Sea quark content of the nucleon from lattice QCD

-A key quantity for the direct detection of neutralino DM -

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arXiv:0806.4744[hep-lat] (PRD78:054502,2008)

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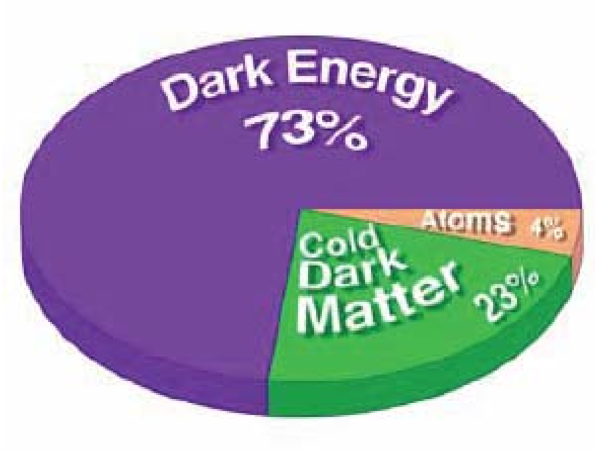
Outline

1. Introduction
2. Basic Methods
3. Lattice calculation
4. Results
5. Comparisons with other results
6. Discussion and summary

1. Introduction

WMAP found existence of Cold Dark Matter

= Evidence of New Physics



Candidates: neutralino, gravitino, axion, ….

In SUSY GUT, neutralino is one of the promising candidates,

but it may not be unique.

Direct detection of the dark matter is important

for finding the property and constituents of DM.

Neutralino(LSP) as DM candidate

- Neutralino is a combination of gaugino and higgsino.

$$\chi = a_1 \tilde{B} + a_2 \tilde{W}_3 + a_3 \tilde{H}_1 + a_4 \tilde{H}_4$$

- Mass and couplings depend on the SUSY breaking parameters, which should be determined by LHC and ILC

- Then one can predict the direct DM detection rate, which can be measured independently by experiment.

A crucial test for identifying the DM constituent.

Detection rate R

$$R \propto \sigma_{\chi N} \times \frac{\rho}{M} \int dv v f(v)$$

DM-Nuclear cross-section

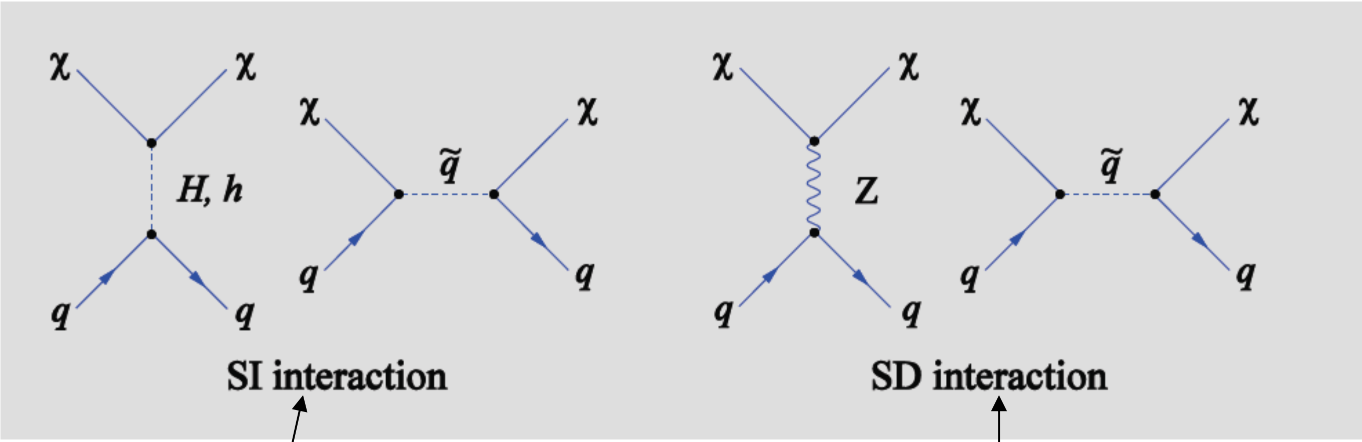
ρ : DM density at the earth
$f(v)$: velocity profile

Determined from SUSY model

Determined from N-body simulation

$\sigma_{\chi N}$ arises from the interaction with quark

- Spin-independent interaction : higgs exchange
- Spin-dependent interaction : Z and squark exchange



Coherent enhancement for large Atomic number

$\sigma_{\chi N}^{\mathrm{SI}} \propto A^2$

No Coherent enhancement

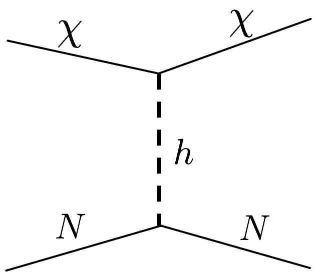
$\sigma_{\chi N}^{\mathrm{SD}} \propto (\text{total spin})$

Spin independent interaction is much larger.

Why sea quark content is important?

- A crucial parameter for the WIMP dark matter detection rate.

The interaction with nucleon is mediated by the higgs boson exchange in the t-channel.



Higgs Yukawa coupling of the proton

$$y_{Hpp} = \frac{m_p}{250\mathrm{MeV}}\left[\frac{2}{27} + \frac{25}{27} f_{T_s}\right] + \cdots$$

heavy quark loop

strange quark

K. Griest, Phys.Rev.Lett.62,666(1988)

Phys,Rev,D38, 2375(1988)

Baltz, Battaglia, Peskin, Wizanksy

Phys. Rev. D74, 103521 (2006).

Sigma term is the parameter to convert

the quark yukawa coupling to nucleon yukawa coupling.

Note: strange quark contribution is dominant.

Nucleon sigma term for strange quark is most important.

What is sigma term ?

- scalar form factor of the nucleon at zero recoil

$$\sigma_{\pi N} \equiv m_{ud}\left(\langle N|\bar{u}u+\bar{d}d|N\rangle - V\langle 0|\bar{u}u+\bar{d}d|0\rangle\right),$$

where $\langle N(p)|N(p')\rangle = (2\pi)^3\delta^3(p-p'),\ \ m_{ud} \equiv \frac{m_u+m_d}{2}$

V : spatial volume

- related quantities

$$y \equiv \frac{2\left(\langle N|\bar{s}s|N\rangle - V\langle 0|\bar{s}s|0\rangle\right)}{\langle N|\bar{u}u+\bar{d}d|N\rangle - V\langle 0|\bar{u}u+\bar{d}d|0\rangle}$$

$$f_{T_s} \equiv \frac{m_s\left(\langle N|\bar{s}s|N\rangle - V\langle 0|\bar{s}s|0\rangle\right)}{m_N}$$

We will denote $\langle N|\bar{q}q|N\rangle - V\langle 0|\bar{q}q|0\rangle$ as $\langle N|\bar{q}q|N\rangle$ for simplicity.

