Kaon Physics with Chiral Quarks

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- The work described below is part of the programme of the RBC & UKQCD collaborations.
- **UKQCD Members:**
  C. Allton, D. Antonio, P. Boyle, D. Brömmel. M. Clark, P. Cooney, L.Del Debbio, M.Donnellan, J. Flynn, A. Hart, R.Horsley, B.Joo, A. Jüttner, A. Kennedy, R. Kenway, C. Kim, C. Maynard, J. Noaki, H. Pedrosa de Lima, B. Pendleton, C. Sachrajda, C.Torres, A. Trivini, R. Tweedie, J. Wennekers, A. Yamaguchi, J. Zanotti
- **RBC Members:**
  Y. Aoki, C. Aubin, T. Blum, M. Cheng, N. Christ, S. Cohen, C. Dawson, T. Doi, K. Hashimoto, T. Ishikawa, T. Izubuchi, C. Jung, M. Li, S. Li, M. Lightman, H. Lin, M. Lin, O. Loktik, R. Mawhinney, S. Ohta, S. Sasaki, E. Scholz, A. Soni, T. Yamazaki

Papers

1. *Physical Results from 2+1 Flavor Domain Wall QCD and SU(2) Chiral Perturbation Theory,*
   C. Allton *et al.*, (32 Authors, 133 pages)
   Phys.Rev. **D78** (2008) 114509; [arXiv:0804.0473 [hep-lat]]
2. $K_{\ell 3}$ *semileptonic form factor from 2+1 flavour lattice QCD,*
   P.A. Boyle, A. Jüttner, R.D. Kenway, C.T. Sachrajda, S. Sasaki, A. Soni, R.J. Tweedie and J.M. Zanotti,
   Phys. Rev. Lett. **100** (2008) 141601; [arXiv:0710.5136 [hep-lat]].
3. *Hadronic form factors in lattice QCD at small and vanishing momentum transfer,*
   P. A. Boyle, J. M. Flynn, A. Juttner, C. T. Sachrajda and J. M. Zanotti,
   JHEP **0705** (2007) 016 [arXiv:hep-lat/0703005].
4. *The pion's electromagnetic form factor at small momentum transfer in full lattice QCD,*
   P.A. Boyle, J.M. Flynn, A. Jüttner, C. Kelly, H. Pedroso de Lima, C.M. Maynard, C.T. Sachrajda and J.M. Zanotti,
   JHEP 0807:112,2008; [arXiv:0804.3971 [hep-lat]].
5. *Neutral kaon mixing from 2+1 flavor domain wall QCD,*
   D. J. Antonio *et al.*, (19 Authors)
   Phys. Rev. Lett. **100** (2008) 032001 [arXiv:hep-ph/0702042].

6. *Non-perturbative renormalization of quark bilinear operators and B_K using domain wall fermions,*
   Y. Aoki *et al.*, (14 Authors, 81 pages)
   Phys. Rev. D78 (2008) 054510 [arXiv:0712.1061 [hep-lat]].
7. *Renormalization of quark bilinear operators in a MOM-scheme with a non-exceptional subtraction point,*
   C. Sturm, Y. Aoki, N. H. Christ, T. Izubuchi, C. T. C. Sachrajda and A. Soni,
   arXiv:0901.2599 [hep-ph].
8. *SU(2) Chiral Perturbation Theory for $K_{\ell 3}$ Decay Amplitudes,*
   J. Flynn and C.T. Sachrajda,
   Nucl. Phys. B812 (2009) 64 [arXiv:0809.1229 [hep-ph]].

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- We use two datasets of DWF with the Iwasaki Gauge Action with a lattice spacing of about 0.114 fm:
  - $24^3 \times 64 \times 16 \quad (L \simeq 2.74\,\text{fm})$
  - $(16^3 \times 32 \times 16 \quad (L \simeq 1.83\,\text{fm})\,)$
- On the 24^3 lattice measurements have been made with 4 values of the light-quark mass: $ma = 0.03\ (m_\pi \simeq 670\,\text{MeV})$; $ma = 0.02\ \ (m_\pi \simeq 555\,\text{MeV})$; $ma = 0.01\ (m_\pi \simeq 415\,\text{MeV})$; $ma = 0.005\ (m_\pi \simeq 330\,\text{MeV})$.
  - (Using partial quenching the lightest pion in our analysis has a mass of about 240 MeV .)

  On the 16^3 lattice results were obtained with $ma = 0.03, 0.02$ and 0.01.
- For the (sea) strange quark we take $m_s a = 0.04$, although a posteriori we see that this is a little too large.
- We are currently generating and analysing an ensemble on a $32^3 \times 64 \times 16$ lattice with $a \simeq 0.081$ fm ($L \simeq 2.6$ fm) with three dynamical masses ($m_\pi \simeq 310,\ 365$ and 420 MeV).

  This will enable us to reduce the discretization errors significantly.

  Some preliminary results were presented at Lattice 2008.

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- $K_{\ell 2}$ Decays:

$$\frac{\Gamma(K\to\mu\bar{\nu}(\gamma))}{\Gamma(\pi\to\mu\bar{\nu}(\gamma))}=\frac{|V_{us}|^2}{|V_{ud}|^2}\,\frac{f_K^2}{f_\pi^2}\,\frac{m_K\left(1-\frac{m_\mu^2}{m_K^2}\right)}{m_\pi\left(1-\frac{m_\mu^2}{m_\pi^2}\right)}\times 0.9930(35)$$

From the experimental ratio of the widths we get:

$$\frac{|V_{us}|^2}{|V_{ud}|^2}\,\frac{f_K^2}{f_\pi^2}=0.07602(23)_{\rm exp}(27)_{\rm RC}\,,\qquad \text{PDG2006}$$

so that a precise determination of f_K/f_π will yield V_{us}/V_{ud} .

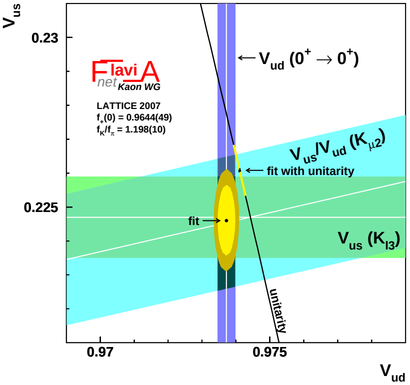
- $K_{\ell 3}$ Decays

$$\Gamma_{K\to\pi\ell\nu}=C_K^2\,\frac{G_F^2 m_K^5}{192\pi^3}\,I\,S_{\rm EW}[1+2\Delta_{SU(2)}+\Delta_{EM}]\,|V_{us}|^2\,|f_+(0)|^2$$

From the experimental measurement of the width we get:

$$|V_{us}|\,f_+(0)=0.2169(9)\,,\qquad \text{PDG2006}$$

so that a precise determination of $f_+(0)$ will yield V_{us}.



