S_4 Flavor Symmetry of Quarks and Leptons in SU(5) GUT

Yusuke Shimizu

Niigata University

Friday 6th February, 2009

YKIS at Kyoto

Collaborated with Hajime Ishimori and Morimitsu Tanimoto @Niigata

Reference:

- S_4 Flavor Symmetry of Quarks and Leptons in SU(5) GUT
  (Hajime Ishimori, Yusuke Shimizu, and Morimitsu Tanimoto, arXiv:0812.5031)

Outline

1. Motivation and Introduction
   - Discrete Flavor Symmetry
   - Discrete Group
2. S_4 Group
3. S_4 flavor model
   - Prototype of S_4 flavor model in SU(5) SUSY GUT
   - Improved S_4 flavor model in SU(5) SUSY GUT
4. Summary and Future work

1. Motivation and Introduction

Neutrinos: Windows to New Physics

Neutrino Oscillations provided important information

- Tiny Neutrino Masses
- Large Neutrino Flavor Mixing

Recent experiments of the neutrino oscillation go into a new phase of precise determination of mixing angles and mass squared differences, in SK, KamLAND, SNO, MINOS, T2K, Double CHOOZ, and etc.

In order to explain large mixing, many authors consider

Non-Abelian Discrete Flavor Symmetry !

The experimental data indicate Tri-bimaximal mixing

- Global fit of experimental data of the neutrino oscillations: (T.Schwetz, et al, hep-ph/0808.2016)

parameter	best fit	2σ	3σ	tri-bimaximal
$\sin^2\theta_{12}$	0.304	0.27-0.35	0.25-0.37	1/3
$\sin^2\theta_{23}$	0.50	0.39-0.63	0.36-0.67	1/2
$\sin^2\theta_{13}$	0.01	0-0.040	0-0.056	0
$\Delta m^2_{\rm sol}$ $[10^{-5}{\rm eV}^2]$	7.65	7.25-8.11	7.05-8.34	*
$\Delta m^2_{\rm atm}$ $[10^{-3}{\rm eV}^2]$	2.40	2.18-2.64	2.07-2.75	*

- Lepton mixing angles:

$$\sin^2\theta_{12} \simeq \frac{1}{3}, \quad \sin^2\theta_{23} \simeq \frac{1}{2}, \quad \sin^2\theta_{13} \simeq 0.$$

Tri-bimaximal mixing !

- Lepton flavor mixing: Tri-bimaximal mixing is written as

$$U_{\text{Tri-bi}} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}.$$

$$|\nu_\alpha\rangle = U_{\alpha i}|\nu_i\rangle$$

- Quark mixing: CKM matrix (PDG 2008)

$$|V_{\text{CKM}}| \simeq \begin{pmatrix} 0.974 & 0.226 & 0.004 \\ 0.226 & 0.973 & 0.04 \\ 0.009 & 0.04 & 1 \end{pmatrix}$$

- Tri-bimaximal mixing matrix can be led from the simple mass matrix:

$$M_\nu = U_{\text{Tri-bi}} \text{diag}(m_1, m_2, m_3) U_{\text{Tri-bi}}^T$$
$$= \frac{m_1+m_3}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \frac{m_2-m_1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{m_1-m_3}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

Non-Abelian discrete symmetry can lead to tri-bimaximal mixing, since non-Abelian discrete symmetry connects different generations.

Typical one: A_4 flavor symmetry

- E. Ma and G. Rajasekaran, Phys. Rev. D 64, 113012 (2001).
- G. Altarelli and F. Feruglio, Nucl. Phys. B 720, 64 (2005); Nucl. Phys. B 741, 215 (2006).

However, it is difficult to explain mixing of both quarks and leptons clearly.

Our purpose:

- Building a new model with non-Abelian discrete flavor symmetry S_4, which explains both lepton mixing and quark mixing.

Typical non-Abelian discrete group

- Symmetry group S_n (number of group elements is $n!$)

	Number of elements	Geometry	Irreducible representations
S_3	6	Regular triangle	$1_S, 1_A, 2$
S_4	24	Octahedron	$1_1, 1_2, 2, 3_1, 3_2$

- Even permutation group A_n (number of group elements is $n!/2$)

	Number of elements	Geometry	Irreducible representations
A_4	12	Tetrahedron	$1, 1', 1'', 3$

- T' group

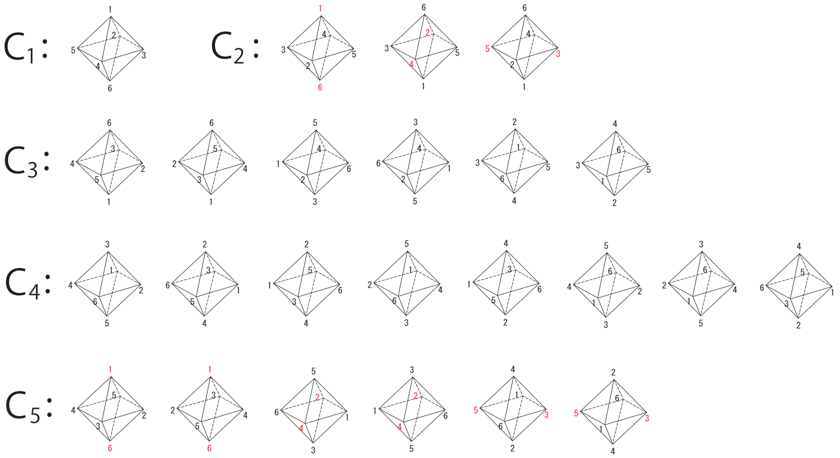
	Number of elements	Geometry	Irreducible representations
T'	24	Double tetrahedron	$1, 1', 1'', 2, 2', 2'', 3$

- A_4 and T' flavor symmetry are successful to get tri-bimaximal mixing.
- It is found that S_4 flavor symmetry can lead to tri-bimaximal mixing.

