Holographic Interface

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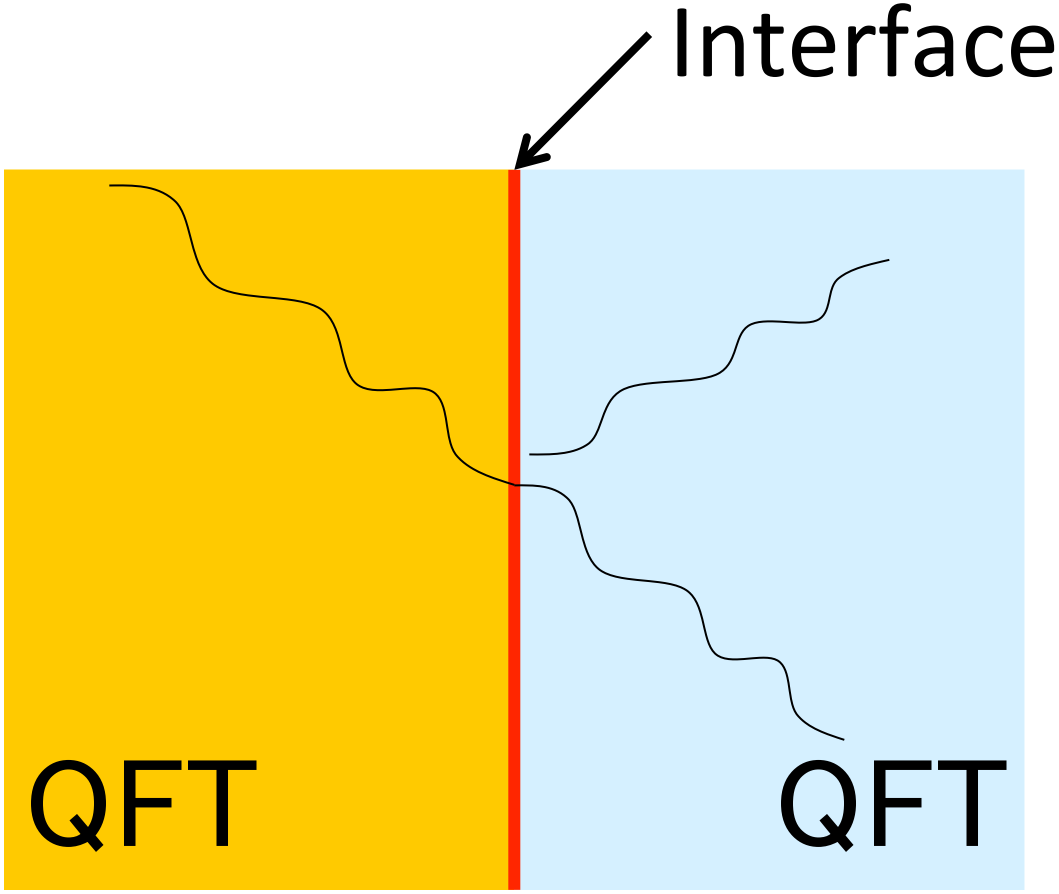
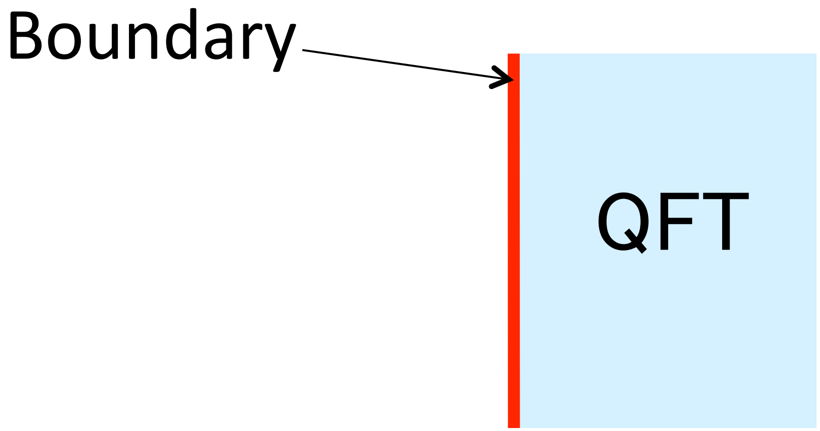
Based on collaboration with
K. Nagasaki and H. Tanida

