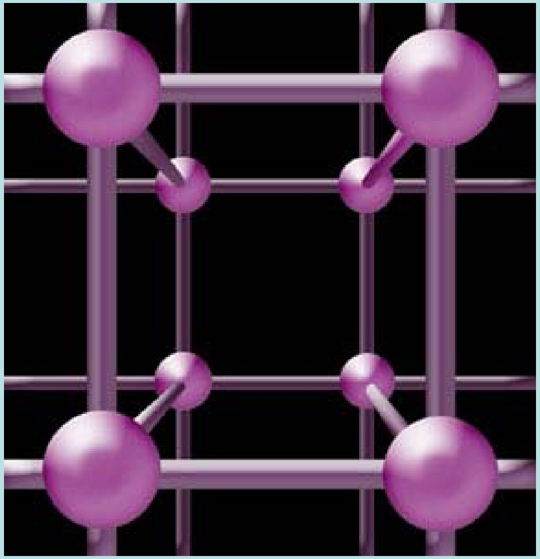
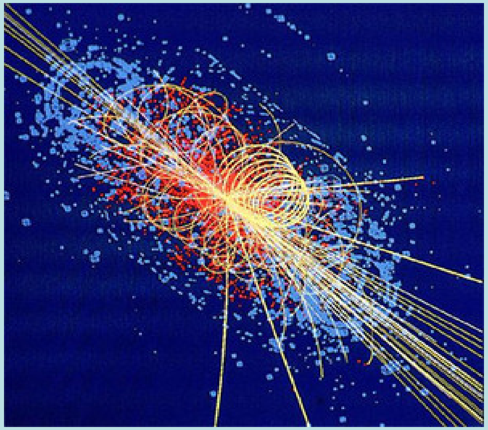
Taking the Lattice Beyond the Standard Model (QCD)



YITP Kyoto
March 4, 2009

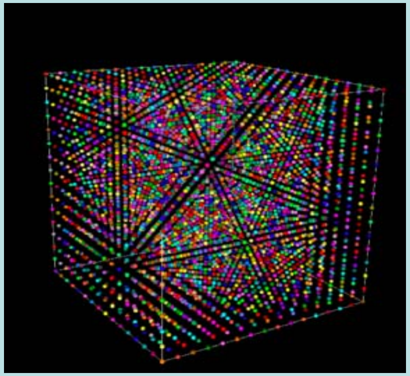
Four good reasons

LHC: Experiments Beyond the Standard Model



Theory: Possibly non-perturbative

Lattice: Non-perturbative methods



Supercomputers: Amazing simulation power

Question : WHERE beyond the standard model?

Disclaimer : I am not a lattice gauge theorist.

1. Collaboration with George Fleming and Ethan Neil (PRL 100, 171607 (2008), hep-ph/0901.3766
2. LSD collaboration

Possible Lattice Directions

- Higg sector of the standard model
  (eg. Triviality bounds) 1980's $\rightarrow$ Present
- Grand unified theories
- Supersymmetry
- New Strong Dynamics
  - Electroweak Breaking
  - New, SM-singlet sector

Strong Dynamics

Electroweak breaking

- Technicolor, ETC, Higgless theories, ADS/CFT approaches
- Near-conformal infrared behavior: walking technicolor

New, SM-singlet Sector

- Conformal infrared behavior
  Unparticles?

QCD-Like Theories (unimaginative?)

- SU(N) Gauge Theories with N_f Massless Fermions (fundamental representation)

- Asymptotically-free (can take lattice spacing to zero)

- Vary N_f and study how the infrared behavior changes (chiral symmetry breaking and confinement versus infrared conformal behavior.)

Conformal Window

↑

Range of N_f Values from the onset of Asymptotic Freedom down to the onset N_{fc} of conformal/chiral-symmetry breaking and confinement. Fermion screening

QCD is NOT Conformal!

1. Standard-Model Singlets with $N_f > N_{fc}$:
   New Conformal Sector?

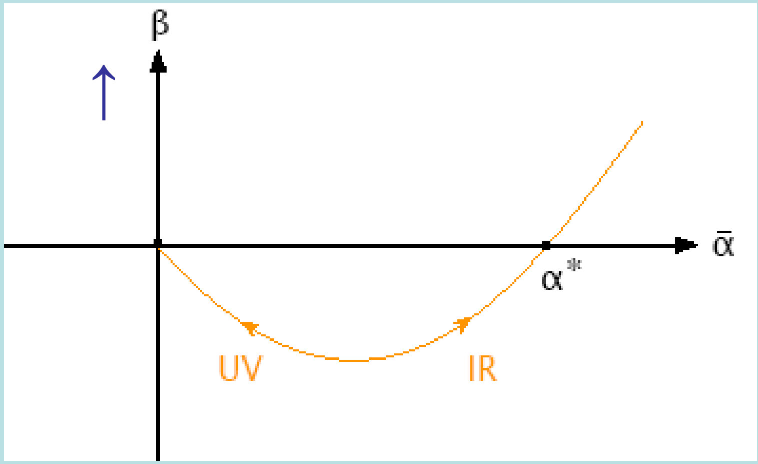
2. Standard-Model Non-singlets with $N_f < N_{fc}$:
   Dynamical Electroweak Symmetry Breaking.
   ($N_f \rightarrow N_{fc}$: Walking Technicolor)?

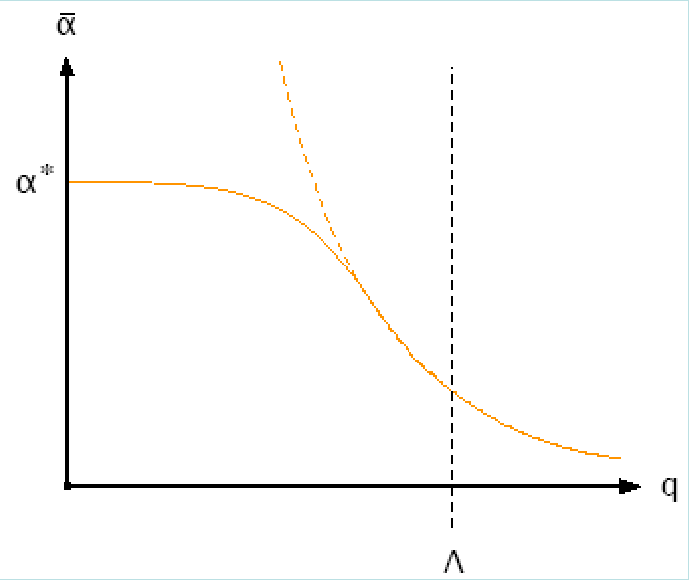
Infrared conformal behavior (IR fixed point) can emerge already in the two-loop β function, depending on the number of massless fermions N_f

Gross and Wilczek, antiquity
Caswell, 1974
Banks and Zaks, 1982
Many Others

Reliable if N_f is just below the value at which asymptotic freedom sets in.

Cartoons





α^* increases as N_f decreases.