Ref: H. Ishimori, Y. Shimizu, and M. Tanimoto, arXiv:0812.5031; F. Bazzocchi and S. Morisi, arXiv:0811.0345.

2. S_4 Group

S_4 group is the symmetry group of octahedron or permutation of four elements. Number of elements is 24, which correspond geometry as:



- Irreducible representations of S_4 are 3_1, 3_2, 2, 1_1, and 1_2.
- Multiplication rules are

$$
\begin{aligned}
&3_1 \times 3_1 = 1_1 + 2 + 3_1 + 3_2 \\
&3_2 \times 3_2 = 1_1 + 2 + 3_1 + 3_2 \\
&3_1 \times 3_2 = 1_2 + 2 + 3_1 + 3_2 \\
&2 \times 3_1 = 3_1 + 3_2 \\
&2 \times 3_2 = 3_1 + 3_2 \\
&2 \times 2 = 1_1 + 1_2 + 2 \\
&\quad \vdots \\
&\text{etc.}
\end{aligned}
$$

- S_4 invariant representation is 1_1.

Multiplication rules of S_4

- Multiplication of 2×2 is given by

$$(a_1, a_2)_2 \times (b_1, b_2)_2 = (a_1 b_1 + a_2 b_2)_{1_1} + (-a_1 b_2 + a_2 b_1)_{1_2} + \begin{pmatrix} a_1 b_2 + a_2 b_1 \\ a_1 b_1 - a_2 b_2 \end{pmatrix}_2$$

- Multiplication of $3_1 \times 3_1$ is

$$\begin{aligned}
&(a_1, a_2, a_3)_{3_1} \times (b_1, b_2, b_3)_{3_1} \\
&= \left(\sum_{j=1}^{3} a_j b_j\right)_{1_1} + \begin{pmatrix} \frac{1}{\sqrt{2}}(a_2 b_2 - a_3 b_3) \\ \frac{1}{\sqrt{6}}(-2a_1 b_1 + a_2 b_2 + a_3 b_3) \end{pmatrix}_2 \\
&\quad + \begin{pmatrix} a_2 b_3 + a_3 b_2 \\ a_1 b_3 + a_3 b_1 \\ a_1 b_2 + a_2 b_1 \end{pmatrix}_{3_1} + \begin{pmatrix} a_3 b_2 - a_2 b_3 \\ a_1 b_3 - a_3 b_1 \\ a_2 b_1 - a_1 b_2 \end{pmatrix}_{3_2}
\end{aligned}$$

3. S_4 flavor model

We consider the model under the framework of SU(5) SUSY GUT.

- Irreducible representations of SU(5) are 1, 5, $\bar{5}$, 10, 45, and etc.

  We write F_i and T_i as $\bar{5}$ and 10 representations of fermions.
  SU(5) Higgs are denoted by H_5 and $H_{\bar{5}}$. (generations: $i = 1,\ 2,\ 3$)

$$\bar{5} : F_1 = \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e \\ \nu_e \end{pmatrix}_L , \quad 10 : T_1 = \begin{pmatrix} 0 & u_3^c & -u_2^c & -u_1 & -d_1 \\ -u_3^c & 0 & u_1^c & -u_2 & -d_2 \\ u_2^c & -u_1^c & 0 & -u_3 & -d_3 \\ u_1 & u_2 & u_3 & 0 & -e^+ \\ d_1 & d_2 & d_3 & e^+ & 0 \end{pmatrix}_L$$

We present the prototype model in SU(5) SUSY GUT

to understand the essence of our model.

- Assignments of S_4 for quarks and leptons.

Particle	(T_1, T_2)	T_3	(F_1, F_2, F_3)	(N_e^c, N_μ^c)	N_τ^c	H_5	$H_{\bar{5}}$
SU(5)	10	10	$\bar{5}$	1	1	5	$\bar{5}$
S_4	2	1_1	3_1	2	1_1	1_1	1_1
Z_4	ω^3	ω^2	ω	1	1	1	1

$T_1 = (q_1, u^c, e^c),\ F_1 = (d^c, l)$

N_e^c, N_μ^c, N_τ^c: Right-handed Majorana neutrinos

- We introduce new scalars χ_i, which are SU(5) gauge singlets.

Coupled	Up	Up	Majorana	Dirac	Charged leptons +	Down type quarks
Scalar	χ_1	(χ_2, χ_3)	(χ_4, χ_5)	(χ_6, χ_7, χ_8)	$(\chi_9, \chi_{10}, \chi_{11})$	$(\chi_{12}, \chi_{13}, \chi_{14})$
SU(5)	1	1	1	1	1	1
S_4	1_1	2	2	3_1	3_1	3_1
Z_4	ω^2	ω^2	1	ω^3	1	ω

- We add Z_4, which controls these scalar couplings. $(\omega = i)$

We can write the superpotential at the leading order in terms of the cut off scale Λ, which is taken to be the Planck scale.
SU(5) invariant superpotential of Yukawa sector respecting S_4 and Z_4 is

$$
\begin{aligned}
w_{SU(5)}^{(0)} = & \, y_1^u (T_1, T_2) \otimes (T_1, T_2) \otimes \chi_1 \otimes H_5/\Lambda \\
& + y_2^u (T_1, T_2) \otimes (T_1, T_2) \otimes (\chi_2, \chi_3) \otimes H_5/\Lambda \\
& + y_3^u T_3 \otimes T_3 \otimes H_5 + M_1 (N_e^c, N_\mu^c) \otimes (N_e^c, N_\mu^c) + M_2 N_\tau^c \otimes N_\tau^c \\
& + y^N (N_e^c, N_\mu^c) \otimes (N_e^c, N_\mu^c) \otimes (\chi_4, \chi_5) \\
& + y_1^D (N_e^c, N_\mu^c) \otimes (F_1, F_2, F_3) \otimes (\chi_6, \chi_7, \chi_8) \otimes H_5/\Lambda \\
& + y_2^D N_\tau^c \otimes (F_1, F_2, F_3) \otimes (\chi_6, \chi_7, \chi_8) \otimes H_5/\Lambda \\
& + y_1 (F_1, F_2, F_3) \otimes (T_1, T_2) \otimes (\chi_9, \chi_{10}, \chi_{11}) \otimes H_{\bar{5}}/\Lambda \\
& + y_2 (F_1, F_2, F_3) \otimes T_3 \otimes (\chi_{12}, \chi_{13}, \chi_{14}) \otimes H_{\bar{5}}/\Lambda ,
\end{aligned}
$$

M_1 and M_2 are mass parameters for right-handed Majorana neutrinos, and Yukawa coupling constants y_i^a and y_i are of order 1.
By using $w_{SU(5)}^{(0)}$, we can discuss mass matrices of quarks and leptons.