Introduction

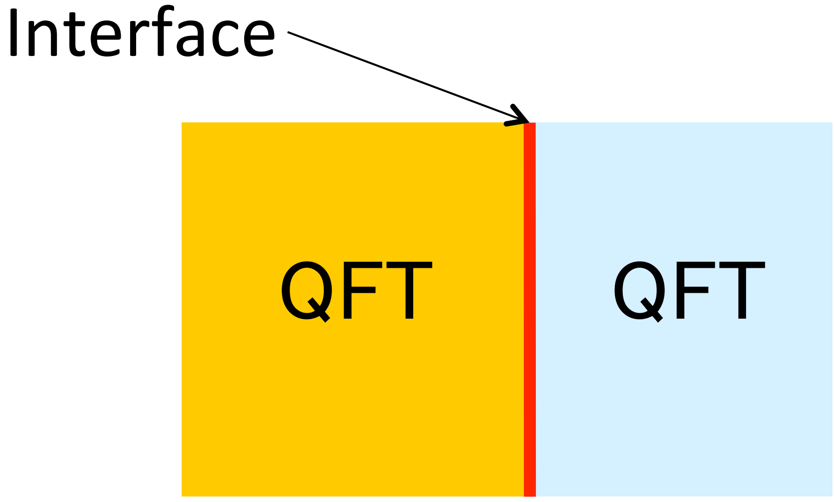

Interface
QFT
QFT

Point of view—Extension of boundary CFT

Boundary CFT



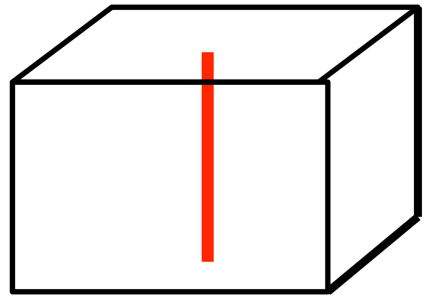
Interface CFT



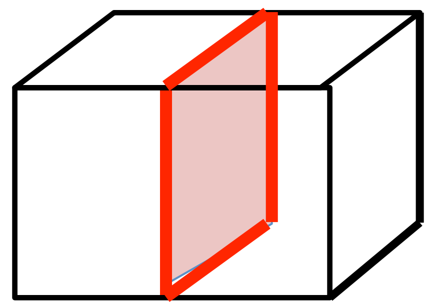
Point of view — test membrane

4 dim

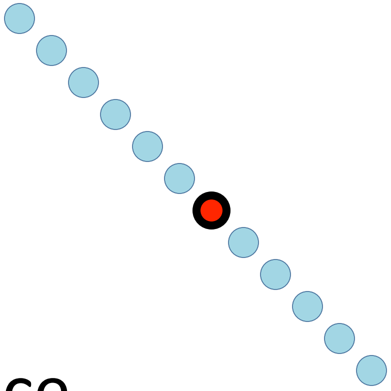
Test particle
(Wilson loop,
’t Hooft loop,...)



Test membrane
(Interface)



Motivation



- Statistical Physics:
  Quantum system with impurity
  Connect two different system by a surface

- Worldsheet CFT in the string theory?

- AdS/CFT correspondence

- Phenomenological model with extra-dimensions

AdS/CFT Correspondence

Certain gravity theory
(String theory)

