Using Quantum Interference to measure the spins of new particles

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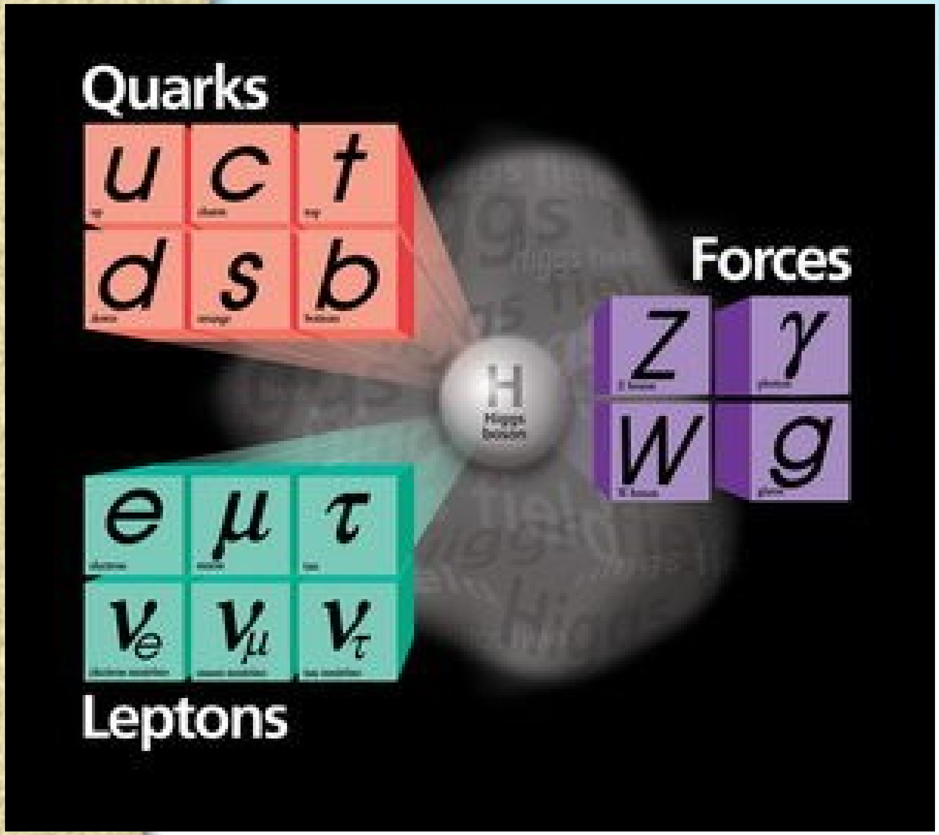
Presentation Outline

- Why is it important to measure spin?
- Measuring spin at the ILC
- Using Quantum Interference of Helicity Amplitudes to measure spin
- Challenge of spin measurement at the LHC
- Randall-Sundrum gravitons
- Application of this technique to the RS graviton case at the LHC

The Standard Model



- Describes Electro-magnetic, Weak and Strong forces
- $SU(3)_C xSU(2)_L xU(1)_Y$ gauge theory
- Higgs acquires a vev at 175 GeV (EWSB)
- Masses of all particles generated by coupling to the Higgs
- $SU(2)_L xU(1)_Y$ broken to $U(1)_{EM}$

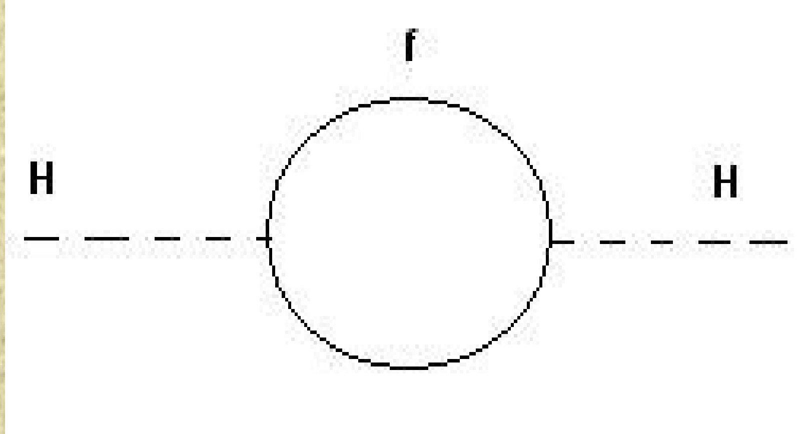
The Standard Model

The Standard Model despite its successes is incomplete!

- Higgs not yet observed
- Hierarchy problem
- Naturalness problem (fine tuning)
- Dark Matter
- Does not include gravity
- Gauge coupling unification?
- ...

Hierarchy/Naturalness Problem

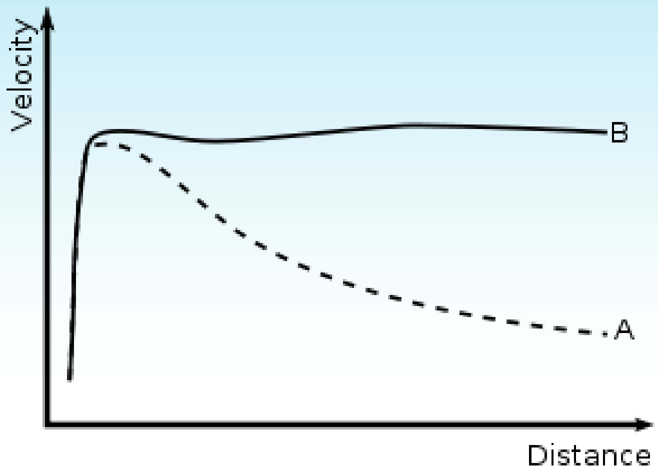
- Why is the EWSB scale so much smaller than the Planck scale?
- Radiative Corrections to the Higgs Mass
- We expect the Higgs Mass to be around ~ 130 GeV
- Need to make the bare mass cancel the mass correction to very fine precision (Fine Tuning/Naturalness Problem)

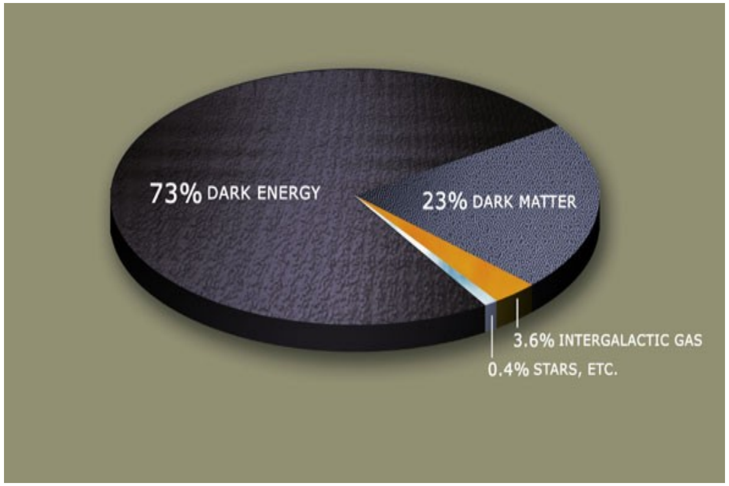


$$\Delta M_h^2 = \frac{-\lambda_f^2}{8\pi^2}\Lambda^2 + \dots$$

Dark Matter

- Evidence from galactic rotation curves, orbital velocity of galaxy clusters, gravitational lensing, anisotropy in the CMB...
- Stable, electromagnetically neutral, weakly interacting particles
- Dark Matter is non-baryonic, cold with mass around the TeV scale
- Dark matter makes up 23% of the Energy content of the universe



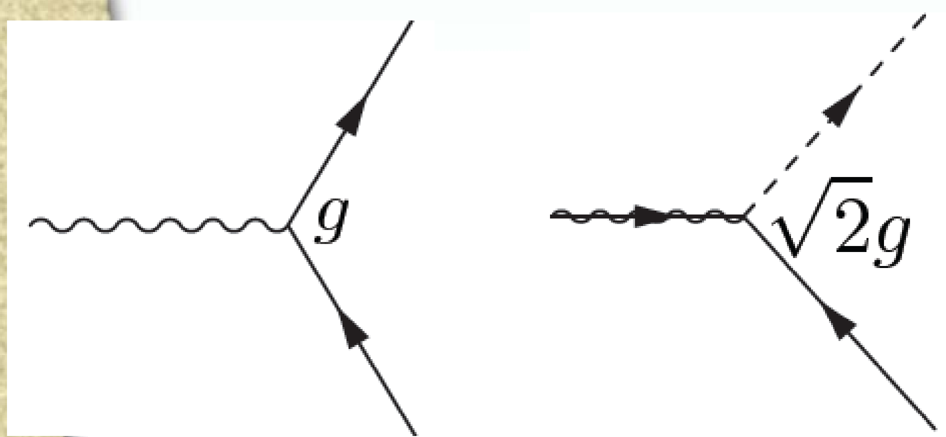


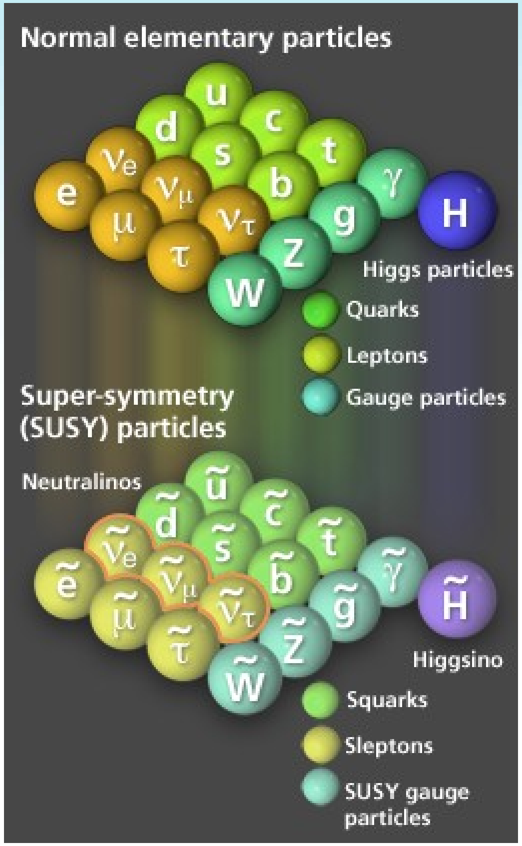
Some Candidate Theories

- Supersymmetry
- Extra Dimensions (UED, Large ED, RS ...)
- Technicolor
- ...

Candidate Theory 1: Supersymmetry

- Supersymmetry is a symmetry that relates the fermionic degrees of freedom to bosonic degrees of freedom
- Supersymmetric version of standard model interactions



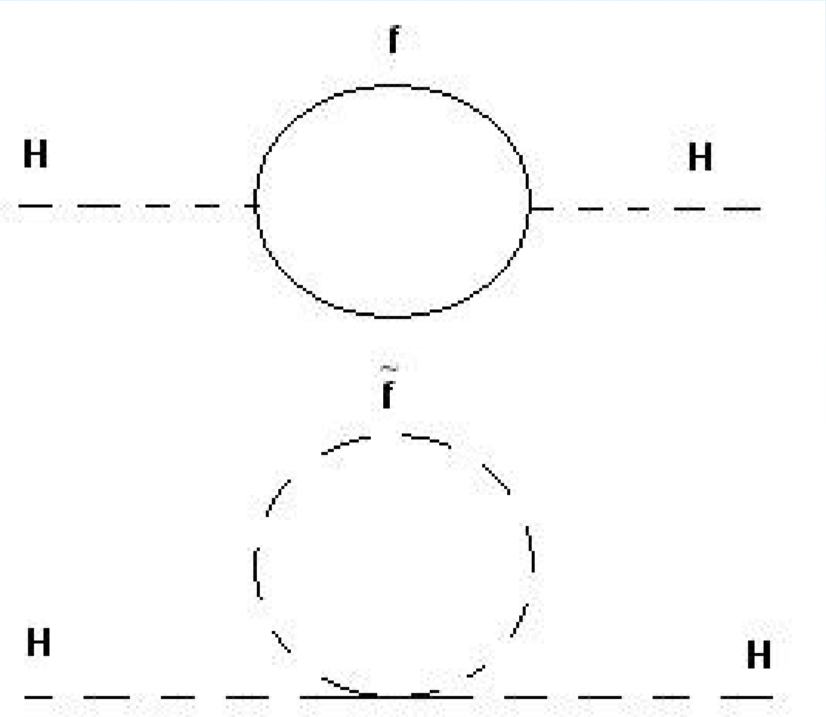


- Minimal Supersymmetric Standard Model (MSSM)
- SUSY can not be an unbroken symmetry because we haven't seen the scalar partners!
- SUSY breaking mechanisms

SUSY at the TeV Scale

Hierarchy Problem

- In SUSY λ_f^2 and λ_s are exactly the same and the quadratic divergence cancels
- $$\Delta M_h^2 = \frac{-\lambda_f^2}{8\pi^2}\Lambda^2 + \frac{\lambda_s}{8\pi^2}\Lambda^2 + \ldots$$
- In the MSSM we are left with a logarithmic correction to the Higgs mass square
- $$\Delta M_h^2 \propto -\lambda_f^2 (m_{\tilde{t}}^2 - m_t^2) \log \frac{\Lambda^2}{m_{\tilde{t}}^2}$$
- Naturalness constrains $m_{\tilde{t}}$ to be less than a TeV to avoid fine tuning at the level of a percent
- Remember that experiments rule out $m_{\tilde{t}}$ < ~100GeV



R-parity and Dark Matter

- Introducing a discrete R-parity symmetry prevents proton decay at the level of renormalizable interactions (any Feynman vertex must have an even number of super-partners)
- As a bonus it prevents the lightest super partner from decaying
- The LSP could be a mix of gaugino and higgsino eigenstates called the neutralino or the gravitino
- We have a stable, electromagnetically neutral particle with mass ~ 1 TeV : Dark Matter Candidate

Features of Supersymmetry

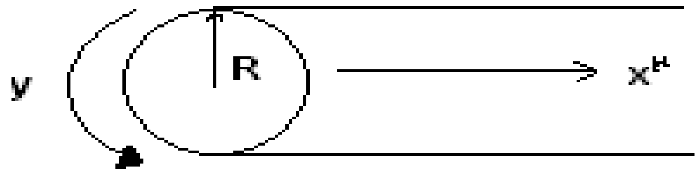
- Symmetry relating bosons and fermions
- Can solve the Naturalness problem
- Suggests a dark matter candidate
- Better gauge coupling unification

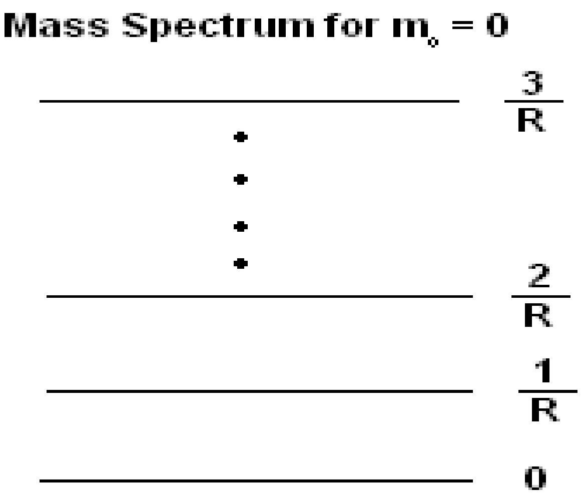
Candidate Theory 2: Extra Dimensions

- Could the universe have more than 3+1 spacelike dimensions?
- Gravity defines space time so must be present in all dimensions
- Need to decide if the SM particles and interactions are present in all dimensions or are confined to a 'brane'
- Universal Extra Dimensions (UED), Other approaches Warped Extra Dimensions, Large Extra Dimensions ...

Universal Extra Dimensions

- Kaluza Klein Theory predicts a tower of KK modes
- Quantized momentum in the extra dimensions shows up as quantized mass in 3+1 dimensions
- $m_n^2 = m_0^2 + n^2/R^2$ for scalars on $R^4 \times S^1$
- Introducing a Z_2 orbifold symmetry gives rise to chiral KK 0 fermions and plays the role of R-parity by preventing the lightest KK 1 mode (LKP) from decaying, giving rise to a dark matter candidate





Summary: The case for TeV physics

- Fermi Coupling $(G_F)^{-1/2} \sim 300$ GeV showed that there is interesting physics at the TeV scale
- SUSY, UED candidate theories at the TeV scale tackle the Naturalness/Hierarchy problem
- Dark Matter is expected at the TeV scale

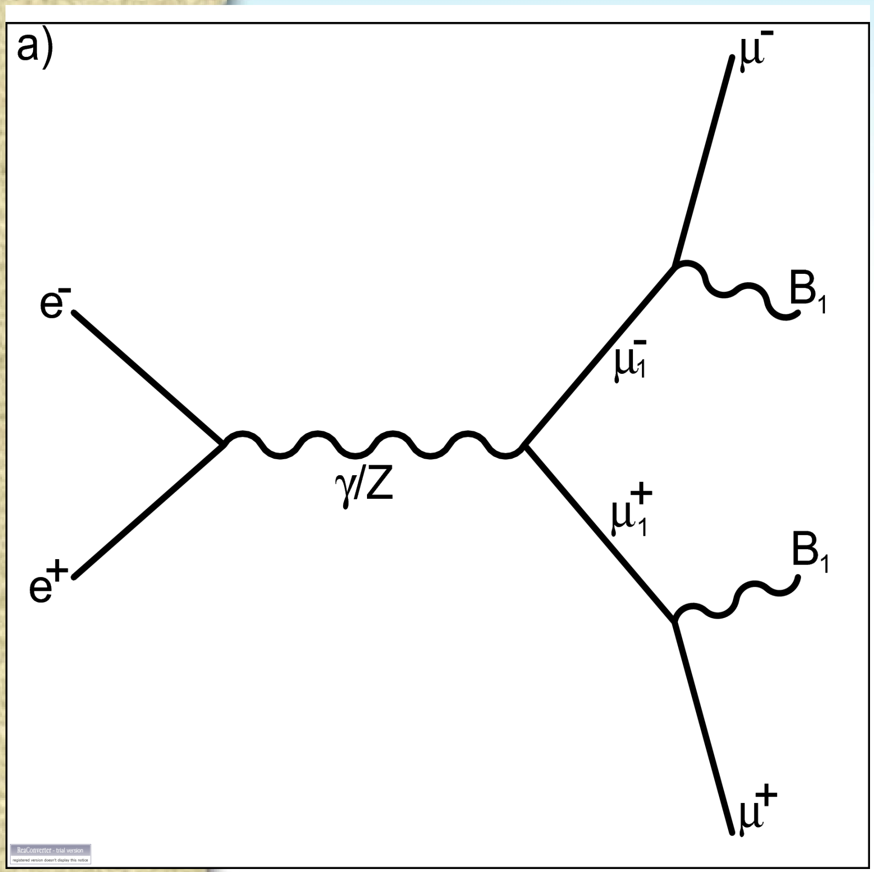
Distinguishing New Physics at the TeV scale