- Superpotential of Yukawa sector respecting $S_4 \times Z_4$ symmetry for charged leptons:

$$w_l = y_1 \left[\frac{e^c}{\sqrt{2}}(l_\mu \chi_{10} - l_\tau \chi_{11}) + \frac{\mu^c}{\sqrt{6}}(-2 l_e \chi_9 + l_\mu \chi_{10} + l_\tau \chi_{11}) \right] h_d/\Lambda$$
$$+ y_2 \tau^c (l_e \chi_{12} + l_\mu \chi_{13} + l_\tau \chi_{14}) h_d/\Lambda.$$

h_d: SU(2) Higgs doublet
(S_4 : $2 \times 3_1 \times 3_1 \times 1_1 \to 1_1$, $1_1 \times 3_1 \times 3_1 \times 1_1 \to 1_1$)

- In terms of VEVs $\langle h_d \rangle = v_d$, $\langle h_u \rangle = v_u$, and $\alpha_i = \langle \chi_i \rangle / \Lambda$, we obtain the mass matrix for charged leptons as

$$M_l = y_1 v_d \begin{pmatrix} 0 & \alpha_{10}/\sqrt{2} & -\alpha_{11}/\sqrt{2} \\ -2\alpha_9/\sqrt{6} & \alpha_{10}/\sqrt{6} & \alpha_{11}/\sqrt{6} \\ 0 & 0 & 0 \end{pmatrix} + y_2 v_d \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \alpha_{12} & \alpha_{13} & \alpha_{14} \end{pmatrix},$$

$2 :\ (e^c,\ \mu^c)$ $\qquad$ $1_1 :\ \tau^c$

To get the left-handed mixing of charged leptons, we consider $M_l^\dagger M_l$. Taking vacuum alignments $(\langle\chi_9\rangle,\langle\chi_{10}\rangle,\langle\chi_{11}\rangle)=(\langle\chi_9\rangle,\langle\chi_{10}\rangle,0)$ and $(\langle\chi_{12}\rangle,\langle\chi_{13}\rangle,\langle\chi_{14}\rangle)=(0,0,\langle\chi_{14}\rangle)$, that is $\alpha_{11}=\alpha_{12}=\alpha_{13}=0$, we obtain

$$M_l^\dagger M_l = v_d^2 \begin{pmatrix} \frac{2}{3}|y_1|^2\alpha_9^2 & -\frac{1}{3}|y_1|^2\alpha_9\alpha_{10} & 0 \\ -\frac{1}{3}|y_1|^2\alpha_9\alpha_{10} & \frac{2}{3}|y_1|^2\alpha_{10}^2 & 0 \\ 0 & 0 & |y_2|^2\alpha_{14}^2 \end{pmatrix} .$$

- Charged lepton masses:

$$m_e^2 \approx \frac{1}{2}|y_1|^2\alpha_9^2 v_d^2 \ , \ m_\mu^2 \approx \frac{2}{3}|y_1|^2\alpha_{10}^2 v_d^2 \ , \ m_\tau^2 = |y_2|^2\alpha_{14}^2 v_d^2 \ .$$

$\alpha_9 \ll \alpha_{10}$

- Charged lepton mixing angles:

$$|\tan\theta_{12}^l| \approx \frac{1}{\sqrt{3}}\frac{m_e}{m_\mu} \approx 2.8\times 10^{-3}, \quad \theta_{13}^l=\theta_{23}^l=0 \ .$$

- For right-handed Majorana neutrinos, the superpotential is given as

$$w_N = M_1(N^c_e N^c_e + N^c_\mu N^c_\mu) + M_2 N^c_\tau N^c_\tau + y^N \left[(N^c_e N^c_\mu + N^c_\mu N^c_e)\chi_4 + (N^c_e N^c_e - N^c_\mu N^c_\mu)\chi_5\right].$$

$(2 \times 2 \rightarrow 1_1 \ , \ 1_1 \times 1_1 \rightarrow 1_1 \ , \ 2 \times 2 \times 2 \rightarrow 1_1)$

- The mass matrix of right-handed Majorana neutrinos:

$$M_N = \begin{pmatrix} M_1 + y^N \alpha_5 \Lambda & y^N \alpha_4 \Lambda & 0 \\ y^N \alpha_4 \Lambda & M_1 - y^N \alpha_5 \Lambda & 0 \\ 0 & 0 & M_2 \end{pmatrix}.$$

- Taking vacuum alignment $(\langle\chi_4\rangle, \langle\chi_5\rangle) = (0, \langle\chi_5\rangle)$, that is $\alpha_4 = 0$, Right-handed Majorana mass matrix of neutrinos turns to

$$M_N = \begin{pmatrix} M_1 + y^N \alpha_5 \Lambda & 0 & 0 \\ 0 & M_1 - y^N \alpha_5 \Lambda & 0 \\ 0 & 0 & M_2 \end{pmatrix}.$$

The mass matrix of right-handed Majorana neutrinos is diagonal.

