}_{r,1,s}(\tau) - \chi^{(m)}_{r,3,s}(\tau) = \widetilde{K}^{(m)}_{r,s}(\tau)\; q^{-\frac{1}{16}} \prod_{n=1}^{\infty} \frac{1-q^{n-\frac{1}{2}}}{1-q^n},$$

R sector

$$\mathrm{ch}^{(m)}_{r,s}(\tau) := \chi^{(m)}_{r,2,s}(\tau) = K^{(m)}_{r,s}(\tau)\; \prod_{n=1}^{\infty} \frac{1+q^{n}}{1-q^n}.$$

$$K^{(m)}_{r,s}(\tau) := \sum_{n\in\mathbb{Z}} \left(q^{\Delta^{(m)}_{n,r,s}} - q^{\Delta^{(m)}_{n,r,-s}} \right), \qquad \Delta^{(m)}_{n,r,s} := \frac{[2m(m+2)n + ms - (m+2)r]^2}{8m(m+2)},$$

$$\widetilde{K}^{(m)}_{r,s}(\tau) := \sum_{n\in\mathbb{Z}} \left((-1)^{\frac{r-s}{2}+mn} q^{\Delta^{(m)}_{n,r,s}} - (-1)^{\frac{r+s}{2}+mn} q^{\Delta^{(m)}_{n,r,-s}} \right).$$

Character relations: (m=3)x(m=3) = (m=10)

Tricritical Ising model ⟺ m=3

$$c_{m=3} = \frac{7}{10}, \quad c_{m=10} = \frac{7}{5} = c_{m=3} + c_{m=3}$$

Tensor product of two representation (m=3) NS (or R) algebra must be decomposed into (m=10)

$$\mathrm{ch}^{(3)}_{1,1}\,\mathrm{ch}^{(3)}_{1,1} = \mathrm{ch}^{(10)}_{1,1} + \mathrm{ch}^{(10)}_{1,5} + \mathrm{ch}^{(10)}_{9,1} + \mathrm{ch}^{(10)}_{9,5},$$

$$\mathrm{ch}^{(3)}_{1,3}\,\mathrm{ch}^{(3)}_{1,1} = \mathrm{ch}^{(10)}_{5,1} + \mathrm{ch}^{(10)}_{5,5},$$

$$\mathrm{ch}^{(3)}_{1,3}\,\mathrm{ch}^{(3)}_{1,3} = \mathrm{ch}^{(10)}_{3,1} + \mathrm{ch}^{(10)}_{3,5} + \mathrm{ch}^{(10)}_{7,1} + \mathrm{ch}^{(10)}_{7,5},$$

etc

$$\widetilde{\mathrm{ch}}^{(3)}_{1,1}\,\widetilde{\mathrm{ch}}^{(3)}_{1,1} = \widetilde{\mathrm{ch}}^{(10)}_{1,1} - \widetilde{\mathrm{ch}}^{(10)}_{1,5} - \widetilde{\mathrm{ch}}^{(10)}_{9,1} + \widetilde{\mathrm{ch}}^{(10)}_{9,5}\,,$$

$$\widetilde{\mathrm{ch}}^{(3)}_{1,3}\,\widetilde{\mathrm{ch}}^{(3)}_{1,1} = \widetilde{\mathrm{ch}}^{(10)}_{5,1} - \widetilde{\mathrm{ch}}^{(10)}_{5,5}\,,$$

$$\widetilde{\mathrm{ch}}^{(3)}_{1,3}\,\widetilde{\mathrm{ch}}^{(3)}_{1,3} = \widetilde{\mathrm{ch}}^{(10)}_{3,1} - \widetilde{\mathrm{ch}}^{(10)}_{3,5} - \widetilde{\mathrm{ch}}^{(10)}_{7,1} + \widetilde{\mathrm{ch}}^{(10)}_{7,5}\,,$$