How well is sigma term known?

Theory	Group		method	$\sigma_{\pi N}$ [MeV]	y
ChPT	Gasser et al.(91)	1-loop	spectrum	44(9)	~ 0.2
	Borasoy-Meissner(96)	1-loop	spectrum	48(10)	0.2(2)
	Borasoy-Meissner(97)	2-loop	spectrum	36(7)	0.21(20)
Lattice	Kuramashi et al(95)	$n_f = 0$ Wilson	3pt/2pt	40-60	0.66(15)
	Dong et al(96)	$n_f = 0$ Wilson	3pt/2pt	50(3)	0.36(3)
	SEASAM(99)	$n_f = 2$ Wilson	3pt/2pt	18(5)	0.59(13)
	UKQCD(02)	$n_f = 2$ Clover	spectrum(unsubtracted)		0.53(12)
			spectrum(subtracted)		-0.30(34)
ChPT+Lattice	Procura et al.(04)	NNLO, $n_f = 2$ Clover	spectrum	49(3)	

$\sigma_{\pi N}$=30-50 MeV from ChPT

y=0-0.2 from ChPT, y=0-0.5 from lattice QCD

The strange quark content has 100% uncertainty.

Theoretical expectation

Down-type Higgs Yukawa coupling

$$y_{NNH_d} \propto \langle N| \underbrace{m_d\bar{d}d}_{20-30\text{MeV}} + \underbrace{m_s\bar{s}s}_{0-360\text{MeV}} + \underbrace{m_b\bar{b}b}_{\sim 60\text{MeV}} |N\rangle \times \tan(\beta)$$

Chiral perturbation theory (ChPT)
+ πN scattering exp. Data
(connected+ disconnected)

Quark model, Lattice
(disconnected)

Trace anomaly formula
(disconnected)

$$\begin{aligned} m_N &= \langle N|T^\mu_\mu|N>, \\ T^{mu}_\mu &= \frac{\beta^{n_f}(\alpha)}{4\alpha}G^2_{\mu\nu} + \sum_{i=1}^{n_f} m_{q_i}\bar{q}_i q_i, \\ \lim_{m_Q\to\infty} &= \langle N|m_Q\bar{Q}Q|N\rangle_{\text{disc.}} = \frac{\beta(\alpha)^{n_f=3+1} - \beta(\alpha)^{n_f=3}}{\beta(\alpha)^{n_f=3}} m_N \end{aligned}$$

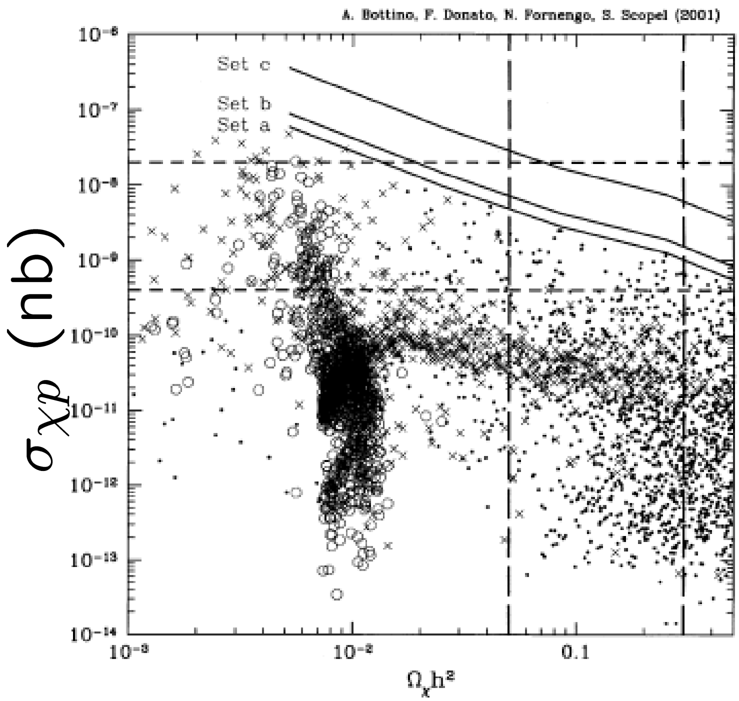
Strange quark contribution can be dominant but with large uncertainty.
(Huge enhancement in the strange quark mass region?)

Naïve question: Can disconnected contribution really so big?

Experimental status

SUSY model prediction of DM cross-section $\sigma_{\chi p}$ choosing $y = 0.2$

Bottino et al. Astroparticle Phys.18(2002)205



Present senstivity

New experiments XMASS, SuperCDMS
can improve the sensitivity
by 2 orders of magnitude.

Determination of y is getting
more and more important.

Our goal

Determine the nucleon sigma term in unquenced QCD using the dynamical quark (overlap fermion), which has an exact chiral symmetry on the lattice.

- The advantage of the exact chiral symmetry
  - No power divergence → subtraction of the vacuum condensate is numerical much more stable
  - No unwanted operator mixing
- In this study, we work in nf=2 unquenched QCD
  nf=2+1 QCD will be studied very soon
- We exploit mass spectrum method (explained later)

2. Basic Methods

Basic Methods

Method 1 : Ratio of 3-pt, 2-pt functions

Define the following ratio of 3-pt,2pt functions

$$
\begin{aligned}
R(t) &= \frac{C_3(t)}{C_2(t)} \\
C_2(t) &\equiv \int d^3x \langle 0|T\left[N(t,\vec{x})N^\dagger(0)\right]|0\rangle \\
C_3(t) &\equiv \int d^3x \int d^4y \langle 0|T\left[N(t,\vec{x})\bar{q}q(t_y,\vec{y})N^\dagger(0)\right]|0\rangle
\end{aligned}
$$

Sigma term can be extracted

from the contribution linear in t as

$$R(t) = \text{const} + t\left[\langle N|\bar{q}q(0)|N\rangle - V\langle 0|\bar{q}q(0)|0\rangle\right] + \cdots$$

Proof: Insert the complete set of states and look at the lowest state

$$
\begin{aligned}
C_2(t) &= e^{-m_N t}\langle 0|N(0)|N\rangle\langle N|N^\dagger(0)|0\rangle + \sum_{n=1} e^{-m_{N_n} t}\langle 0|N(0)|N_n\rangle\langle N_n|N^\dagger(0)|0\rangle \\
C_3(t) &= \int d^3x d^3y \int_0^t dt_y \sum_{n,m}\langle 0|N(t,\vec{x})|n\rangle\langle n|\bar{q}q(t_y,\vec{y})|m\rangle\langle m|N^\dagger(0)|0\rangle \\
&\quad + \int d^3x d^3y \int_t^T dt_y \sum_{n,m}\langle 0|\bar{q}q(t_y,\vec{y})|n\rangle\langle n|N(t,\vec{x})|m\rangle\langle m|N^\dagger(0)|0\rangle \\
&= t e^{-m_N t}\langle 0|N(0)|N\rangle\langle N|\bar{q}q(0)|N\rangle\langle N|N^\dagger(0)|0\rangle \\
&\quad + V\,(T-t)\,e^{-m_N t}\langle 0|S(0)|0\rangle\langle 0|N(0)|N\rangle\langle N|N^\dagger(0)|0\rangle \\
&\quad + \text{ excited states} + \text{ contact terms}
\end{aligned}
$$

$$
\boxed{\Rightarrow \qquad R(t) = \text{const} + t\,[\langle N|S(0)|N\rangle - V\langle 0|S(0)|0\rangle] + \text{ exponentially suppressed terms}}
$$