$$f_+^{K\pi}(0) \;=\; 0.9644(33)(34)$$
$$\Rightarrow\; |V_{us}| = 0.2247(12)$$

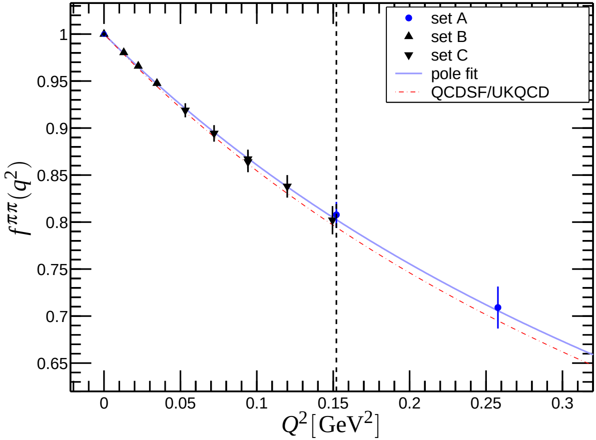
$$\frac{f_K}{f_\pi} \;=\; 1.198(10)$$
$$\Rightarrow\; |V_{us}| = 0.2241(24)$$

A.Jüttner, Lattice 2007

Our final result from the $K_{\ell 3}$ project is

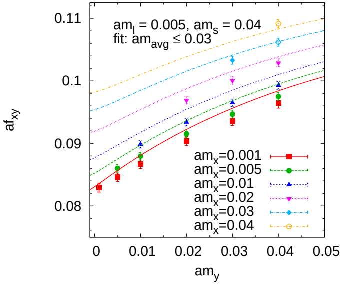
$$f_+^{K\pi}(0) = 0.964(5)\,.$$

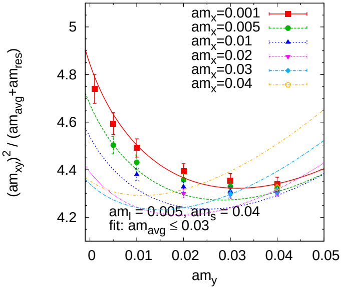
Electromagnetic form factor of a pion with mass 330 MeV

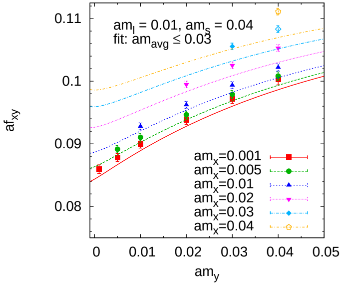


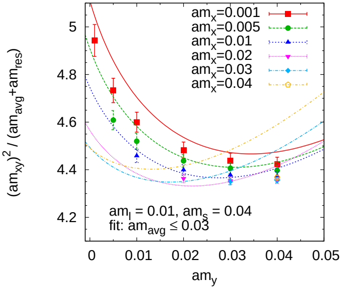
- Much effort is being devoted to the comparison of our data to the predictions of ChPT and PQChPT (in which the masses of the valence and sea quarks are different - S.R.Sharpe and N.Shoresh, [hep-lat/0006017]) to the meson masses and decay constants.
- Since (after tuning) m_s can be kept at the physical value we can use either SU(3) or SU(2) ChPT.
- Not surprisingly, fitting the complete range of our data to ChPT are poor gives poor χ^2/dof.
- For meson masses $\lesssim 415$ MeV the fits are good,
- but ....

Results – NLO $SU(3) \times SU(3)$ fit is bad for cut $am_{avg} < 0.03$









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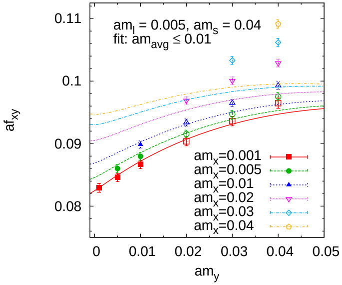
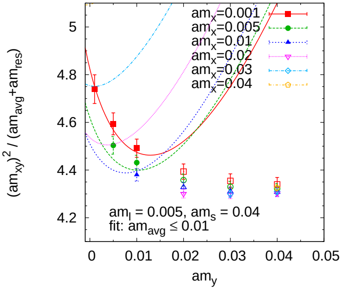
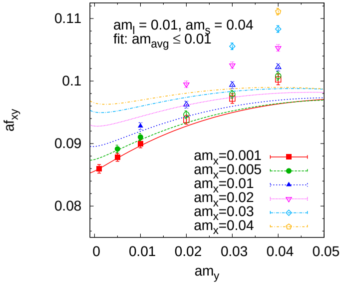
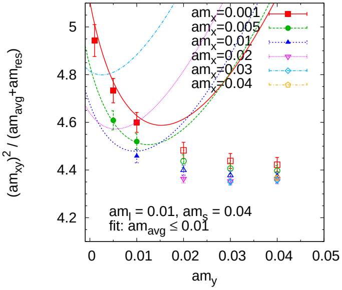
Results – NLO $SU(3) \times SU(3)$ fit is good for cut $am_{avg} < 0.01$









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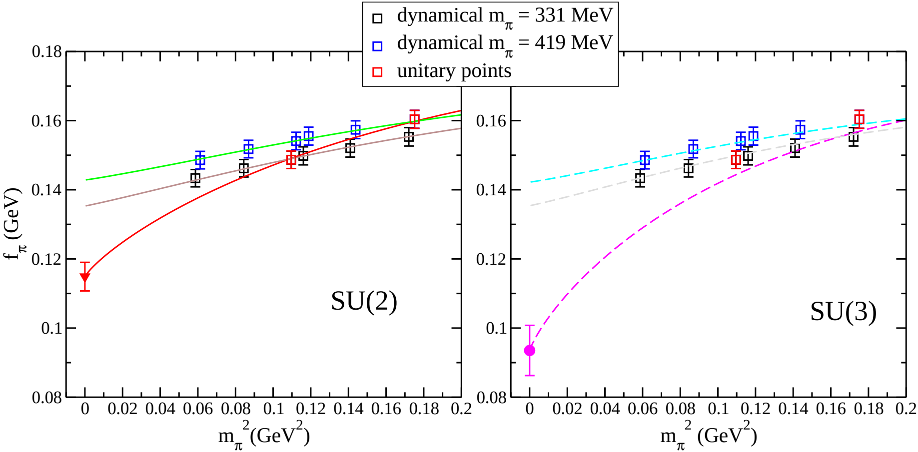


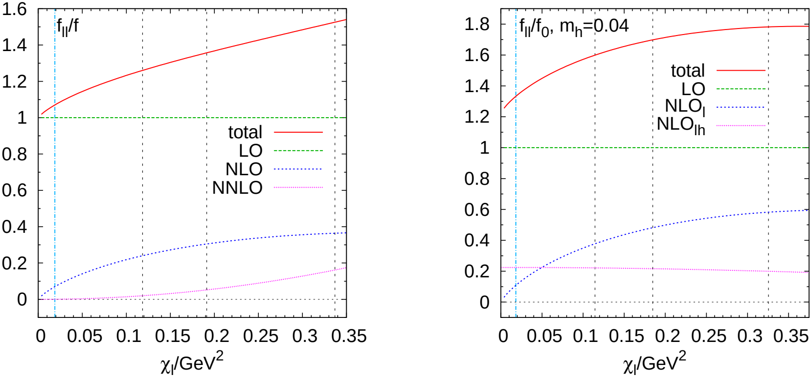
$$f_\pi/f \simeq 1.08\,, \quad f/f_0 = 1.23(6)\,, \quad (f/f_0{}_{\,\mathrm{MILC}} = 1.15(5)^{+13}_{-3})\,,$$

$$f_\pi = (124.1 \pm 3.6_{\,\mathrm{stat}} \pm 6.9_{\,\mathrm{syst}})\mathrm{MeV} \simeq (124.1 \pm 7.8)\mathrm{MeV}$$

- The large value of f_π/f_0 (and even larger values of f_P/f_0 of 1.6 or so) lead us to present our results based on $SU(2)\times SU(2)$ ChPT.