When N_f reaches N_{fc}, α^* becomes large enough to trigger chiral symmetry breaking and confinement.

Not necessarily accessible in perturbation theory

Questions

1. Value of N_{fc}?

2. Order of the phase transition?

2. Correlation functions and anomalous dimensions inside the Conformal window

3. Physical states below and near the transition?
   Implications for electroweak precision studies (S parameter etc)? Implications for the LHC?

N_{fc} in SU(N) QCD

- Degree-of-Freedom Inequality (Cohen, Schmaltz, TA 1999). Fundamental rep:

  $N_{fc} \leq \; 4\,N[1 - 1/18N^2 + \ldots] \qquad (N \geq 3)$

- Gap-Equation Studies, Instantons (1996): $N_{fc} \cong 4\,N$

- Lattice Simulation (Iwasaki et al, Phys Rev D69, 014507 2004):

  $6 < N_{fc} < 7 \qquad$ For $N = 3$

Some Quasi-Perturbative Studies of the Conformal Window in QCD-like Theories

1. Gap – Equation studies in the mid 1990s

2. V. Miransky and K. Yamawaki hep-th/9611142 (1996)

3. E. Gardi, G. Grunberg, M. Karliner hep-ph/9806462 (1998)

4. E. Gardi and G. Grunberg JHEP/004A/1298 (2004)

   "The IRFP is perturbative in the entire conformal window"

5. Kurachi and Shrock, hep-ph/0605290

6. H. Terao and A. Tsuchiya arXiv:0704.3659 [hep-ph] (2007)

Lattice-Simulation Study of the Extent of the Conformal window in an SU(3) Gauge Theory with Dirac Fermions in the Fundamental Representation

Fleming, Neil, TA

Some Previous Lattice Work with Many Light Fermions

1. Brown et al (Columbia group) Phys. Rev. D12, 5655 (1992). $N_f = 8$

2 R. Mahwinney, hep/lat 9701030 (1) $\left(N_f \to 4\right)$
Nucl.Phys.Proc.Suppl.83:57-66,2000. e-Print: hep-lat/0001032

3. C. Sui, Flavor dependence of quantum chromodynamics. PhD thesis, Columbia University, New York, NY, 2001. UMI-99-98219

4. Iwasaki et al, Phys. Rev, D69, 014507 (2004)

Focus: Gauge Invariant and Non-Perturbative Definition of the Running Coupling Deriving from the Schroedinger Functional of the Gauge Theory

ALPHA Collaboration: Luscher, Sommer, Weisz, Wolff, Bode, Heitger, Simma, …

Using Staggered Fermions as in

U. Heller, Nucl. Phys. B504, 435 (1997)
Miyazaki & Kikukawa

Focus on N_f = multiples of 4:

16: Perturbative IRFP

12: IRFP “expected”, Simulate

8 : IRFP uncertain , Simulate

4 : Confinement, ChSB

The Shroedinger Functional

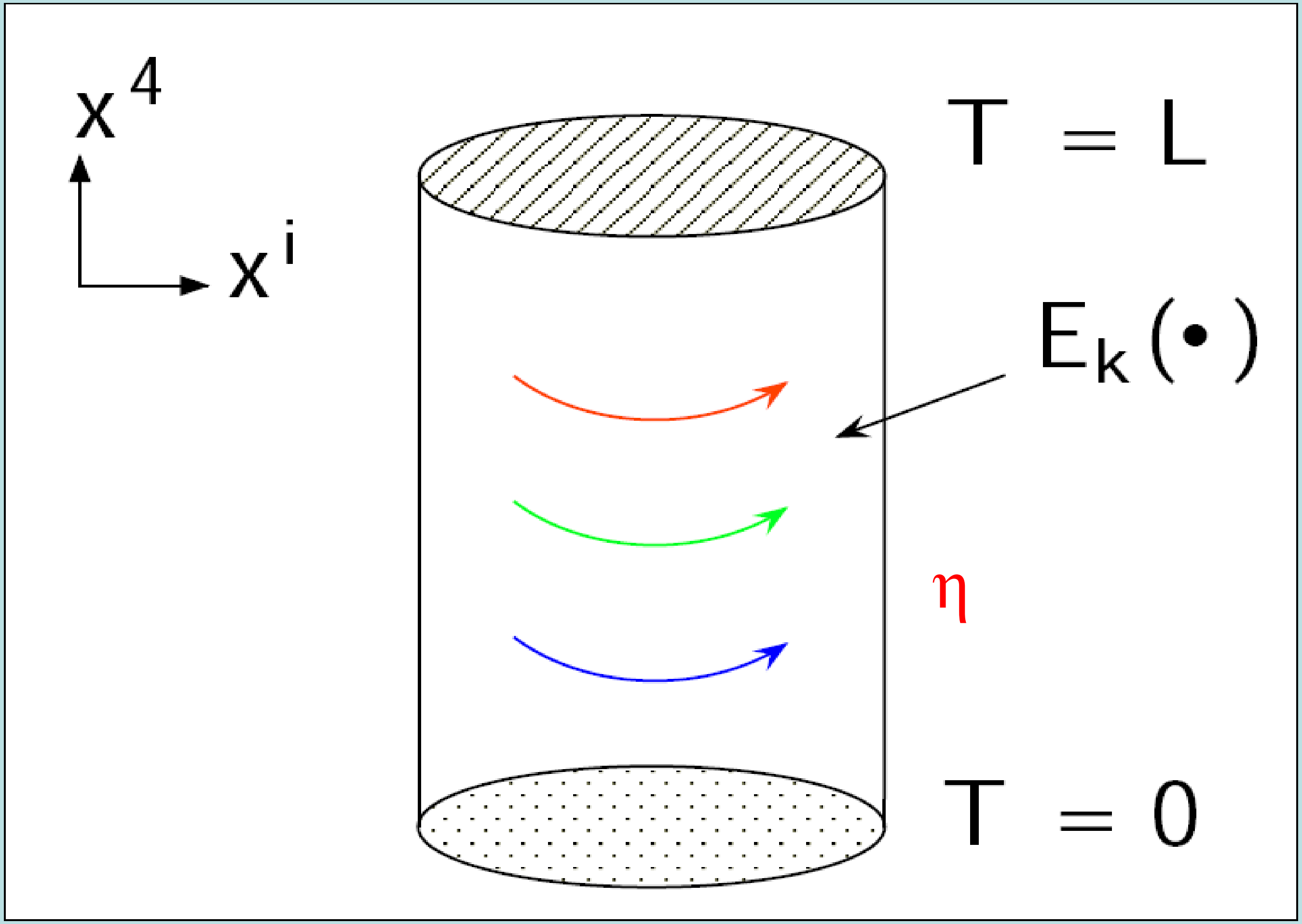
- Transition amplitude from a prescribed state at t=0 to one at t=T (Dirichlet BC).
  (m = 0)
- Euclidean path integral with Dirichlet BC in time and periodic in space (L) to describe a constant chromo-electric background field.