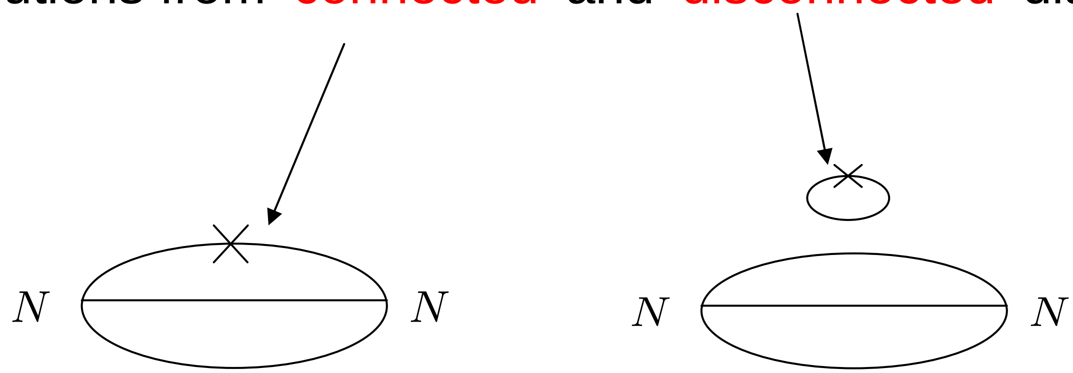
Basic Methods

Method 2: nucleon mass spectrum

Feynman - Hellman theorem

$$\frac{dm_N}{dm_q} = \langle N|\bar{q}q|N\rangle$$

Moreover, partial derivatives with respect to the valence and sea quark masses give contributions from ‘connected’ and ‘disconnected’ diagrams.



$$\frac{\partial m_N(m_{\mathrm{val}}, m_{\mathrm{sea}})}{\partial m_{\mathrm{val}}} = \langle N|\bar{q}q|N\rangle_{\mathrm{conn.}} \qquad \frac{\partial m_N(m_{\mathrm{val}}, m_{\mathrm{sea}})}{\partial m_{\mathrm{sea}}} = \langle N|\bar{q}q|N\rangle_{\mathrm{disc.}}$$

Proof: differentiate 2pt function

Differentiate the 2-pt function

$$\frac{dC_2(t)}{dm_q} \equiv \int d^3x \int d^4y \langle 0|T\left[N(t,\vec{x})\left(-\bar{q}q(t_y,\vec{y})\right)N^\dagger(0)\right]|0\rangle$$
$$-\int d^3x \int d^4y \langle 0|T\left[N(t,\vec{x})N^\dagger(0)\right]|0\rangle\langle 0|\left(-\bar{q}q(t_y,\vec{y})\right)|0\rangle$$
$$= -C_3(t) + C_2(t) \times TV\langle 0|\bar{q}q(0)|0\rangle$$

Then take the ratio with 2-pt function

$$\frac{d}{dm_q} ln\left(C_2(t)\right) = -R(t) + TV\langle 0|\bar{q}q(0)|0\rangle$$
$$\text{LHS} = -\frac{dm_N}{dm_q} t + \text{ exponentially suppressed terms}$$
$$\text{RHS} = -\left[\langle N|\bar{q}q(0)|N\rangle - V\langle 0|(0)|0\rangle\right] t + \text{ exponentially suppressed terms}$$

The term linear in t gives the Feynman-Hellman theorem

Ratio method vs spectrum method

- They treat identical quantities: the t-linear term of R(t).

  The only difference is that one take the derivative with respect quark mass before or after the path-integral. No fundamental advantage or disadvantage.

- In practice, the contamination from excited states is the source of systematic error

- Spectrum method automatically gives

  the measurement of S for all spacetime points.

3. Lattice Calculation

Recent simlations in unquenched QCD

Many unquenched simulations are performed or starting now.
In addition to rooted staggered by MILC collab.,
Wilson-type fermions and Ginsparg-Wilson fermions are in progress.
Important for cross-check and theoretically clean

Group	Action	n_f	a (fm)	m_π (MeV)
MILC	Staggered	2+1	0.09, 0.12	$\geq$300
Del Debbio et al.	Wilson, O(a)-imp Wilson	2	0.052-0.075	$\geq$ 300
PACS-CS	O(a)-imp Wilson	2+1	0.07, 0.10, 0.12	$\geq$210
ETMC	twisted Wilson	2	0.075, 0.096	$\geq$270
JLQCD	Overlap	2 (2+1)	0.11	$\geq$300
RBC UKQCD	Domain wall	2+1	0.09-0.13	$\geq$ 310

Chiral symmetry on the lattice

- Nielsen-Ninomiya's theorem

  Nielsen and Ninomiya, Nucl.Phys.B185(1981) 20

  Consider a lattice fermion action $S_F = \bar{\psi} D \psi$ satisfying

  - Translational invariance $D\gamma_5 + \gamma_5 D = 0$
  - Chiral symmetry:
  - Hermiticity
  - Locality

  Then, doublers must exist

Wilson fermion : broken chiral symmetry, symmetry recovered in continuum.

Staggered fermion: 4 spinors $\otimes$ 4 "tastes" (doublers)

to apply QCD (u,d,s) one must take "the fourth root trick" $det(D) \sim det(D_{staggered})^{1/4}$

Very dangerous compromise! Even locality is doubtful.