Summary

- $SU(3) \times SU(3)$ chiral fits to the pseudoscalar masses and decay constants work well, but only at very light masses.
- Perhaps going to NNLO would increase the range of the good fits, but the number of new LECs is too large for the data which we have (other collaborations are trying to use at least the analytical terms and we have also tried this).
- In view of the large chiral corrections which we find for the pseudoscalar decay constant we prefer to present our results using SU(2) ChPT where possible.
- The dynamics of the interaction between the Lattice and ChPT communities has changed radically.
  - We continue of course to use ChPT to guide the chiral extrapolations.
  - The data is becoming sufficiently accurate that (at least some of) the Low Energy Constants of ChPT are being evaluated with unprecedented precision.
    (This is not surprising since we have the powerful tool of being able to vary the quark masses.)
  - I make an important additional point below.



SU(2) SU(3)

- SU(2) – Only the NNLO analytic terms are included in this fit.
- SU(3) –

$$f_{ll} = f_0 \left\{ 1 + \frac{24}{f_0^2} L_4^{(3)} \bar{\chi} + \frac{8}{f_0^2} L_5^{(3)} \chi_l - \frac{1}{16\pi^2 f_0^2} \left[\frac{\chi_l + \chi_h}{2} \log \frac{\chi_l + \chi_h}{2\Lambda_\chi^2} + 2\chi_l \log \frac{\chi_l}{\Lambda_\chi^2} \right] \right\}$$

- NLO terms of order of several 10%s are present.

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$$m_\pi^2 = \chi_l\left\{1+\frac{\chi_l}{16\pi^2f^2}\left(64\pi^2 l_3^r + \log\left[\frac{\chi_l}{\Lambda_\chi^2}\right]\right)\right\} \equiv \chi_l\left\{1-\frac{\chi_l}{16\pi^2f^2}\bar{l}_3\right\}$$

$$f_\pi = f\left\{1+\frac{m_\pi^2}{8\pi^2f^2}\left(16\pi^2 l_4^r - \log\left[\frac{m_\pi^2}{\Lambda_\chi^2}\right]\right)\right\} \equiv f\left\{1+\frac{m_\pi^2}{8\pi^2f^2}\bar{l}_4\right\}$$

- "Phenomenological Indirect Determinations":

$\bar{l}_3 = 2.9 \pm 2.4$, Gasser&Leutwyler (1984); $\bar{l}_4 = 4.4 \pm 0.2$, Colangelo, Gasser, Leutwyler (2001)

G.Colangelo – Kaon2007

- Lattice Determinations:

Collaboration	Paper	$\bar{l}_3$	$\bar{l}_4$
MILC	hep-lat/0611024	0.60(12)	3.9(5)
MILC	arXiv:0710.1118	2.85(7)(?)	–
RBC/UKQCD	arXiv:0804.3971	3.13(33)(24)	4.43(0.14)(77)
PACS-CS	arXiv:0810.0351	3.14(23)	4.09(19)
Del Debbio et al.	hep-lat/0610059	3.0(5)(1)	–
ETM	hep-lat/0701012	3.44(8)(35)	4.61(4)(11)
JLQCD/TWQCD	arXiv:0806.0894	$3.38(40)(24)(^{+31}_{-0})$	$4.12(35)(30)(^{+31}_{-0})$

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    (This is not surprising since we have the powerful tool of being able to vary the quark masses.)
  - I make an important additional point below.

- Applying $SU(2)\times SU(2)$ χPT transformations to kaons, only the u and d quarks transform $\Rightarrow$ χPT formalism must be extended.
- Roessl has introduced the corresponding Lagrangian for the interactions of kaons and pions in order to study $K\pi$ scattering near threshold.

A.Roessl, hep-ph/9904230

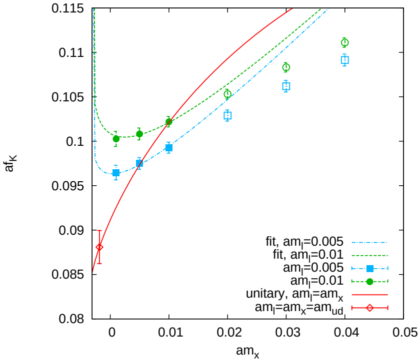
- There are overlaps with Heavy Meson Chiral Perturbation Theory, but an important difference is that $m_{K^*}\neq m_K$, whereas in the heavy quark limit $m_{B^*}=m_B$.

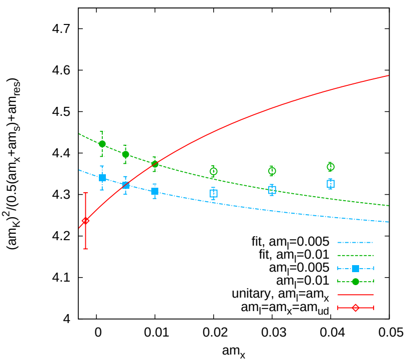
M.B.Wise, Phys.Rev **D45** (1992) 2188

G.Burdman and J.Donoghue, Phys.Lett. **B280** (1992) 287

- We have derived the chiral behaviour of m_K^2, f_K and B_K in the unitary and partially quenched theories and have used the results in our phenomenological studies.
- m_s is considered to be of $O(\Lambda_{\rm QCD})$ so that the expansion is in m_π^2/m_K^2 as well as m_π^2/Λ_χ^2.
  m_K^2/Λ_χ^2 effects however, are fully absorbed into the LECs of $SU(2)\times SU(2)$ χPT.

- For f_K and m_K^2 we use PQ $SU(2)\times SU(2)$ χPT keeping the light valence quark $am_{ud} < 0.01$ and $am_s = 0.04$.





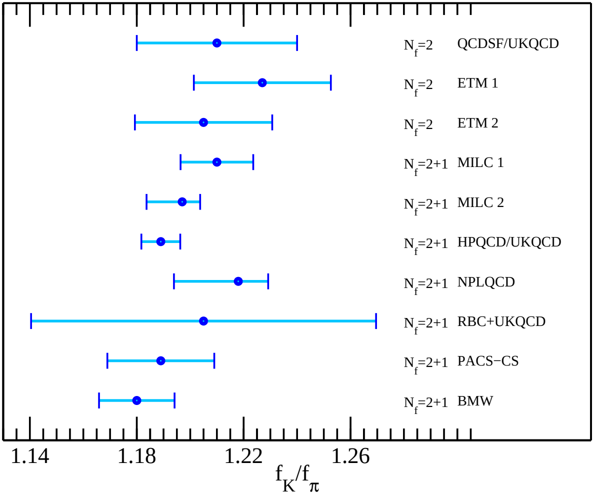
Our preliminary result is

$$f_K/f_\pi = 1.205(18)_{\mathrm{stat}}(62)_{\mathrm{syst}} \qquad (\rightarrow 1.205(18)_{\mathrm{stat}}(40)_{\mathrm{syst}})$$

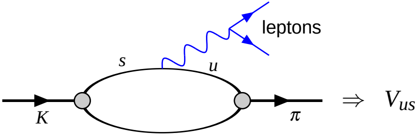
to be compared with A.Jüttner's best lattice value of 1.198(10).