- For Dirac neutrinos, the superpotential is given as

$$w_D = y_1^D \left[\frac{N_e^c}{\sqrt{2}}(l_\mu \chi_7 - l_\tau \chi_8) + \frac{N_\mu^c}{\sqrt{6}}(-2 l_e \chi_6 + l_\mu \chi_7 + l_\tau \chi_8) \right] h_u/\Lambda$$
$$+ y_2^D N_\tau^c (l_e \chi_6 + l_\mu \chi_7 + l_\tau \chi_8) h_u/\Lambda.$$

$(2 \times 3_1 \times 3_1 \times 1_1 \rightarrow 1_1\ ,\ 1_1 \times 3_1 \times 3_1 \times 1_1 \rightarrow 1_1)$

- Taking vacuum alignment $(\langle\chi_6\rangle, \langle\chi_7\rangle, \langle\chi_8\rangle) = (\langle\chi_6\rangle, \langle\chi_6\rangle, \langle\chi_6\rangle)$, Dirac mass matrix of neutrinos turns to

$$M_D = y_1^D \alpha_6 v_u \begin{pmatrix} 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ -2/\sqrt{6} & 1/\sqrt{6} & 1/\sqrt{6} \\ 0 & 0 & 0 \end{pmatrix} + y_2^D \alpha_6 v_u \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{pmatrix},$$

$2 :\ (N_e^c,\ N_\mu^c)$ $\qquad$ $1_1 :\ N_\tau^c$

- The origin of tri-bimaximal is Dirac neutrino mass matrix in S_4.
  (In A_4, the origin is Right-handed Majorana neutrino mass matrix.)

By using the seesaw mechanism $M_\nu = M_D^T M_N^{-1} M_D$,
the left-handed Majorana neutrino mass matrix is given

$$M_\nu = \begin{pmatrix} a+\frac{2}{3}b & a-\frac{1}{3}b & a-\frac{1}{3}b \\ a-\frac{1}{3}b & a+\frac{1}{6}b+\frac{1}{2}c & a+\frac{1}{6}b-\frac{1}{2}c \\ a-\frac{1}{3}b & a+\frac{1}{6}b-\frac{1}{2}c & a+\frac{1}{6}b+\frac{1}{2}c \end{pmatrix},$$

$$a = \frac{(y_2^D \alpha_6 v_u)^2}{M_2}, \qquad b = \frac{(y_1^D \alpha_6 v_u)^2}{M_1 - y^N \alpha_5 \Lambda}, \qquad c = \frac{(y_1^D \alpha_6 v_u)^2}{M_1 + y^N \alpha_5 \Lambda}.$$

The neutrino mass matrix is decomposed as

$$M_\nu = \frac{b+c}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \frac{3a-b}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{b-c}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}.$$

As well known, the neutrino mass matrix with the tri-bimaximal mixing is expressed in terms of neutrino mass eigenvalues m_1, m_2 and m_3 as

$$M_\nu = \frac{m_1+m_3}{2}\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}+\frac{m_2-m_1}{3}\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix}+\frac{m_1-m_3}{2}\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix}.$$

In our model, neutrino mass matrix is

$$M_\nu = \frac{b+c}{2}\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}+\frac{3a-b}{3}\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix}+\frac{b-c}{2}\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix}.$$

Therefore, our neutrino mass matrix M_ν gives the tri-bimaximal mixing matrix $U_{\text{tri-bi}}$ and mass eigenvalues as follows:

$$m_1 = b\ , \quad m_2 = 3a\ , \quad m_3 = c\ .$$

Estimate of $\alpha_5 = \langle\chi_5\rangle/\Lambda$ and $\alpha_6 = \langle\chi_6\rangle/\Lambda$

Tri-bimaximal mixing is obtained independent of magnitudes of α_i, however α_5 and α_6 are key parameters to give $\Delta m^2_{\rm atm}$ and $\Delta m^2_{\rm sol}$.

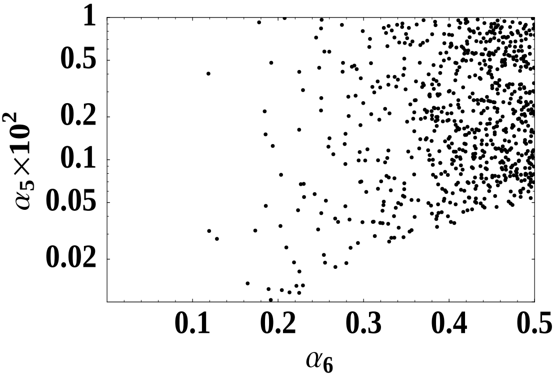
Defining parameters $\mu_0 = v_u/\Lambda$, $\lambda_1 = M_1/\Lambda$ and $\lambda_2 = M_2/\Lambda$, and taking $y_1^D = y_2^D$ for convenience, observed values $\Delta m^2_{\rm atm}$ and $\Delta m^2_{\rm sol}$ are expressed

$$\Delta m^2_{atm} = -\frac{4y^N\alpha_5\lambda_1}{(\lambda_1^2 - y^{N2}\alpha_5^2)^2}\,(y_1^D\alpha_6)^4\mu_0^2 v_u^2,$$

$$\Delta m^2_{sol} = \frac{9(\lambda_1^2 - y^N\alpha_5)^2 - \lambda_2^2}{\lambda_2^2(\lambda_1 - y^N\alpha_5)^2}\,(y_1^D\alpha_6)^4\mu_0^2 v_u^2.$$

Putting experimental values of $\Delta m^2_{\rm atm} = (2.1 - 2.8)\times 10^{-3}{\rm eV}^2$ and $\Delta m^2_{\rm sol} = (7.1 - 8.3)\times 10^{-5}{\rm eV}^2$, we can estimate magnitudes of α_5 and α_6 in the case of the normal neutrino mass hierarchy.

- We show the numerical result.
  Random plot in $\alpha_6 - \alpha_5$ plane



- $\alpha_6 \geq 0.1, \quad \alpha_5 \approx 10^{-4} - 10^{-2}$.