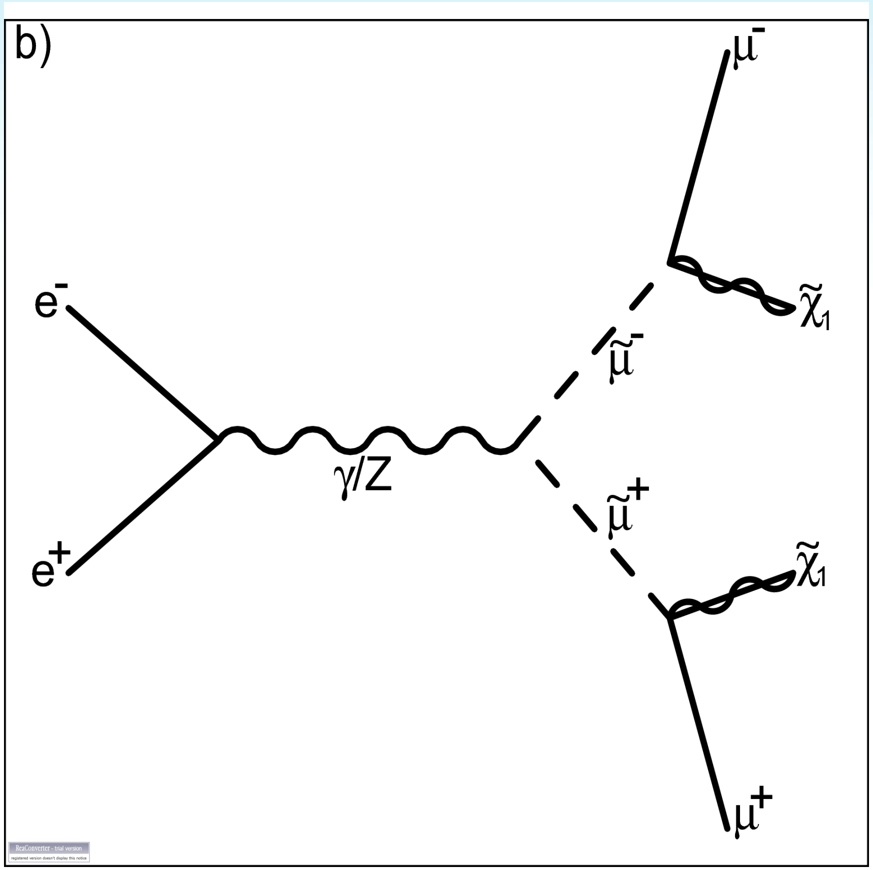
- Clear signal of new physics is new particles around the ~300 GeV scale
- Cannot rely on the mass spectrum of new particles alone to distinguish between various theories (also KK2 modes)
- We could have particles with identical quantum numbers except for spins
- To conclusively establish a theory we need to measure the spins of the new particles

UED vs SUSY

Spin-1/2

Spin-0





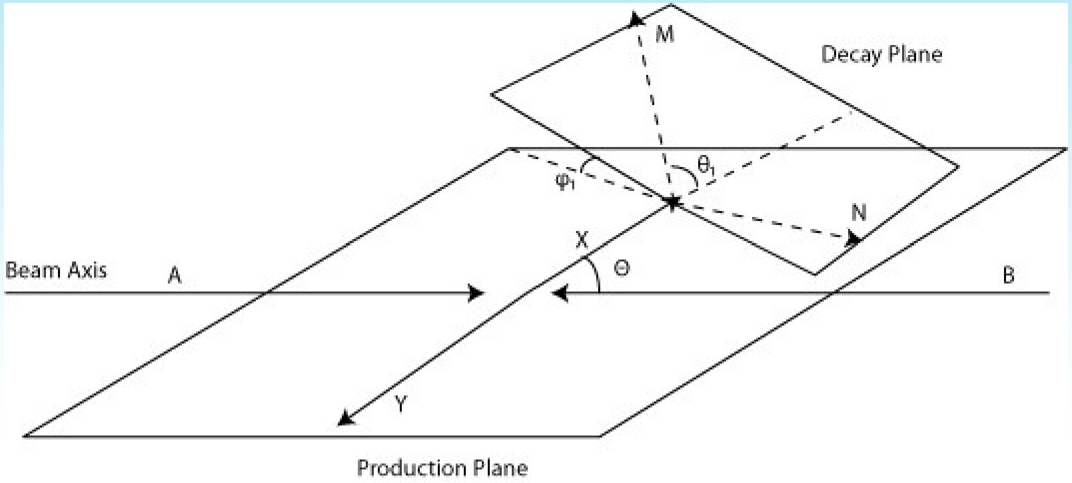
The International Linear Collider

- $e^- e^+$ beam in a linear accelerator
- ~ 500-1000 GeV center of mass energy
- Integrated luminosity of at least 500 fb^{-1}
- Still in the planning stages, scheduled for late 2010's?

ILC Discussion

- Some spin measurement techniques
- Our model independent spin measurement technique
- Application of our technique to test SUSY/UED
  - scalars/spinors
  - spinors/vectors
- What works and what doesn't

Collider Physics Angles



- θ is the production angle
- θ_i is the polar angle of decay
- φ_i is the azimuthal angle of decay
- φ_i is invariant to boosts of the parent particle. In particular it is the same in the lab frame, as it is in the rest frame of the parent particle

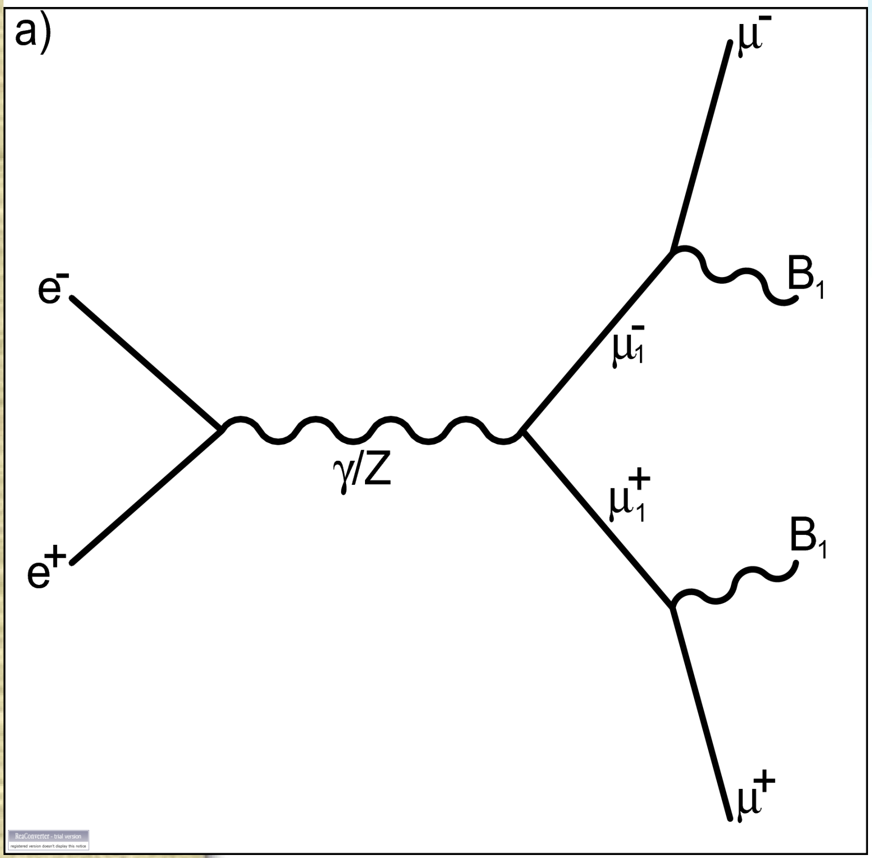
Spin Measurement Techniques

More possibilities at a linear collider (control over center of mass energy)

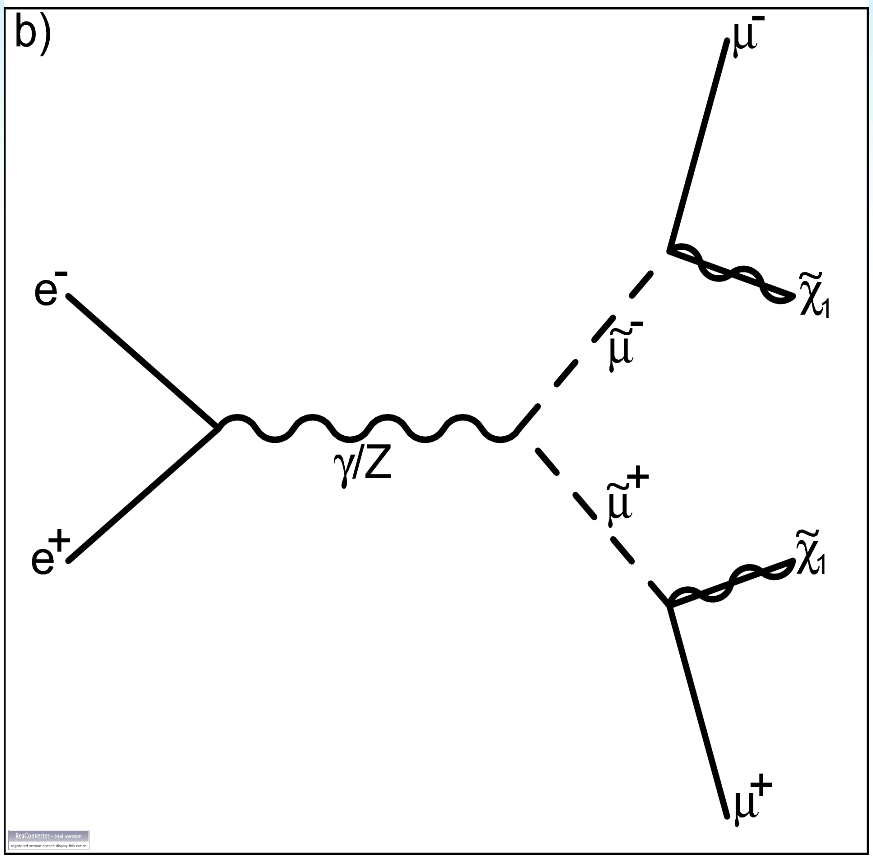
- Threshold scans distinguish scalars from spinors or vector bosons
  - Scalar cross section rises like β^3 whereas the spinor/vector cross section rises like β
  - Cannot be used at a hadron collider
  - Cannot distinguish between spin 1 and spin ½
- Polar angular dependence in decay
  - Requires knowledge of final state spins
  - Requires chiral couplings → introduces model dependence

UED vs SUSY

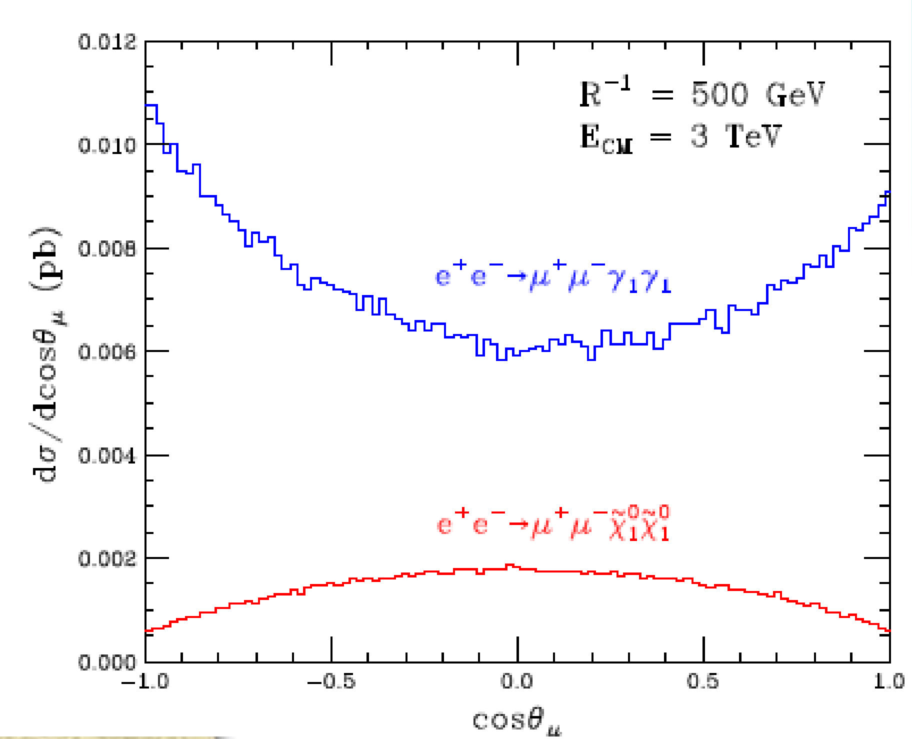
Spin-1/2

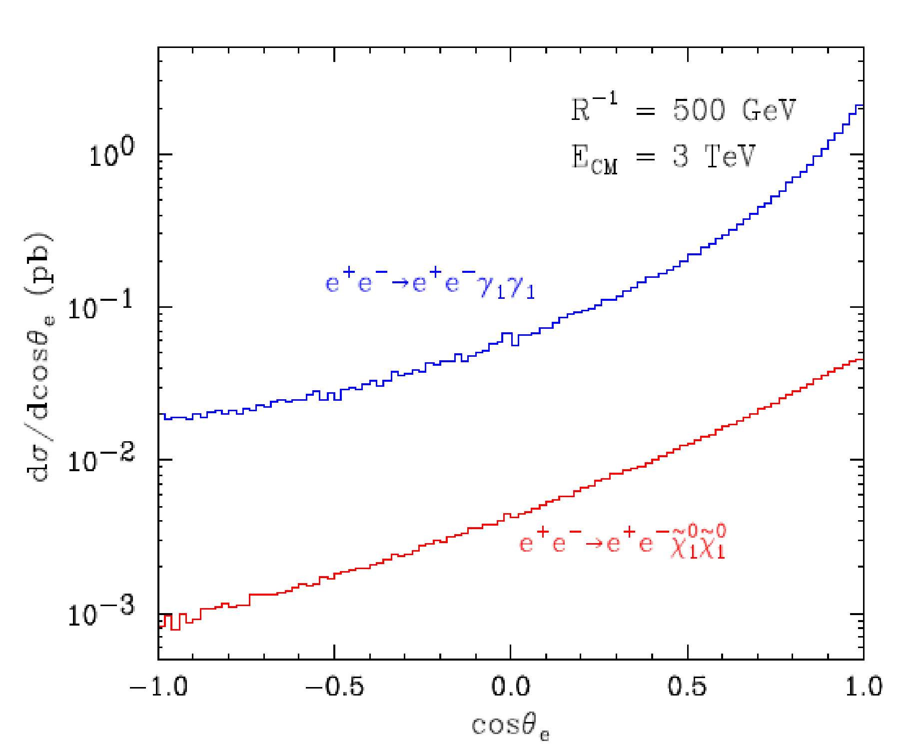


Spin-0



- Differential cross section w.r.t. production angle
  - s-channel : Scalars → $\sin^2\theta$, Spinors → $1+\frac{E^2-m^2}{E^2+m^2}\cos^2\theta$
  - Model dependence in the form of t-channel may introduce a forward peak which is similar for both spin statistics





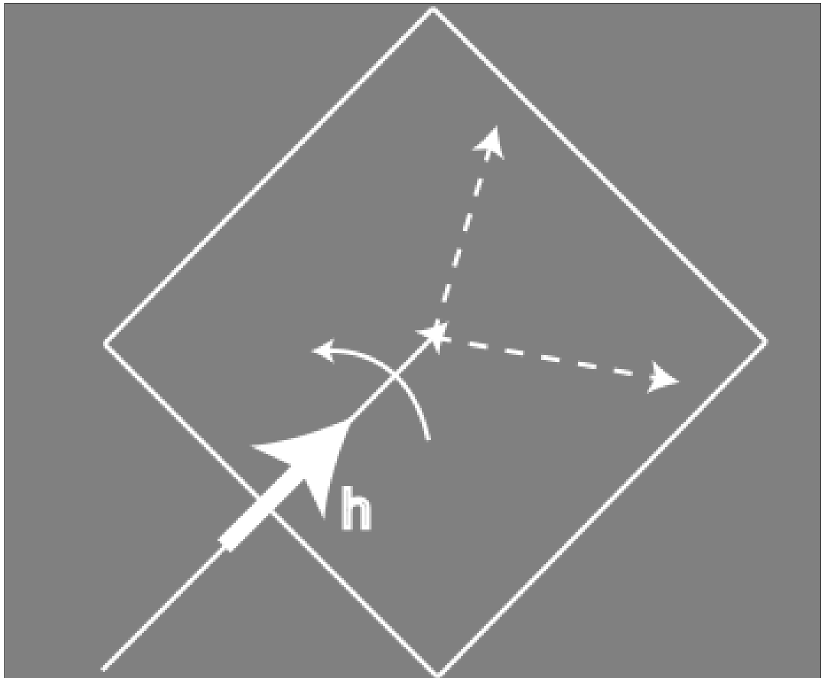
hep-ph/0502041 M.Battaglia et al

Model Independent Technique for Measuring Spins

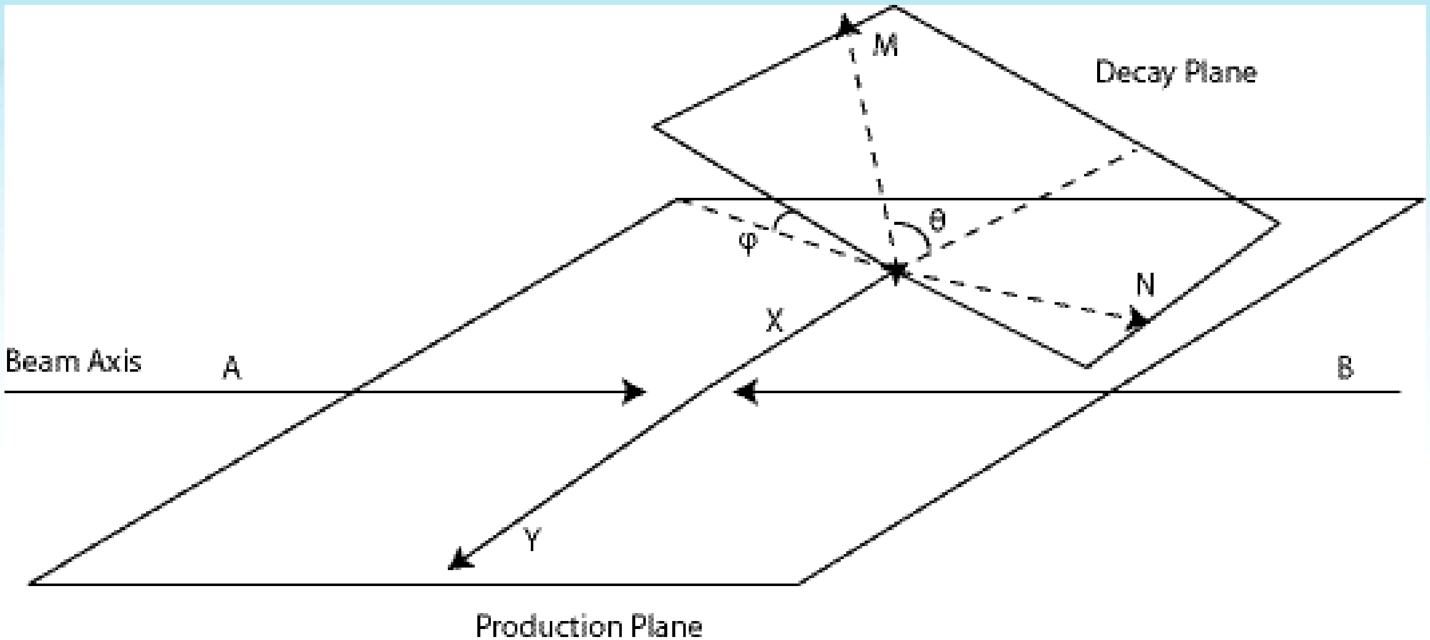
Back to Fundamentals

- Spin is a type of angular momentum
- Angular momentum generates rotations $U(\vec{n},\varphi)=e^{i\frac{\vec{J}.\vec{n}}{\hbar}\varphi}$
- We can isolate spin from orbital angular momentum by considering the component of angular momentum in the direction of motion of a particle