$$Z\left[W,\zeta,\bar{\zeta};W',\zeta',\bar{\zeta}'\right]=$$

η ↓

$$\int\left[DUD\chi D\bar{\chi}\right]e^{-S_G(W,W')-S_F(W,W',\zeta,\bar{\zeta},\zeta',\bar{\zeta}')}$$

Picture



Schroedinger Functional (SF) Running Coupling on Lattice

Define: $$\frac{1}{\overline{g}^2(L,T)} \equiv \frac{-1}{k} \frac{\partial}{\partial \eta} \log Z \Big|_{\eta=0},$$

$$= \frac{1}{g_0^2} + 0(1) + 0\left(g_0^2\right) + \ldots$$

Response of system to small changes in the background field.

$$k = 12\left(\frac{L}{a}\right)^2 \left[\sin\left(\frac{2\pi a^2}{3LT}\right) + \sin\left(\frac{\pi a^2}{3LT}\right)\right]$$

SF Running Coupling

Then, to remove the O(a) bulk lattice artifact

$$\frac{1}{\bar{g}^2(L)} = \frac{1}{2}\left[\frac{1}{\bar{g}^2(L, L-a)} + \frac{1}{\bar{g}^2(L, L+a)}\right]$$

Depends on only one large scale L!

Perturbation Theory

$$L\frac{\partial}{\partial L}\bar{g}^2(L)=\beta\left(\bar{g}^2(L)\right)=b_1\bar{g}^4(L)+b_2\bar{g}^6(L)+b_3\bar{g}^8(L)+\ldots.$$

$$b_1=\frac{2}{(4\pi)^2}\left(11-\frac{2}{3}N_f\right),\qquad b_2=\frac{2}{(4\pi)^4}\left(102-\frac{38}{3}N_f\right)$$

$$b_3=b_3^{\overline{MS}}+\frac{b_2c_2}{2\pi^2}-\frac{b_1\left(c_3-c_2\right)}{8\pi^2}$$

$$b_3^{\overline{MS}}=\frac{2}{(4\pi)^6}\left[\frac{2857}{2}-\frac{5033}{18}N_f+\frac{325}{54}N_f^2\right]$$

At three loops

$N_f = 16$ IRFP at $g^{*2}_{SF} = 0.47$ $\left(\frac{\bar{g}^2}{4\pi^2} \approx .01 \right)$

$N_f = 12$ IRFP at $g^{*2}_{SF} = 5.18$ $\left(\frac{\bar{g}^2}{4\pi^2} \approx .13 \right)$

$N_f \leq 8$ No perturbative IRFP

Lattice Simulations

MILC Code (Heller)

Staggered Fermions

$$N_f = 8, 12$$

Range of Lattice Couplings g_0^2 (= 6/β) and Lattice Sizes L/a → 20

O(a) Lattice Artifacts due to Dirichlet Boundary Conditions

Statistical and Systematic Error

"Don't
Mis-underestimate."

1. Numerical-simulation error

2. Interpolating-function error

3. Continuum-extrapolation error

Statistics Dominates

Lattice Data

$N_f = 8$

$\bar{g}^2(L)$	L/a				
β	6	8	10	12	16
4.550	$13.0(^{+0.4}_{-0.5})$	$17.18(^{+75}_{-58})$			
4.560	$10.66(^{+59}_{-62})$				
4.570	$11.19(^{+41}_{-52})$				
4.580	$10.14(^{+28}_{-25})$				
4.590	$10.11(^{+50}_{-41})$				
4.600	$9.75(^{+20}_{-21})$	$11.6(^{+1.3}_{-1.3})$	$14.46(^{+73}_{-58})$	$16.9(^{+2.8}_{-1.6})$	
4.650	$7.859(^{+110}_{-97})$	$9.64(^{+29}_{-25})$	$11.24(^{+37}_{-33})$	$17.5(^{+1.4}_{-1.0})$	
4.700	$6.90(^{+13}_{-13})$	$8.13(^{+57}_{-52})$	$10.87(^{+79}_{-55})$		
4.800	$5.621(^{+90}_{-86})$	$6.54(^{+25}_{-22})$	$7.1(^{+0.2}_{-0.2})$	$7.6(^{+0.5}_{-0.3})$	$9.69(^{+190}_{-86})$
4.900	$4.758(^{+52}_{-52})$	$5.16(^{+18}_{-16})$			
5.000	$4.201(^{+65}_{-54})$	$4.67(^{+15}_{-14})$	$4.98(^{+15}_{-14})$	$5.60(^{+30}_{-29})$	$6.32(^{+57}_{-39})$
5.100	$3.785(^{+42}_{-39})$	$4.214(^{+100}_{-90})$			
5.200	$3.379(^{+39}_{-38})$	$3.676(^{+72}_{-52})$			
5.300	$3.087(^{+24}_{-24})$	$3.308(^{+39}_{-33})$			
5.400	$2.970(^{+27}_{-30})$	$3.145(^{+44}_{-37})$			
5.500	$2.724(^{+23}_{-22})$	$2.962(^{+34}_{-33})$	$3.121(^{+86}_{-44})$	$3.347(^{+84}_{-73})$	$3.361(^{+55}_{-63})$
5.600	$2.603(^{+11}_{-11})$	$2.794(^{+44}_{-30})$			
5.700	$2.443(^{+8}_{-11})$	$2.590(^{+13}_{-12})$			
5.800	$2.3334(^{+82}_{-90})$	$2.490(^{+15}_{-14})$			
5.830	$2.2860(^{+110}_{-99})$	$2.456(^{+15}_{-15})$	$2.534(^{+13}_{-12})$	$2.643(^{+25}_{-20})$	$2.836(^{+44}_{-35})$
5.900	$2.2240(^{+87}_{-88})$	$2.340(^{+13}_{-11})$			
6.000	$2.1375(^{+56}_{-54})$	$2.263(^{+11}_{-10})$			
6.100	$2.0579(^{+64}_{-65})$	$2.161(^{+9}_{-10})$			
6.200	$1.9776(^{+43}_{-48})$	$2.0684(^{+80}_{-89})$			
6.300	$1.9129(^{+67}_{-55})$	$2.0003(^{+85}_{-80})$			
6.400	$1.8426(^{+91}_{-53})$	$1.9230(^{+57}_{-55})$			