Deviation from tri-bimaximal mixing

$$M_\nu = \begin{pmatrix} a+\frac{2}{3}b & a-\frac{1}{3}b & a-\frac{1}{3}b \\ a-\frac{1}{3}b & a+\frac{1}{6}b+\frac{1}{2}c & a+\frac{1}{6}b-\frac{1}{2}c \\ a-\frac{1}{3}b & a+\frac{1}{6}b-\frac{1}{2}c & a+\frac{1}{6}b+\frac{1}{2}c \end{pmatrix} + \mathcal{O}(\alpha_4).$$

Tri-bimaximal mixing requires $(\langle\chi_4\rangle, \langle\chi_5\rangle) = (0, \langle\chi_5\rangle)$; $(\alpha_4 = 0)$.
If $\langle\chi_4\rangle \neq 0$ $(\alpha_4 \neq 0)$, mixing matrix deviates from tri-bimaximal.
After rotating M_ν by the tri-bimaximal mixing matrix, we obtain off-diagonal elements in the neutrino mass matrix due to $\alpha_4 = \langle\chi_4\rangle/\Lambda$:

$$U_{\text{tri-bi}}^{\text{T}} M_\nu U_{\text{tri-bi}} = \hat{M}_\nu = \alpha_6^2 v_u^2 \begin{pmatrix} \frac{y_1^{D\,2}(M_1+y^N\alpha_5\Lambda)}{M_1^2-y^{N\,2}(\alpha_4^2+\alpha_5^2)\Lambda^2} & 0 & -\frac{y_1^{D\,2}y^N\alpha_4\Lambda}{M_1^2-y^{N\,2}(\alpha_4^2+\alpha_5^2)\Lambda^2} \\ 0 & \frac{3y_2^{D\,2}}{M_2} & 0 \\ -\frac{y_1^{D\,2}y^N\alpha_4\Lambda}{M_1^2-y^{N\,2}(\alpha_4^2+\alpha_5^2)\Lambda^2} & 0 & \frac{y_1^{D\,2}(M_1-y^N\alpha_5\Lambda)}{M_1^2-y^{N\,2}(\alpha_4^2+\alpha_5^2)\Lambda^2} \end{pmatrix}.$$

Then the mixing angle $\delta\theta_{13}^\nu$, which diagonalizes this mass matrix, is

$$\tan 2\delta\theta_{13}^\nu = \frac{\alpha_4}{\alpha_5}, \quad \delta\theta_{12}^\nu = \delta\theta_{23}^\nu = 0.$$

- We obtain non-vanishing U_{e3}.

$$|U_{e3}| = \frac{2}{\sqrt{6}}|\delta\theta_{13}^{\nu}| \approx \frac{1}{\sqrt{6}}\left|\frac{\alpha_4}{\alpha_5}\right| ,$$
$$|U_{e2}| = \frac{1}{\sqrt{3}} ,$$
$$|U_{\mu 3}| \approx \left|\frac{1}{\sqrt{2}} + \frac{1}{2\sqrt{6}}\frac{\alpha_4}{\alpha_5}\right| .$$

- This prediction is testable in the future neutrino precise experiments.

Quark sector

- For up type quarks, the superpotential of the Yukawa sector with $S_4 \times Z_4$ is given as

$$w_u = y_1^u (u^c q_1 + c^c q_2)\chi_1 h_u/\Lambda + y_2^u \left[(u^c q_2 + c^c q_1)\chi_2 + (u^c q_1 - c^c q_2)\chi_3\right] h_u/\Lambda + y_3^u t^c q_3 h_u .$$

$(2 \times 3_1 \times 3_1 \times 1_1 \to 1_1 \ , \ 1_1 \times 3_1 \times 3_1 \times 1_1 \to 1_1)$

- Taking vacuum alignment $(\langle\chi_2\rangle, 0)$, and condition $y_1^u\langle\chi_1\rangle = y_2^u\langle\chi_2\rangle$:

$$M_u = v_u \begin{pmatrix} y_1^u\alpha_1 & y_1^u\alpha_1 & 0 \\ y_1^u\alpha_1 & y_1^u\alpha_1 & 0 \\ 0 & 0 & y_3^u \end{pmatrix}, \quad U_u = \begin{pmatrix} \cos 45^\circ & \sin 45^\circ & 0 \\ -\sin 45^\circ & \cos 45^\circ & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$m_u = 0, \quad m_c = 2y_1^u v_u \alpha_1, \quad m_t = y_3^u v_u \simeq 170\text{GeV} .$$

Remark

- $y_1^u\langle\chi_1\rangle = y_2^u\langle\chi_2\rangle$ is not guaranteed in general.
  In Improved model in the next section, we do not need this condition.

- For down type quarks, $M_d = M_l^T$, because we consider $H_{\bar{5}}$ in SU(5).

$$M_d^\dagger M_d = v_d^2 \begin{pmatrix} \frac{1}{2}|y_1|^2\alpha_{10}^2 & \frac{1}{2\sqrt{3}}|y_1|^2\alpha_{10}^2 & 0 \\ \frac{1}{2\sqrt{3}}|y_1|^2\alpha_{10}^2 & \frac{1}{6}|y_1|^2(4\alpha_9^2+\alpha_{10}^2) & 0 \\ 0 & 0 & |y_2|^2\alpha_{14}^2 \end{pmatrix} .$$

- Then, the down type quark masses are given as

$$m_d^2 = m_e^2 \approx \frac{1}{2}|y_1|^2\alpha_9^2 v_d^2 \ , \ m_s^2 = m_\mu^2 \approx \frac{2}{3}|y_1|^2\alpha_{10}^2 v_d^2 \ , \ m_b^2 = m_\tau^2 \approx |y_2|^2\alpha_{14}^2 v_d^2 \ .$$

- The mass matrix is diagonalized by the orthogonal matrix U_d:

$$U_d = \begin{pmatrix} \cos 60^\circ & \sin 60^\circ & 0 \\ -\sin 60^\circ & \cos 60^\circ & 0 \\ 0 & 0 & 1 \end{pmatrix} ,$$

Small α_9 is neglected to get this mixing.