Ginsparg-Wilson fermion

- Ginsparg-Wilson relation

Ginsparg and Wilson, Phys.Rev.D 25(1982) 2649.

$$D\gamma_5 + \gamma_5 D = aD\gamma_5 D$$

Exact chiral symmetry on the lattice (index theorem)

Hasenfratz, Laliena and Niedermayer, Phys.Lett. B427(1998) 125

Luscher, Phys.Lett.B428(1998)342.

$$\begin{aligned} \psi &\rightarrow \psi + i\gamma_5(1 - aD)\psi = \psi + i\hat{\gamma}_5\psi \\ \bar{\psi} &\rightarrow \bar{\psi} + i\bar{\psi}\gamma_5 \end{aligned}$$

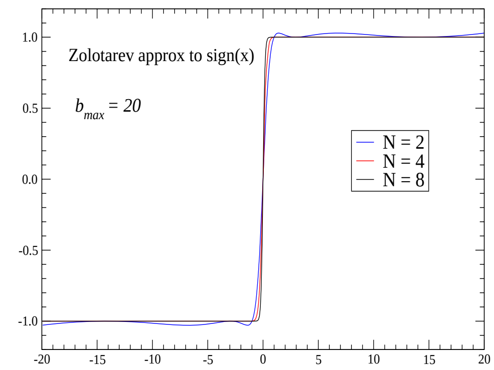
- Overlap fermion (explicit construction)

$$D = \frac{1}{a}\left[1 + \gamma_5 sign(H_W)\right], \qquad H_W \equiv \gamma_5(D_W + M_0)$$

D_W : Wilson Dirac op., $\qquad M_0$: negative mass

Problems（all related to the zeros of Hw）

- We make rational approximation with completely controlled error except near zero mode.



$$sign(H_W) = \frac{H_W}{\sqrt{H_W^2}} \sim H_W(p_0 + \sum_{l=1}^{N} \frac{p_l}{H_W^2 + q_l})$$

- D makes a discontinous jump when an eigenmode of Hw crosses zero. Hybrid Monte Carlo breaks down.

- A method to cure this problem has been developed. One has to monitor the zero crossing at much higher precision and include correction terms at the exact point of crossing. (Hopelessly huge numerical cost)

Strategy by JLQCD

Topology conserving Det(Hw) term

Fukaya, Vranas, Fukaya et al. hep-lat/0607020

Introduce negative heavy mass wilson fermion as a UV regulator field, whose mass is exactly the same as that appears in Dov. Infrared physics is unchanged.

$$Z = \int DU \frac{det(H_W^2)}{det(H_W^2+\mu^2)} det(D_{\mathrm{ov}})^2 e^{-S}$$

This term should kill the breakdown of locality topology change, and blow-up of numerical cost simultaneouly.

Status of JLQCD2 GW project

KEK BlueGene (10 racks, 57.3 TFlops)

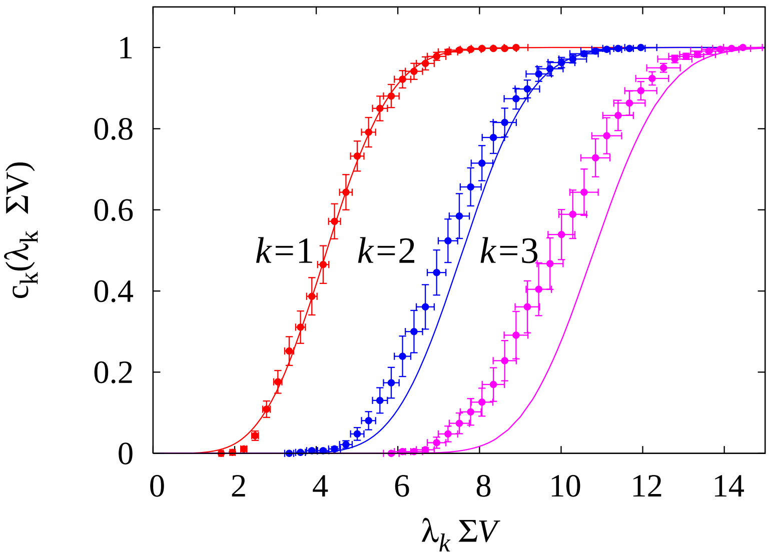
- Started on March 1, 2006
- 1rack=1024 nodes
- PowerPC440(700MHz,2.8Gflops)
- 1node=2CPU, 4MB L3 cache, 512MB memory
- network= 3D torus(half-rack) (8x8x8) +global tree
- 24^3x48 Wilson fermion inversion sustained speed = 28% of the peak speed

16^3x32 … slightly lower sustained speed

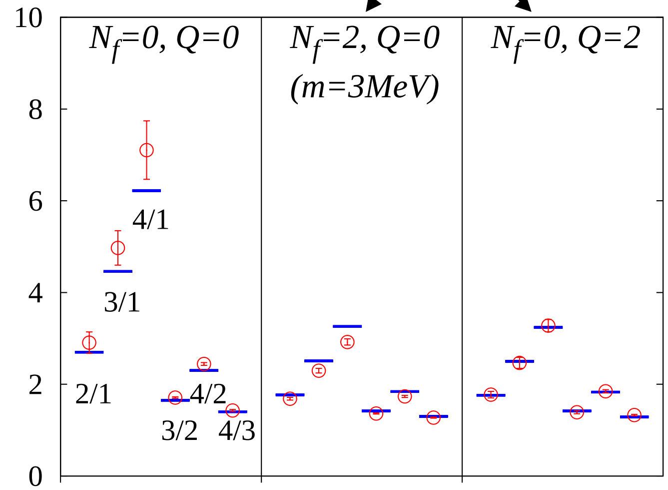
QCD in ϵ regime (Run1)

- Eigenmode distribution is consistent with Chiral Random Matrix model up to finite volume corrections.

flavor-topology duality



Cumulative distribution of low eigenvalues



Low Eigenvalue ratios

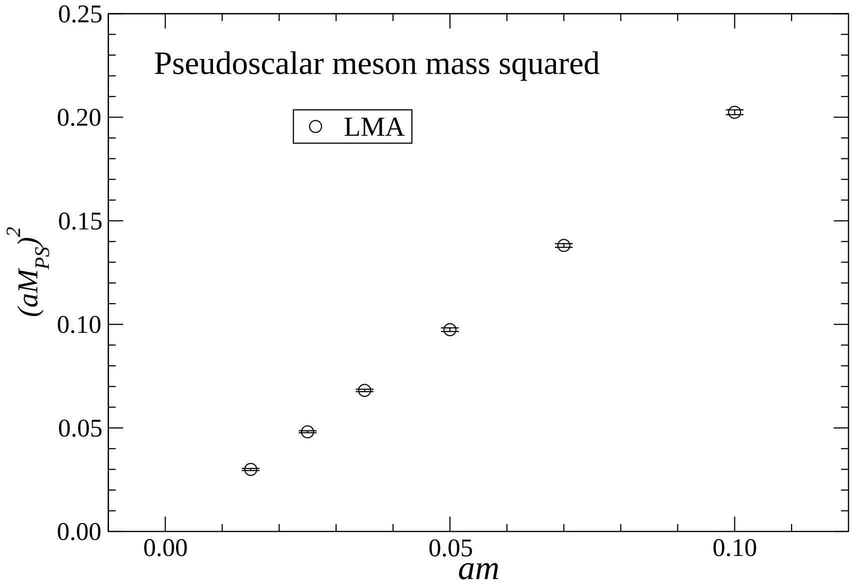
$$\Sigma^{\overline{MS}}_{n_f=2}(2GeV) = (251 \pm 7 \pm 11\text{MeV})^3$$