⟺

Quantum field theory without gravity

Brane, etc

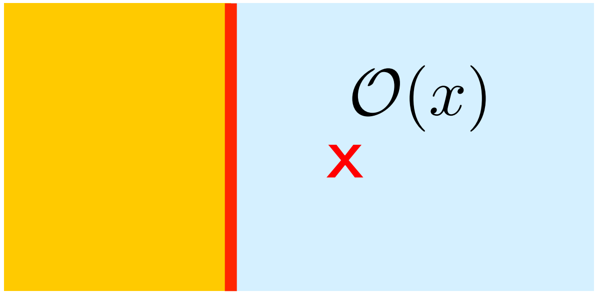
⟺

Interface

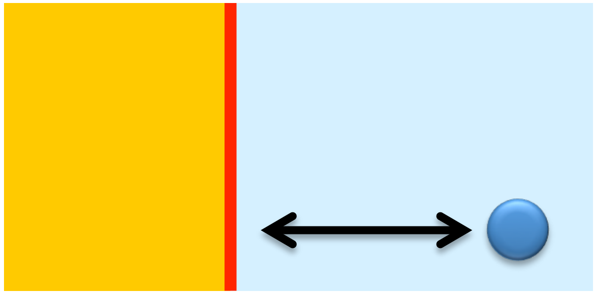
Want to check

Physical quantities

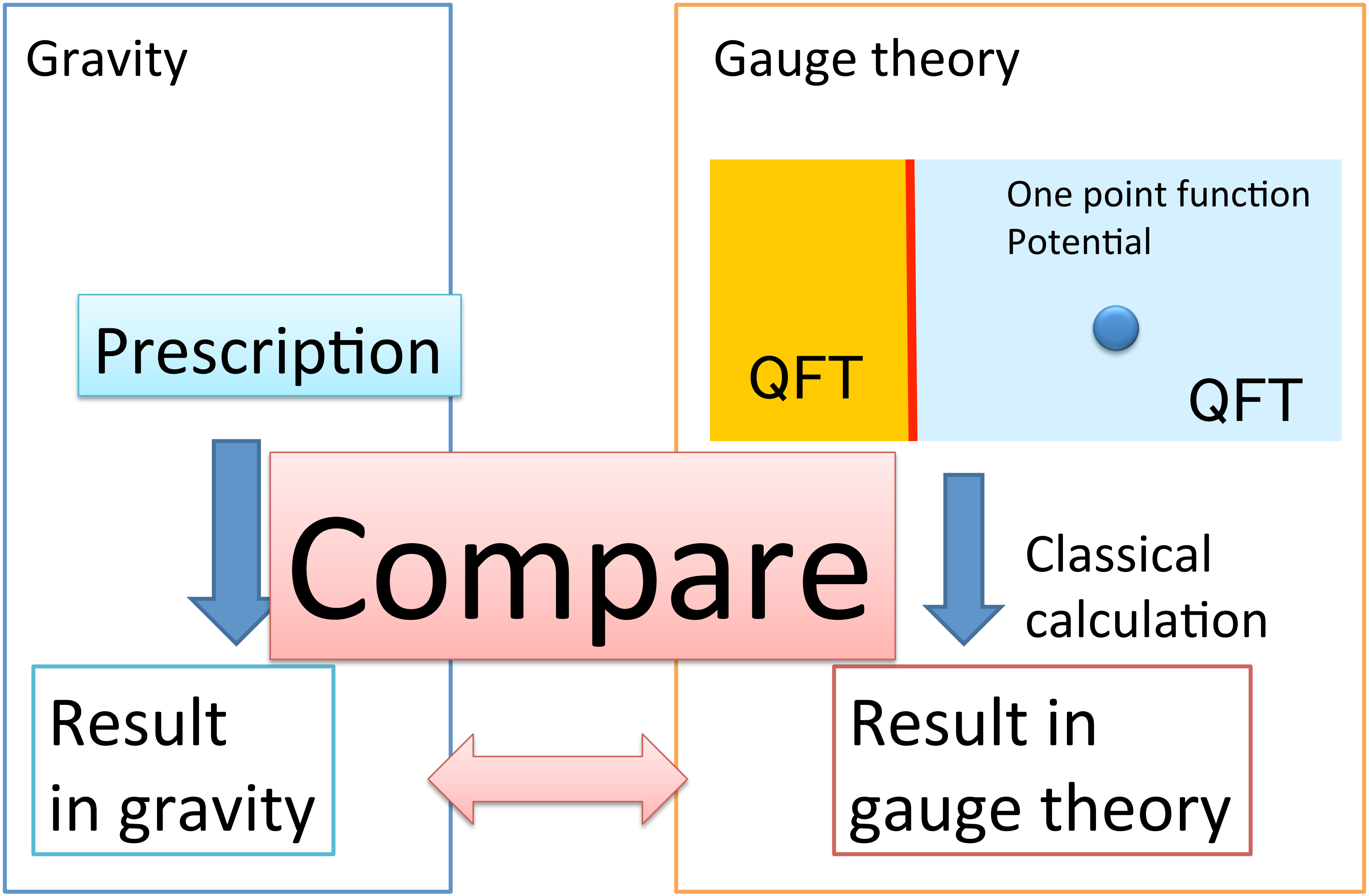
One point function of local operators



Potential between interface and particle



Strategy


Gravity
Gauge theory
One point function
Potential
QFT
QFT
Prescription
Compare
Classical
calculation
Result
in gravity
Result in
gauge theory

Result

Agree nontrivially

② Gravity

$\lambda \gg 1$

① Gauge theory

$\lambda \ll 1$

Why agree？

Gauge theory side

N=4 Super Yang-Mills

Fields $A_\mu, \ \psi, \ \phi_i, \quad i = 4, \cdots, 9$

Adjoint rep. of the gauge group SU(N)

$$S = \frac{2}{g^2} \int d^4x \, \mathrm{tr} \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} D_\mu \phi_i D^\mu \phi_i + \frac{i}{2} \bar{\psi} \Gamma^\mu D_\mu \psi + \frac{1}{2} \bar{\psi} \Gamma^i [\phi_i, \psi] + \frac{1}{4} [\phi_i, \phi_j][\phi_i, \phi_j] \right]$$

Large N limit

g : gauge coupling

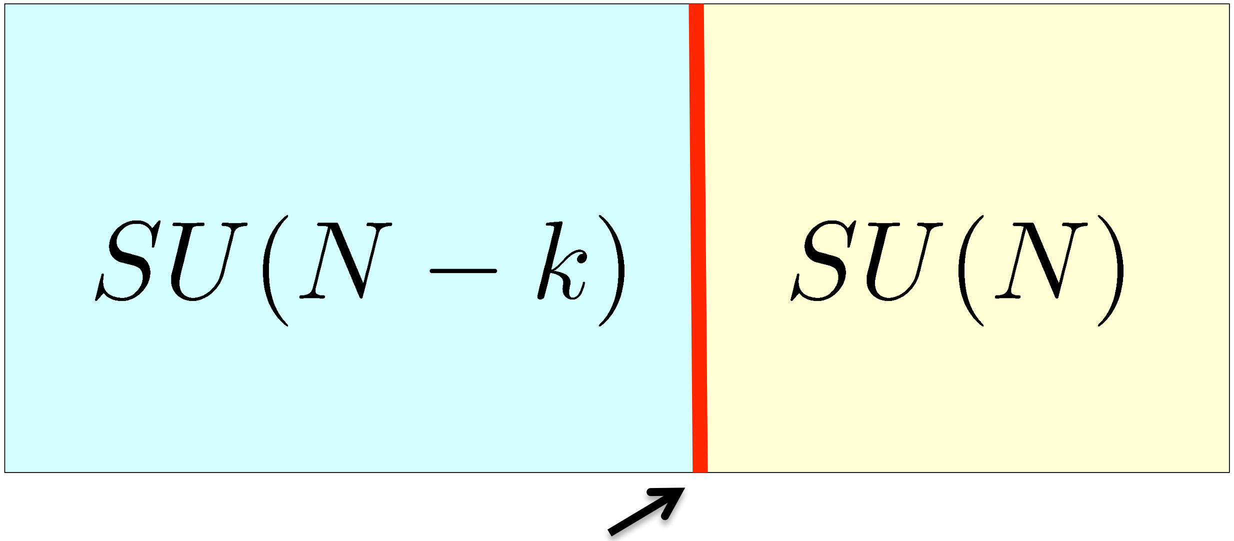
$$\lambda := g^2 N$$

Large N limit

Fix λ $\quad$ $N \to \infty$

$$(g \to 0)$$

1/2 BPS Interface



Junction condition (boundary condition) here

Use **Fuzzy funnel background**

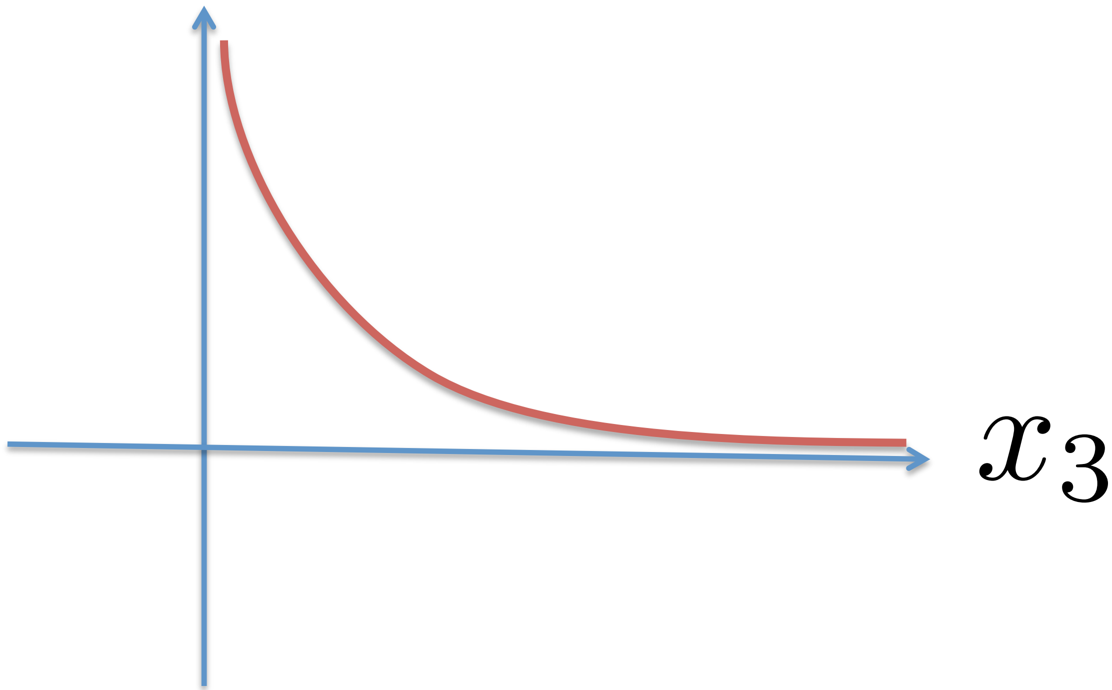
[Constable, Myers, Tafjord],
[Gaiotto, Witten]