Now we can estimate CKM matrix: $\theta_{12}^{\mathrm{CKM}} = \theta_{12}^{d} - \theta_{12}^{u} = 60^\circ - 45^\circ = 15^\circ$

$$V^{CKM} = U_u^\dagger U_d = \begin{pmatrix} \cos 15^\circ & \sin 15^\circ & 0 \\ -\sin 15^\circ & \cos 15^\circ & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Summary up to here

- In the framework of the SU(5) SUSY GUT, the prototype model with S_4 flavor symmetry leads to tri-bimaximal mixing.
- The mixing angle between the first and the second generations of quarks is 15°.

Problems

- Charged lepton masses are the same ones of down type quarks.
- The experimental data V_{us}^{CKM} is $13^\circ(0.226)$. predict: 15°
- The experimental data V_{cb}^{CKM} and $V_{ub}^{CKM}$are non-vanishing but small.

We improve the prototype model

to get observed quark/lepton masses and CKM mixing.

- Add SU(5) 45-dimensional Higgs h_{45}.
- Add SU(5) singlet scalars (χ_i').

Coupled	Up type	Charged leptons +	Down type quarks
particle	(χ_2', χ_3')	$(\chi_9', \chi_{10}', \chi_{11}')$	h_{45}
SU(5)	1	1	45
S_4	$\mathbf{2}$	$\mathbf{3}_1$	$\mathbf{1}_1$
Z_4	ω^3	ω^2	ω^2

h_{45}, $(\chi_9', \chi_{10}', \chi_{11}')$: Different masses of charged leptons and down quarks

- Superpotential of Yukawa sector is

$$w_{SU(5)} = w_{SU(5)}^{(0)} + w_{SU(5)}^{(1)},$$

$$w_{SU(5)}^{(1)} = y_4^u (T_1, T_2) \otimes T_3 \otimes (\chi_2', \chi_3') \otimes H_5 + y_1' (F_1, F_2, F_3) \otimes (T_1, T_2) \otimes (\chi_9', \chi_{10}', \chi_{11}') \otimes h_{45}.$$

- Neutrino is just same as prototype model.

Taking vacuum alignments $(\langle\chi_9\rangle, \langle\chi_{10}\rangle, 0)$, $(0, 0, \langle\chi_{14}\rangle)$, $(\langle\chi'_9\rangle, \langle\chi'_{10}\rangle, 0)$.

- For charged leptons:

$$M_l = v_d \begin{pmatrix} 0 & (y_1\alpha_{10} - 3\bar{y}_1\alpha'_{10})/\sqrt{2} & 0 \\ -2(y_1\alpha_9 - 3\bar{y}_1\alpha'_9)/\sqrt{6} & (y_1\alpha_{10} - 3\bar{y}_1\alpha'_{10})/\sqrt{6} & 0 \\ 0 & 0 & y_2\alpha_{14} \end{pmatrix}.$$

$$m_e^2 \approx \frac{1}{2}|y_1\alpha_9 - 3\bar{y}_1\alpha'_9|^2 v_d^2,\ m_\mu^2 \approx \frac{2}{3}|y_1\alpha_{10} - 3\bar{y}_1\alpha'_{10}|^2 v_d^2,\ m_\tau^2 \approx |y_2|^2\alpha_{14}^2 v_d^2,$$

$$|\theta^l_{12}| = \left| -\frac{y_1\alpha_9 - 3\bar{y}_1\alpha'_9}{2(y_1\alpha_{10} - 3\bar{y}_1\alpha'_{10})} \right| \approx \frac{1}{\sqrt{3}}\frac{m_e}{m_\mu} \approx 2.8 \times 10^{-3},\ \theta^l_{23} = 0,\ \theta^l_{13} = 0.$$

- For down type quarks:

$$M_d = v_d \begin{pmatrix} 0 & -2(y_1\alpha_9 + \bar{y}_1\alpha'_9)/\sqrt{6} & 0 \\ (y_1\alpha_{10} + \bar{y}_1\alpha'_{10})/\sqrt{2} & (y_1\alpha_{10} + \bar{y}_1\alpha'_{10})/\sqrt{6} & 0 \\ 0 & 0 & y_2\alpha_{14} \end{pmatrix}.$$

There are differences in coefficients of α'_9, α'_{10} in M_l, M_d^T.

We consider $M_d^\dagger M_d$ as follows:

$$v_d^2 \begin{pmatrix} \frac{1}{2}|y_1\alpha_{10}+\bar{y}_1\alpha'_{10}|^2 & \frac{1}{2\sqrt{3}}|y_1\alpha_{10}+\bar{y}_1\alpha'_{10}|^2 & 0 \\ \frac{1}{2\sqrt{3}}|y_1\alpha_{10}+\bar{y}_1\alpha'_{10}|^2 & \frac{1}{6}(4|y_1\alpha_9+\bar{y}_1\alpha'_9|^2+|y_1\alpha_{10}+\bar{y}_1\alpha'_{10}|^2) & 0 \\ 0 & 0 & |y_2|^2\alpha_{14}^2 \end{pmatrix}.$$

After rotating $M_d^\dagger M_d$ by U_d ($\theta_{12}^d = 60^\circ,\ \theta_{23}^d = \theta_{13}^d = 0$), it turns to be

$$v_d^2 \begin{pmatrix} \frac{1}{2}|y_1\alpha_9+\bar{y}_1\alpha'_9|^2 & -\frac{1}{2\sqrt{3}}|y_1\alpha_9+\bar{y}_1\alpha'_9|^2 & 0 \\ -\frac{1}{2\sqrt{3}}|y_1\alpha_9+\bar{y}_1\alpha'_9|^2 & \frac{1}{6}(|y_1\alpha_9+\bar{y}_1\alpha'_9|^2+4|y_1\alpha_{10}+\bar{y}_1\alpha'_{10}|^2) & 0 \\ 0 & 0 & |y_2|^2\alpha_{14}^2 \end{pmatrix}.$$