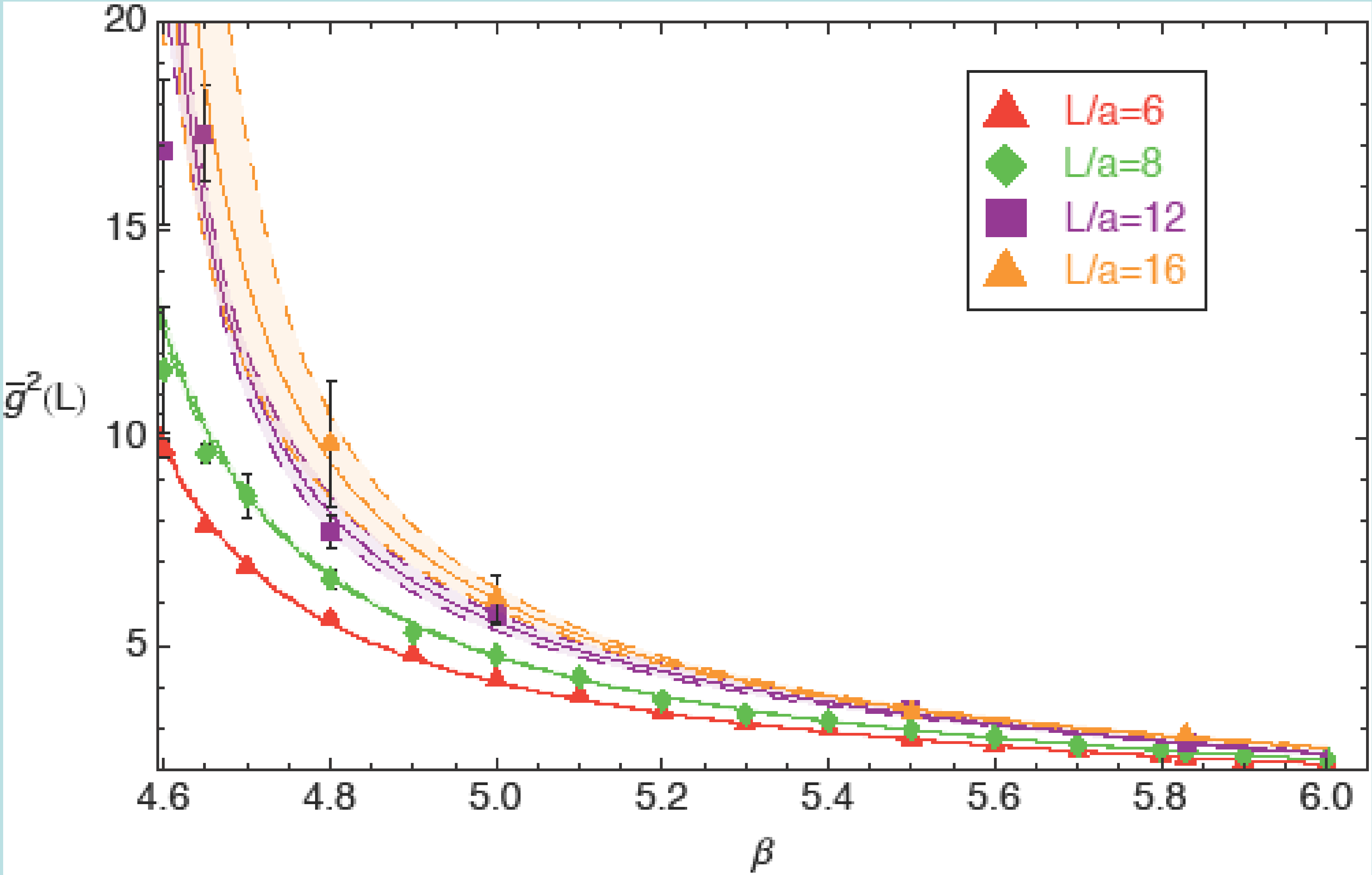
Perturbation Theory

$$\frac{1}{\overline{g}^2(\beta, L/a)} = \frac{1}{g_0^2}\left[1 + O(g_0^2)\right] = \frac{\beta}{6}\left[1 + O(\frac{1}{\beta})\right]$$

Polynomial Fit N_f = 8, 12

$$\frac{1}{\overline{g}^2(\beta, L/a)} = \frac{\beta}{6}\left[1 - \sum_{i=1}^{n} c_{i,L/a}\left(\frac{6}{\beta}\right)^i\right]$$

$N_f = 8$ Data with Fits



Fit Parameters $N_f = 8$

	L/a			
param	6	8	12	16
$c_{1,L/a}$	$0.4655(^{+98}_{-99})$	$0.4966(^{+57}_{-56})$	$33(^{+27}_{-29})$	$1.00(^{+18}_{-19})$
$c_{2,L/a}$	$-0.17(^{+12}_{-12})$	$-0.194(^{+48}_{-38})$	$-120(^{+110}_{-100})$	$-0.98(^{+38}_{-37})$
$c_{3,L/a}$	$1.18(^{+58}_{-54})$	$0.82(^{+10}_{-12})$	$170(^{+140}_{-150})$	$0.59(^{+19}_{-19})$
$c_{4,L/a}$	$-3.2(^{+1.2}_{-1.4})$	$-1.02(^{+12}_{-10})$	$-105(^{+93}_{-85})$	0
$c_{5,L/a}$	$4.5(^{+1.6}_{-1.5})$	$0.454(^{+36}_{-40})$	$24(^{+19}_{-21})$	0
$c_{6,L/a}$	$-3.26(^{+84}_{-93})$	0	0	0
$c_{7,L/a}$	$0.92(^{+21}_{-20})$	0	0	0
χ^2/dof	1.50	2.09	6.52	1.72
N_{dof}	55	44	2	2

Renormalization Group (Step Scaling)