1/2BPS → Nahm equation

$$\partial_3 \phi_i = -\frac{i}{4}\epsilon_{ijk}[\phi_j, \phi_k] \qquad i,j,k = 4,5,6$$

Solution $\phi_i = -\frac{1}{x_3} t_i \oplus 0_{N-k}$

$t_i, \quad i = 4,5,6$ k dim irrep of SU(2)



Path integral with the boundary condition that fields go to the solution in $x_3 \to 0$

One point function

Chiral primary operator

$$O_\Delta(x) = C^{I_1,\ldots,I_\Delta}\mathrm{Tr}[\phi_{I_1}\ldots\phi_{I_\Delta}]$$

Traceless symmetric

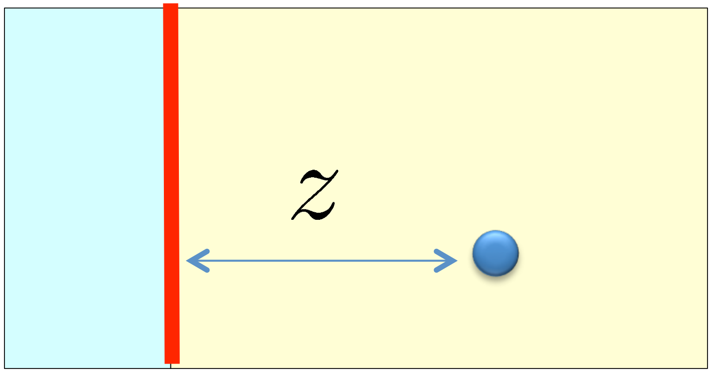
Evaluate one point function classically

$$O_\Delta(x) = C^{I_1,\dots,I_\Delta}\mathrm{Tr}[\phi_{I_1}\dots\phi_{I_\Delta}]$$

Just substitute

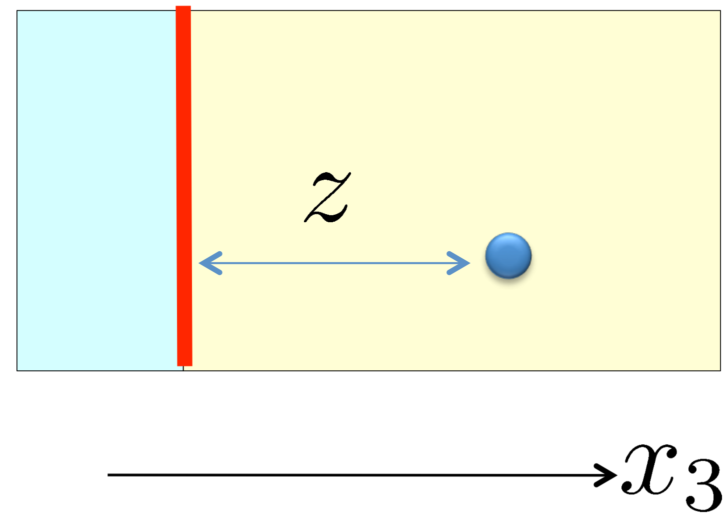
$$\phi_i = -\frac{1}{x_3}t_i \oplus 0_{N-k}\,, \qquad i = 4,5,6$$

One point function --- result



$$\langle O_\Delta(x)\rangle_I = \frac{(2\pi^2)^{\Delta/2}}{\lambda^{\Delta/2}\sqrt{\Delta}} C_{\Delta/2}(k^2-1)^{\Delta/2} k \frac{1}{z^\Delta}$$