Then, down type quark masses are given as

$$m_d^2 \approx \frac{1}{2}|y_1\alpha_9+\bar{y}_1\alpha'_9|^2 v_d^2\ ,\ \ m_s^2 \approx \frac{2}{3}|y_1\alpha_{10}+\bar{y}_1\alpha'_{10}|^2 v_d^2\ ,\ \ m_b^2 \approx |y_2|^2\alpha_{14}^2 v_d^2\ ,$$

- The mixing angle θ^d_{12} is $60^\circ + \delta\theta^d_{12}$, where

$$\delta\theta^d_{12} = -\frac{\sqrt{3}|y_1\alpha_9 + \bar{y}_1\alpha'_9|^2}{4|y_1\alpha_{10} + \bar{y}_1\alpha'_{10}|^2} = -\frac{m_d^2}{\sqrt{3}m_s^2} \approx -1.5\times 10^{-3}.$$

Therefore, θ^d_{12} is almost 60°.

We add a comment on θ^d_{23} and θ^d_{13}, which vanish in our scheme because of $\alpha_{11} = \alpha'_{11} = \alpha_{12} = \alpha_{13} = 0$.

- Non-vanishing α_{11}, α_{13} lead to

$$\theta^d_{13} \approx -\frac{y_1\alpha_{11}}{\sqrt{2}y_2\alpha_{14}}, \qquad \theta^d_{23} \approx \frac{2(y_1\alpha_{10} + \bar{y}_1\alpha'_{10})\alpha_{13} - y_1\alpha_{11}\alpha_{14}}{\sqrt{6}y_2\alpha_{14}^2},$$

These mixing angles are expected to be tiny as far as $\alpha_{14} \gg \alpha_{11},\ \alpha_{13}$.

For up type quarks, the mass matrix is

$$M_u = v_u \begin{pmatrix} y_1^u\alpha_1 + y_2^u\alpha_3 & y_2^u\alpha_2 & y_4^u\alpha_2' \\ y_2^u\alpha_2 & y_1^u\alpha_1 - y_2^u\alpha_3 & y_4^u\alpha_3' \\ y_4^u\alpha_2' & y_4^u\alpha_3' & y_3^u \end{pmatrix}.$$

After rotating M_u by U_u ($\theta_{12}^u = 45^\circ,\ \theta_{23}^u = \theta_{13}^u = 0$):

$$\hat{M}_u = U_u^T M_u U_u = v_u \begin{pmatrix} y_1^u\alpha_1 - y_2^u\alpha_2 & y_2^u\alpha_3 & \frac{y_4^u}{\sqrt{2}}(\alpha_2' - \alpha_3') \\ y_2^u\alpha_3 & y_1^u\alpha_1 + y_2^u\alpha_2 & \frac{y_4^u}{\sqrt{2}}(\alpha_2' + \alpha_3') \\ \frac{y_4^u}{\sqrt{2}}(\alpha_2' - \alpha_3') & \frac{y_4^u}{\sqrt{2}}(\alpha_2' + \alpha_3') & y_3^u \end{pmatrix}.$$

Take vacuum alignments $\langle\chi_1\rangle = 0,\ (0, \langle\chi_3\rangle),\ \langle\chi_2'\rangle(1,1)$.

- Since $\alpha_1 = \alpha_2 = 0$, $\alpha_2' = \alpha_3'$, up type quark mass matrix turns to simple one (so-called Fritzsch-type mass matrix).

$$\hat{M}_u \simeq v_u \begin{pmatrix} 0 & y_2^u\alpha_3 & 0 \\ y_2^u\alpha_3 & 0 & \sqrt{2}y_4^u\alpha_2' \\ 0 & \sqrt{2}y_4^u\alpha_2' & y_3^u \end{pmatrix}.$$

As well known, the complex phases in the Fritzsch-type mass matrix can be removed by the phase matrix P as $P^\dagger \hat{M}_u P$;

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{-i\rho} & 0 \\ 0 & 0 & e^{-i\sigma} \end{pmatrix}.$$

Therefore, up type quark masses are

$$m_u = \left| \frac{y_3^u {y_2^u}^2 \alpha_3^2}{2{y_4^u}^2 {\alpha_2'}^2} \right| v_u, \quad m_c = \left| -\frac{2{y_4^u}^2}{y_3^u} {\alpha_2'}^2 \right| v_u \quad m_t = |y_3^u| v_u .$$

The mixing matrix to diagonalize $\hat{M}_u$, V_F ($M_u^\mathrm{diagonal} = V_\mathrm{F}^\dagger P^\dagger \hat{M}_u P V_\mathrm{F}$), is

$$V_\mathrm{F} \approx \begin{pmatrix} 1 & \sqrt{\frac{m_u}{m_c}} & -\sqrt{\frac{m_u}{m_t}} \\ -\sqrt{\frac{m_u}{m_c}} & 1 & \sqrt{\frac{m_c}{m_t}} \\ \sqrt{\frac{m_u}{m_t}} & -\sqrt{\frac{m_c}{m_t}} & 1 \end{pmatrix}.$$

- Up type quark and down type quark mixing matrices U_u and U_d:

$$U_u = \begin{pmatrix} \cos 45^\circ & \sin 45^\circ & 0 \\ -\sin 45^\circ & \cos 45^\circ & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{-i\rho} & 0 \\ 0 & 0 & e^{-i\sigma} \end{pmatrix} \begin{pmatrix} 1 & \sqrt{\frac{m_u}{m_c}} & -\sqrt{\frac{m_u}{m_t}} \\ -\sqrt{\frac{m_u}{m_c}} & 1 & \sqrt{\frac{m_c}{m_t}} \\ \sqrt{\frac{m_u}{m_t}} & -\sqrt{\frac{m_c}{m_t}} & 1 \end{pmatrix},$$

$$U_d = \begin{pmatrix} \cos 60^\circ & \sin 60^\circ & 0 \\ -\sin 60^\circ & \cos 60^\circ & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

Phase matrix is inserted in U_u, where ρ and σ are arbitrary at present.