Syst. error dominated by lack of continuum extrapolation ($\rightarrow$ chiral extrapolation).

f_K/f_π Compendium



A.Jüttner and Flavianet Kaon Averaging Group



$$\langle \pi(p_\pi)\,|\bar{s}\gamma_\mu u\,|K(p_K)\rangle = f_0(q^2)\,\frac{M_K^2-M_\pi^2}{q^2}q_\mu + f_+(q^2)\left[(p_\pi+p_K)_\mu - \frac{M_K^2-M_\pi^2}{q^2}q_\mu\right]$$

where $q \equiv p_K - p_\pi$.
To be useful in extracting V_{us} we require $f_0(0)=f_+(0)$ to better than about 1% precision.

$$\chi\text{PT} \Rightarrow f_+(0) = 1+f_2+f_4+\cdots \quad \text{where} \quad f_n = O(M^n_{K,\pi,\eta}).$$

Reference value $f_+(0)=0.961\pm0.008$ where $f_2=-0.023$ is relatively well known from χPT and $f_4, f_6, \cdots$ are obtained from models. Leutwyler & Roos (1984)

1% precision of $f^+(0)$ is conceivable because it is actually $1-f^+(0)$ which is computed:

Bećirević et al. [hep-ph/0403217] based on S.Hashimoto et al. [hep-ph/9906376] for $B\to D$ Decays

- The starting point is the evaluation of the matrix elements at $q^2_{\rm max}$, i.e. with the pion and kaon at rest:

$$\frac{\langle\pi|\bar s\gamma_4 u|K\rangle\langle K|\bar u\gamma_4 s|\pi\rangle}{\langle\pi|\bar u\gamma_4 u|\pi\rangle\langle K|\bar s\gamma_4 s|K\rangle}=\left[f_0(q^2_{\rm max})\right]^2\,\frac{(m_K+m_\pi)^2}{4m_Km_\pi}\,.$$

$f_0(q^2_{\rm max})$ is obtained with excellent precision.

	$16^3\times 32$		$24^3\times 64$	
am_{ud}	$q^2_{\rm max}$ (GeV^2)	$f_0(q^2_{\rm max})$	$q^2_{\rm max}$ (GeV^2)	$f_0(q^2_{\rm max})$
0.03	0.00233(4)	1.00035(3)	0.00235(4)	1.00029(6)
0.02	0.01178(24)	1.00241(19)	0.01152(20)	1.00192(34)
0.01	0.03475(66)	1.01436(81)	0.03524(62)	1.00887(89)
0.005	–	–	0.06070(107)	1.02143(132)

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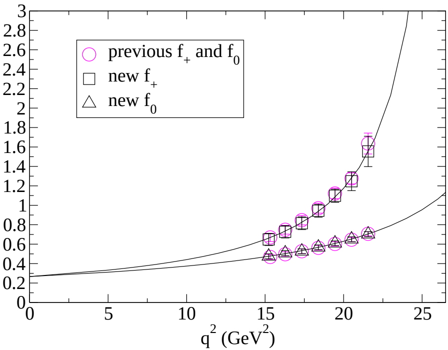
- In the SU(2) chiral limit, $m_{ud} = 0$, we have the Callan-Treiman Relation

$$f_0(q^2_{\rm max}) = \frac{f_K}{f_\pi} \simeq 1.26.$$

- We have investigated whether the difference of the numbers in the table and 1.26 can be understood using SU(2) ChPT. J.Flynn & CTS
  - The one-loop chiral logarithms have a large coefficient and are of the correct size to account for the difference. However they have the wrong sign!
  - There are linear and quadratic terms in m_π.
    They cannot be calculated in SU(2) ChPT, but estimating the LECs by converting results from SU(3) ChPT suggests that these terms have the correct sign and magnitude to account for the difference.



E.Gulez et al.
hep-lat/0601021

- The Callan-Treiman relation can be generalised to the $f^{\,0}$ form factor for other flavours (and in the static theory) in the SU(2) Chiral limit.

$$f^0_{D\to\pi}(q^2_{\rm max}) \xrightarrow[m_\pi^2\to 0]{} \frac{f^{(D)}}{f} \qquad \text{and} \qquad f^0_{B\to\pi}(q^2_{\rm max}) \xrightarrow[m_\pi^2\to 0]{} \frac{f^{(B)}}{f}\,,$$

	$16^3 \times 32$		$24^3 \times 64$	
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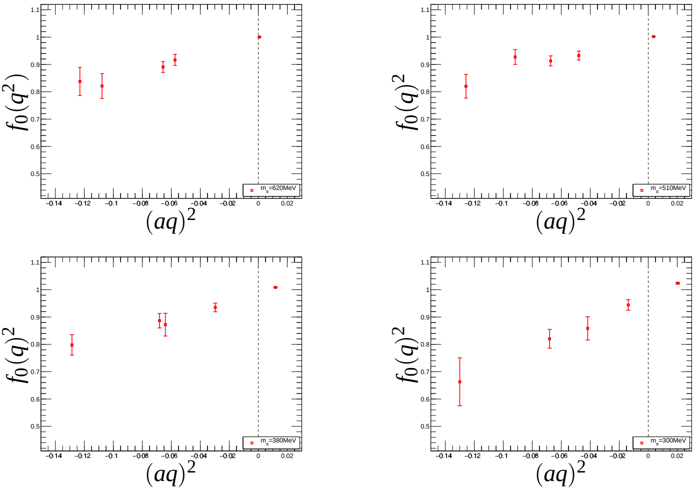
- Having obtained $f_0(q^2_{\rm max})$ we need to extrapolate in q^2 and m_{ud}.
- Note that for heavier values of m_{ud}, $q^2_{\rm max}$ is close to zero.
- Conventionally the q^2 extrapolation is done by calculating the form factors with

$$|\vec{p}_K| \quad \text{or} \quad |\vec{p}_\pi| \quad = \quad p_{\rm min} \quad \text{or} \quad \sqrt{2}\,p_{\rm min}\,,$$

where $p_{\rm min} = 2\pi/L$ and L is the spatial extent of the lattice.

$f_0(q^2)$ for the four values of m_{ud}



- There are a number of ChiPT-motivated extrapolation ansatze available. For our central values we use a simultaneous fit to the q^2 and chiral behaviour:

$$f_0(q^2, m_\pi^2, m_K^2) = \frac{1 + f_2 + (m_K^2 - m_\pi^2)^2 (A_0 + A_1 (m_K^2 + m_\pi^2))}{1 - q^2/(M_0 + M_1 (m_K^2 + m_\pi^2))^2}\,,$$

  where f_2 is known and A_0, A_1, M_0, M_1 are fit parameters.

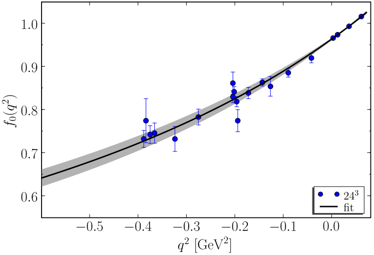
- The spread of results obtained with this simultaneous above, the polynomial fit

$$\begin{aligned} f_0(q^2, m_\pi^2, m_K^2) &= 1 + f_2 + (m_K^2 - m_\pi^2)^2 (A_0 + A_1 + A_2 (m_K^2 + m_\pi^2)) \\ &+ (A_3 + (2A_0 + A_1)(m_K^2 + m_\pi^2))\, q^2 + (A_4 - A_0 + A_5 (m_K^2 + m_\pi^2))\, q^4\,, \end{aligned}$$

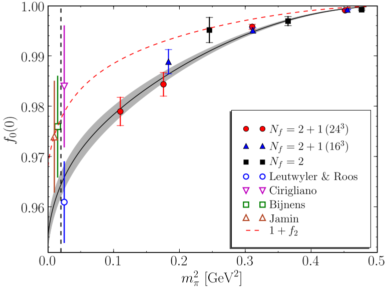
  and the *z-fit* form of Hill (hep-ph/0607108) are used to estimate the systematic errors.