H. Fukaya et al. [JLQCD&TWQCD]

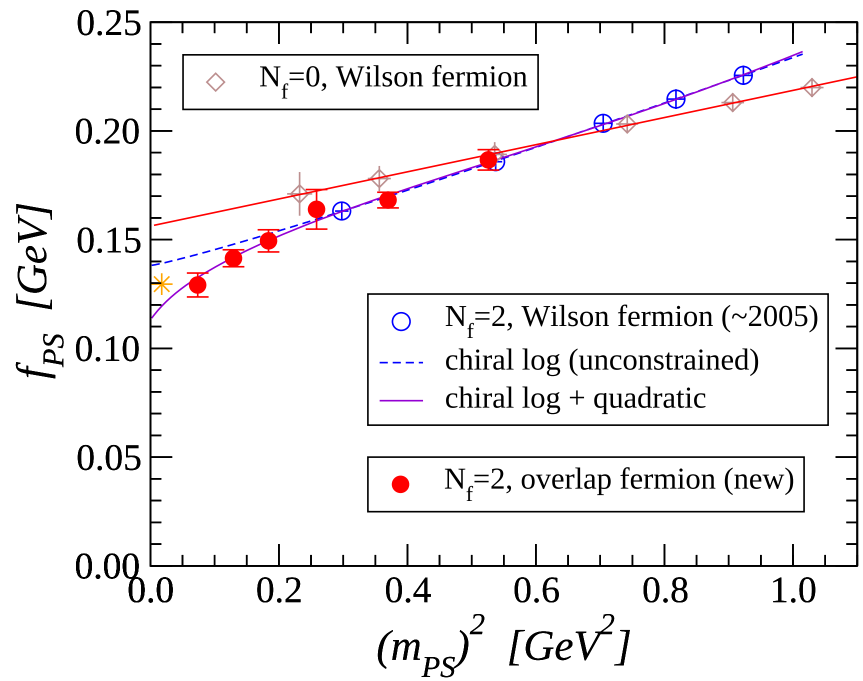
Phys. Rev. Lett. 98, 172001 (2007)

QCD in normal regime (Run2)

Nf=2



Quark mass dependence of the pion mass



Quark mass dependence of the decay const

Dynamical overlap projects by JLQCD collab.

- Run 1 (epsilon-regime) Nf=2: 16^3x32, a=0.11fm

  ε-regime (m_{sea} ~ 3MeV)
  - 1,100 trajectories with length 0.5
  - *20-60 min/traj on BG/L 1024 nodes*
  - Q=0

- Run 2 (p-regime) Nf=2: 16^3x32, a=0.12fm

  6 quark masses covering (1/6~1) m_s
  - 10,000 trajectories with length 0.5
  - *20-60 min/traj on BG/L 1024 nodes*
  - Q=0, Q=−2,−4 (m_{sea} ~ m_s/2)

Nucleon sigma term

- Run 3 (p-regime) Nf=2+1 : 16^3x48, a=0.11fm (in progress)
  - 2 strange quark masses around physical m_s
  - 5 ud quark masses covering (1/6~1)m_s
  - Trajectory length = 1
  - About 2 hours/traj on BG/L 1024 nodes

Numerical simulation

Measurement of the nucleon 2pt function

- 6pts(sea) and 9pts(valence) for quark masses
- Low mode averaging is employed (#eigenmodes=100)

$$
\begin{aligned}
a^{-1} &= 1.67\ \text{GeV} \\
am_{\text{val}} &= 0.015, 0.025, 0.035, 0.050, 0.060, \\
&\quad 0.070, 0.080, 0.090, 0.100
\end{aligned}
$$

Nf=2 overlap fermion configurations

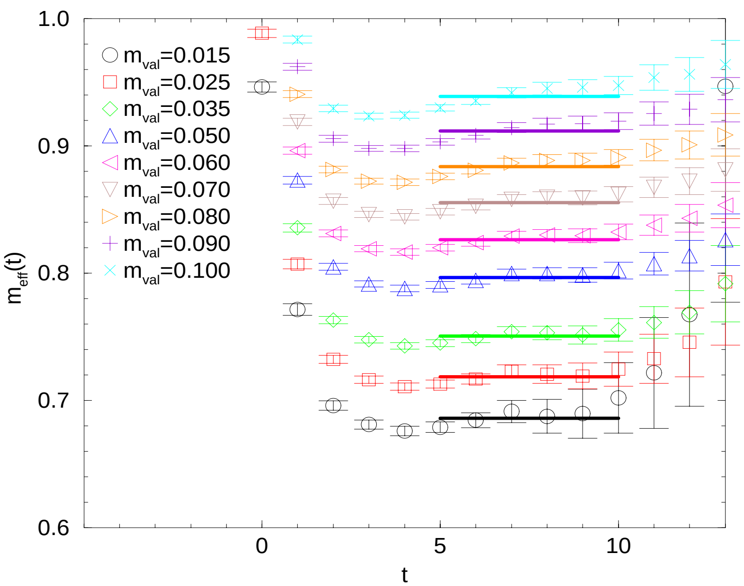
- 16^3 x 32, a=0.12 fm, L=1.9 fm
- 6 values of sea quark mass
- fixed topology
- At Q=0 accumulated 10,000 trajectories

am_{sea}	m_π [GeV]	confs
0.015	0.3063(20)	500
0.025	0.3905(14)	500
0.035	0.4635(14)	500
0.050	0.5549(14)	500
0.070	0.6608(11)	500
0.100	0.7993(15)	500

4. Results

Results

- Nucleon masses from 2-pt functions



Effective mass plot for amq=0.035 Solid lines are the mass from the fit

Chiral extrap. (unitary point)

—extraction of nucleon sigma term —

- Fit without lightest quark mass data(5pts)
  - several fit forms to study chiral extrapolation errors
- Fit with finite volume correction (5 and 6pts)
  - fits including finite volume effects estimated by ChPT.

ChPT Fit of nucleon mass spectrum

Fit formula with Heavy Baryon chiral perturbation theory

$$\text{Fit 0}: \quad m_N = m_0 - 4c_1 m_\pi^2 - \frac{3g_A^2}{32\pi f_\pi^2} m_\pi^3 + e_1 m_\pi^4$$

$$\text{Fit I}: \quad m_N = m_0 - 4c_1 m_\pi^2 - \frac{3g_A^2}{32\pi f_\pi^2} m_\pi^3 + [e_1 - \frac{3g_A^2}{64\pi^2 f_\pi^2 m_0}(1 + 2\log\frac{m_\pi}{\mu})] m_\pi^4 + \frac{3g_A^2}{256\pi f_\pi^2 m_0^2} m_\pi^5$$

$$\text{Fit II,III}: \quad m_N = m_0 - 4c_1 m_\pi^2 - \frac{3g_A^2}{32\pi f_\pi^2} m_\pi^3 + [e_1 - \frac{3}{64\pi^2 f_\pi^2}(\frac{g_A^2}{m_0} - \frac{c_2}{2}) - \frac{3}{32\pi^2 f_\pi^2}(\frac{g_A^2}{m_0} - 8c_1 + c_2 + 4c_3)\log\frac{m_\pi}{\mu}] m_\pi^4 + \frac{3g_A^2}{256\pi f_\pi^2 m_0^2} m_\pi^5$$

c.f. E. E. Jenkins et. al., PLB255,558 (1991)