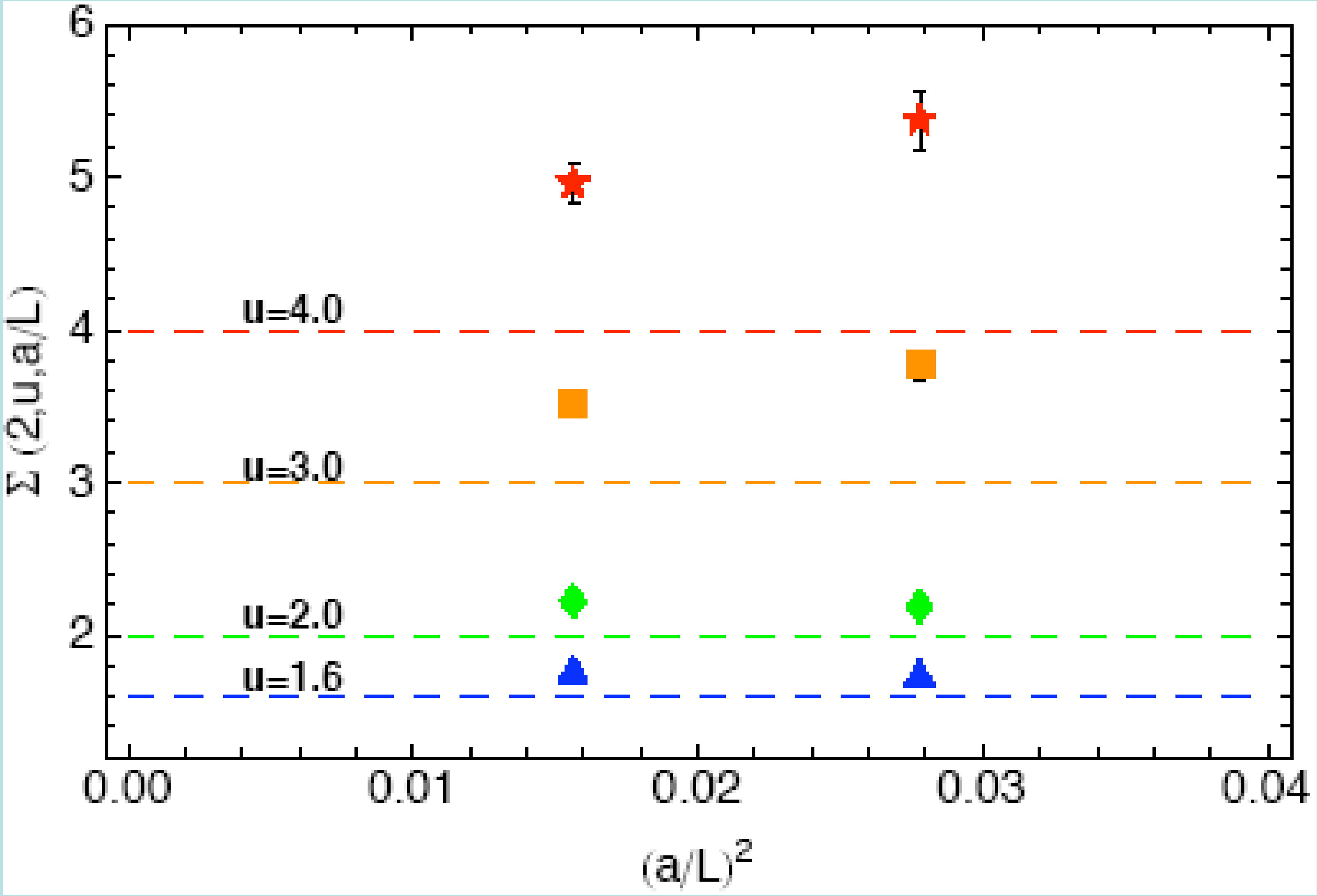
$$\bar{g}^2\left(g_0^2, \frac{a}{L}\right) = \bar{g}^2\left(\bar{g}^2(L_0), \frac{L}{L_0}, \frac{a}{L_0}\right)$$

$$g_0^2 \xrightarrow{a\to 0} 1/\ln\left(L_0/a\right)$$

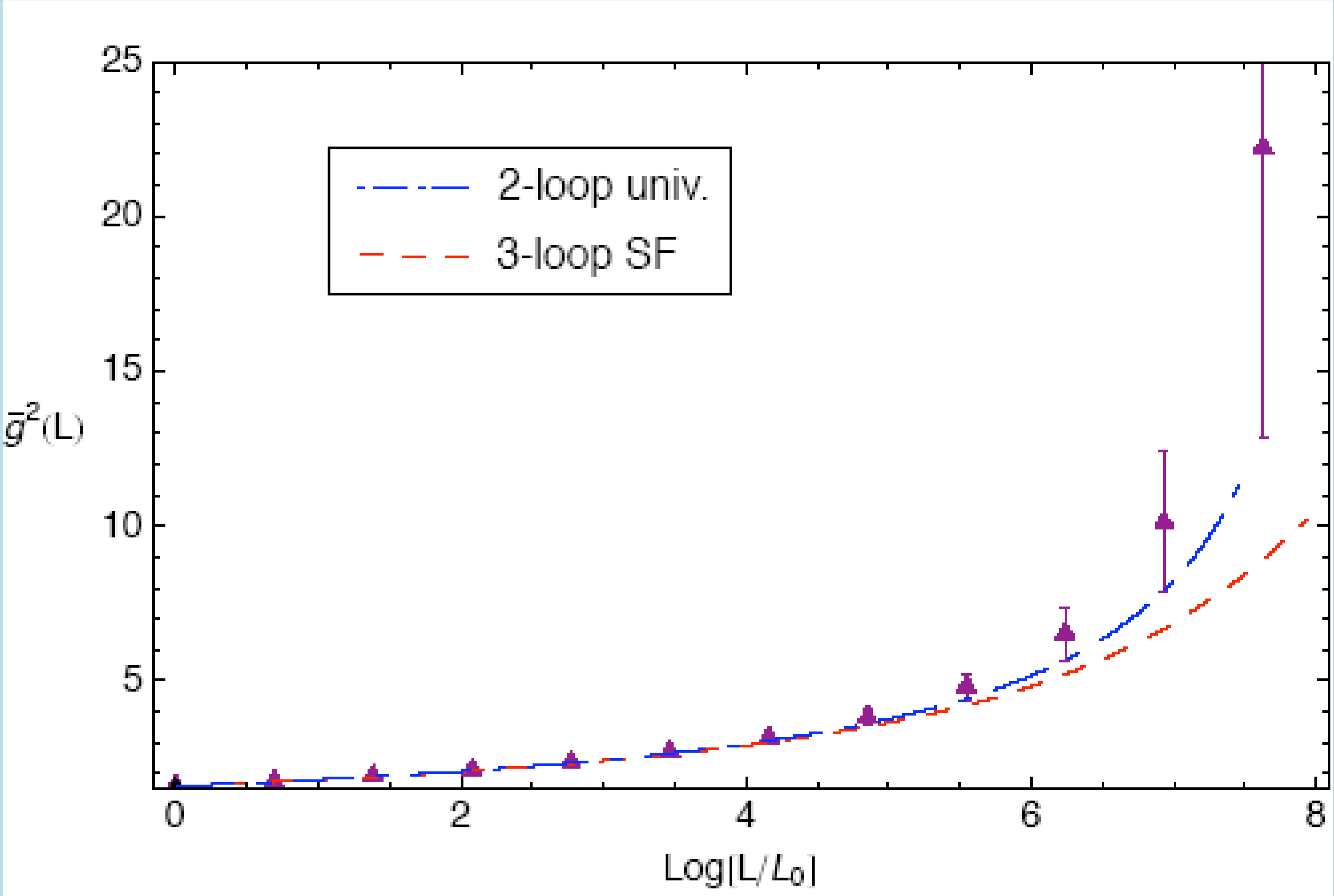
$$\xrightarrow{a/L_0\to 0} \bar{g}^2\left(\bar{g}^2(L_0), \frac{L}{L_0}\right) \equiv \bar{g}^2\left(\frac{L}{L_0}\right) \qquad \left(\frac{L}{L_0} = 2\right)$$

$$\bar{g}^2\left(\bar{g}^2(L), \frac{L'}{L}, \frac{a}{L}\right) \xrightarrow{\frac{a}{L}\to 0} \bar{g}^2\left(\frac{L'}{L}\right) \qquad \left(\frac{L'}{L} = 2\right)$$

N_f=8 Extrapolation Curve



N_f = 8 Continuum Running



N_f = 8 Features

1. No evidence for IRFP or even inflection point up through $\bar{g}^2(L) \approx 20$.

2. Exceeds rough estimate $\left(\alpha_c^* / \pi \approx 1/4\right)$ of strength required to break chiral symmetry, and therefore produce confinement. Must be confirmed by direct lattice simulations.