- It would be particularly interesting to have the NNLO results in a form useful for these extrapolations. Bijnens et al. – Work in Progress.

q^2 and Chiral Extrapolations – Cont.

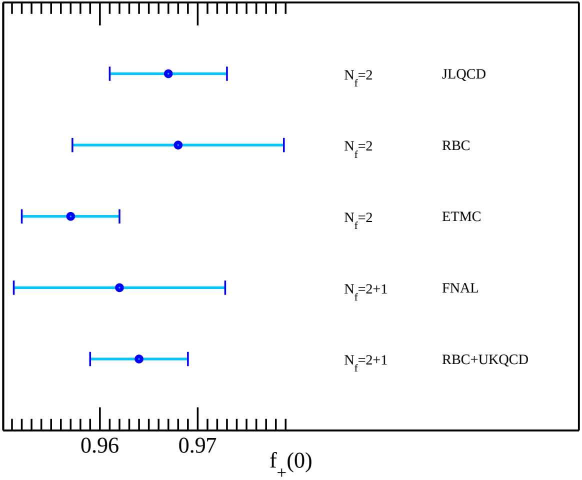


- Simultaneous pole fit to our 24^3 data.
- $f_0(q^2, m_\pi^{\rm latt}, m_K^{\rm latt}) - f_0(q^2, m_\pi^{\rm phys}, m_K^{\rm phys})$ has been subtracted from the lattice data.
- Fit shown is at physical masses.



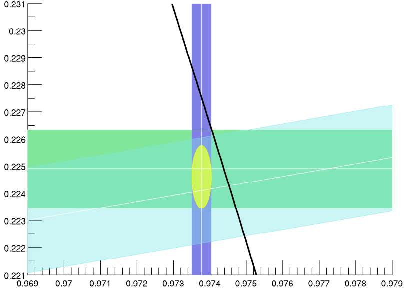
- $f_0(0)$ as a function of the pion masses with the simultaneous fit.
- $1+f_2$ is not a good approximation to $f_0(0)$.

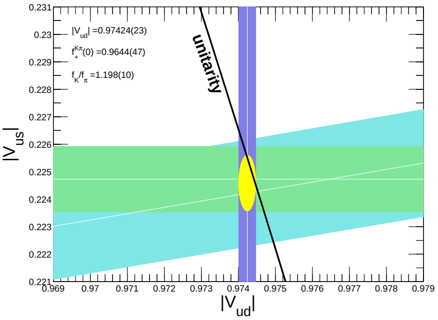
Comparison with Other Calculations



- Our final answer:

$$f_0(0) = 0.964(5)$$





$$V_{ud} = 0.97372(10)(15)(19)$$

W.Marciano, Kaon2007

$$V_{ud} = 0.97424(23)$$

I.Towner and J.Hardy, CKM(2008)

Courtesy of Flavianet Kaon WG and A.Jüttner

- The uncertainties on $|V_{ud}|^2$ and $|V_{us}|^2$ are comparable!

Improvements – Eliminating the Interpolation in q^2

- The momentum resolution with conventional methods is very poor:

  On the present lattice:

$$L = 24a \quad \text{with} \quad a^{-1} = 1.73\,\text{GeV} \quad \Rightarrow \quad \frac{2\pi}{L} = .45\,\text{GeV}$$

- Using twisted boundary conditions

$$q(x_i + L) = e^{i\theta_i} q(x_i)$$

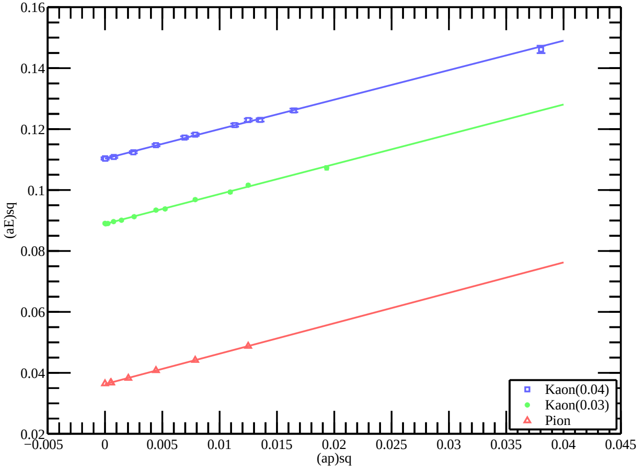
  the momentum spectrum is modified (relative to periodic bcs)

$$p_i = n_i \frac{2\pi}{L} + \frac{\theta_i}{L}\,.$$

- For quantities which do not involve Final State Interactions (e.g. masses, decay constants, form-factors) the Finite-Volume corrections are exponentially small also with Twisted BC's. CTS & G. Villadoro (2004)
- Moreover they are also exponentially small for *partially twisted boundary conditions* in which the sea quarks satisfy periodic BC's but the valence quarks satisfy twisted BC's. CTS & G. Villadoro (2004); Bedaque & Chen (2004)

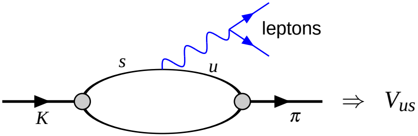
We do not need to perform new simulations for every choice of $\{\theta_i\}$.

Examples of Dispersion Relations with Twisted Boundary Conditions



$$\left(\frac{2\pi}{24}\right)^2 = 0.0685\,.$$

- For a detailed study using older $N_f = 2$ improved Wilson configurations see: J.Flynn, A.Jüttner, CTS, Phys.Lett.B632 (2006) 313 [hep-lat/0506016].



- By tuning the twisting angles appropriately it is possible to calculate the matrix element at $q^2 = 0$ directly (or at any other required value of q^2) .

  P.A.Boyle, J.M.Flynn, A.Jüttner, CTS, and J.M.Zanotti, [hep-lat/0703005]

- By calculating:

$$\langle \pi(\vec{0}) | V_4 | K(\vec{\theta}_K) \rangle \quad \text{with} \quad |\vec{\theta}_K| = L\sqrt{\left[\frac{(m_K^2 + m_\pi^2)}{2m_\pi}\right]^2 - m_K^2}$$

and

$$\langle \pi(\vec{\theta}_\pi) | V_4 | K(\vec{0}) \rangle \quad \text{with} \quad |\vec{\theta}_\pi| = L\sqrt{\left[\frac{(m_K^2 + m_\pi^2)}{2m_K}\right]^2 - m_\pi^2}$$

we obtain the form factors directly at $q^2 = 0$.

Improvements – Eliminating the Interpolation in q^2 Cont.

- The feasibility of this method was demonstrated on a subset of configurations on a $16^3 \times 32$ lattice at two values of m_{ud}.