M. Procura et. al. PRD69, 034505(2004)

g_A: axial coupling of nucleon

Phenomenological value: $g_A = 1.267$

I : $O(p^3)$

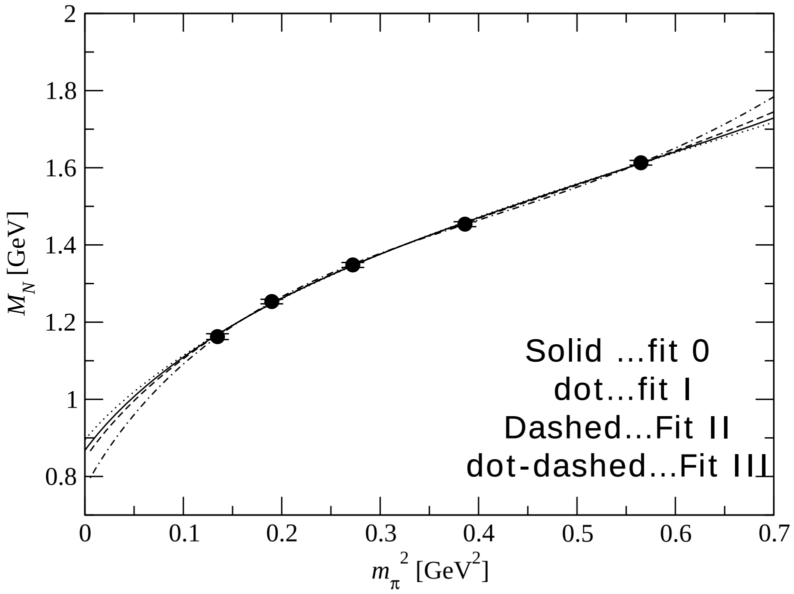
II : $O(p^4)$with input $c_2 = 3.2[\mathrm{GeV}^{-1}]$, $c_3 = -3.4[\mathrm{GeV}^{-1}]$

III : $O(p^4)$with input $c_2 = 3.2[\mathrm{GeV}^{-1}]$, $c_3 = -4.7[\mathrm{GeV}^{-1}]$

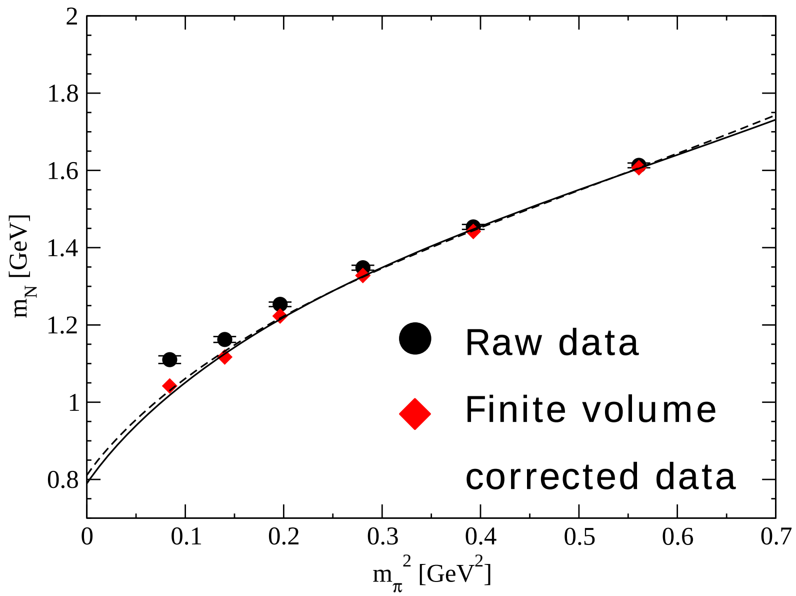
(0 : simplified version of Fit I)

Fit results with and without finite volume corrections

raw data



Finite volume corrected(Fit 0)



Nicely fit to the ChPT formula without lightest point.

Fit uncertainty is O(10)%.

Successful with all data point

Results of sigma term

[MeV]	$\sigma_{\pi N}$ (5 pt)	$\sigma_{\pi N}$ with FSE (5 pt)	(6 pt)
Fit 0	52.2(1.8)	56.7(1.8)	55.1(1.5)
Fit I	45.1(1.7)	48.9(1.7)	47.2(1.5)
Fit II	56.5(1.2)	59.5(1.2)	58.2(1.0)
Fit III	71.8(1.2)	75.1(1.2)	72.7(1.0)

1. The systematic error is mainly the chiral extrap. error.
2. Finite volume effect (FVE) is sub-leading (~ 9%).
3. We quate final results from Fit 0(FVE uncorrected).

$$\sigma_{\pi N} \;=\; 52(2)_{\mathrm{stat}}\left({}^{+20}_{-7}\right)_{\mathrm{extrap.}}\left({}^{+5}_{-0}\right)_{\mathrm{FVE}}\,[\mathrm{MeV}]$$

PQChPT fit (partially quenched data points)

——extraction of y parameter ——

- Fit with partially quenched ChPT (5 X 8 data points)
  - consistency check of the unitary point fit
  - interpolation to the strange quark mass.
- Separate extraction of connected and disconnected contributions

PQChPT fit function

J.W. Chen et al.,PRD65,094001(2002)

S.R. Beane et al.NPA709,319 (2002)

$$\boxed{\begin{aligned} m_N = \; & B_{00} + B_{10} m_{val} + B_{01} m_{sea} + B_{11} m_{sea} m_{val} + B_{20} m_{val}^2 + B_{02} m_{sea}^2 \\ & -\frac{1}{16\pi f_\pi^2}\Big\{\frac{g_A^2}{12}\left[-7(m_\pi^{vv})^3 + 16(m_\pi^{vs})^3 + 9m_\pi^{vv}(m_\pi^{ss})^2\right] \\ & +\frac{g_1^2}{12}\left[-19(m_\pi^{vv})^3 + 10(m_\pi^{vs})^3 + 9m_\pi^{vv}(m_\pi^{ss})^2\right] \\ & +\frac{g_1 g_A}{3}\left[-13(m_\pi^{vv})^3 + 4(m_\pi^{vs})^3 + 9m_\pi^{vv}(m_\pi^{ss})^2\right]\Big\} \end{aligned}}$$