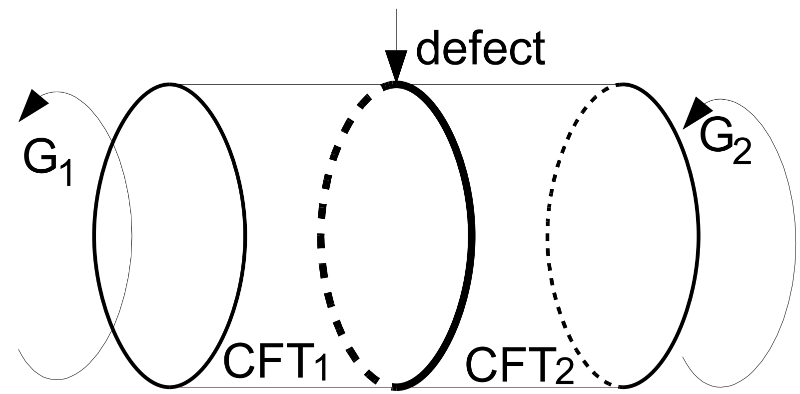
$$\mathrm{ch}^{(3)}_{1,2}\,\mathrm{ch}^{(3)}_{1,2} = \mathrm{ch}^{(10)}_{3,4} + \mathrm{ch}^{(10)}_{7,4}\,,$$

$$\mathrm{ch}^{(3)}_{1,4}\,\mathrm{ch}^{(3)}_{1,2} = \mathrm{ch}^{(10)}_{5,4}\,,$$

$$\mathrm{ch}^{(3)}_{1,4}\,\mathrm{ch}^{(3)}_{1,4} = \mathrm{ch}^{(10)}_{1,4} + \mathrm{ch}^{(10)}_{9,4}\,,$$

Supersymmetric folding trick

$$G^1(z) - \bar{G}^1(\bar{z}) = G^2(z) - \bar{G}^2(\bar{z})$$



Folding

$$G(z) := G^1(z) + \bar{G}^2(\bar{z})$$

$G^1(z)$ and $\bar{G}^2(\bar{z})$ must have the same spin structure (NS or R)

Otherwise, supersymmetry is broken

Folded theory is the tensor product with spin structure aligned

Toroidal partition function

Tensor product (with spin structure aligned)

$$Z_{\mathrm{tri}\otimes\mathrm{tri}} = \frac{1}{4}\sum_{r_1,s_1,r_2,s_2\in \mathrm{NS}} \left[|\chi^{(3)}_{r_1,1,s_1}\chi^{(3)}_{r_2,1,s_2} + \chi^{(3)}_{r_1,3,s_1}\chi^{(3)}_{r_2,3,s_3}|^2 + |\chi^{(3)}_{r_1,1,s_1}\chi^{(3)}_{r_2,3,s_2} + \chi^{(3)}_{r_1,3,s_1}\chi^{(3)}_{r_2,1,s_2}|^2 \right]$$

$$+ \frac{1}{4}\sum_{r_1,s_1,r_2,s_2\in \mathrm{R}} 2|\chi^{(3)}_{r_1,2,s_1}\chi^{(3)}_{r_2,2,s_2}|^2.$$

$$= \sum_{t=1,3} \left[|\chi^{(10)}_{1,t,1} + \chi^{(10)}_{1,t,7} + \chi^{(10)}_{9,t,1} + \chi^{(10)}_{9,t,7}|^2 + |\chi^{(10)}_{3,t,1} + \chi^{(10)}_{3,t,7} + \chi^{(10)}_{7,t,1} + \chi^{(10)}_{7,t,7}|^2 \right.$$

$$\left. + 2|\chi^{(10)}_{5,t,1} + \chi^{(10)}_{5,t,7}|^2 \right] + 2|\chi^{(10)}_{1,2,4} + \chi^{(10)}_{1,2,8}|^2 + 2|\chi^{(10)}_{3,2,4} + \chi^{(10)}_{3,2,8}|^2 + 4|\chi^{(10)}_{5,2,4}|^2$$

$$= \frac{1}{2}\sum_{\substack{r+s+t=\mathrm{odd}\\ \bar{r}+\bar{s}+t=\mathrm{odd}}} N^{D_6}_{r\bar{r}} N^{E_6}_{s\bar{s}} \chi^{(10)}_{r,t,s}\bar{\chi}^{(10)}_{\bar{r},t,\bar{s}}$$