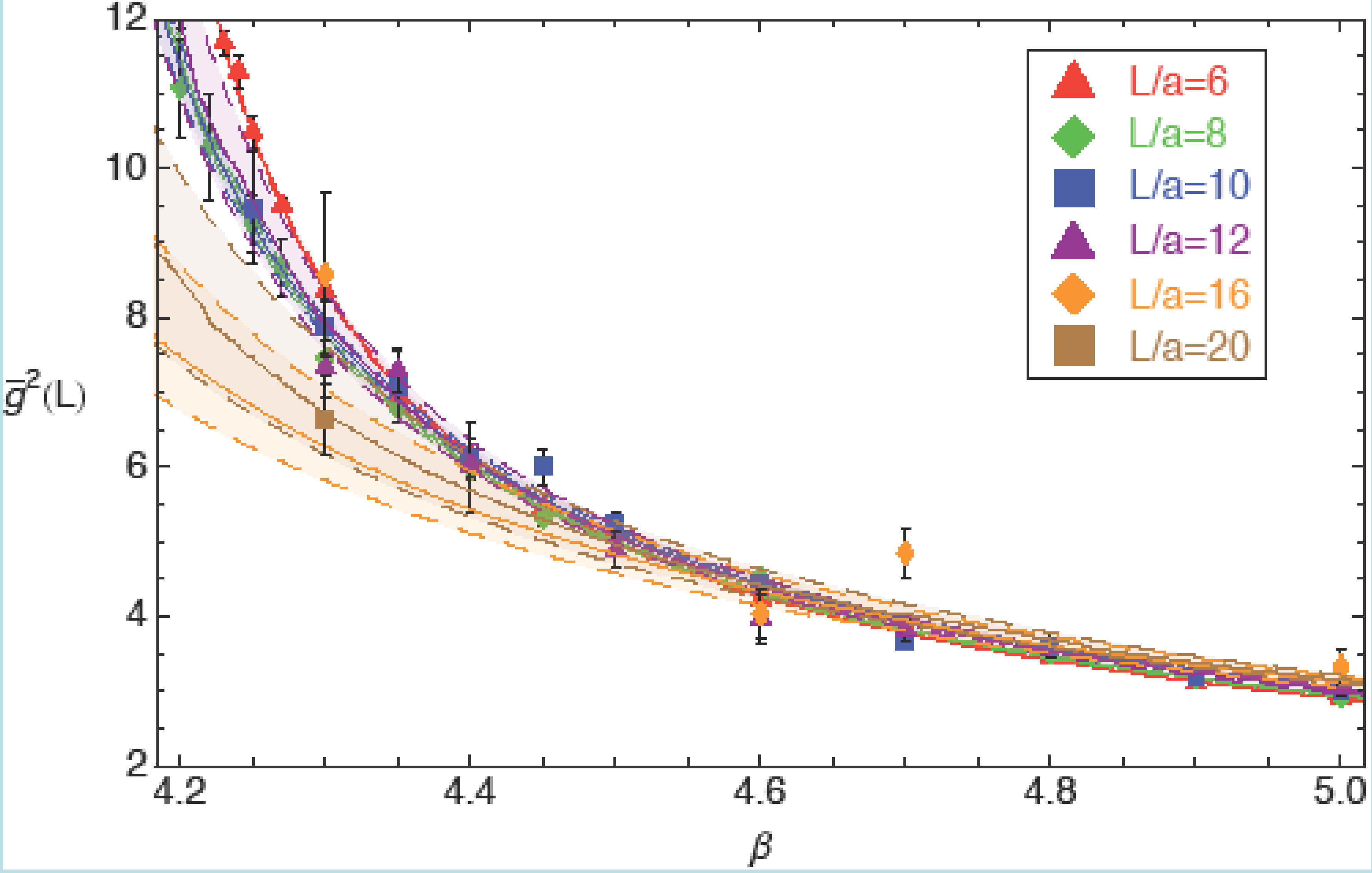
3. Rate of growth exceeds 3 loop perturbation theory.

4. Behavior similar to quenched theory [ALPHA N.P. Proc. Suppl. 106, 859 (2002)] and N_f=2 theory [ALPHA, N.P. B713, 378 (2005)], but slower growth as expected.

Lattice Data
N_f = 12

$\bar{g}^2(L)$	L/a					
β	6	8	10	12	16	20
4.100	$67.6(^{+5.3}_{-4.7})$	$25.5(^{+1.4}_{-1.2})$				
4.130	$30.9(^{+1.2}_{-1.2})$	$20.0(^{+1.5}_{-1.2})$				
4.150	$23.1(^{+1.3}_{-1.2})$	$14.2(^{+1.1}_{-0.9})$	$15.54(^{+98}_{-87})$			
4.170	$19.0(^{+0.7}_{-0.6})$					
4.180	$17.74(^{+49}_{-52})$	$13.38(^{+100}_{-86})$	$12.8(^{+0.9}_{-0.7})$			
4.190	$15.43(^{+38}_{-36})$					
4.200	$14.82(^{+25}_{-23})$	$10.92(^{+66}_{-55})$	$12.8(^{+1.6}_{-1.2})$			
4.210	$13.53(^{+28}_{-27})$					
4.220	$12.39(^{+30}_{-32})$	$10.4(^{+0.8}_{-0.8})$				
4.230	$11.65(^{+23}_{-25})$					
4.240	$11.21(^{+28}_{-24})$					
4.250	$10.51(^{+25}_{-21})$	$9.11(^{+35}_{-36})$	$9.44(^{+110}_{-75})$			
4.270	$9.44(^{+13}_{-13})$	$8.62(^{+35}_{-35})$				
4.300	$8.28(^{+18}_{-17})$	$7.4(^{+0.3}_{-0.3})$	$7.95(^{+39}_{-35})$	$7.32(^{+41}_{-42})$	$8.8(^{+1.3}_{-1.0})$	$6.71(^{+80}_{-49})$
4.350	$6.96(^{+12}_{-13})$	$6.72(^{+23}_{-21})$	$6.80(^{+38}_{-37})$	$7.52(^{+35}_{-34})$		
4.400	$6.080(^{+85}_{-71})$	$6.04(^{+13}_{-13})$	$6.07(^{+30}_{-23})$	$5.91(^{+54}_{-47})$		
4.450	$5.456(^{+59}_{-62})$	$5.3(^{+0.1}_{-0.1})$	$5.86(^{+22}_{-23})$			
4.500	$5.170(^{+94}_{-92})$	$5.03(^{+10}_{-11})$	$5.26(^{+15}_{-12})$	$5.01(^{+27}_{-19})$		
4.600	$4.253(^{+60}_{-50})$	$4.48(^{+10}_{-12})$	$4.399(^{+100}_{-75})$	$3.90(^{+31}_{-31})$	$3.84(^{+20}_{-16})$	
4.700	$3.830(^{+69}_{-52})$	$3.737(^{+52}_{-45})$	$3.664(^{+130}_{-90})$	$3.81(^{+28}_{-31})$	$4.9(^{+0.6}_{-0.5})$	
4.800	$3.475(^{+28}_{-27})$	$3.477(^{+45}_{-43})$	$3.60(^{+20}_{-13})$			
4.900	$3.107(^{+27}_{-25})$	$3.196(^{+46}_{-39})$	$3.207(^{+64}_{-52})$			
5.000	$2.890(^{+24}_{-19})$	$2.918(^{+38}_{-31})$	$3.020(^{+76}_{-62})$	$3.024(^{+94}_{-70})$	$3.12(^{+25}_{-18})$	
5.100	$2.739(^{+36}_{-27})$	$2.819(^{+78}_{-69})$	$2.804(^{+20}_{-19})$			
5.200	$2.542(^{+13}_{-10})$	$2.576(^{+27}_{-22})$				
5.300	$2.474(^{+33}_{-29})$	$2.469(^{+26}_{-20})$	$2.531(^{+26}_{-21})$			