  P.A.Boyle, J.M.Flynn, A.Jüttner, CTS, and J.M.Zanotti

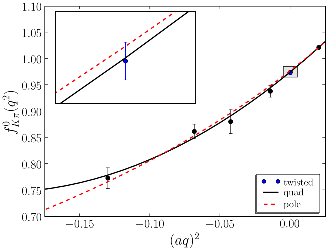
- We are currently using partially twisted boundary conditions to get $f_0(0)$ for our lightest quark mass ($ma = 0.005$) directly at $q^2 = 0$.

$$f_0(0) = 0.9774(35)\ \text{(pole fit)} \quad \text{and} \quad f_0(0) = 0.9749(59)\ \text{(quadratic fit)}$$

quoted in the paper. Our preliminary result computing the form-factor directly at $q^2 = 0$ is

$$f_0(0) = 0.974(4)\,, \qquad f_-(0) = -0.113(12)$$

J.M.Flynn et al, arXiV:0812.6265 [hep-lat]



- In our final result for the physical $f_0(0) = 0.9644(33)(34)(14)$, 34 was due to the model dependence.

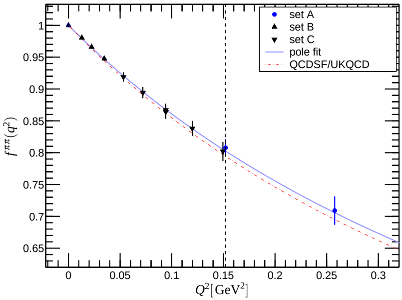
EM Form Factor of a Pion with Mass 330 MeV

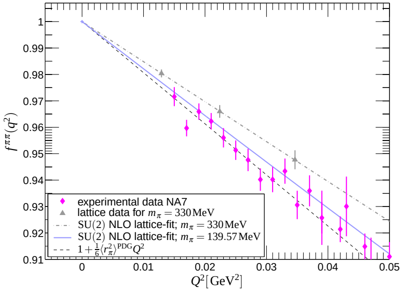
- Twisted boundary conditions were previously applied to $K_{\ell 3}$ decays (although not directly at $q^2 = 0$) in a quenched simulation.

  D.Guadagnoli, F.Mescia and S,Simula, [hep-lat/0512020]

- We have studied the electromagnetic form factor of a *pion* with mass 330 MeV at small momentum transfers and use NLO ChPT to determine the form factor of a physical pion.

The Pion EM Form Factor at Small q^2.





- NLO ChPT has one LEC (l_6^r) which governs the q^2 and m_π^2 behaviour.

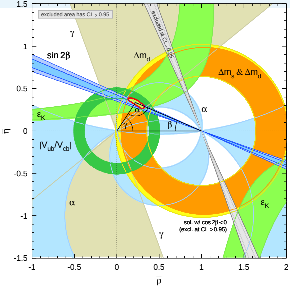
$$q^2 \text{ behaviour of FF at } m_\pi = 330\,\mathrm{MeV} \;\Rightarrow\; l_6^r \;\Rightarrow\; \text{Physical FF}\,.$$

- We find: P.A.Boyle et al., arXiv:0804.3971

$$\langle r_\pi^2\rangle = 0.418(31)\,\mathrm{fm}^2\,,$$

to be compared to the PDG value $\langle r_\pi^2\rangle = 0.452(11)\,\mathrm{fm}^2$.

B_K



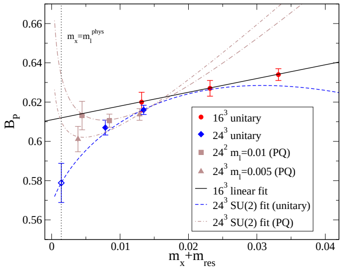
- $|\varepsilon_K| = (2.232 \pm 0.007)\,10^{-3}$ 0.3% precision
- B_K known to 16% precision $\Rightarrow$ physics information severely limited by theoretical uncertainty.

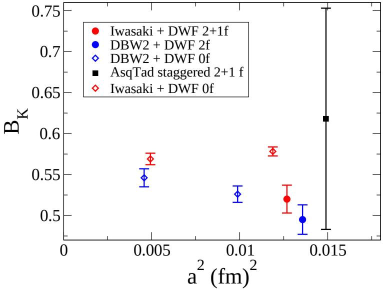
G.Sciolla (Kaon 2007)

- Flavour and Chiral symmetry properties of DWF well suited to this calculation.
- $\Delta S = 2$ operator renormalises multiplicatively and is renormalized nonperturbatively.
- Again it is found that $SU(2)_L \times SU(2)_R$ (PQ)ChPT should be used:

$$B_K = B_0^{(K)} \left\{ 1 + \frac{b_1 \chi_l}{f^2} + \frac{b_2 \chi_x}{f^2} - \frac{\chi_l}{32\pi^2 f^2} \log \frac{\chi_x}{\Lambda_\chi^2} \right\}$$

- Kaons with $m_s \neq m_d$ are used and the chiral behaviour in m_d is fit successfully.

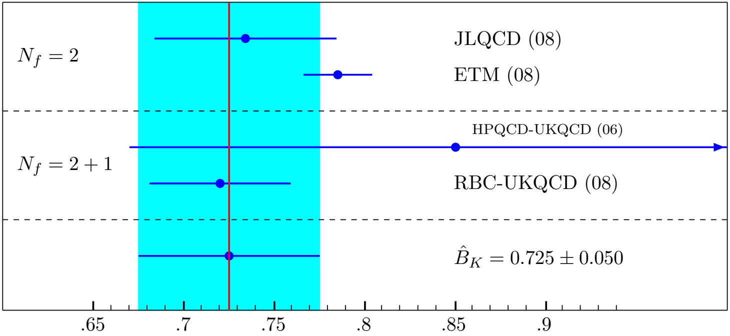




$$B_K^{\overline{\rm MS}}(2\,{\rm GeV}) = 0.524(10)(28)\,, \qquad \hat{B}_K = 0.720(13)(37)\,.$$

RBC/UKQCD

- Systematic error includes estimates of finite-volume effects, discretization errors, interpolation to the physical strange quark mass and ChPT.
- Calculation at a second lattice spacing (in progress) will reduce the estimated 4% discretization error (which is the largest component of the quoted systematic error).



L.Lellouch, Lattice 2008

$$\hat{B}_K = 0.720(13)(37)\,.$$

- This low result for $\hat{B}_K$ is contributing to the tension in the Unitarity Triangle.
- For example, the recent analysis of Lunghi and Soni, [arXiv:0803.4340] takes

$$\hat{B}_K = 0.720(13)(37),\ \xi = \frac{f_{B_s}\sqrt{\hat{B}_{B_s}}}{f_{B_d}\sqrt{\hat{B}_{B_d}}} = 1.20 \pm 0.06 \text{ and } |V_{cb}| = (40.6 \pm 0.6)\cdot 10^{-3}$$

(PDG 2008 quote $V_{cb} = (41.2 \pm 1.1)10^{-3}$) to predict

$$\sin(2\beta)^{\text{prediction}}_{\text{no } V_{ub}} = 0.87(9)$$

.

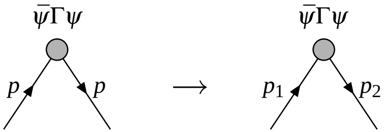
- The direct experimental result is $\sin(2\beta)^{\text{ direct}} = 0.681(0.025)$.
- In order to reduce the discrepancy to 1σ they would need either

$$\hat{B}_K \to 0.96(4) \quad \text{or} \quad \xi \to 1.37(6) \quad \text{or} \quad |V_{cb}| \to (44.3 \pm 0.6)\cdot 10^{-3}\,.$$

- By including $V_{ub} = (37.2 \pm 2.7)\cdot 10^{-4}$, the prediction becomes $\sin(2\beta)^{\text{prediction}}_{\text{with } V_{ub}} = 0.75(4)$.