$(m_\pi^{vv})^2 = Am_{val}$, $(m_\pi^{vs})^2 = \frac{A}{2}(m_{val} + m_{sea})$, $(m_\pi^{ss})^2 = Am_{sea}$, A: constant

g_A, g_1: axial couplings of nucleon

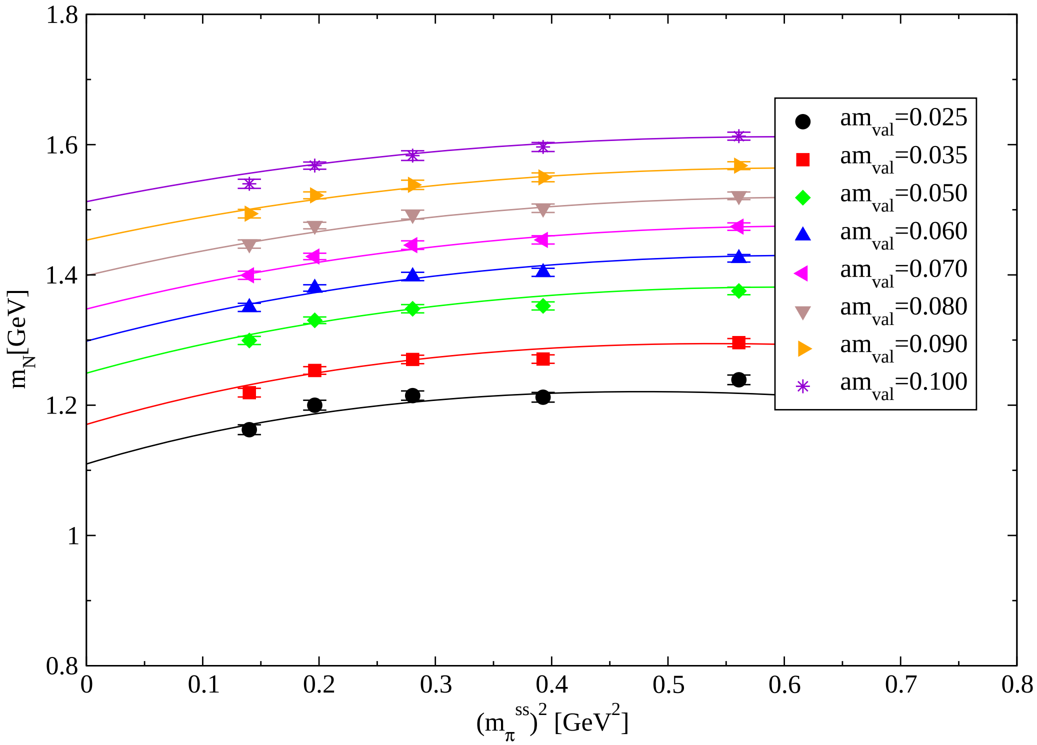
Phenomenological values:

$g_A = 1.267$, $g_1 = -(0.4 - 0.6)$

- Fit a: 6 parameters $B_{00}, B_{01}, B_{10}, B_{11}, B_{20}, B_{02}$ with fixed g_A, g_1.
- Fit b: 7 parameters $B_{00}, B_{01}, B_{10}, B_{11}, B_{20}, B_{02}, g_1$ with fixed g_A.
- Fit b: 8 parameters $B_{00}, B_{01}, B_{10}, B_{11}, B_{20}, B_{02}, g_A, g_1$.

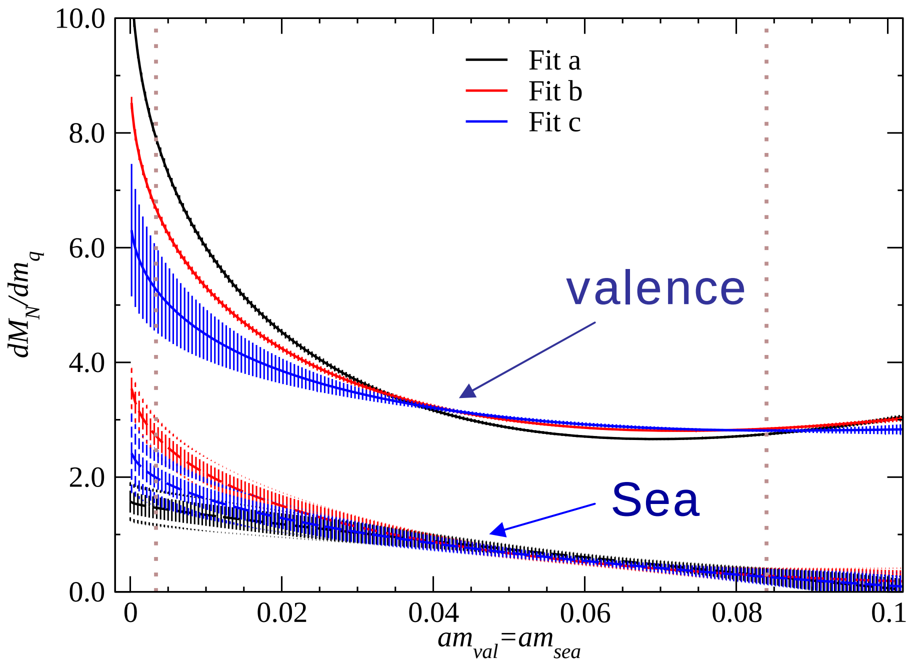
Fit results (PQChPT)

	B_{00} [GeV]	B_{01} [GeV^{-1}]	B_{10} [GeV^{-1}]	B_{11} [GeV^{-3}]	B_{20} [GeV^{-3}]	B_{02} [GeV^{-3}]	g_1	g_A	χ^2/d.o.f.
Fit a	0.87(2)	0.47(10)	3.37(3)	-0.94(2)	3.77(2)	0.17(15)	[-0.66]	[1.267]	1.82
Fit b	0.86(2)	1.13(11)	2.71(4)	0.97(8)	1.81(15)	0.25(15)	-0.378	[1.267]	1.28
Fit c	0.92(2)	0.76(23)	1.98(39)	0.43(31)	0.95(43)	-0.03(23)	-0.29(5)	0.93(22)	1.28



PQChPT fit works very well.
It gives consistent results
with the unitary point fit.

Connected and disconnected contributions at $m_{\text{val}} = m_{\text{sea}}$



fit	m_q	$\partial M_N/\partial m_{\text{val}}$	$\partial M_N/\partial m_{\text{sea}}$
Fit (a)	0.0034	7.92(8)	1.47(32)
Fit (b)	0.0034	6.68(8)	2.72(33)
Fit (c)	0.0034	5.3(8)	1.99(50)
Fit (a)	0.084	2.75(3)	0.28(14)
Fit (b)	0.084	2.84(3)	0.28(14)
Fit (c)	0.084	2.81(3)	0.26(14)

The disconnected contribution (sea quark content) is always smaller than the connected contribution (valence quark content).

Connected and disconnected contributions at $m_{\mathsf{val}} = m_{\mathsf{sea}}$

- Strictly speaking, it is not possible to extract the strange quark content within two-flavor QCD.