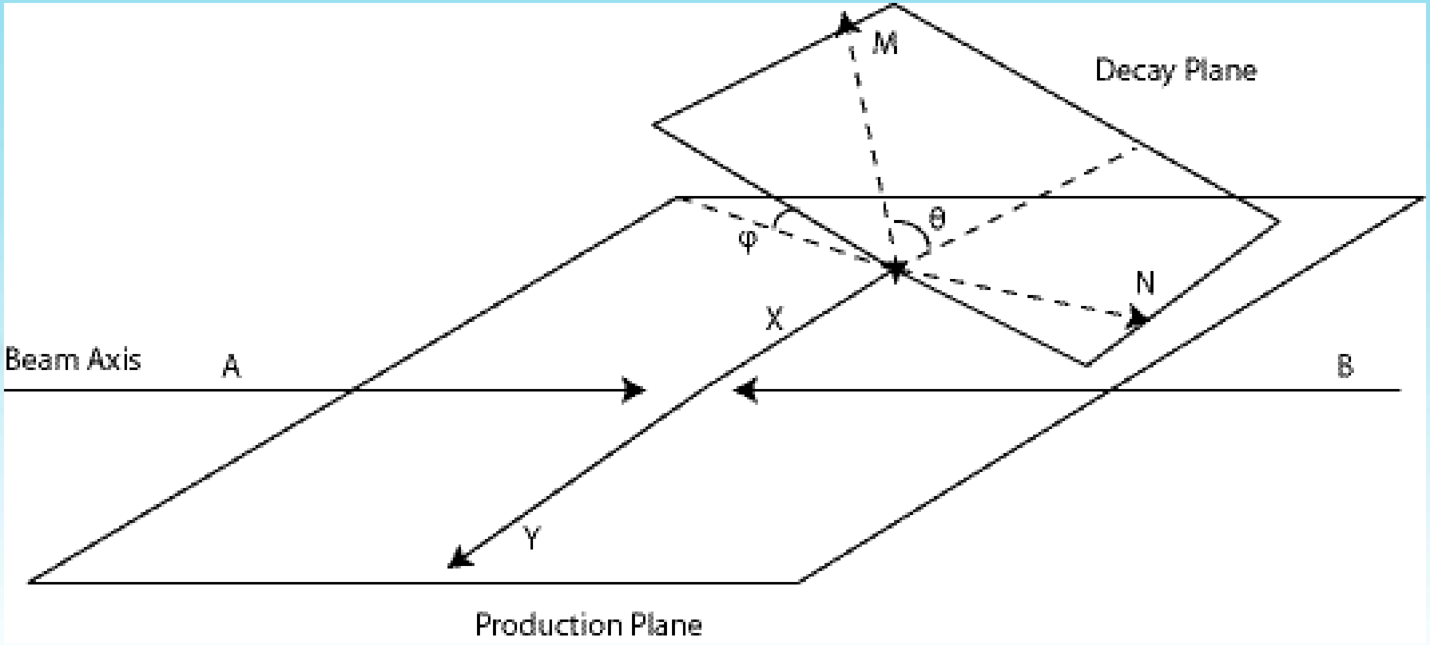
$$J_z = \vec{J}\cdot\hat{p} = (\vec{s}+\vec{r}\times\vec{p})\cdot\hat{p} = \vec{s}\cdot\hat{p} = h$$



Model Independent Technique for Measuring Spins



- Production followed by decay of a new particle
- Two planes to consider: Production and Decay planes
- Rotating the decay plane about the +z axis by an angle $\varphi \rightarrow$ action of this rotation on the matrix element of the decay must be equivalent to the action of rotation on the parent particle by φ.



$$\mathcal{M}_{prod} = \langle X, Y | \mathcal{H}_{prod} | A, B \rangle \quad \mathcal{M}_{decay}(\phi) = \langle M, N, \phi | \mathcal{H}_{decay} | X \rangle$$

$$\mathcal{M}_{decay}(\phi) = \langle M, N(\phi = 0) | e^{+iJ_z\phi} \mathcal{H}_{decay} | X \rangle$$

$$J_z = \vec{J} \cdot \hat{p} = (\vec{s} + \vec{r} \times \vec{p}) \cdot \hat{p} = \vec{s} \cdot \hat{p} = h$$

$$\mathcal{M}_{decay}(\phi) = e^{+ih\phi} \mathcal{M}_{decay}(\phi = 0)$$

Quantum Interference of Helicity States

Vector Boson

$$\begin{aligned} \mathcal{M}_+ &\propto e^{i\phi_1} \\ \mathcal{M}_0 &\propto 1 \\ \mathcal{M}_- &\propto e^{-i\phi_1} \end{aligned}$$

Spinor

$$\begin{aligned} \mathcal{M}_\uparrow &\propto e^{i\phi_1/2} \\ \mathcal{M}_\downarrow &\propto e^{-i\phi_1/2} \end{aligned}$$

- If multiple helicity states are produced this phase dependence is observable

$$\frac{d\sigma}{d\phi} \propto |\sum_h \mathcal{M}_{prod} e^{+ih\phi} \mathcal{M}_{decay}(\phi = 0)|^2$$

- True within … akly coupled” physics)
- As a result of interference the differential cross-section develops a cos(nφ) dependence, where $n = h_{max} - h_{min} = 2s$.

The Bottom Line

Scalar: $$\frac{d\sigma}{d\varphi} = A_0$$

Spinor: $$\frac{d\sigma}{d\varphi} = A_0 + A_1 \cos(\varphi)$$

Vector boson: $$\frac{d\sigma}{d\varphi} = A_0 + A_1 \cos(\varphi) + A_2 \cos(2\varphi)$$

Tensor (spin-2): $$\frac{d\sigma}{d\varphi} = A_0 + A_1 \cos(\varphi) + A_2 \cos(2\varphi) + A_3 \cos(3\varphi) + A_4 \cos(4\varphi)$$

Look for the highest cosine mode to determine the spin!*

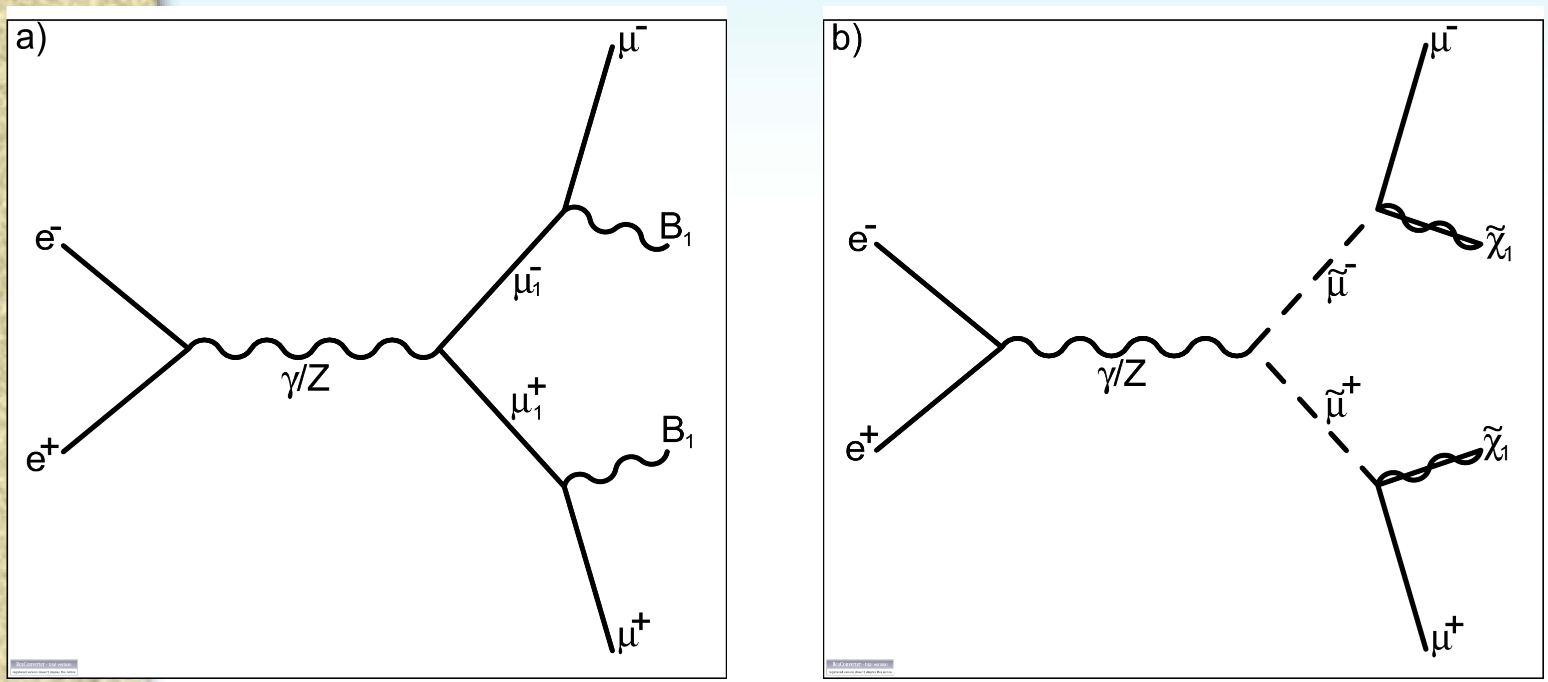
*(Can set a lower bound on the spin of a particle)

- This argument is based entirely on Quantum Mechanical principles, to actually compute the coefficients requires Feynman diagrams!

Spin Measurement at ILC

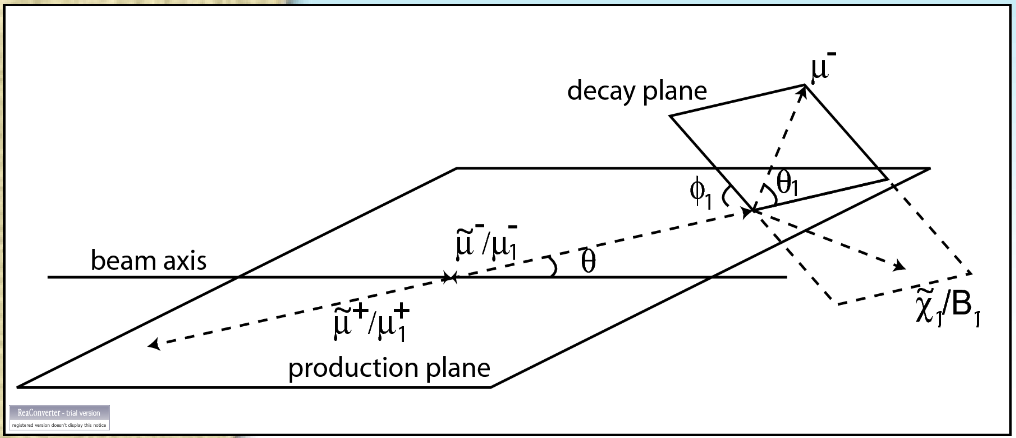
M.R. Buckley, H. Murayama , W. Klemm, V. Rentala arXiv:0711.0364 [hep-ph]

- Typical pal pair production processes followed by 2 body decay
- 2 body → 2 body → 4 body final state



- Characteristic signal is $\mu^+ \mu^-$ and missing energy (LKP/LSP) – fairly generic to most extensions of the SM
- Need to be able to reconstruct the momenta of the parent particle

2-fold ambiguity



- **θ is the production angle**
- **θ_i,φ_i are the decay angles in the lab frame**
- **φ_i are the same in the rest frame of the parent particle**

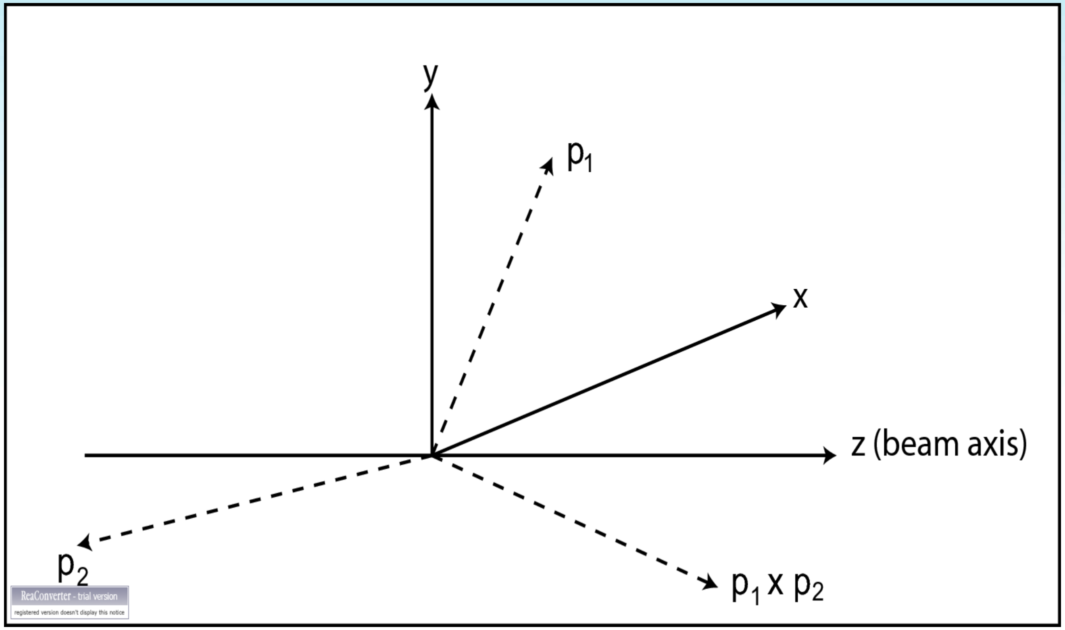
- Knowns: Outgoing lepton momenta, incoming energy-momentum, masses of all particles
- Unknowns: Missing Particles 4-momentum for a total of 8 unknowns
- Equations:
  - Overall energy momentum conservation: 4 equations
  - 4 mass shell constraints for the parent/missing particles = 4 equations

8 equations and 8 unknowns!

But mass-shell constraints are quadratic! Kinematic reconstruction leads to a true and a false solution.

Reconstruction

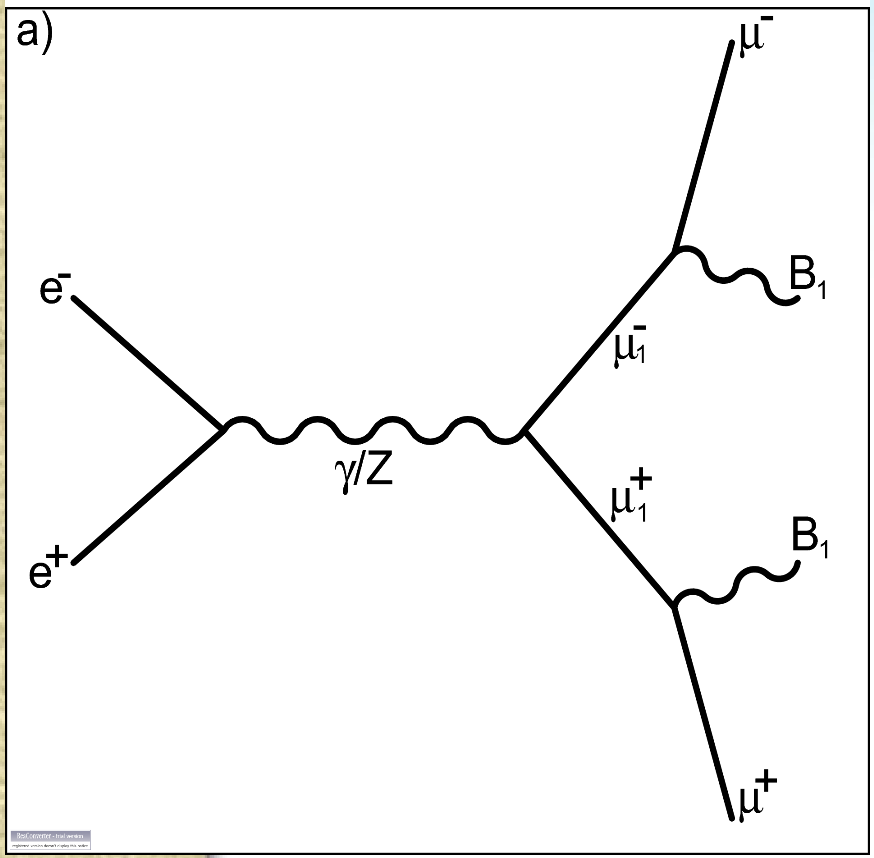
- Projection of the parent particle momentum on the p_1 and p_2 axes can be solved for in terms of the known parameters
- Magnitude of the parent particle momentum is known
- Two fold ambiguity in finding the projection along the $p_1 \times p_2$ axis (involves taking a square root)