Direct product of two tricritical Ising model
(with spin structure aligned)

→

$c=7/5$

D_6-E_6 modular invariant

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Naive boundary states in D6-E6 theory

The D6-E6 theory has 36 diagonal primary states

⇒ 36 Ishibashi states

⇒ 36 consistent boundary states

Problem

These 36 consistent boundary states do not include "no defect" boundary state.

Consistent set of defects

- Satisfies Cardy condition
- Includes “no defect”

Ishibashi states $|(r,t,s)_{10}\rangle\rangle$

Define

$$|(a,b,NS)\rangle\rangle = \sum_{r+s=\text{even},s<6} \frac{\psi_a^r\psi_b^s}{\sqrt{S_{1r}S_{1s}}}(|(r,1,s)_{10}\rangle\rangle + |(r,3,s)_{10}\rangle\rangle), \quad \tau(a)+\tau(b)=\text{odd}$$

$$|(a,b,\overline{NS})\rangle\rangle = \sum_{r+s=\text{even},s<6} \frac{\psi_a^r\psi_b^s}{\sqrt{S_{1r}S_{1s}}}(|(r,1,s)_{10}\rangle\rangle - |(r,3,s)_{10}\rangle\rangle), \quad \tau(a)+\tau(b)=\text{even}$$

$$|(a,b,R)\rangle\rangle = \sum_{r+s=\text{odd},s<6} 2^{1/4}\frac{\psi_a^r\psi_b^s}{\sqrt{S_{1r}S_{1s}}}|(r,2,s)_{10}\rangle\rangle, \quad \tau(a)+\tau(b)=\text{odd}$$

$$|(a,b,\overline{R})\rangle\rangle = \sum_{r+s=\text{odd},s<6} 2^{1/4}\frac{\psi_a^r\psi_b^s}{\sqrt{S_{1r}S_{1s}}}|(r,\bar{2},s)_{10}\rangle\rangle, \quad \tau(a)+\tau(b)=\text{odd}$$

Following 18 boundary states are a consistent set.

$$|(a,b)\rangle_{A\pm} = \frac{1}{2}|(a,b),\overline{NS}\rangle + \frac{1}{\sqrt{2}}|(a,\sigma(b)),NS\rangle \pm |(a,\sigma(b)),R\rangle$$

where, $(a,b) = \{1,3,5,6\} \times \{3\}, \{2,4\} \times \{6\}$

$$|(a,b)\rangle_{B} = |(a,b),\overline{NS}\rangle + \frac{1}{\sqrt{2}}|(a,\sigma^{-1}(b)),NS\rangle$$

where, $(a,b) = \{1,3,5,6\} \times \{1\}, \{2,4\} \times \{2\}$

$|(2,6)\rangle_{A+}$ is the "no defect" boundary state

Cardy condition can be checked

Transmitting ratio

[Quella, Runkel, Watts]

$$\mathcal{T} = \frac{2}{c_1 + c_2}(R_{12} + R_{21}) \qquad R_{ij} = \frac{\langle 0 | L_2^i \overline{L}_2^j | b \rangle}{\langle 0 | b \rangle}$$

T=0 for purely reflective defect

T=1 for purely transmissive defect

Transmitting ratio of our defects

$$\mathcal{T} = 1 \quad : \quad |(2,6)\rangle_{A_\pm}, \quad |(4,6)\rangle_{A_\pm}$$

$$\mathcal{T} = 0 \quad : \quad |(1,1)\rangle_{B}, \quad |(3,1)\rangle_{B}, \quad |(5,1)\rangle_{B}, \quad |(6,1)\rangle_{B}$$

$$\mathcal{T} = \frac{3}{3+\sqrt{3}} \quad : \quad |(1,3)\rangle_{A_\pm}, \quad |(3,3)\rangle_{A_\pm}, \quad |(5,3)\rangle_{A_\pm}, \quad |(6,3)\rangle_{A_\pm}$$

$$\mathcal{T} = \frac{\sqrt{3}}{3+\sqrt{3}} \quad : \quad |(2,2)\rangle_{B}, \quad |(4,2)\rangle_{B}$$

Summary

We consider superconformal defects in tricritical Ising model using folding trick.