Conclusions and Prospects

- I have presented a selection of the phenomenological lattice studies being undertaken in kaon physics by RBC/UKQCD.
- The lattice community is beginning to make strong contact with Chiral Perturbation Theory and to determine the *low energy constants* with unprecedented precision.
- The RBC/UKQCD research programme is now moving onto a finer lattice $\Rightarrow$ information about the continuum extrapolation.
    - One technical improvement will be to perform the RI-Mom NPR at non-exceptional momenta, Y.Aoki, Lattice 2008

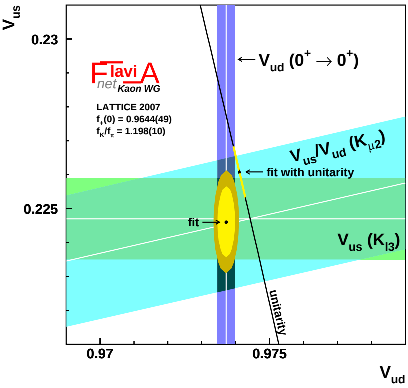


$$p_1^2 = p_2^2 = (p_1 - p_2)^2$$

-
    - B_K is also being renormalized at non-exceptional momenta.
    - This not only eliminates difficulties associated with the *pion-pole* but reduces infrared effects.

- We will continue to extend the range of quantities being computed: $\{K \to \pi\pi$ decays; Heavy Quark Physics.$\}$
  - For $I = 2$ final states, by using twisted boundary conditions it is possible to calculate $E_{\pi\pi}$ and hence the scattering phase-shift for a range of momenta and hence obtain the derivative of the phase-shift and compute the Lellouch-Lüscher finite-volume corrections. C.h.Kim and CTS
- In the medium term we are moving towards a target simulation of $a = 0.06$ fm, $L = 4$ fm, $m_\pi = 195$ MeV.
- Selected Physics Results:

$$B_K^{\overline{\text{MS}}}(2\,\text{GeV}) = 0.524(10)(28), \qquad \hat{B}_K = 0.720(13)(37)\,.$$



$$f_+^{K\pi}(0) = 0.9644(33)(34)$$
$$\Rightarrow |V_{us}| = 0.2247(12)$$

$$\frac{f_K}{f_\pi} = 1.198(10)$$
$$\Rightarrow |V_{us}| = 0.2241(24)$$

A.Jüttner, Lattice 2007

Our final result from the $K\ell 3$ project is

$$f_+^{K\pi}(0) = 0.964(5)\,.$$

P.A.Boyle et al. [RBC&UKQCD Collaborations – arXiv:0710.5136 [hep-lat]]

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$$
\begin{aligned}
f &= 114.8\,(4.1)_{\rm stat}\,(8.1)_{\rm syst}\,\mathrm{MeV}\\
B^{\overline{\rm MS}}(2\,\mathrm{GeV}) &= 2.52\,(0.11)_{\rm stat}\,(0.23)_{\rm ren}\,(0.12)_{\rm syst}\,\mathrm{GeV}\\
\Sigma^{\overline{\rm MS}}(2\,\mathrm{GeV}) &= \left(255\,(8)_{\rm stat}\,(8)_{\rm ren}\,(13)_{\rm syst}\,\mathrm{MeV}\right)^3\\
\bar{l}_3 &= 3.13\,(0.33)_{\rm stat}\,(0.24)_{\rm syst}\\
\bar{l}_4 &= 4.43\,(0.14)_{\rm stat}\,(0.77)_{\rm syst}\\
m_{ud}^{\overline{\rm MS}}(2\,\mathrm{GeV}) &= 3.72\,(0.16)_{\rm stat}\,(0.33)_{\rm ren}\,(0.18)_{\rm syst}\,\mathrm{MeV}\\
m_{s}^{\overline{\rm MS}}(2\,\mathrm{GeV}) &= 107.3\,(4.4)_{\rm stat}\,(9.7)_{\rm ren}\,(4.9)_{\rm syst}\,\mathrm{MeV}\\
m_s/m_{ud} &= 28.8\,(0.4)_{\rm stat}\,(1.6)_{\rm syst}\\
f_\pi &= 124.1\,(3.6)_{\rm stat}\,(6.9)_{\rm syst}\,\mathrm{MeV}\\
f_K &= 149.6\,(3.6)_{\rm stat}\,(6.3)_{\rm syst}\,\mathrm{MeV}\\
f_K/f_\pi &= 1.205\,(0.018)_{\rm stat}\,(0.062)_{\rm syst}\\
B_K^{\overline{\rm MS}}(2\,\mathrm{GeV}) &= 0.524\,(0.010)_{\rm stat}\,(0.013)_{\rm ren}\,(0.025)_{\rm syst}
\end{aligned}
$$

Supplementary Slides

- Lattice simulations are performed for fixed bare input parameters $g(a)$, $m_u = m_d$ (in the isospin limit) and m_s .

  Three physical quantities are therefore needed to determine the *physical* values of these bare parameters (we take m_π, m_K and m_{Ω^-}).
- Simulations are performed with m_{ud} larger than the physical values and the results are extrapolated to the physical limit.

  Increased computing resources and improvements in algorithms $\Rightarrow$ now dynamical simulations with $m_\pi \simeq 300$ MeV are the norm and the situation is rapidly improving.
- m_s can be kept at the physical value (after tuning).
- Chiral Perturbation Theory (χPT) is a key ingredient in performing the extrapolation in m_{ud}, raising the questions of:
  - How reliable is it?
  - What are the values of the *Low Energy Constants*?
  - $SU(3)\times SU(3)$ or $SU(2)\times SU(2)$?
- The use of *Partially Quenched* simulations, in which the masses of the valence and sea quarks are different $\Rightarrow$ the use of PQχPT.

S.R.Sharpe and N.Shoresh, [hep-lat/0006017]

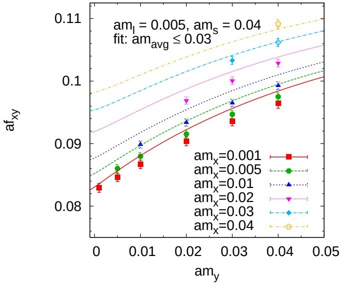
- Approximate chiral symmetry of QCD $\Rightarrow$
  effective theory of pseudo-goldstone bosons of chiral symmetry breaking $\Rightarrow$
  systematic expansion in powers of $M^2_{(\pi,K,\eta)}/\Lambda^2_\chi$ (up to *chiral logarithms*).
- For example, at one-loop order:

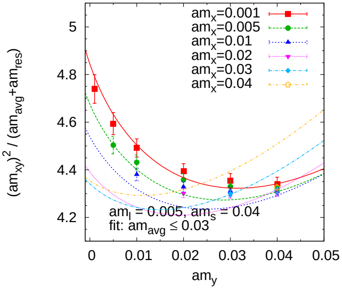
$$
\begin{aligned}
m_\pi^2 &= \chi_{ud}\left\{1+\frac{48}{f_0^2}(2L_6-L_4)\bar{\chi}+\frac{16}{f_0^2}(2L_8-L_5)\chi_{ud}\right.\\
&\qquad\left.+\frac{1}{24\pi^2 f_0^2}\left(\frac{3}{2}\chi_{ud}\log\left[\frac{\chi_{ud}}{\Lambda_\chi^2}\right]-\frac{1}{2}\chi_\eta\log\left[\frac{\chi_\eta}{\Lambda_\chi^2}\right]\right)\right\},\\
f_\pi &= f_0\left\{1+\frac{24}{f_0^2}L_4\bar{\chi}+\frac{8}{f_0^2}L_5\chi_{ud}\right.\\
&\qquad\left.-\frac{1}{16\pi^2 f_0^2}\left(2\chi_{ud}\log\left[\frac{\chi_{ud}}{\Lambda_\chi^2}\right]+\frac{\chi_{ud}+\chi_s}{2}\log\left[\frac{\chi_{ud}+\chi_s}{2\Lambda_\chi^2}\right]\right)\right\}.
\end{aligned}
$$