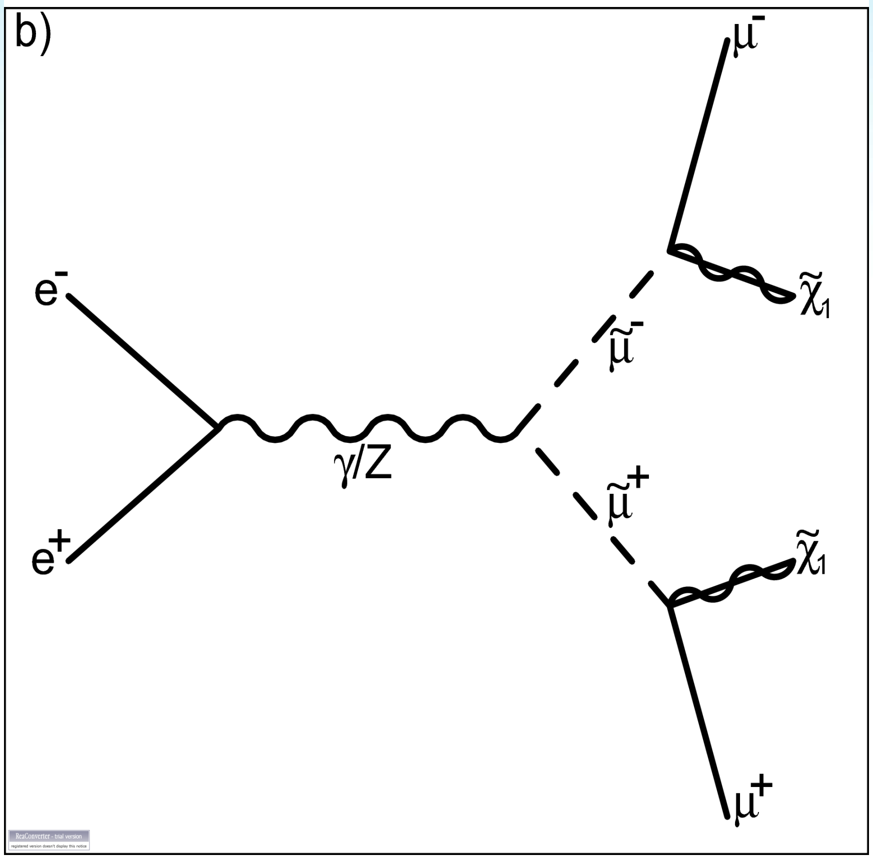


Scalars vs Spinors

Spinors → $A_0 + A_1 \cos\varphi$

Scalars → A_0 (flat)





Mass Spectrum

mSUGRA point SPS3: m_0 = 90 GeV, $m_{1/2}$= 400 GeV, A_0 = 0, tan β = 10, μ > 0
MUED: n = 1, R^{-1} = 300 GeV, Λ= $20R^{-1}$, M_{Higgs}= 120 GeV

	SPS3	MUED
$\tilde{\chi}_1^0 / B_1$	161 GeV	302 GeV
$\tilde{l}_R / l_{1R}$	181 GeV	304 GeV
$\tilde{l}_L / l_{1L}$	289 GeV	309 GeV
$\tilde{\chi}_1^\pm / W_1^\pm$	306 GeV	327 GeV
$\tilde{\nu}_L / \nu_{1L}$	276 GeV	309 GeV

Backgrounds

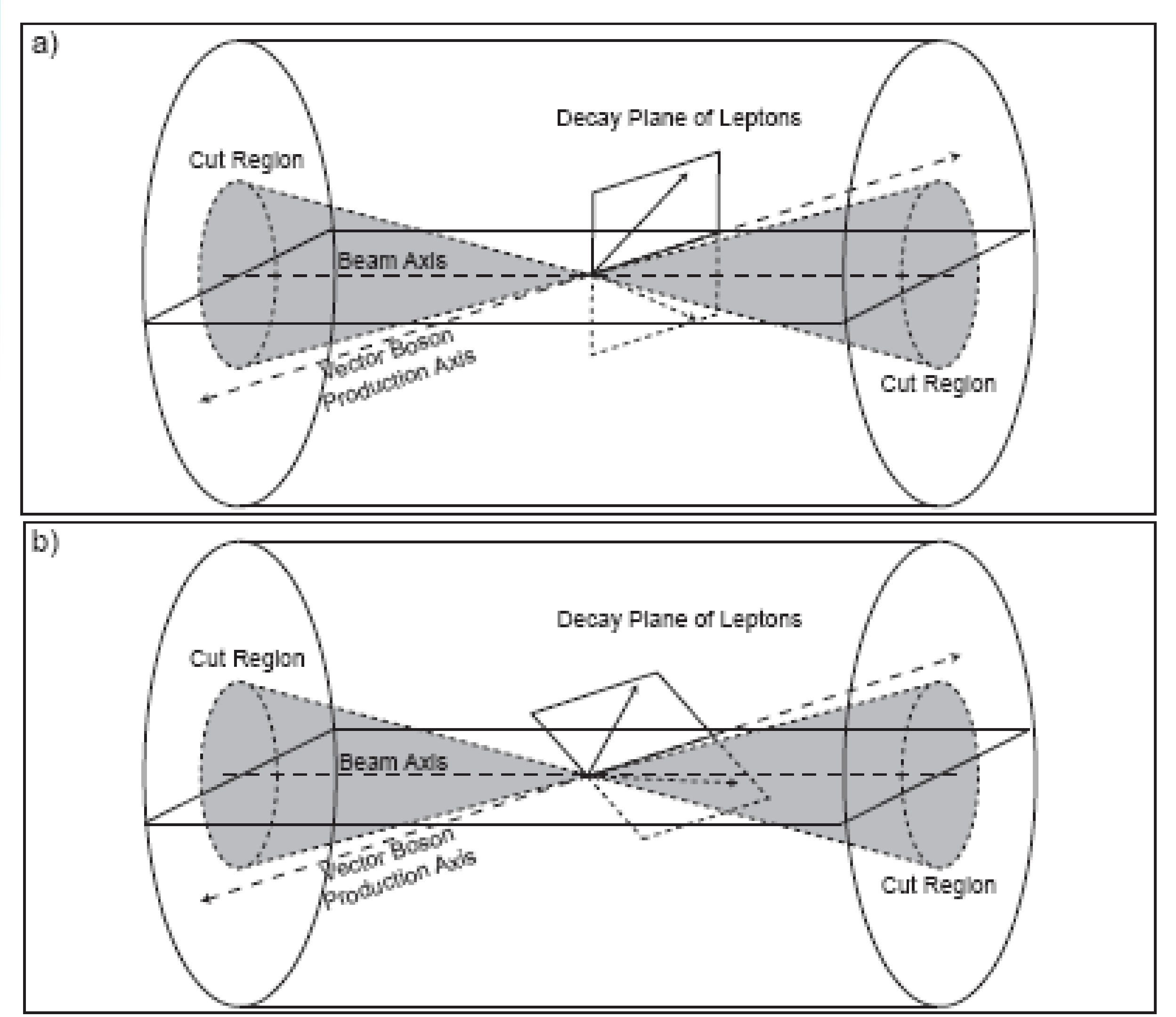
- Standard Model:
  - 2 photon background
  - W+, W- production with leptonic decay
  - ZZ production with l^+l^- and $\nu\nu$
- Model dependent background
  - $\tilde{\chi}_1^{\pm}/W_1^{\pm}$ production followed by decay to muons and $\tilde{\nu}/\nu_1$

Cuts

1. **Cutting on background**
   - Kinematic cut on the invariant mass of muon pairs can greatly reduce SM background
   - More efficient cuts obtained by requiring successful reconstruction of the parent momentum (quantity under the square root must be positive)
2. **Detector cuts**
   - $|\eta| < 2.5$ for both visible muons and for missing p_T

Cuts can introduce new angular dependencies!

Cuts destroy rotational invariance



Matthew R. Buckley , Beate Heinemann , William Klemm, Hitoshi Murayama arXiv:0804.0476 [hep-ph]

Software Tools

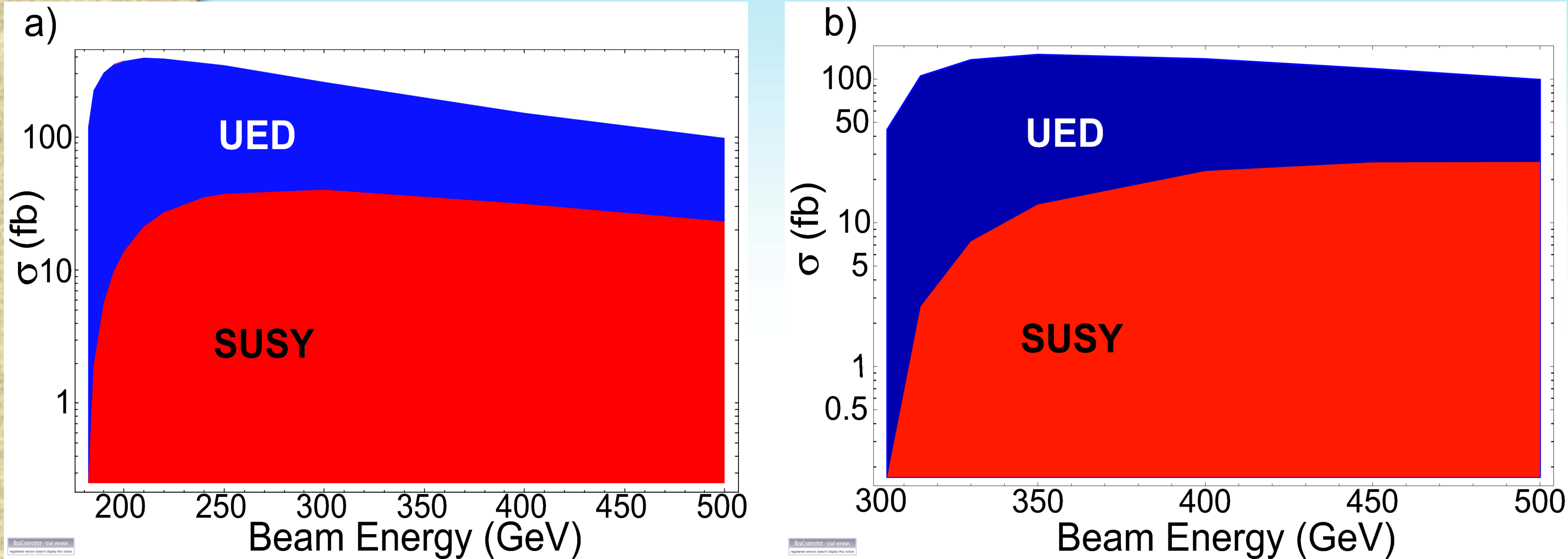
- HELAS: "HELicity Amplitude Subroutines for Feynman diagram calculation" used to get differential cross-section

  (H. Murayama, I. Watanabe, Kaoru Hagiwara, 1992)

- BASES: adaptive Monte Carlo package to integrate the differential distributions

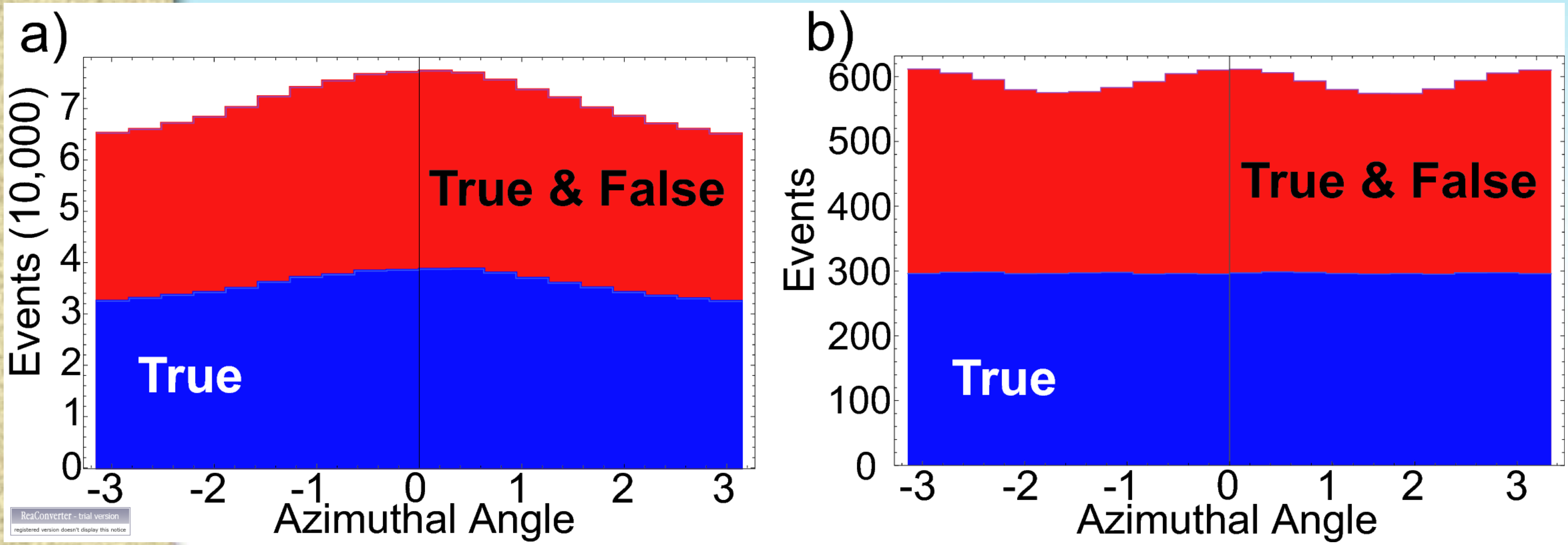
  (S. Kawabata, 1986)

Total Cross-section



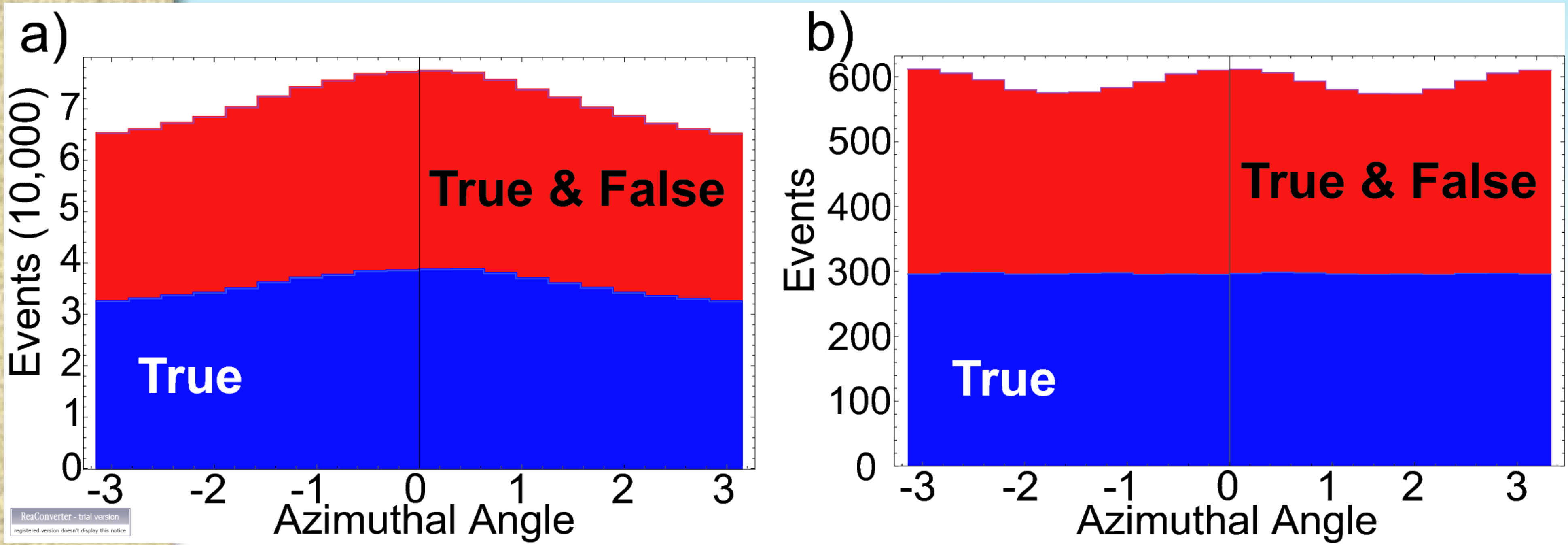
- Figure a) is the cross-section for smuon/kkmuon production and decay using the SPS3 mass spectrum
- Figure b) uses the MUED mass spectrum

Differential Cross-section

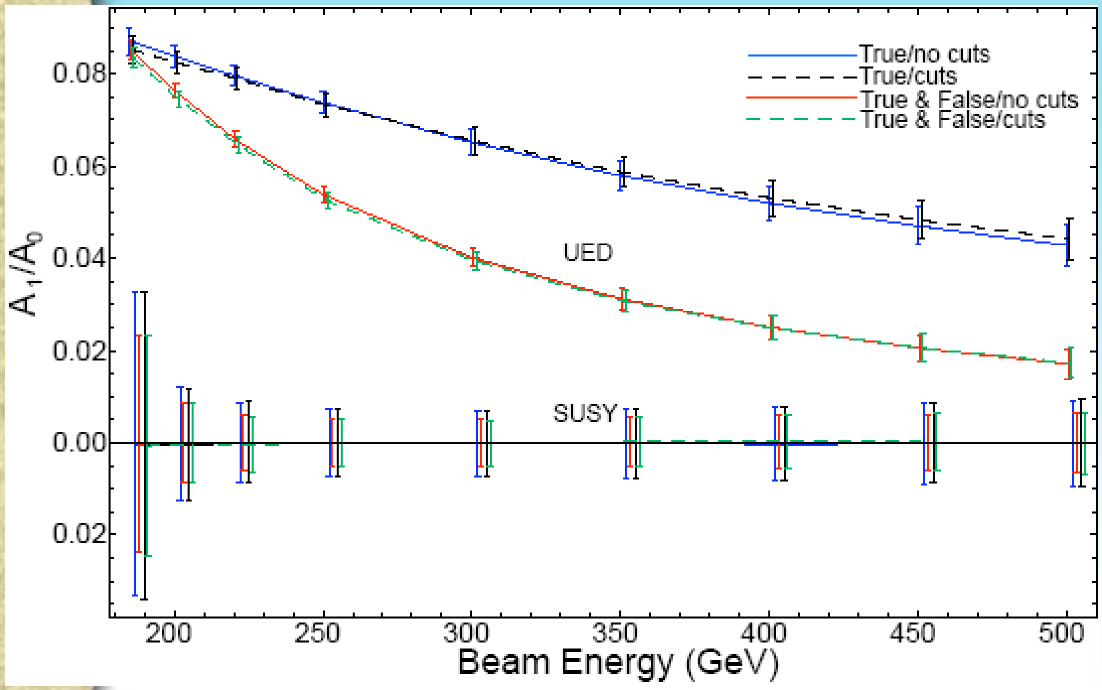


- a) shows the UED (kkmuon) distribution for E_{cm} = 370GeV and luminosity of 500 fb^{-1}
- b) shows the SUSY (smuon) distribution