Test particle



Introduce test particle

⇕

Wilson loop

$$W(z) = \mathrm{tr} P \exp \int_{x_3=z} (iA_0 - \phi_4)$$

Coupling to the scalar field

Potential $V(z)$

Time interval $T \to \infty$

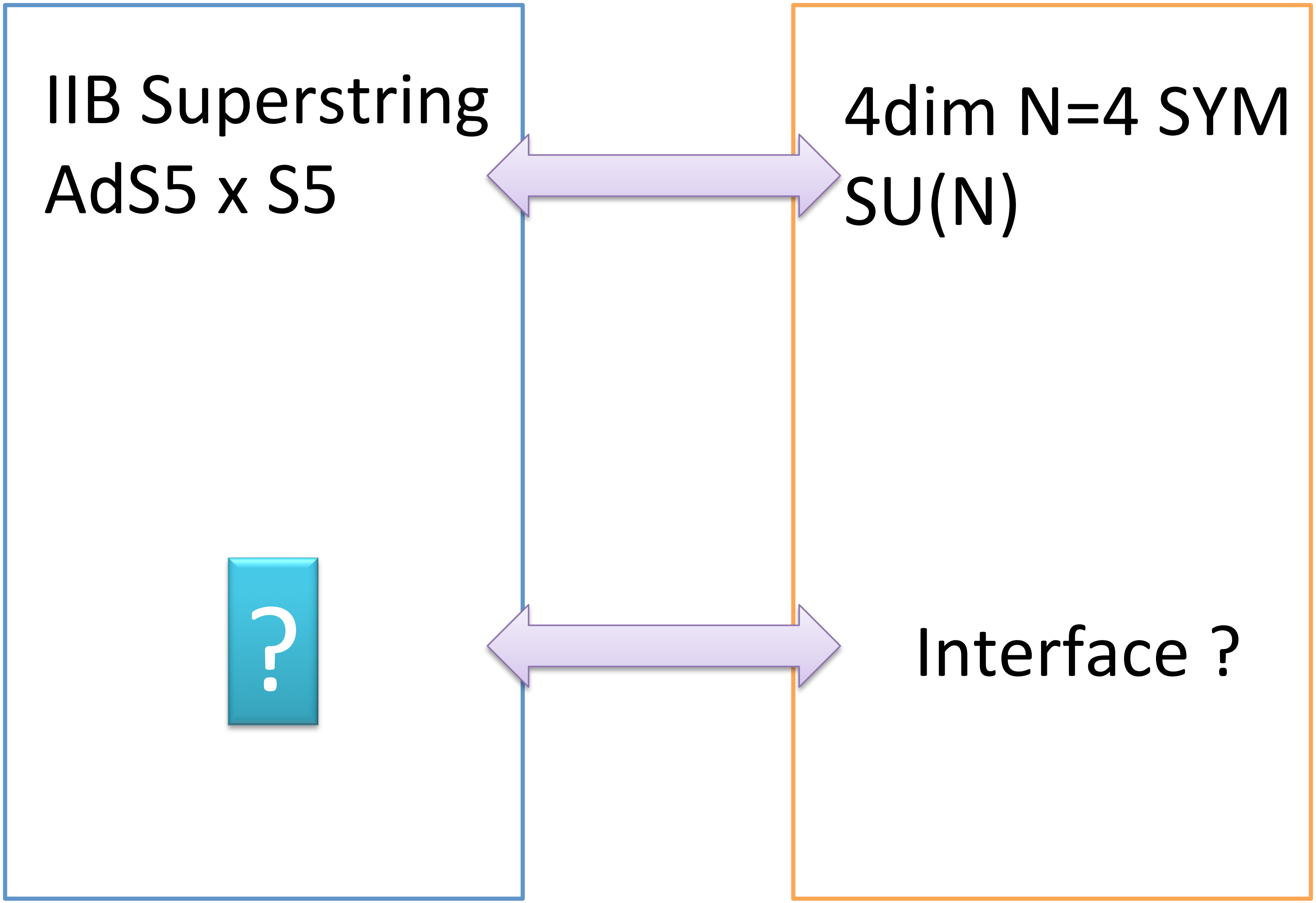
$$\langle W(z) \rangle \cong \exp(-TV(z))$$

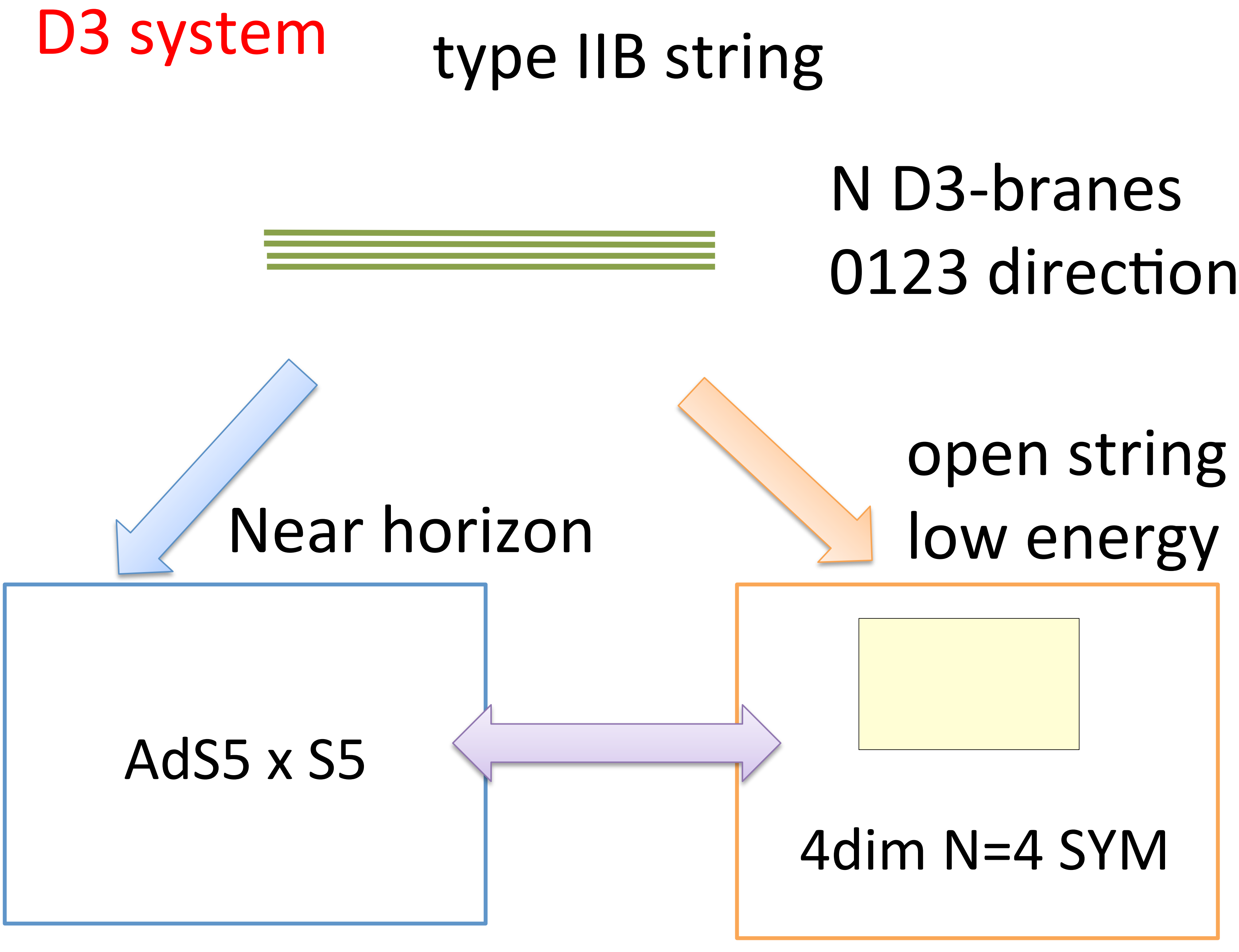
Evaluate Wilson loop clasically $\lambda \ll 1$