  For the final result, we should wait for 2+1-flavor QCD result (coming soon).

- We present semi-quenched estimate of the y parameter

Semi quenched estimate of y

$$y \equiv \frac{\left.\frac{dM_N}{dm_{sea}}\right|_{m=m_s}}{\left.\left(\frac{dM_N}{dm_{val}}+\frac{dM_N}{dm_{sea}}\right)\right|_{m=m_{ud}}}$$

$$\Downarrow$$

$$y^{N_f=2} = 0.030(16)_{\mathsf{stat.}}\left({}^{+6}_{-8}\right)_{\mathsf{extrap.}}\left({}^{+1}_{-2}\right)_{m_s}$$

5. Comparison with other results

Comparison with other results

– Our results of $\sigma_{\pi N}$ is consistent with ChPT .
– Finite Volume correction is controllable.

– Previous lattice result of sea/valence is larger than 1.
  Our result is 0~0.3.
– ChPT predicts $y = 0 - 0.4$
– Previous lattice results $y \sim 0.5$ due to large sea quark contribution without removing lattice artifact.
– After removing lattice artifact previous y is -0.3(3)
– Our result gives $y \sim 0.03$

Uncertainties in y parameter

- ChPT: Low Energy Constants (higher order).
- Previous lattice calculations (Wilson type fermion).

Mixing of connected and disconnected contributions

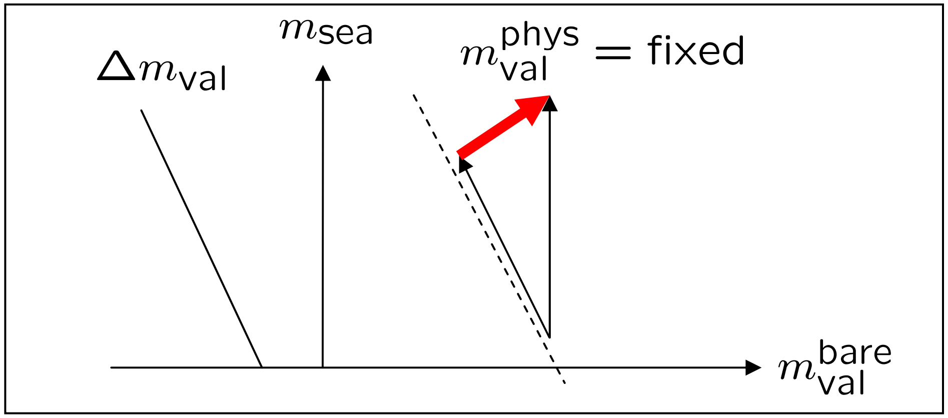
(Matrix methods and spectrum methods) due to lattice artifact.

The most crucial uncertainty is the additive mass shift.

c.f. C. Michael et.al. Nucl. Phys. Proc. Suppl. 106, 293 (2002)

Sea quark mass derivative with fixed bare valence quark mass is contaminated by physical valence quark mass derivative Which is unwanted lattice artifact (red arrow).

Spectrum methods with Wilson type fermions



Operator mixing due to Wilson fermion artifact

If there is an additive mass shift there can be operator mixing which should be subtracted (but not subtracted except for UKQCD) for disconnected diagram.

$$\begin{aligned}&\left(\bar{\psi}_{\mathsf{sea}}\psi_{\mathsf{sea}}\right)^{\mathsf{lat}}\\ =\ & C_0 I + Z_S\left[\left(\bar{\psi}_{\mathsf{sea}}\psi_{\mathsf{sea}}\right)^{\bar{M}S} + \frac{\partial\Delta m_q}{\partial m_{\mathsf{sea}}}\left(\bar{\psi}_{\mathsf{val}}\psi_{\mathsf{val}}\right)^{\bar{M}S}\right] + O(a)\end{aligned}$$

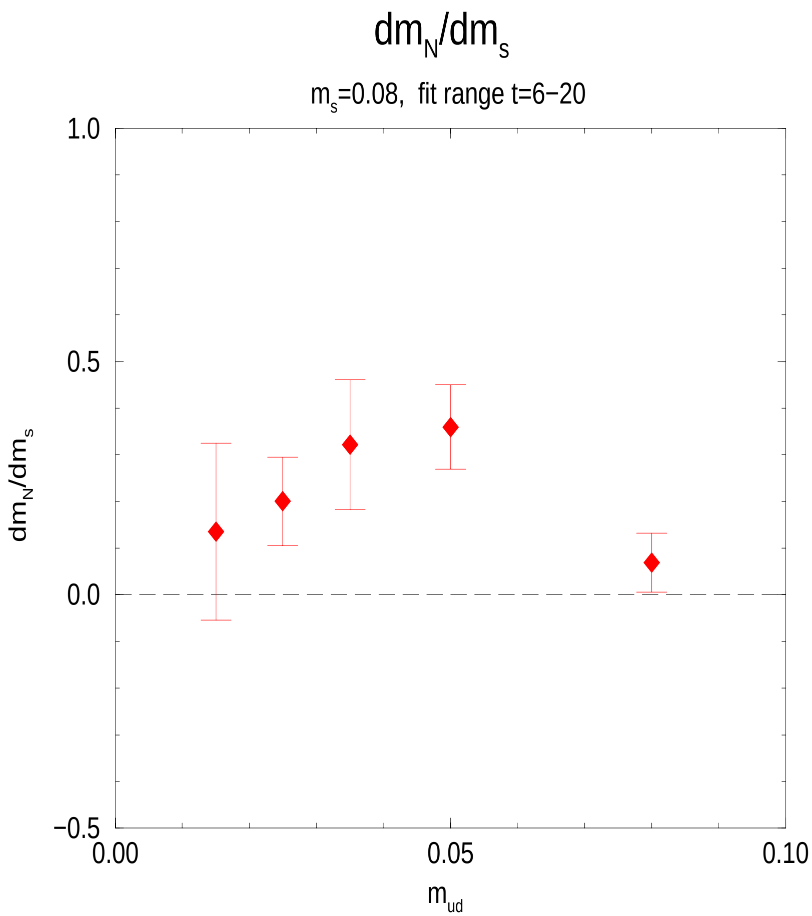
Subtracting this mixing effect by using the sea quark mass dependence of the quark mass shift, the disconnected contribution becomes tiny (consistent with zero).

6. Discussion and summary

Discussion and summary

- We studied the nucleon mass spectrum for nf=2 unquenched QCD using exactly chirally symmetric dynamical fermion.
- It is expected that our calculation is free from dangerous lattice artifacts (power divergence, operator mixing)
- Our result is consistent with ChPT prediction.
- We found disconnected (strange quark content) part is tiny.
- We pointed out that the descrepancies from previous lattice calculation can come from artifact in Wilson fermion.

Preliminary results in 2+1 flavor QCD



Consistent with 2-flavor QCD
suggesting smaller values
for y parameter