Some comments

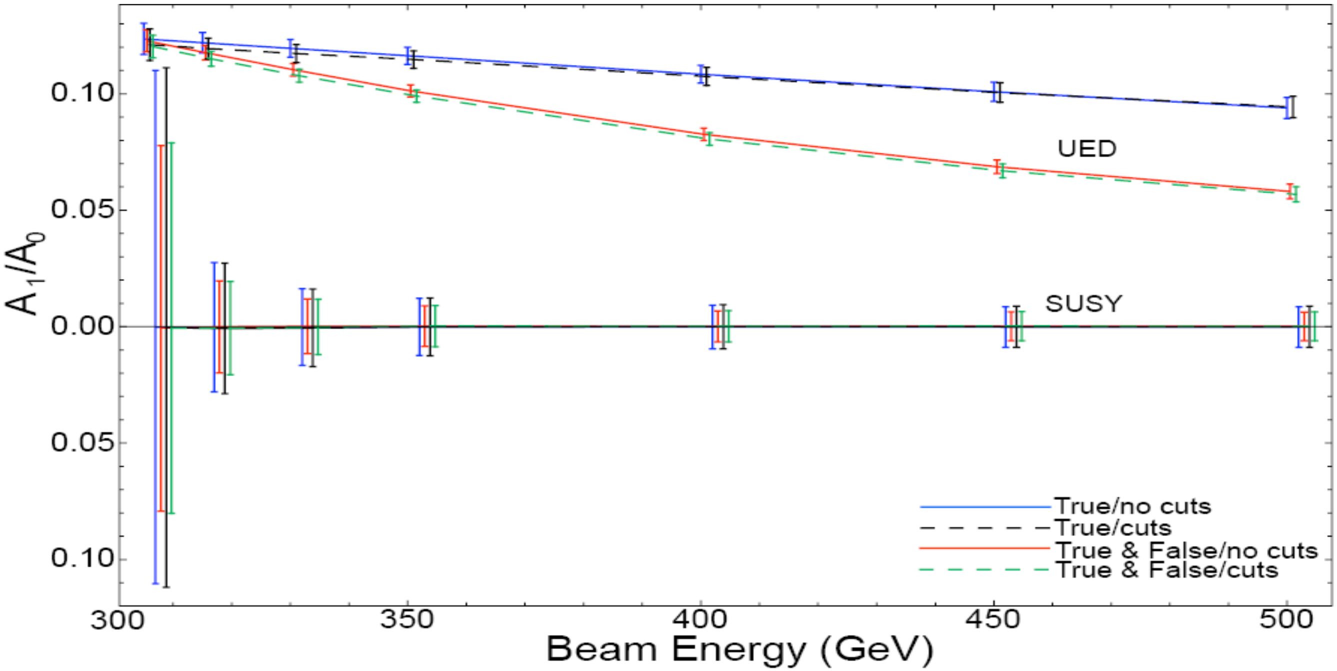


- Unexpected cos 2φ dependence develops due to the false solution and rapidity cuts for both SUSY and UED
- Unimportant for distinguishing scalar versus higher spin states but becomes important when distinguishing spinors and vectors (more on this later)
- Fits are made to $A_0 + A_1 \cos(\varphi) + A_2 \cos(2\varphi)$

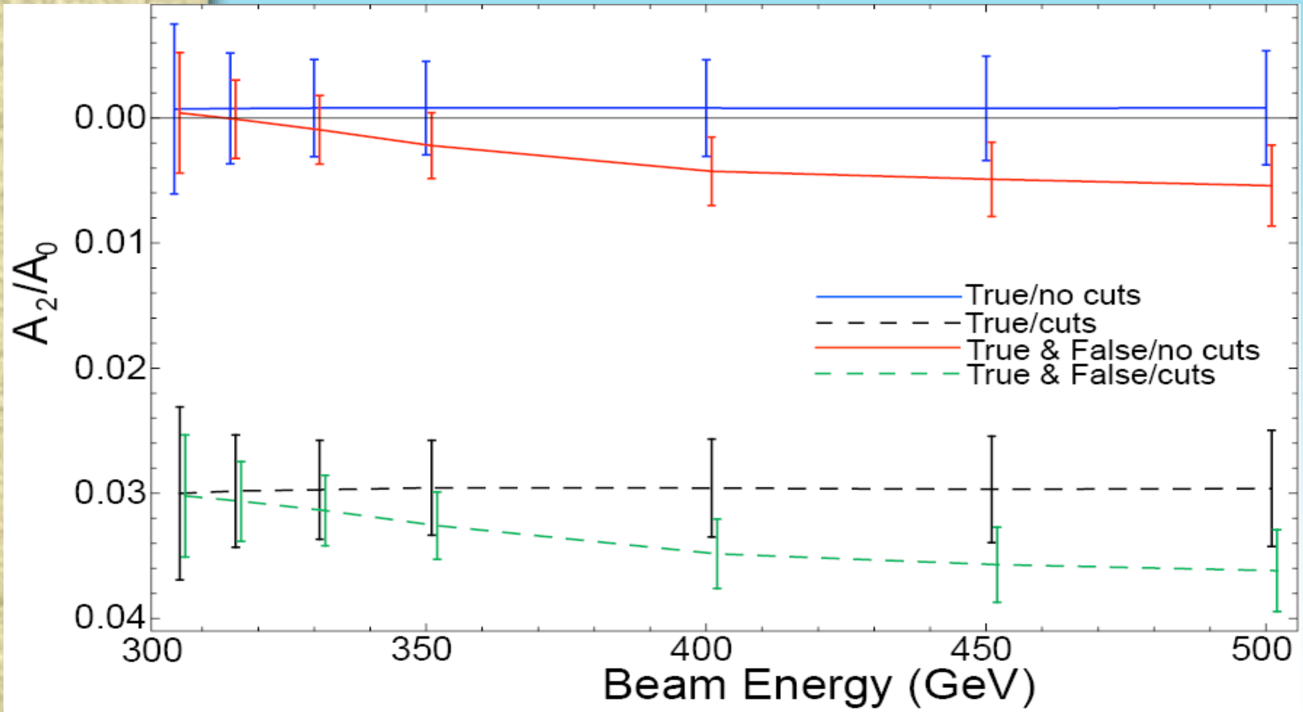


← Using SPS3

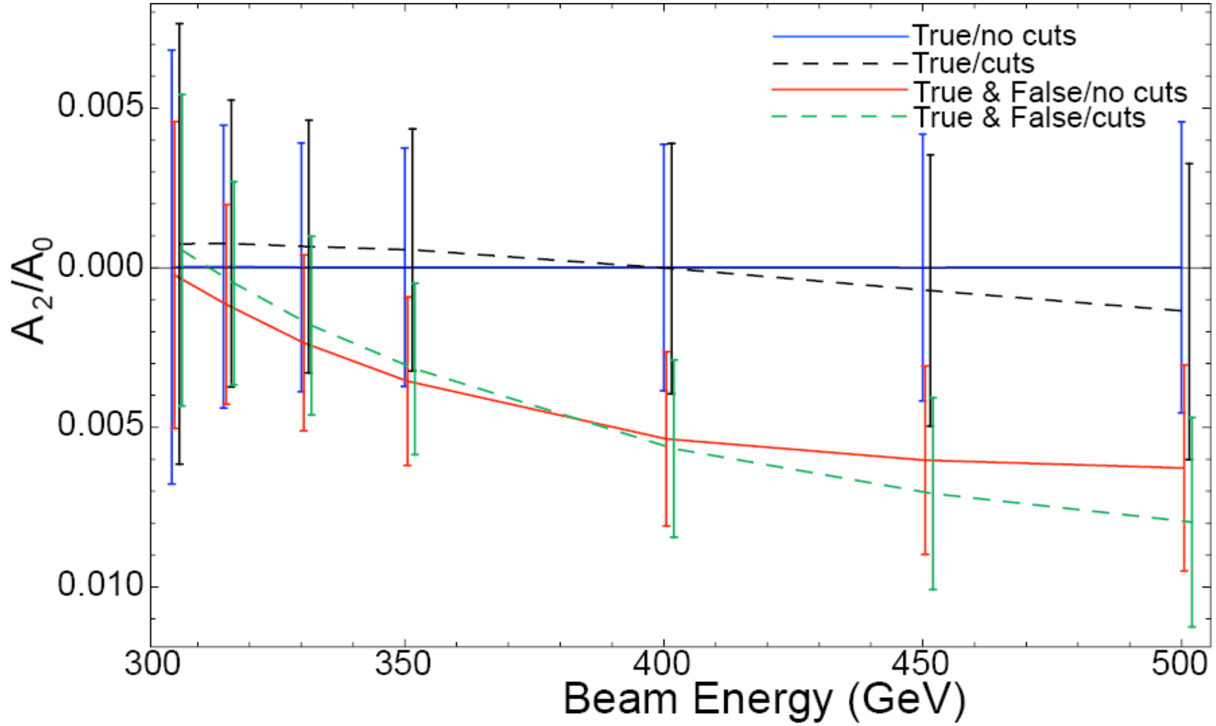
Using MUED →



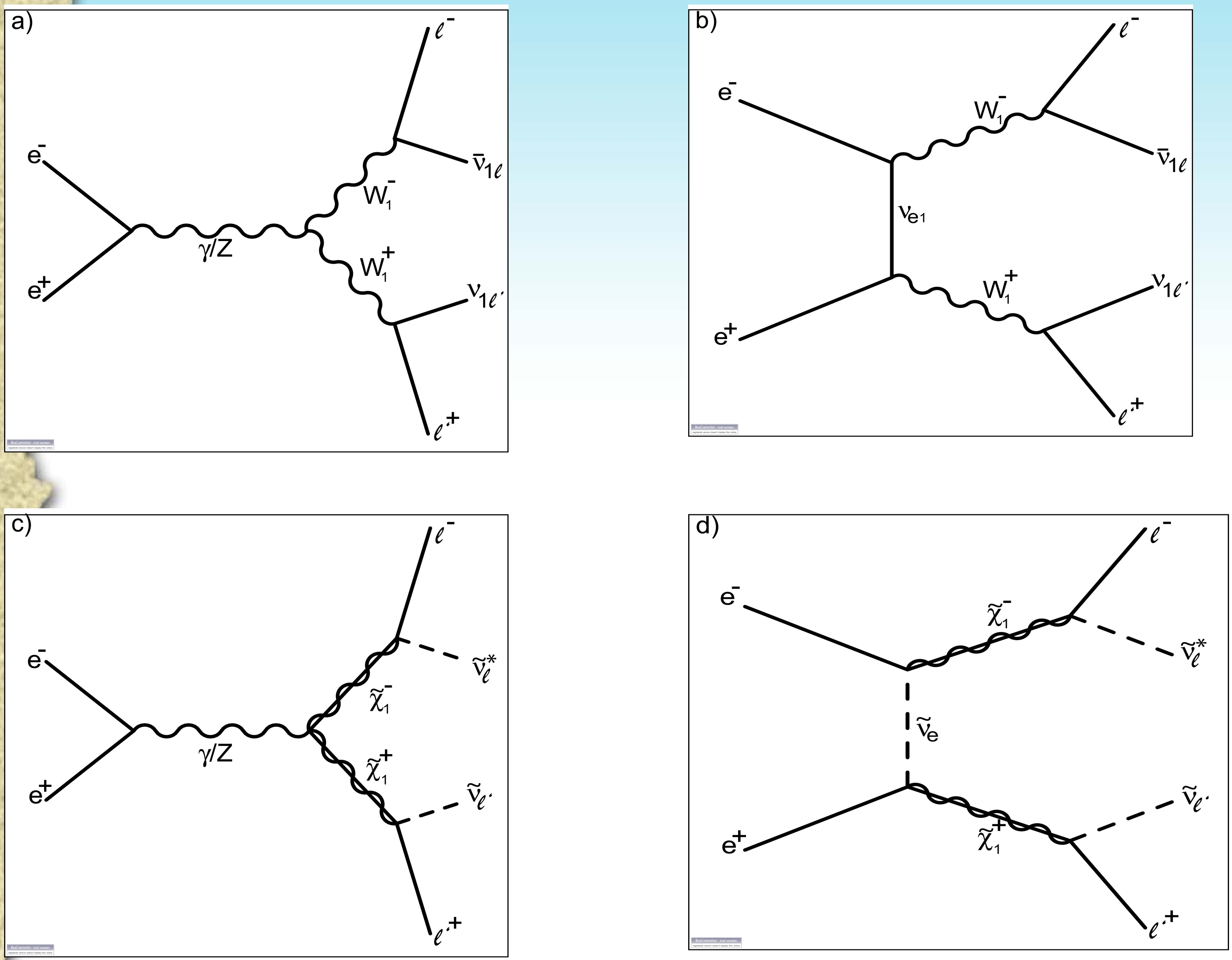
← UED (spinor) pair production using the SPS3 parameter point



After correcting for detector cuts →

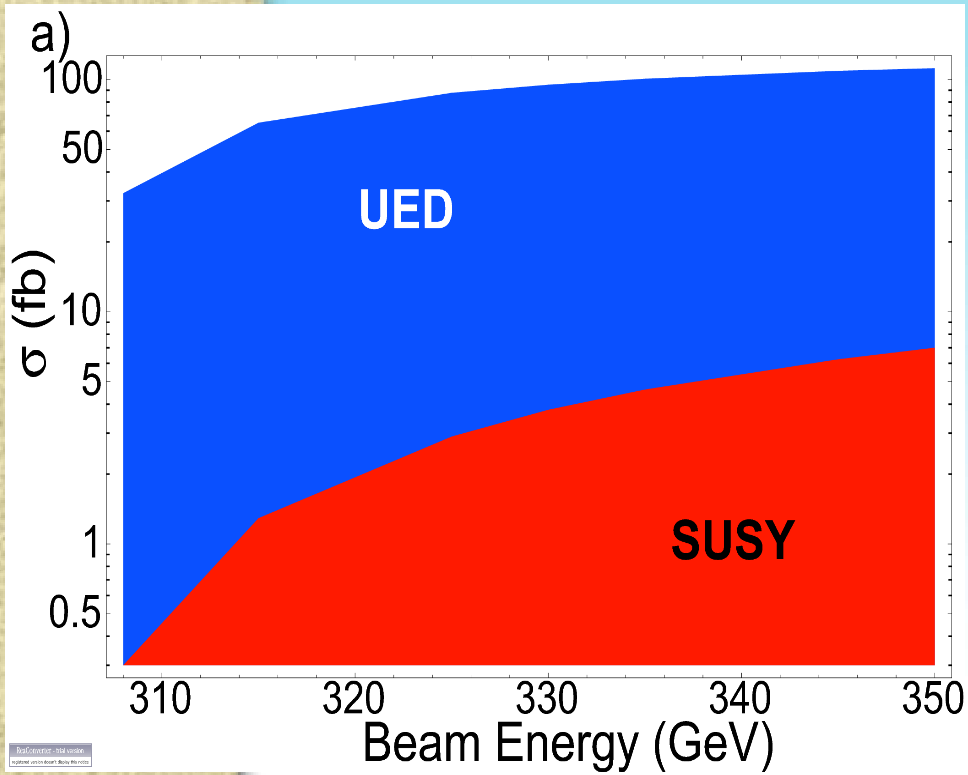


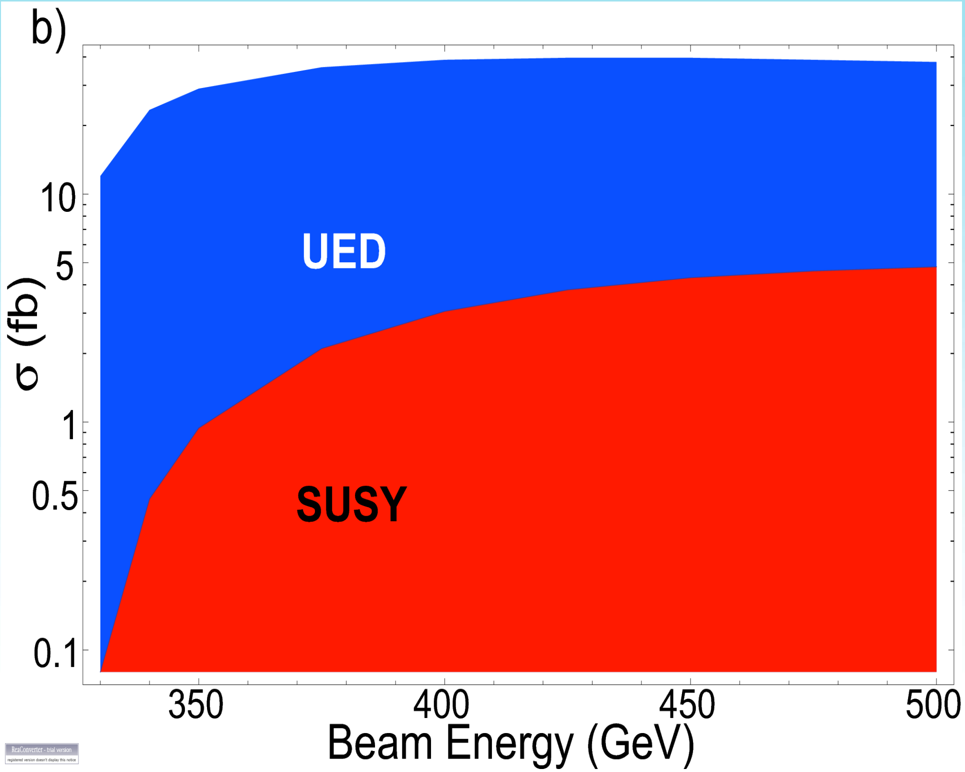
Spinor vs Vector



Backgrounds and Cuts

- Background
  - Standard Model: 2 photon, W^+W^- and ZZ production
  - Model Dependent: $\tilde{\chi}_2^0\tilde{\chi}_2^0 / W_1^3 W_1^3$ $\tilde{\ell}^-\tilde{\ell}^+ / \ell_1^- \ell_1^+$
- Cuts
  - Successful reconstruction cuts background significantly
  - $\eta < 2.5$ cuts on charged lepton and missing momentum