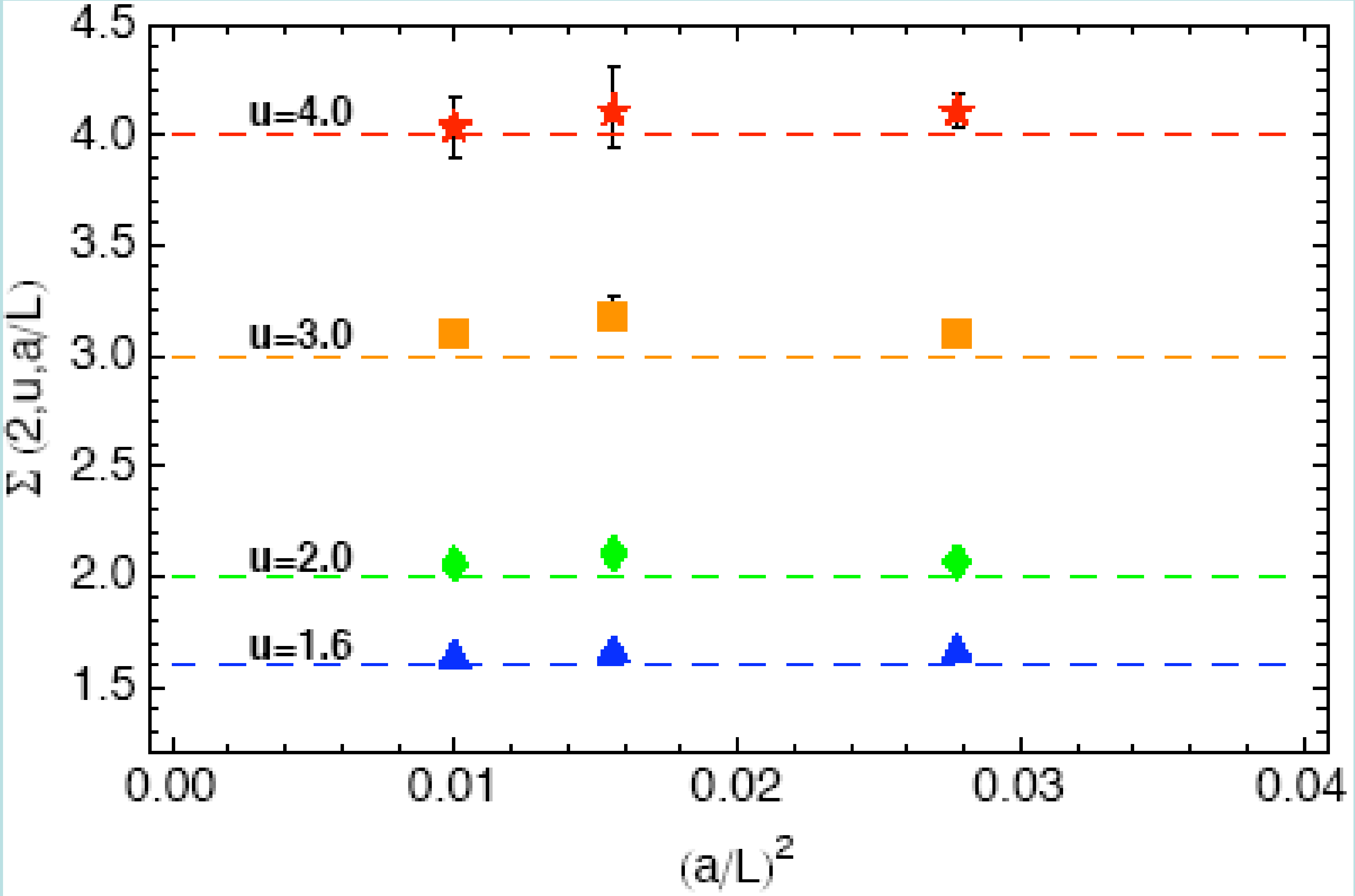
$N_f = 12$ Data with Fits



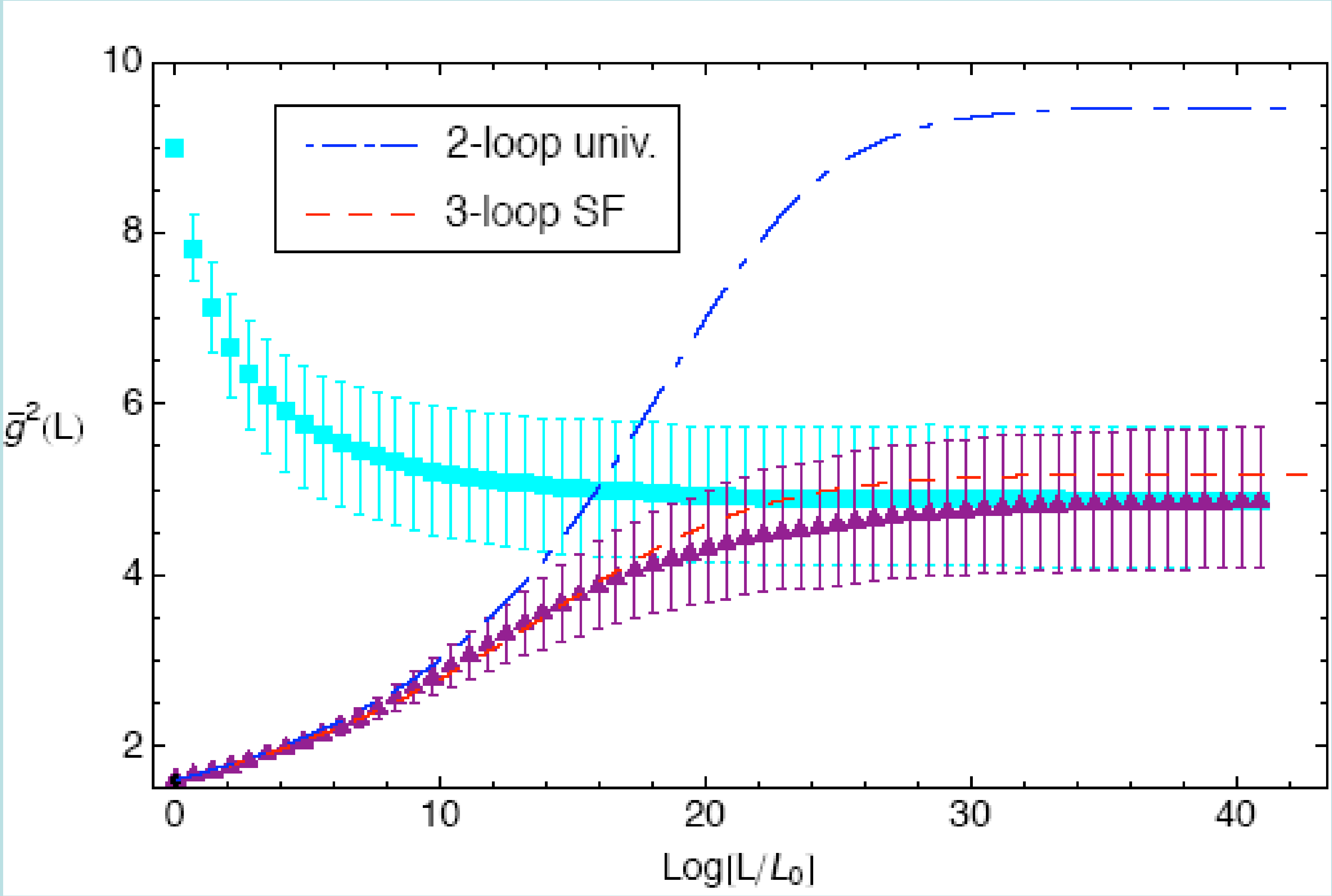
Fit Parameters $N_f = 12$

	L/a					
param	6	8	10	12	16	20
$c_{1,L/a}$	$0.381(^{+12}_{-12})$	$0.4053(^{+60}_{-60})$	$0.4181(^{+83}_{-77})$	$0.405(^{+12}_{-11})$	$0.450(^{+35}_{-40})$	$0.541(^{+80}_{-84})$
$c_{2,L/a}$	$-0.09(^{+14}_{-13})$	$-0.178(^{+41}_{-43})$	$-0.206(^{+59}_{-61})$	$-0.095(^{+89}_{-100})$	$-0.110(^{+88}_{-80})$	$-0.27(^{+17}_{-16})$
$c_{3,L/a}$	$0.59(^{+56}_{-58})$	$0.731(^{+96}_{-95})$	$0.77(^{+15}_{-15})$	$0.61(^{+26}_{-24})$	$0.135(^{+46}_{-49})$	$0.204(^{+78}_{-81})$
$c_{4,L/a}$	$-1.4(^{+1.2}_{-1.1})$	$-0.859(^{+83}_{-89})$	$-0.85(^{+15}_{-15})$	$-0.81(^{+26}_{-28})$	0	0
$c_{5,L/a}$	$1.8(^{+1.2}_{-1.2})$	$0.354(^{+27}_{-26})$	$0.336(^{+51}_{-46})$	$0.345(^{+96}_{-91})$	0	0
$c_{6,L/a}$	$-1.27(^{+61}_{-62})$	0	0	0	0	0
$c_{7,L/a}$	$0.36(^{+13}_{-12})$	0	0	0	0	0
χ^2/dof	1.63	1.62	1.47	1.58	1.68	1.27
N_{dof}	55	53	36	16	8	1

N_f=12 Extrapolation Curve



N_f = 12 Continuum Running



Approach to Fixed Point

$$\overline{\beta}\left(\overline{g}^2(L)\right) \simeq \gamma\left[\overline{g}_*^2 - \overline{g}^2(L)\right]$$

$$\overline{g}^2(L) \to \overline{g}_*^2 - \frac{\text{const}}{L^\gamma}$$

$$Fit: \gamma = 0.15 \pm^{0.03}_{0.04}$$

$$3 - loop: \gamma = 0.296$$

Conclusions

1. First lattice evidence that for an SU(3) gauge theory with N_f Dirac fermions in the fundamental representation

   $8 < N_{fc} < 12$

2. N_f=12: Relatively weak IRFP

3. N_f=8: Confinement and chiral symmetry breaking.

Employing the Schroedinger functional running coupling defined <u>at</u> the box boundary L

Things to Do

1. Refine the simulations at N_f = 8 and 12 and examine other values such as N_f =10.

2. Study the phase transition as a function of N_f.

3. Consider other gauge groups and representation assignments for the fermions

   Sannino et al