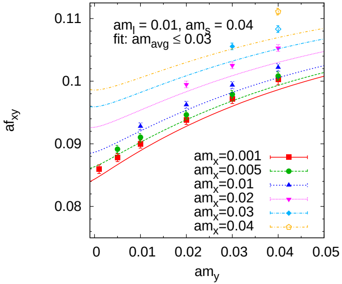
where $\chi_i = 2B_0 m_i$ ($i = ud, s$), $\chi_\eta = \frac{1}{3}(\chi_{ud}+2\chi_s)$ and $\bar{\chi} = \frac{1}{3}(2\chi_{ud}+\chi_s)$.

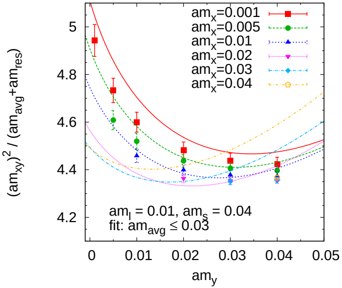
Do such formulae represent our data?

Results – NLO $SU(3) \times SU(3)$ fit is bad for cut $am_{avg} < 0.03$









Results – NLO $SU(3) \times SU(3)$ fit is good for cut $am_{avg} < 0.01$

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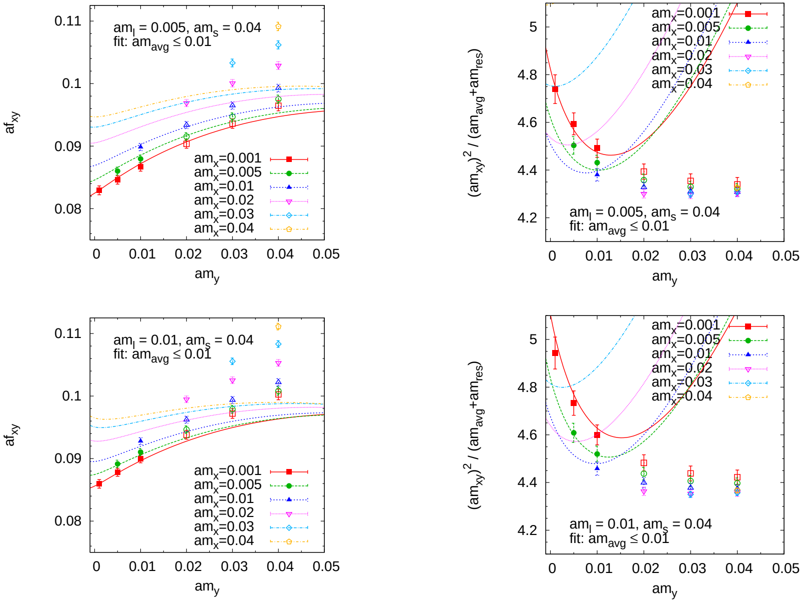

- $SU(3) \times SU(3)$ chiral fits to the pseudoscalar masses and decay constants work well, but only at very light masses.
- Perhaps going to NNLO would increase the range of the good fits, but the number of new LECs is too large for the data which we have (other collaborations are trying to use at least the analytical terms and we have also tried this - see below).
- We find for $\Lambda_\chi = m_\rho$:

$2L_8 - L_5$	L_5	$2L_6 - L_4$	L_4
$2.4(5) \cdot 10^{-4}$	$8.7(10) \cdot 10^{-4}$	$0.0(4) \cdot 10^{-4}$	$1.4(8) \cdot 10^{-4}$

and $af_0 = 0.054(4)$ and $aB_0 = 2.35(16)$.

- The fits can also be performed using $SU(2) \times SU(2)$ chiral perturbation theory in the range $m_{avg} < 0.01$. This treats the heavy strange quark mass correctly.

	aB	af	$\bar{l}_3$	$\bar{l}_4$
$SU(2) \times SU(2)$	2.41(6)	0.067(2)	3.1(3)	4.4(2)
$SU(3) \times SU(3)$ conv.	2.46(8)	0.066(2)	2.9(3)	4.1(1)

$$m_\pi^2 = \chi_l \left\{ 1 + \frac{\chi_l}{16\pi^2 f^2} \left(64\pi^2 l_3^r + \log\left[\frac{\chi_l}{\Lambda_\chi^2}\right] \right) \right\} \equiv \chi_l \left\{ 1 - \frac{\chi_l}{16\pi^2 f^2} \bar{l}_3 \right\}$$

$$f_\pi = f \left\{ 1 + \frac{m_\pi^2}{8\pi^2 f^2} \left(16\pi^2 l_4^r - \log\left[\frac{m_\pi^2}{\Lambda_\chi^2}\right] \right) \right\} \equiv f \left\{ 1 + \frac{m_\pi^2}{8\pi^2 f^2} \bar{l}_4 \right\}$$

- “Phenomenological Indirect Determinations":

$\bar{l}_3 = 2.9 \pm 2.4$, Gasser&Leutwyler (1984); $\bar{l}_4 = 4.4 \pm 0.2$, Colangelo, Gasser, Leutwyler (2001)

G.Colangelo – Kaon2007

- Lattice Determinations:

Collaboration	Paper	$\bar{l}_3$	$\bar{l}_4$
MILC	hep-lat/0611024	0.60(12)	3.9(5)
MILC	arXiv:0710.1118	2.85(7)(?)	–
RBC/UKQCD	arXiv:0804.3971	3.13(33)(24)	4.43(0.14)(77)
PACS-CS	arXiv:0810.0351	3.14(23)	4.09(19)
Del Debbio et al.	hep-lat/0610059	3.0(5)(1)	–
ETM	hep-lat/0701012	3.44(8)(35)	4.61(4)(11)

Ref.	$f_+(0)$	Δf	m_π [GeV]	a [fm]	N_f
Leutwyler & Roos (1984)	0.961(8)	-0.016(8)			
Bijnens& Talavera (2003)	0.978(10)	+0.001(10)			
Cirigliano et al. (2005)	0.984(12)	+0.007(12)			
Jamin, Oller & Pich (2004)	0.974(11)	-0.003(11)			
Becirevic et al. (2005)	0.960(5)(7)	-0.017(5)(7)	$\gtrsim 0.5$	0.07	0
Dawson et al. (2006)	0.968(9)(6)	-0.009(9)(6)	$\gtrsim 0.49$	0.12	2
Okamoto et al. (2004)	0.962(6)(9) †	-0.015(6)(9)†	‡	‡	2+1
Tsutsui et al. (2005)	0.967(6) †	-0.010(6)†	$\gtrsim 0.55$	0.09	2
Brommel et al. (2007)	0.965(2) $^\dagger_{\rm stat}$	-0.012(2)$^\dagger_{\rm stat}$	$\gtrsim 0.5$	0.08	2
This work	0.964(5)	-0.013(5)	$\gtrsim 0.33$	0.114	2+1

- Summary of ChPT-based and lattice results.
- † Results in conference proceedings only.
- ‡ Information not provided.