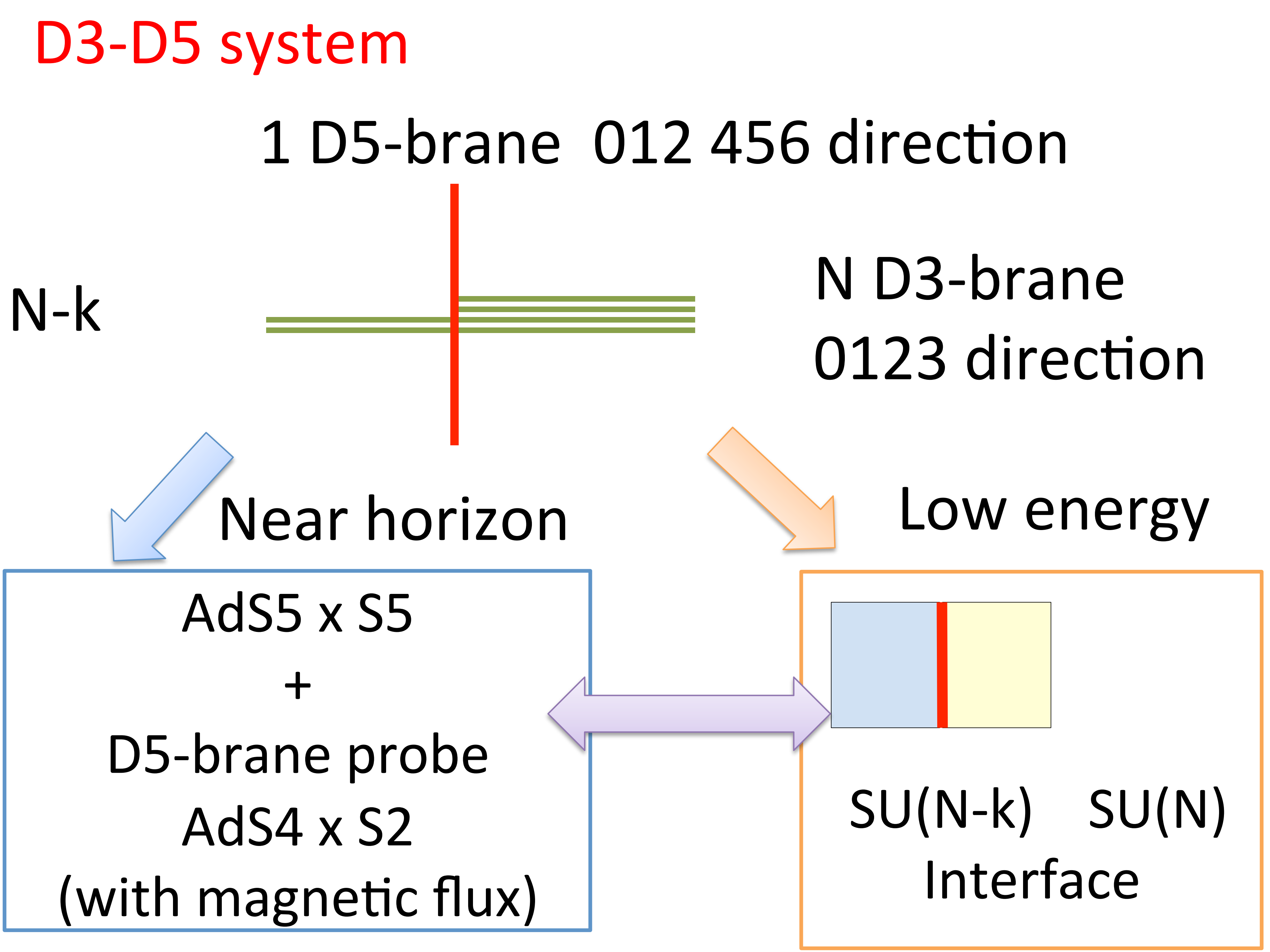
$$V(z) = -\frac{k-1}{2z}$$

Gravity side

AdS/CFT correspondence

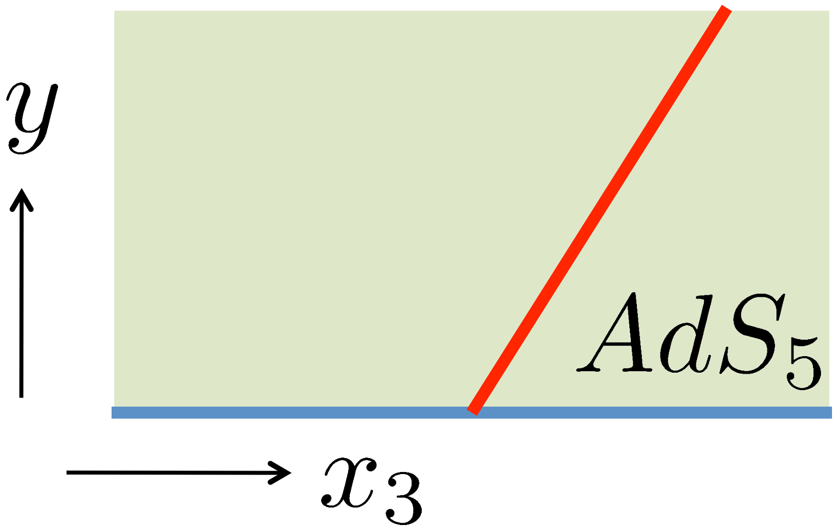


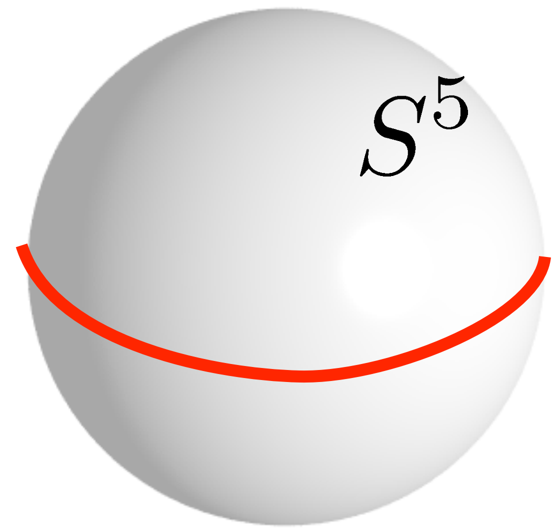

D3 system
type IIB string
N D3-branes
0123 direction
Near horizon
open string
low energy
AdS5 x S5
4dim N=4 SYM


D3-D5 system
1 D5-brane 012 456 direction
N-k
N D3-brane
0123 direction
Near horizon
Low energy
AdS5 x S5
+
D5-brane probe
AdS4 x S2
(with magnetic flux)
SU(N-k) SU(N)
Interface

Description in the gravity side [Karch, Randall]

$$ds^2_{AdS_5} = \frac{1}{y^2}(dy^2 + \eta_{\mu\nu}dx^\mu dx^\nu)$$





D5-brane

$$x_3 = \kappa y$$

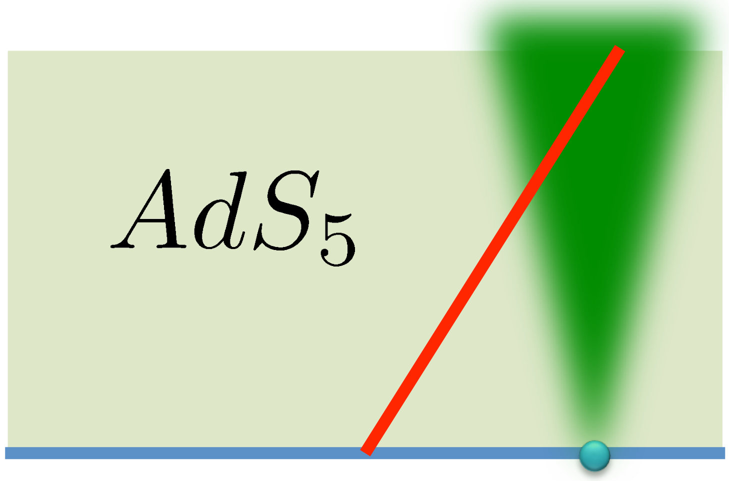
$$(AdS_4)$$

$$S^2 \subset S^5$$

magnetic flux $k = \frac{\sqrt{\lambda}\kappa}{2}$

One point function

GKPW

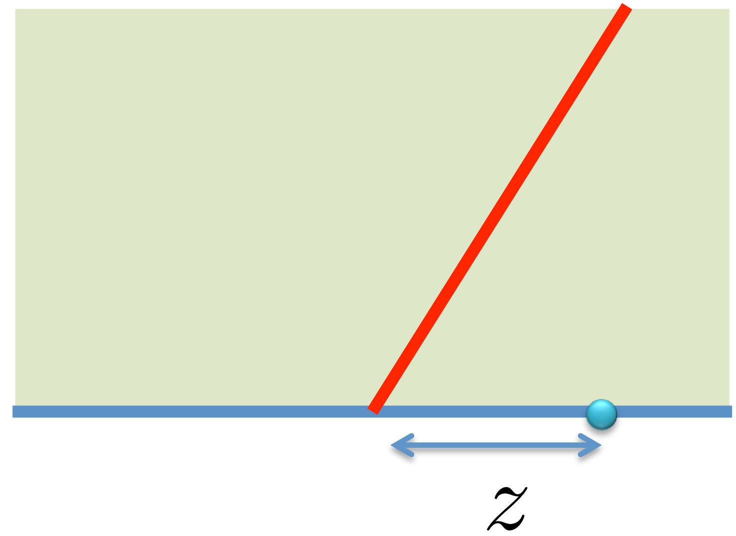