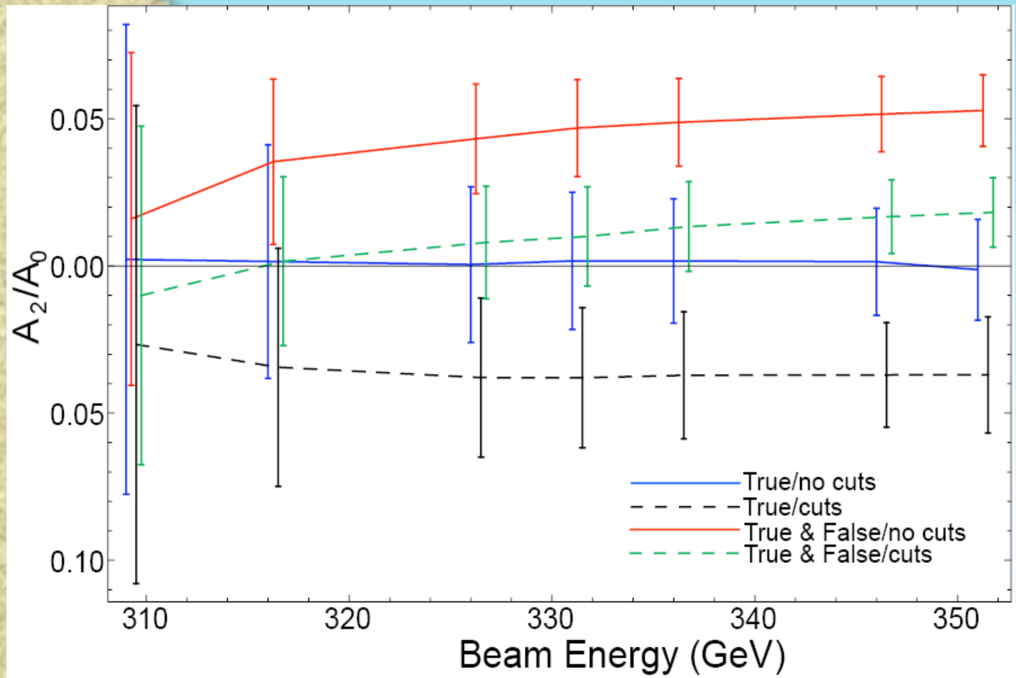




Using SPS3 mass spectrum

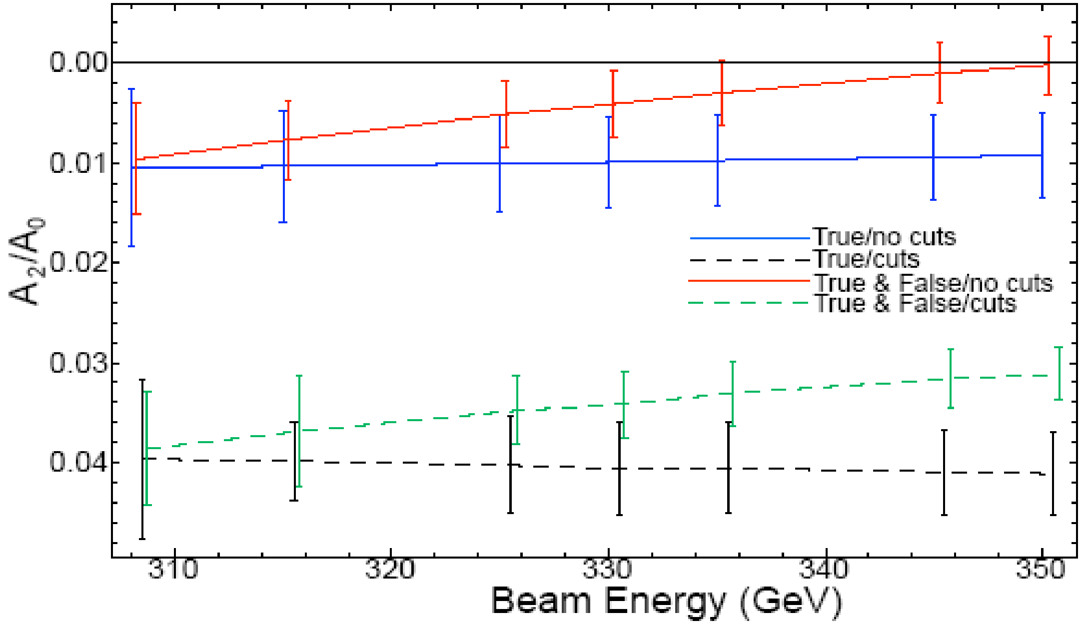
Using MUED mass spectrum

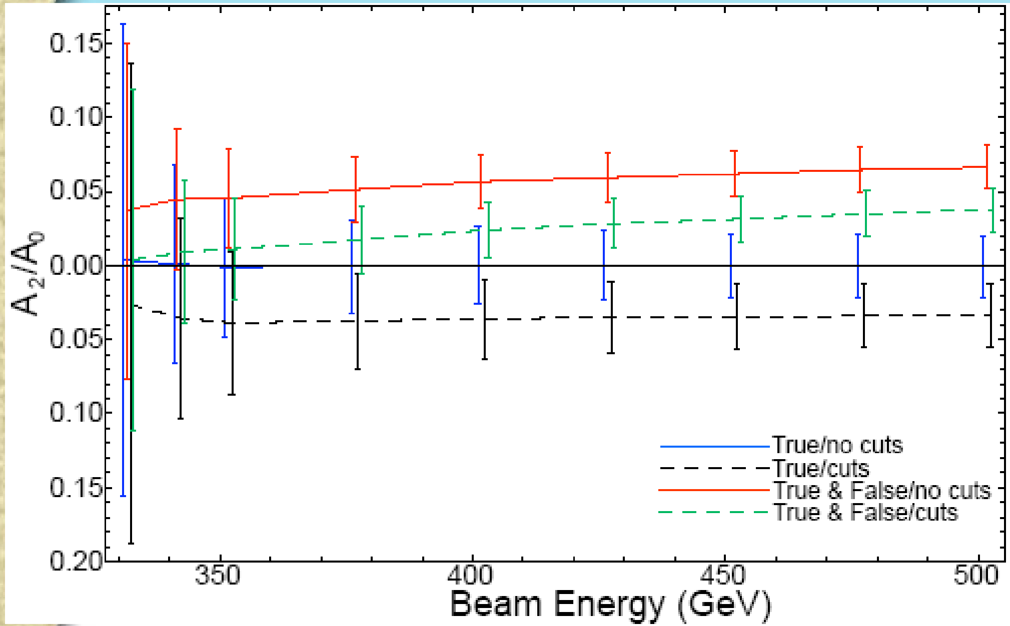
- **Using 1000 fb^{-1} of integrated luminosity (smaller cross-section)**
- **Fit is made to** $A_0 + A_1 \cos(\varphi) + A_2 \cos(2\varphi)$



← **SUSY chargino (spinor) production Using SPS3**

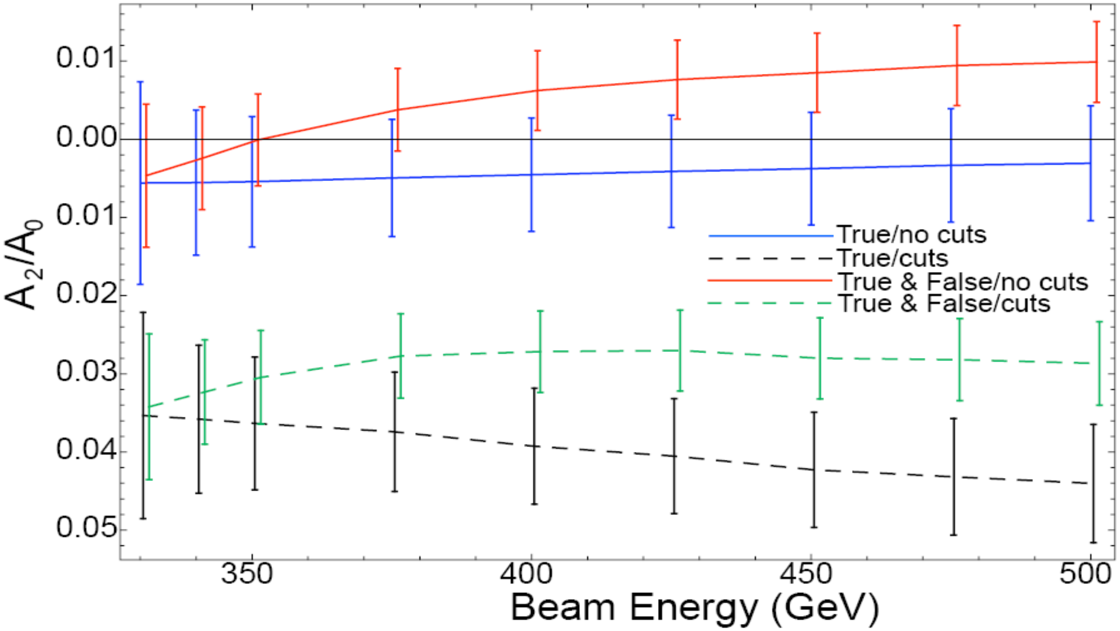
MUED KKW (vector) production using SPS3 →

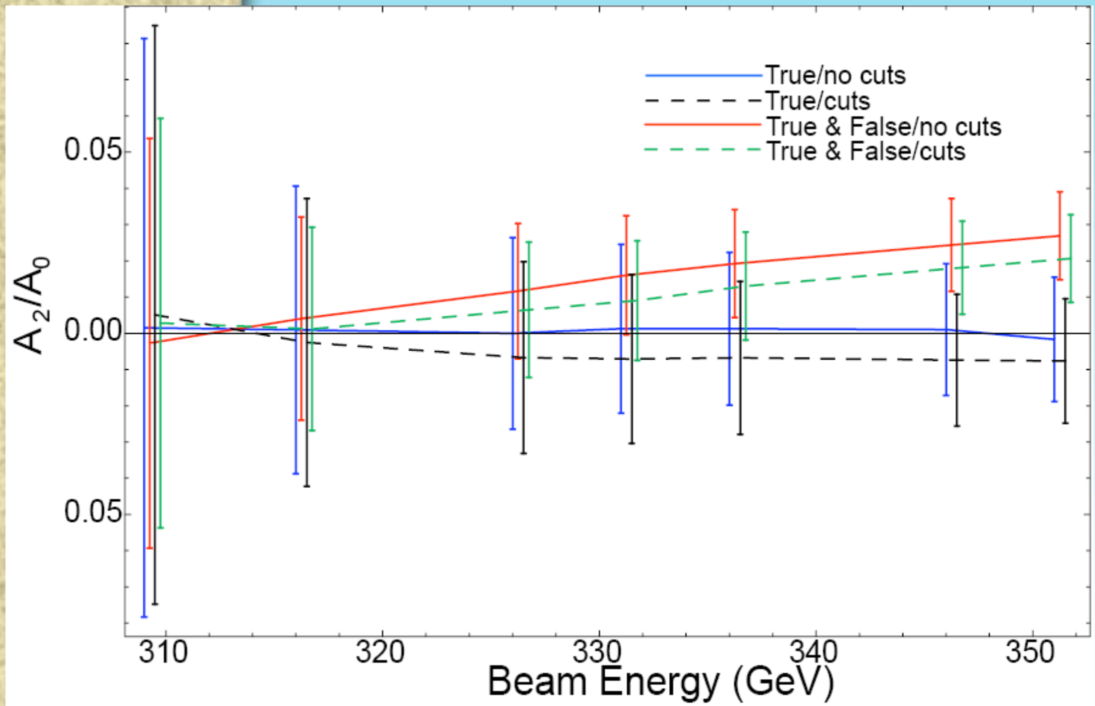




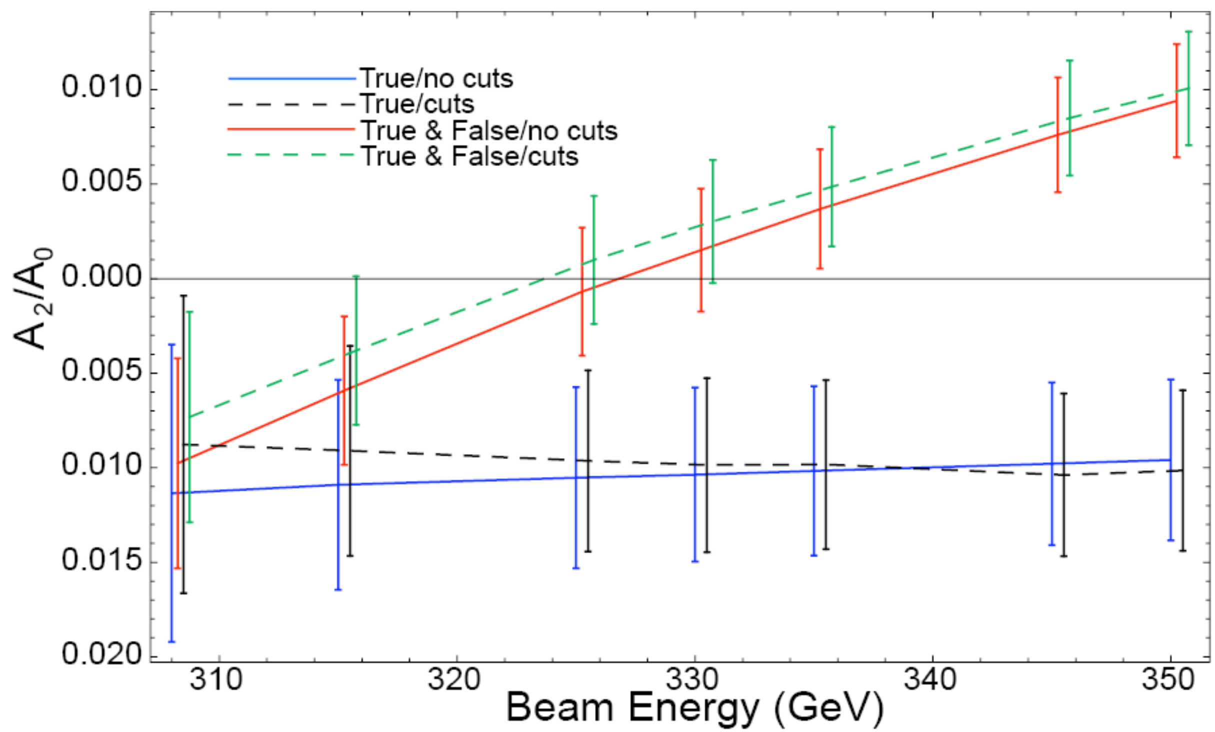
← **SUSY chargino (spinor) production using MUED mass spectrum**

UED KKW (vector) production using MUED mass spectrum →





← SUSY chargino (spinor) production Using SPS3 adjusted to account for cut effects

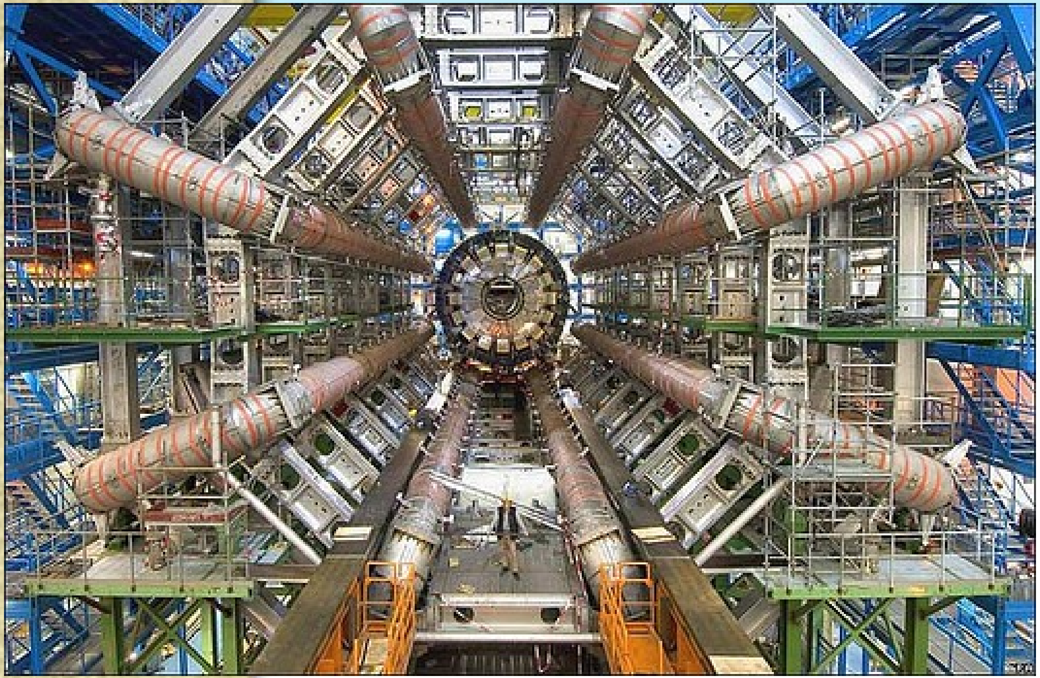


MUED KKW (vector) production using SPS3 adjusted to account for cut effects→

Conclusions

- This technique can be used to differentiate scalars/spinors at ILC
- Inability to distinguish spinors/vectors not a fundamental flaw in the technique unlike threshold measurements
- Look at $\Delta\varphi$ dependence instead of φ_1 or φ_2 dependence. $\Delta\varphi$ is the same for the true and false solutions! **(M. R. Buckley, S. Y. Choi , K. Mawatari, H. Murayama arXiv:0811.3030 [hep-ph])**
- Need to look at other processes
  - No missing energy
  - Further along the decay chain

The Large Hadron Collider



Description

- Proton-Proton beams of 7 TeV each circulated in a ring
- Luminosity of 10-100 fb^{-1} per year
- Scheduled to go online later this year

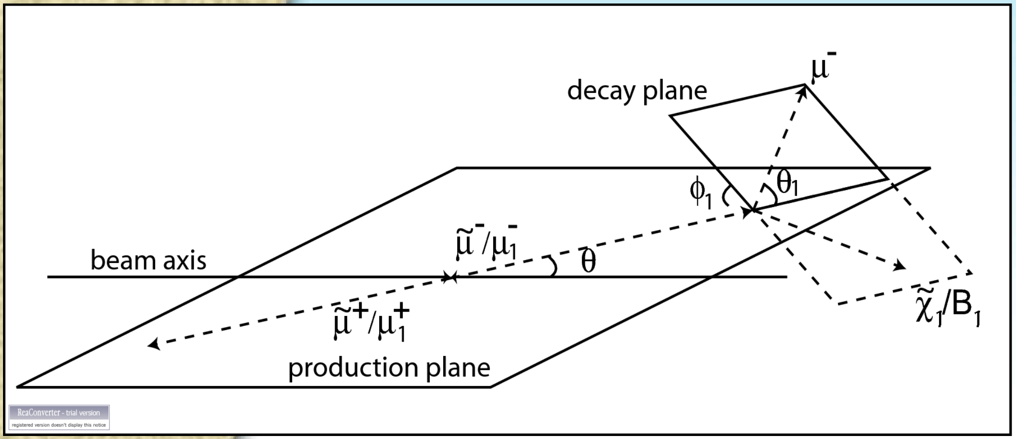
Advantages of the LHC

- High center of mass energy
- High luminosity

Disadvantages of a Hadron Collider

- "Messy" signal
- A-priori unknown center of mass energy and momentum
- Initial states are unknown

Many-fold ambiguity at LHC



- θ **is the production angle**
- θ_i,φ_i **are the decay angles in the lab frame**
- φ_i **are the same in the rest frame of the parent particle**

- Knowns: Outgoing lepton momenta, incoming energy-momentum, masses of all particles
- Unknowns: Missing Particles 4-momentum for a total of 8 unknowns
- Equations:
  - Overall energy momentum conservation: 4 equations
  - 4 mass shell constraints
  - CENTER OF MASS ENERGY AND MOMENTUM RELATIVE TO THE LAB FRAME

8 equations and 8 unknowns + 2 MORE NEW UNKNOWNS!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Spin measurement at the LHC
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Spin Reconstruction at the LHC

- At the LHC this is an even more difficult problem than at a Linear Collider
- Spin measurement relies on looking at the angular dependence of differential cross sections
- Requires reconstructing all momenta
- Edge methods in transverse mass (MT2) used to determine the mass but these do not give the right kinematics.
- Some proposals to measure spin in these schemes (MAOS reconstruction)*

***W.S. Cho, K.C., Y.G. Kim, C.B. Park (KAIST)**

arXiv:0810.4853 [hep-ph] and in preparation

Randall-Sundrum Graviton spin?