- Folded theory= (N=1 minimal model, m=10, D6-E6)

- Criterion of consistent set of defects

  - Includes “no defect”
  - Satisfies Cardy condition

- We found a consistent set of 18 defects

  4 purely reflective

  4 purely transmissive

  10 intermediate

Thank you

Modular invariants

$$\sum_{r,\bar{r}} N^{D_6}_{r,\bar{r}} \chi_r \chi_{\bar{r}} = |\chi_1 + \chi_3 + \chi_5 + \chi_7|^2 + 2|\chi_5|^2,$$

$$\sum_{r,\bar{r}} N^{E_6}_{r,\bar{r}} \chi_r \chi_{\bar{r}} = |\chi_1 + \chi_7|^2 + |\chi_4 + \chi_8|^2 + |\chi_5 + \chi_{11}|^2.$$

Coefficients of the boundary states

D6

$$\frac{\psi_a^r}{\sqrt{S_{1r}}}$$

$a\backslash r$	1	3	5	5′	7	9
1	$\sqrt{2A_-}$	$\sqrt{2A_+}$	$5^{-1/4}$	$5^{-1/4}$	$\sqrt{2A_+}$	$\sqrt{2A_-}$
2	1	1	0	0	-1	-1
3	$\sqrt{40A_+^3}$	$\sqrt{40A_-^3}$	$-5^{-1/4}$	$-5^{-1/4}$	$\sqrt{40A_-^3}$	$\sqrt{40A_+^3}$
4	$\sqrt{20}A_+$	$-\sqrt{20}A_-$	0	0	$\sqrt{20}A_-$	$-\sqrt{20}A_+$
5	$\sqrt{2A_+}$	$-\sqrt{2A_-}$	$2\cdot 5^{1/4}A_+$	$-2\cdot 5^{1/4}A_-$	$-\sqrt{2A_-}$	$\sqrt{2A_+}$
6	$\sqrt{2A_+}$	$-\sqrt{2A_-}$	$-2\cdot 5^{1/4}A_-$	$2\cdot 5^{1/4}A_+$	$-\sqrt{2A_-}$	$\sqrt{2A_+}$

E6

$$\frac{\psi_a^s}{\sqrt{S_{1s}}}$$

$b\backslash s$	1	4	5	7	8	11
1	$\frac{1}{\sqrt{2}}$	$2^{-1/4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	$2^{-1/4}$	$\frac{1}{\sqrt{2}}$
2	$4\sqrt{3}B_+^2$	$2^{-1/4}$	$4\sqrt{3}B_-^2$	$-4\sqrt{3}B_-^2$	$-2^{-1/4}$	$-4\sqrt{3}B_+^2$
3	$4\sqrt{6}B_+^2$	0	$-4\sqrt{6}B_-^2$	$-4\sqrt{6}B_-^2$	0	$4\sqrt{6}B_+^2$
4	$4\sqrt{3}B_+^2$	$-2^{-1/4}$	$4\sqrt{3}B_-^2$	$-4\sqrt{3}B_-^2$	$2^{-1/4}$	$-4\sqrt{3}B_+^2$
5	$\frac{1}{\sqrt{2}}$	$-2^{-1/4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	$-2^{-1/4}$	$\frac{1}{\sqrt{2}}$
6	1	0	-1	1	0	-1

$$A_\pm := \frac{5\pm\sqrt{5}}{20} \qquad B_\pm := \frac{1}{2}\sqrt{\frac{3\pm\sqrt{3}}{6}}$$