$$\langle \exp[\int d^4x s_0(x)\mathcal{O}(x)]\rangle \cong \exp(-I_{\rm classical}(s_0))$$

$$I = I_{DBI} + I_{WZ}$$

Preliminary result

$$\langle \mathcal{O} \rangle = \frac{A_\Delta}{z^\Delta}$$



$$A_\Delta = \frac{\sqrt{\lambda}2^{\Delta/2}\Gamma(\Delta+\frac{1}{2})}{\pi^{3/2}\sqrt{\Delta}\Gamma(\Delta)} C_{\Delta/2} \int_0^\infty du \frac{u^{\Delta-2}}{[(\kappa u-1)^2+u^2]^{\Delta+1/2}}$$

Comparison to the gauge theory side

Gauge

$$\langle O_\Delta(x)\rangle_I = \frac{(2\pi^2)^{\Delta/2}}{\lambda^{\Delta/2}\sqrt{\Delta}} C_{\Delta/2} k^{\Delta+1} \frac{1}{z^\Delta}$$

(k : Large)

Agree

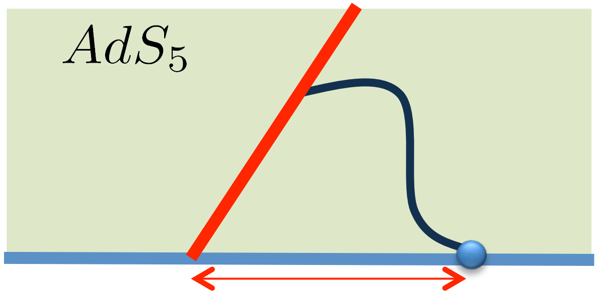
Gravity

$$\langle O_\Delta(x)\rangle_I = \frac{(2\pi^2)^{\Delta/2}}{\lambda^{\Delta/2}\sqrt{\Delta}} C_{\Delta/2} k^{\Delta+1} \frac{1}{z^\Delta}$$