- RS case: Fully reconstructible! No missing energy. Spin measurement easier.
- Unique signature! → cos(4ø) mode

$$\frac{d\sigma}{d\varphi} = A_0 + A_1 \cos(\varphi) + A_2 \cos(2\varphi) + A_3 \cos(3\varphi) + A_4 \cos(4\varphi)$$

Warped extra dimensions

- RS-1 Model (4+1 dimensional space time, one dimension compactified)
- Two branes (TeV and Planck brane)
- 5D bulk cosmological constant which is related to the brane vacuum potentials

$$V_{hid} = -V_{vis} = 24M^3k, \quad \Lambda = -24M^3k^2$$

- Solving Einstein's equations leads to the metric

$$ds^2 = e^{-2kr_c\phi}\eta_{\mu\nu}dx^\mu dx^\nu + r_c^2 d\phi^2$$

- Mass scales on the TeV brane are exponentially suppressed relative to mass scales on the Planck brane

$$v \equiv e^{-kr_c\pi} v_0$$

- Solves the Hierarchy problem!

Randall-Sundrum Gravitons

- Quantizing the gravitational perturbations about this metric gives rise to a tower of KK modes in the effective 3+1 dimensional theory on the TeV brane.
- The 0-mode is the regular massless graviton which has an interaction suppressed by the Planck mass. The higher modes are massive and interact with strength $\Lambda = e^{-kr_c\pi}\bar{M}_{pl}$

$$\mathcal{L}_{int} = -\frac{1}{\Lambda}\sum_n G^{(n)\mu\nu}T_{\mu\nu} \qquad T_{\mu\nu} = -\eta_{\mu\nu}\mathcal{L}_{SM} + 2\frac{\delta\mathcal{L}_{SM}}{\delta g_{\mu\nu}}\bigg|_{g_{\mu\nu}=\eta_{\mu\nu}}$$

- The massive KK modes have masses given by $m_n = x_n\Lambda\frac{k}{\bar{M}_{pl}}$

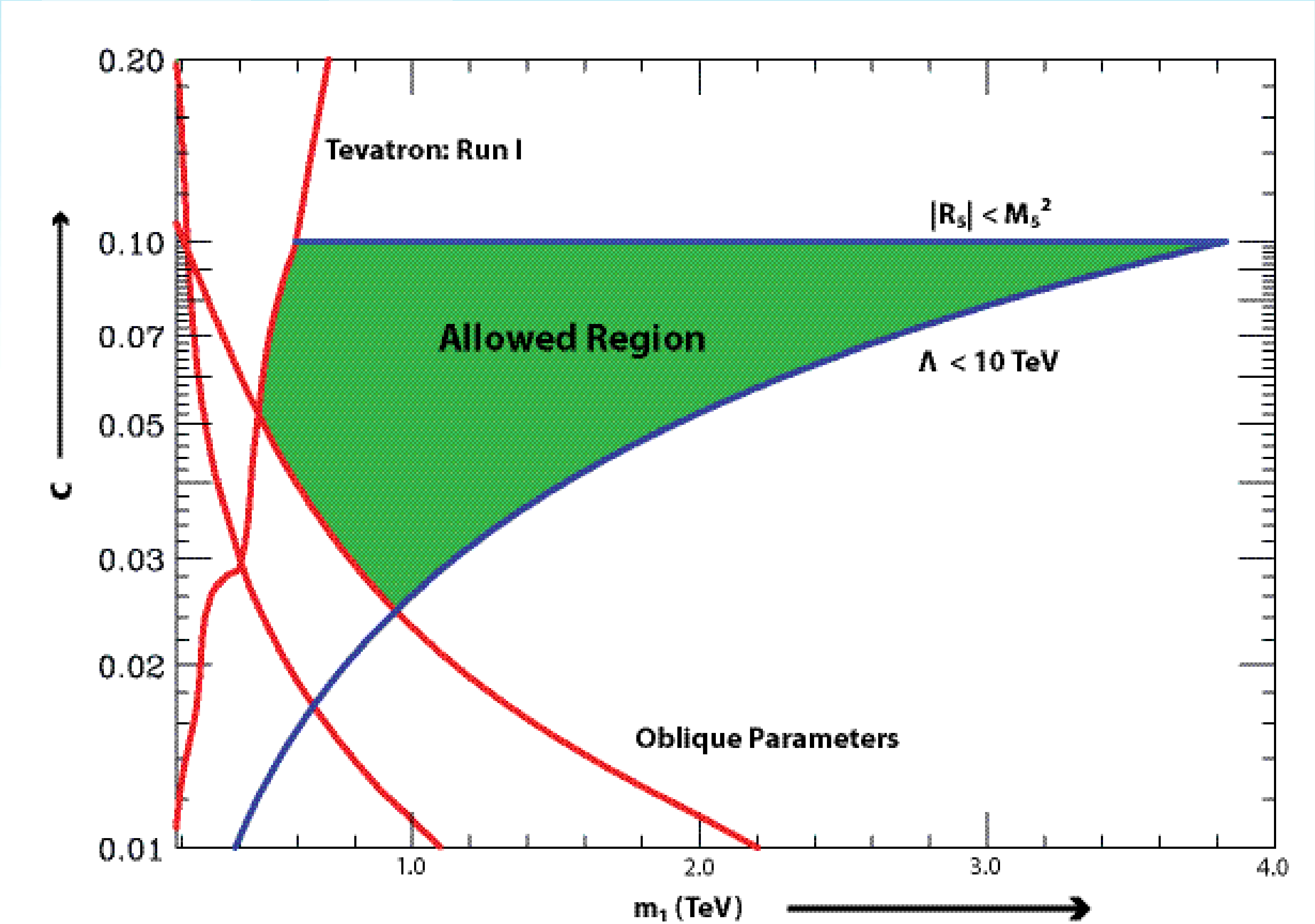
Randall-Sundrum Model

Features:

- Smoking gun: masses in the ratio of the zeros of the J_1 Bessel function
- Introduces gravitational interactions at TeV scale... unique!
- Solves the hierarchy problem
- However... no discrete symmetry KK parity/R parity etc. Implies no missing energy signatures!

Parameter Space

$c \equiv \frac{k}{M_{pl}}$ m_1



hep-ph/0006041 H. Davoudiasl, J.L. Hewett, T.G. Rizzo

Plan for the rest of the talk

- Currently proposed technique to measure the spin of a KK graviton
- Our model independent spin measurement technique using quantum interference
- Results of applying this technique
- Comparison of Techniques

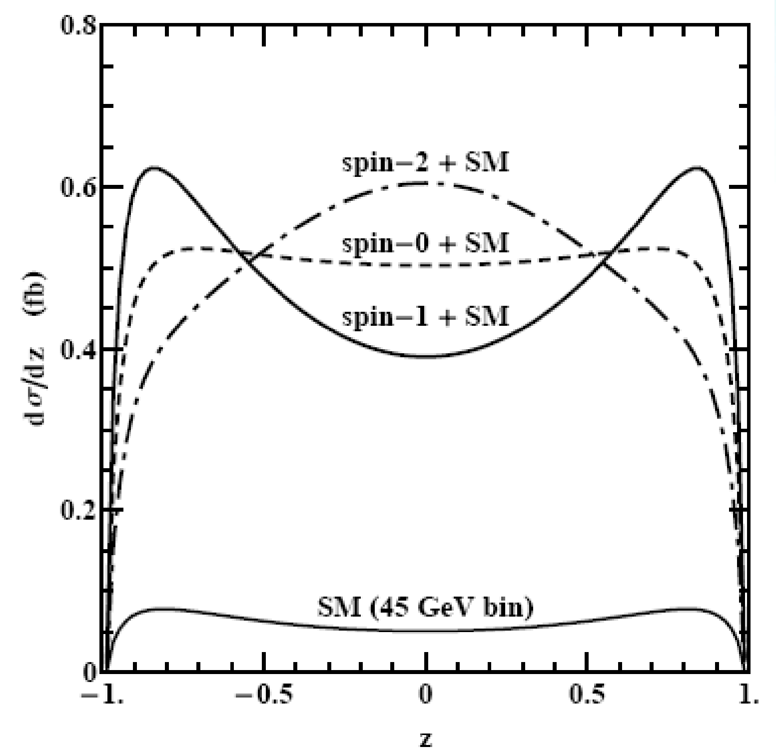
Current Technique

- Consider resonant graviton production followed by decay into a lepton pair

$$q\bar{q} \to G \to l^+l^-$$

$$gg \to G \to l^+l^-$$

$$\frac{d\sigma}{d\cos\theta} = A\cos^4\theta + B\cos^2\theta + C$$



arXiv:0805.2734 P. Osland, A.A. Pankov, N. Paver, A.V. Tsytrinov

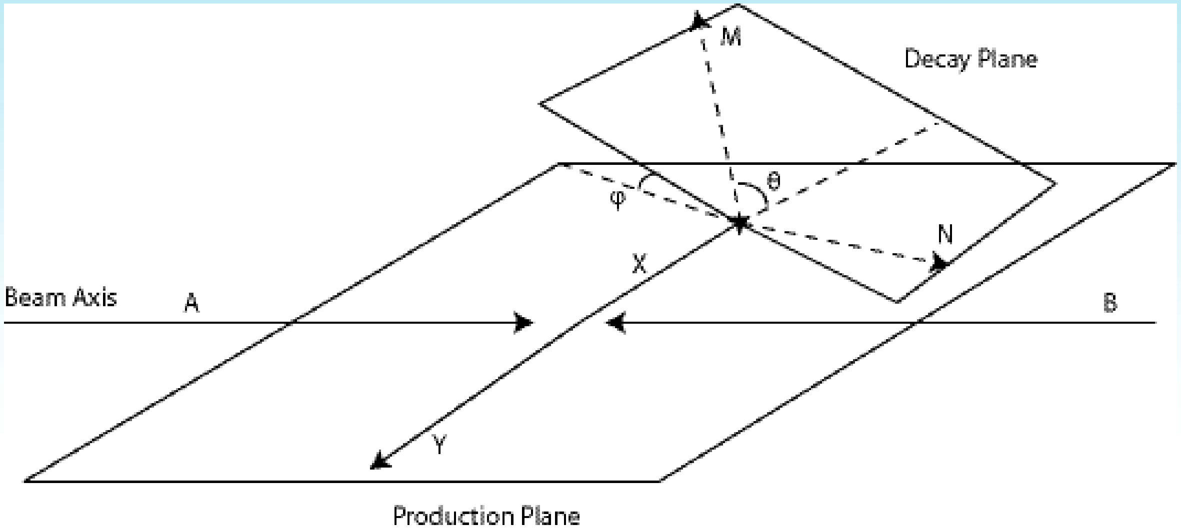
Partonic Processes

- Process

$$gg \longrightarrow Gg$$

$$q(\bar{q})\, g \to G\, q(\bar{q})$$

$$q\,\bar{q} \longrightarrow G\, g.$$



- SM background

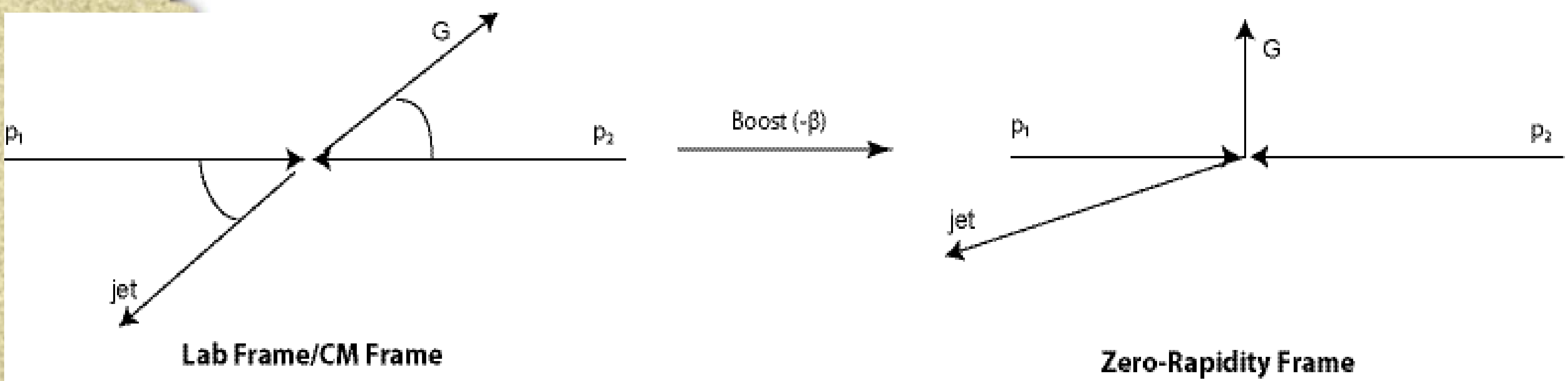
  Through an offshell Z, γ

- Finally decay to $e^+ e^-$ pair

  Background is from spin-1 particles. No contribution to the 4-mode! ... but contributes to the overall normalization of the cross-section.

Zero-Rapidity Frame

- Choose a frame which maximizes $S_4 \equiv |A_4/A_0|$
- CM frame found to have a larger value than lab frame, but error in reconstruction dependent on jet resolution
- Use ZR frame instead. Signal found to be even better!



Cuts

- Cuts on the jet: $|\eta| < 2.5$ GeV, $p_T > 20$ GeV
- Mass window cut (from ATLAS e+e- resolution)

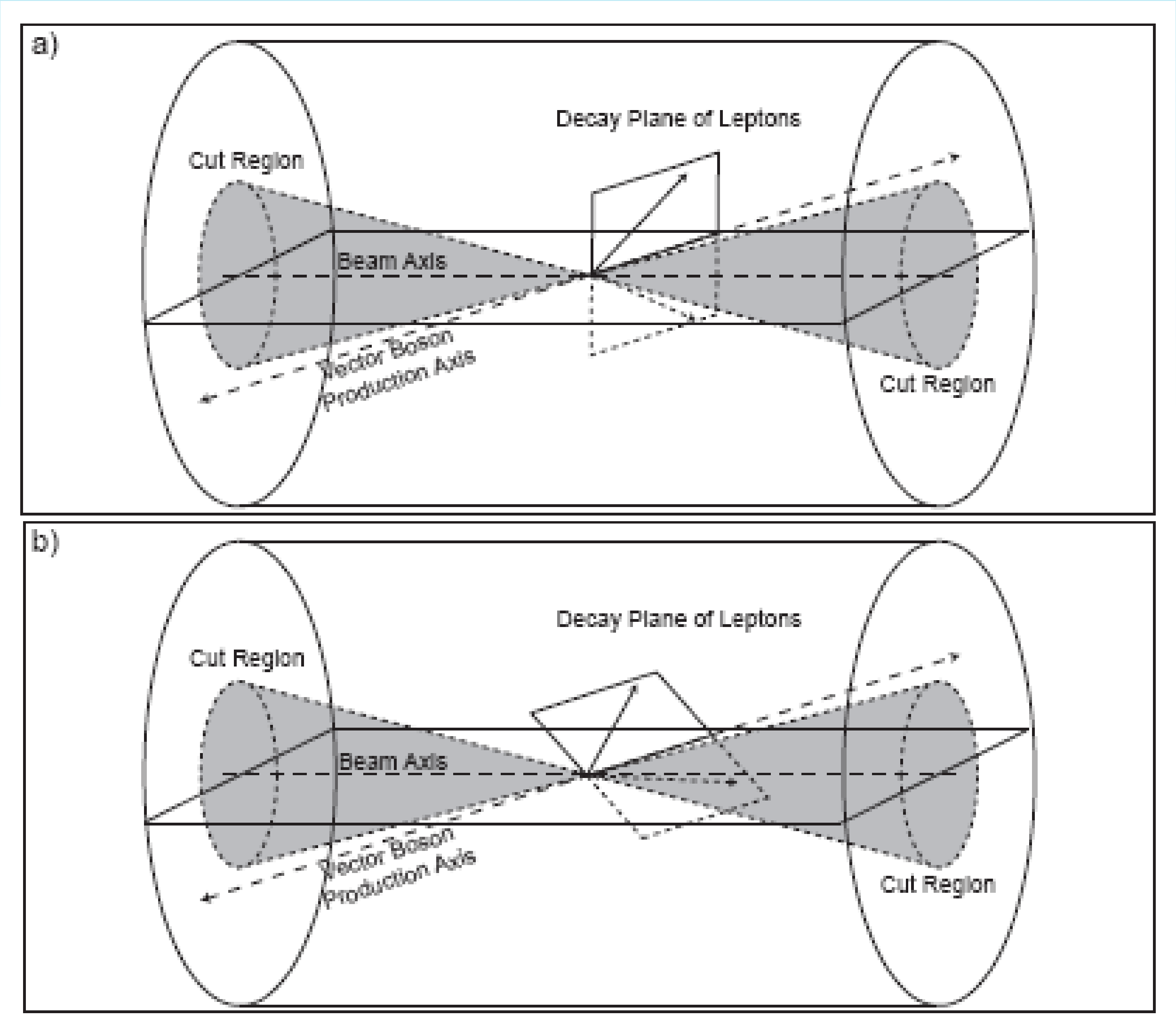
$$\Delta M = 24\,(0.625M + M^2 + 0.0056)^{1/2}\ \mathrm{GeV}.$$

- Cuts on the leptons: $p_{T1} > 10$ GeV and $p_{T2} > 20$ GeV
- Lepton isolation cut:

$$\Delta r \equiv \sqrt{(\Delta\eta)^2 + \Delta\phi^2} > 0.7$$

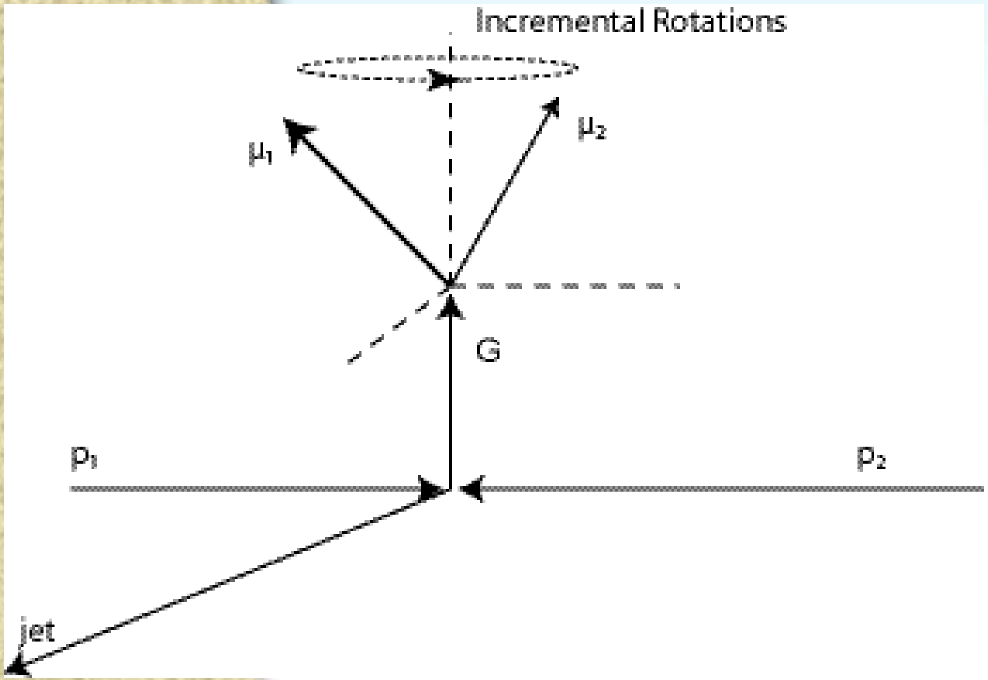
- These cuts are not rotationally invariant!