4. Examine physical quantities such as the static potential (Wilson loop) and correlation functions. LSD

5. Examine chiral symmetry breaking directly:

<ψ ψ> at zero temperature LSD

6. Apply to BSM Physics. Is S naturally small as $N_f \rightarrow N_{fc}$ due to approximate parity doubling?

$$S\left(m_{H,ref}\right)=4\int_0^\infty \frac{ds}{s}\left\{\left[\mathrm{Im}\,\Pi_{VV}(s)-\mathrm{Im}\,\Pi_{AA}(s)\right]-\frac{1}{48\pi}\left[1-\left(1-\frac{m_{H,ref}}{s}\right)^3\theta\left(s-m_{H,ref}^2\right)\right]\right\}$$

LSD

LSD **Collaboration**

Lattice Strong Dynamics

J. C. Osborn
Argonne National Laboratory

R. Babich, R. C. Brower, M. A. Clark, C. Rebbi, D. Schaich
Boston University

M. Cheng, T. Luu, R. Soltz, P. M. Vranas
Lawrence Livermore National Laboratory

T. Appelquist, G. T. Fleming, E. T. Neil
Yale University

Rapidly Growing Activity

Lattice-2008 Applications beyond QCD

Thomas DeGrand	Exploring the phase diagram of sextet QCD
Albert Deuzeman	The physics of eight flavours
Michael Endres	Numerical simulation of N=1 supersymmetric Yang-Mills theory
Philipp Gerhold	Higgs mass bounds from a chirally invariant lattice Higgs-Yukawa model with overlap fermions
Joel Giedt	Domain Wall Fermion Lattice Super Yang Mills
Ari Hietanen	Spectrum of SU(2) gauge theory with two fermions in the adjoint representation
Kieran Holland	Probing technicolor theories with staggered fermions
Xiao-Yong Jin	Lattice QCD with Eight Degenerate Quark Flavors
Biagio Lucini	Orientifold Planar Equivalence: The Chiral Condensate
Ethan Neil	The Conformal Window in SU(3) Yang-Mills
Daniel Nogradi	Nearly conformal electorweak sector with chiral fermions
Elisabetta Pallante	Searching for the conformal window
Agostino Patella	Fermions in higher representations. Some results about SU(2) with adjoint fermions.
Benjamin Svetitsky	Nonperturbative infrared fixed point in sextet QCD