(κ : Large)

Potential

[Rey, Yee], [Maldacena]



$$\langle W \rangle \sim \exp(-S_{\text{classical}})$$

Worldsheet on-shell action

Valid in $N \to \infty, \quad \lambda \gg 1$

Result

$$V(z) = -\frac{\sqrt{\lambda}}{2\pi z}\left(\frac{\kappa}{(1+\kappa^2)^{1/4}} + E(\varphi_1, i) - F(\varphi_1, i)\right)^2$$

E, F : incomplete elliptic integral of 1st and 2nd kind

$$\sin\varphi_1 = (1+\kappa^2)^{-1/4} \qquad \kappa = \frac{\pi k}{\sqrt{\lambda}}$$

※ $\frac{1}{z}$ behavior (determined only by symmetry)

Comparison to the gauge theory side

Gauge

$$V(z) = -\frac{k-1}{2z} \cong -\frac{k}{2z} \qquad (k: \text{Large})$$

Agree

Gravity

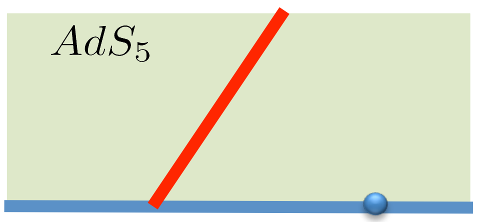
$$V(z) \cong -\frac{k}{2z} \qquad (\kappa: \text{Large})$$

Summary

One point function

AdS5 x S5

probe D5-brane

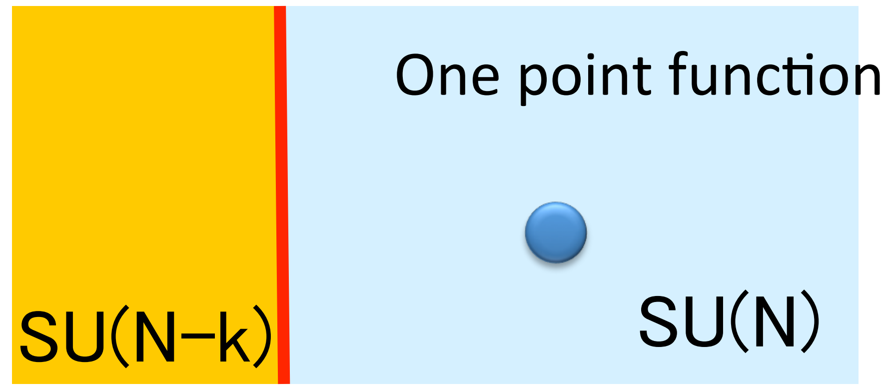


GKPW

$$\langle O_\Delta(x)\rangle_I = \frac{(2\pi^2)^{\Delta/2}}{\lambda^{\Delta/2}\sqrt{\Delta}} C_{\Delta/2} k^{\Delta+1} \frac{1}{z^\Delta} + \cdots$$

Agree

N=4 SYM



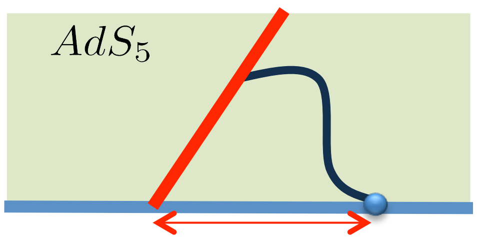
Classical calculation

$$\langle O_\Delta(x)\rangle_I = \frac{(2\pi^2)^{\Delta/2}}{\lambda^{\Delta/2}\sqrt{\Delta}} C_{\Delta/2} k^{\Delta+1} \frac{1}{z^\Delta}$$

Potential

AdS5 x S5

probe D5-brane

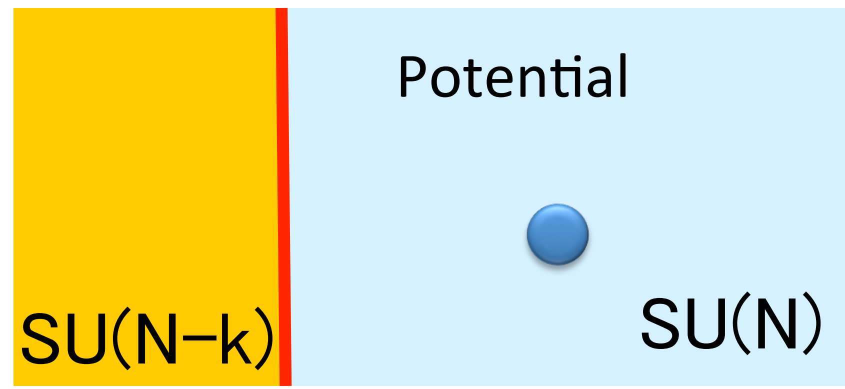


On-shell action of the strign

$$V(z) = -\frac{k}{2z} + \cdots$$

Agree

N=4 SYM



Classical calculation

$$V(z) \cong -\frac{k}{2z}$$

Why agree?

Calculation in gravity side

Valid in $\lambda \gg 1$

?

Calculation in gauge theory side

Valid in $\lambda \ll 1$

Gravity side

$$V(z) = -\frac{k}{2z}\left(1 + \frac{1}{6\pi^2}\frac{\lambda}{k^2} + O(\frac{\lambda^2}{k^4})\right)$$

Positive power series in λ since k is large.

cf [Berenstein, Maldacena, Nastase]

Future problem

$$V(z) = -\frac{k}{2z}\left(1 + \frac{1}{6\pi^2}\frac{\lambda}{k^2} + O(\frac{\lambda^2}{k^4})\right)$$

Can reproduce from the perturbative calculation in the gauge theory side?

Thank you