Rotationally invariant Cuts



Matthew R. Buckley , Beate Heinemann , William Klemm, Hitoshi Murayama arXiv:0804.0476 [hep-ph]

Rotationally invariant Cuts



For each event:

- Boost the event to the ZR frame.
- Make a small rotation.
- Boost back to the lab frame to check that this passes the cuts.
- If it doesn't pass the cuts, throw out the event.
- If it does, repeat.
- If the event survives a full 360^{o} rotation, keep it.

Software Tools used

- Helas with spin 2-particles

  K. Hagiwara, J. Kanzaki, Q. Li, K. Mawatari, 2008

- BASES (adaptive monte-carlo)
- LHApdf (cteq6l)

Results from Simulation

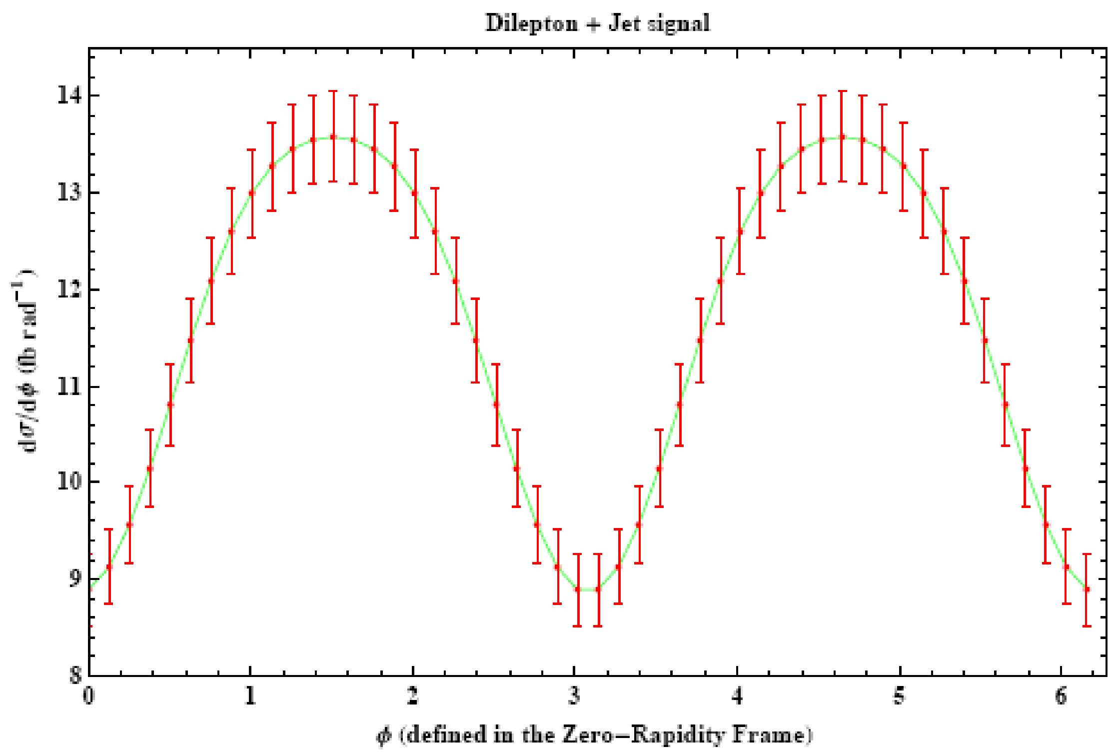


FIG. 5: Differential distribution ($\frac{d\sigma}{d\phi}$) for $m_1 = 1$ TeV and c = 0.05. A strong $\cos(2\phi)$ mode can be seen but there is also a $\cos(4\phi)$ component. The fit is shown in green. The error bars correspond to Gaussian errors for a luminosity of 500 fb^{-1}

- The green curve shows the differential distribution
- 2-mode is easily visible. Is there a 4-mode?
- How do we extract information about it?

H. Murayama and V. Rentala (in preparation)

Extracting the coefficients

$$x_i \equiv \frac{1}{\text{Binsize}} \int_{\frac{2\pi(i-1)}{2n}}^{\frac{2\pi i}{2n}} \frac{d\sigma}{d\phi} d\phi$$

$$= \frac{1}{2\pi/2n} \int_{\frac{2\pi(i-1)}{2n}}^{\frac{2\pi i}{2n}} [\sum_{j=0}^{n-1} A_j \cos(j\phi)$$

$$+ \sum_{j=1}^{n} B_j \sin(j\phi)] d\phi$$

- Linear relationship between binned values and coefficients $x_i = p_{ij} y_j$
  - Matrix with entries as shown below

$$\int_{\frac{2\pi(i-1)}{2n}}^{\frac{2\pi i}{2n}} \cos(j\phi) d\phi \text{ or } \int_{\frac{2\pi(i-1)}{2n}}^{\frac{2\pi i}{2n}} \sin(j\phi) d\phi.$$

- Invert the matrix to recover the coefficients!

Error simulation

- Assume Gaussian errors in each bin

$$(\Delta x_j = x_j \frac{\sqrt{N_j}}{N_j}) \qquad N_j = \mathcal{L}\sigma \frac{x_j}{\sum x_j}$$

- Use the inverted matrix relation y = q.x to find the errors in the coefficients

$$q_{ij} = p_{ij}^{-1} \qquad \Delta S_i = \sqrt{\sum_j \left(\frac{q_{ij}}{A_0} - \frac{S_i}{A_0} q_{0j}\right)^2 \Delta x_j^2}$$

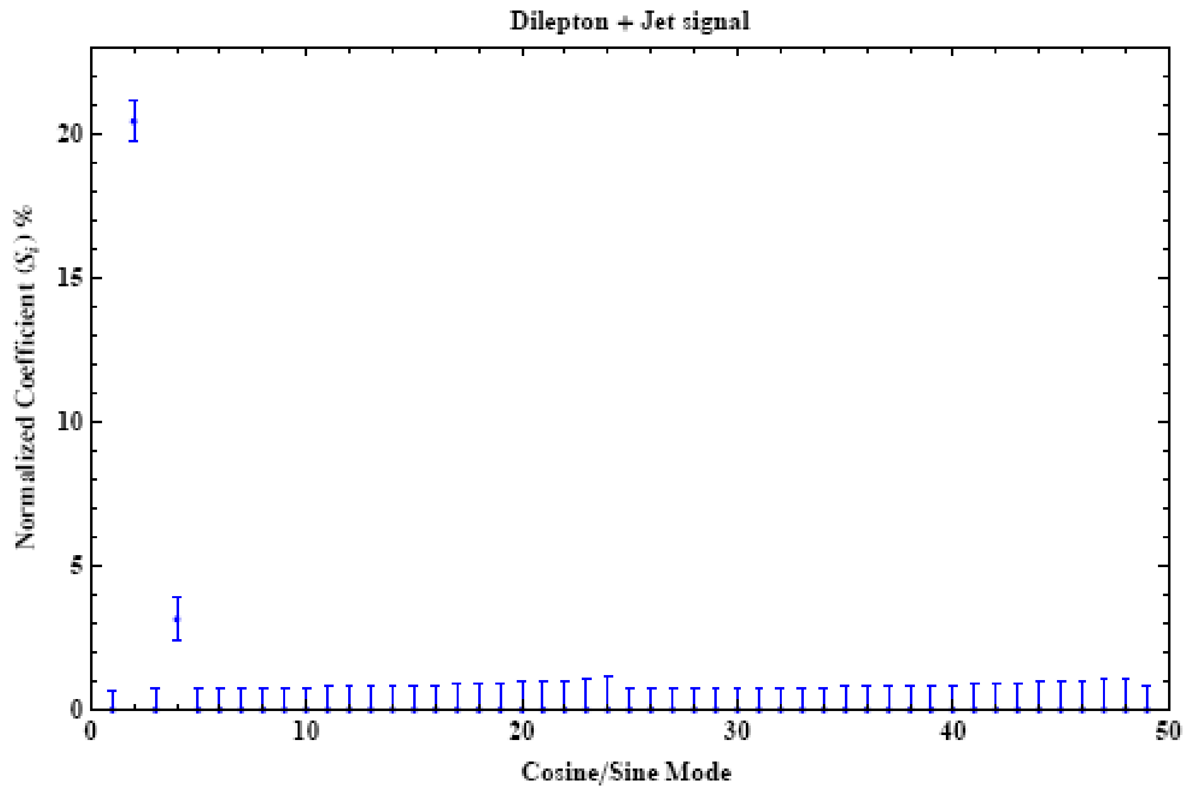


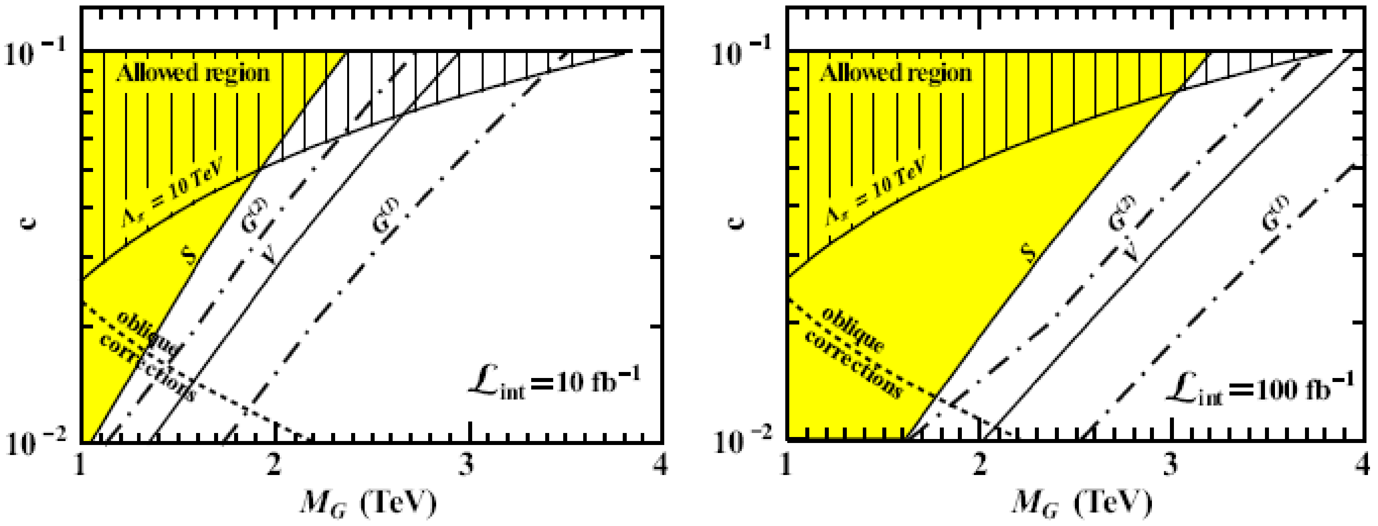
FIG. 6: Fitted cosine coefficients of the binned differential cross-section shown in Figure 5 corresponding to 50 bins. The first 25 modes label the normalized cosine modes, the next 25 show the sine modes. The large 0-mode which would be 100% is not shown)

- Can see a cos(4Ø) mode in addition to the cos(2Ø) mode! (with about 3% strength)
- Error in this example are ~ 20%

m_1 (TeV)	c	10 fb^{-1}	100 fb^{-1}	500 fb^{-1}
0.75	0.1	0.43	0.14	0.06
1.0	0.01	8.03	2.54	1.14
1.0	0.02	3.97	1.26	0.56
1.0	0.05	1.65	0.52	0.23
1.0	0.1	0.93	0.29	0.13
1.5	0.1	5.42	1.71	0.77
2.0	0.1	23.52	7.44	3.32

TABLE III: Statistical Error $\Delta S_4/S_4$ for different integrated luminosities.

- $\Delta S_4/S_4$ < 0.20 corresponds to 5-sigma accuracy
- $\Delta S_4/S_4$ < 0.5 corresponds to 2-sigma accuracy
- $\Delta S_4/S_4$ > 1, imply consistency with zero (no discovery)
- If one includes muons error falls by a factor $\sqrt{2}$



	Discovery		Identification	
$\mathcal{L}_{int}$	$c = 0.01$	$c = 0.1$	$c = 0.01$	$c = 0.1$
10 fb^{-1}	1.7 TeV	3.5 TeV	1.1 TeV	2.4 TeV
100 fb^{-1}	2.5 TeV	4.6 TeV	1.6 TeV	3.2 TeV

arXiv:0805.2734 P. Osland, A.A. Pankov, N. Paver, A.V. Tsytrinov

arXiv:0805.2734 P. Osland, A.A. Pankov, N. Paver, A.V. Tsytrinov

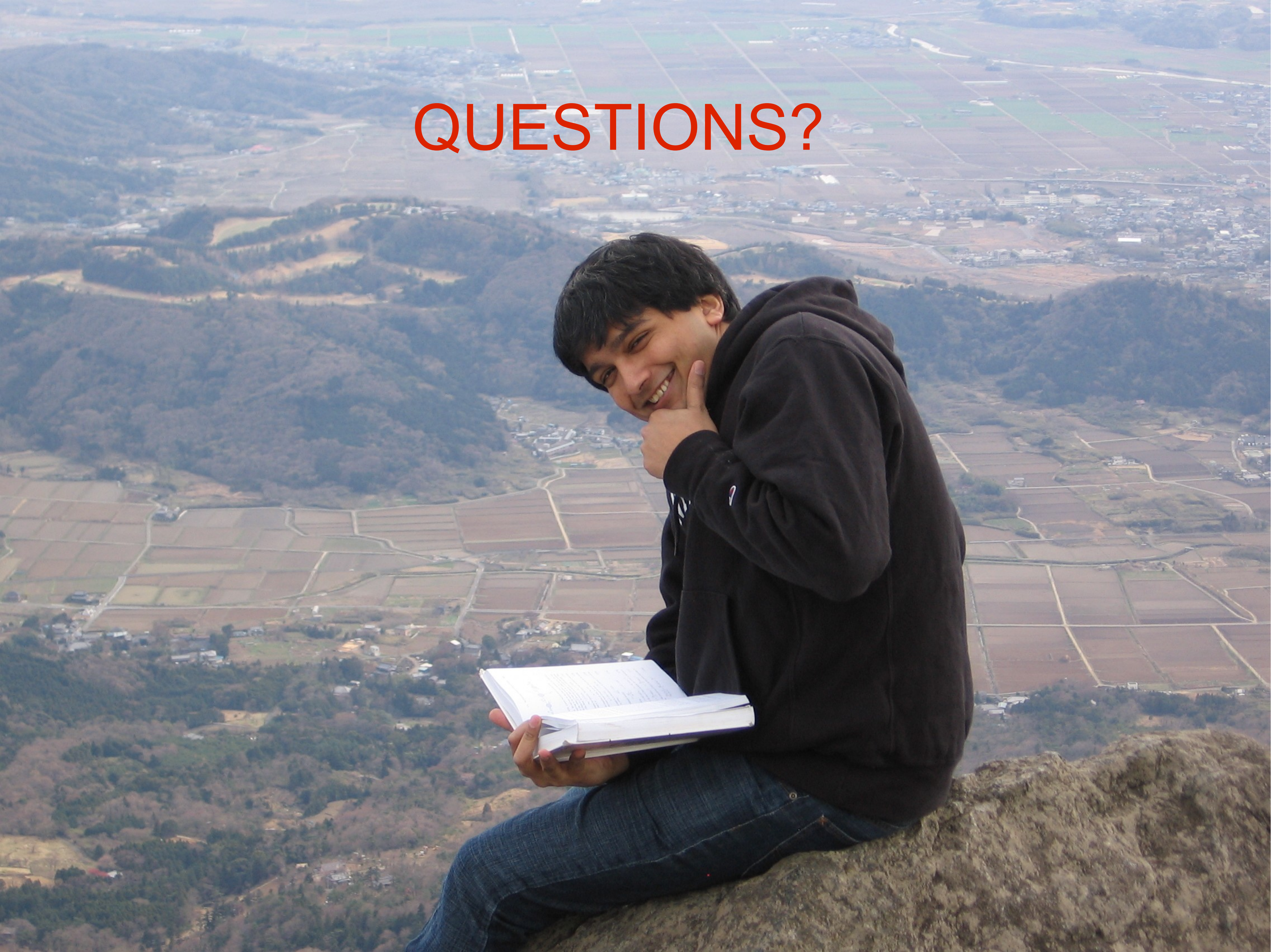
m_1 (TeV)	c	10 fb^{-1}	100 fb^{-1}	500 fb^{-1}
0.75	0.1	0.07	0.02	0.01
1.0	0.01	1.30	0.41	0.18
1.0	0.02	0.62	0.19	0.09
1.0	0.05	0.25	0.08	0.04
1.0	0.1	0.14	0.04	0.02
1.5	0.1	0.39	0.12	0.06
2.0	0.1	0.93	0.29	0.13

TABLE IV: Statistical Error $\Delta S_2/S_2$ for different integrated luminosities.

- Even with low statistics can distinguish scalar and graviton

Summary

- ~3% signal in S_4 for values of $m_1 < 1$ TeV and large values of the coupling $c \sim 0.1$.
- Error in measurement only dependent on statistics but cross-section drops rapidly
- Can distinguish scalars from spin-2 objects easily even with low luminosities!
- Important complementary, model-independent determination of spin possible with large integrated luminosity


QUESTIONS?