- CKM matrix:

$$V^{CKM} = U_u^\dagger U_d = \begin{pmatrix} 1 & -\sqrt{\frac{m_u}{m_c}} & \sqrt{\frac{m_u}{m_t}} \\ \sqrt{\frac{m_u}{m_c}} & 1 & -\sqrt{\frac{m_c}{m_t}} \\ -\sqrt{\frac{m_u}{m_t}} & \sqrt{\frac{m_c}{m_t}} & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\rho} & 0 \\ 0 & 0 & e^{i\sigma} \end{pmatrix} \begin{pmatrix} \cos 15^\circ & \sin 15^\circ & 0 \\ -\sin 15^\circ & \cos 15^\circ & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

- The CKM mixing elements are given as

$$\left|V_{us}^{\text{CKM}}\right| = \left|\sin 15^\circ - \cos 15^\circ \sqrt{\frac{m_u}{m_c}} e^{i\rho}\right|,$$

$$\left|V_{cb}^{\text{CKM}}\right| = \sqrt{\frac{m_c}{m_t}}, \qquad \left|V_{ub}^{\text{CKM}}\right| = \sqrt{\frac{m_u}{m_t}}$$

- Putting experimental values of quark masses and $\rho = 50^\circ$.
(Red colors denote experimental data of 3σ.)

$$\left|V_{us}^{\text{CKM}}\right| = 0.226(\text{Input}),$$

$$\left|V_{cb}^{\text{CKM}}\right| = 0.048(0.04117 - 0.4180),$$

$$\left|V_{ub}^{\text{CKM}}\right| = 0.003(0.00311 - 0.00407).$$

Non-vanishing θ_{13}^d, θ_{23}^d improve predictions.
Including phase σ, we can discuss the CP violation.

Let us discuss vacuum alignments

In our model, we need vacuum alignments of scalar fields χ_i of S_4 doublets and triplets. Vacuum alignments are summarized at the leading order as follows:

$$\chi_1 = 0,\ (\chi_2, \chi_3) = (0,1),\ (\chi_2', \chi_3') = (1,1),\ (\chi_4, \chi_5) = (0,1),\ (\chi_6, \chi_7, \chi_8) = (1,1,1)$$
$$(\chi_9, \chi_{10}, \chi_{11}) = (0,1,0),\ (\chi_9', \chi_{10}', \chi_{11}') = (0,1,0),\ (\chi_{12}, \chi_{13}, \chi_{14}) = (0,0,1),$$

where magnitudes are given in arbitrary units. Non-vanishing m_e and m_d require tiny deviations from zeros for χ_9 and χ_9', which could be realized in the next leading order.

- Magnitudes of VEVs

Putting typical values of quark masses, $M_2 = 10^{16}$GeV, and $\tan\beta = 3$ with taking 1 for Yukawa couplings, we have

$$\alpha_2' \sim 0.03, \quad \alpha_3 \sim 1\times 10^{-4}, \quad \alpha_5 \sim 10^{-4} - 10^{-2}, \quad \alpha_6 \geq 0.1,$$
$$\alpha_{10} \sim 8\times 10^{-4}, \quad \alpha_{10}' \sim 2\times 10^{-4}, \quad \alpha_{14} \sim 0.02.$$
$$\alpha_i = \mathcal{O}(0.1) \sim \mathcal{O}(10^{-4})$$

Potential analysis

We present the scalar potential to discuss the vacuum alignment.
The SU(5)$\times S_4 \times Z_4$ invariant superpotential is given as

$$
\begin{aligned}
w = {}& \mu_1(\chi_1)^2_{1_1} + \mu_2(\chi_2,\chi_3)^2_2 + \mu_3(\chi_4,\chi_5)^2_2 \\
&+ \mu_4(\chi_9,\chi_{10},\chi_{11})^2_{3_1} + \mu_5(\chi_9',\chi_{10}',\chi_{11}')^2_{3_1} + \mu_6(\chi_6,\chi_7,\chi_8)_{3_1} \otimes (\chi_{12},\chi_{13},\chi_{14})_{3_1} \\
&+ \eta_1(\chi_1)_{1_1} \otimes (\chi_2,\chi_3)_2 \otimes (\chi_4,\chi_5)_2 + \eta_2(\chi_1)_{1_1} \otimes (\chi_6,\chi_7,\chi_8)^2_{3_1} \\
&+ \eta_3(\chi_1)_{1_1} \otimes (\chi_{12},\chi_{13},\chi_{14})^2_{3_1} + \eta_4(\chi_1)_{1_1} \otimes (\chi_9,\chi_{10},\chi_{11})_{3_1} \otimes (\chi_9',\chi_{10}',\chi_{11}')_{3_1} \\
&+ \eta_5(\chi_2,\chi_3)^2_2 \otimes (\chi_4,\chi_5)_2 + \eta_6(\chi_2,\chi_3)_2 \otimes (\chi_6,\chi_7,\chi_8)^2_{3_1} \\
&+ \eta_7(\chi_2,\chi_3)_2 \otimes (\chi_9,\chi_{10},\chi_{11})_{3_1} \otimes (\chi_9',\chi_{10}',\chi_{11}')_{3_1} + \eta_8(\chi_2,\chi_3)_2 \otimes (\chi_{12},\chi_{13},\chi_{14})^2_{3_1} \\
&+ \eta_9(\chi_2',\chi_3') \otimes (\chi_6,\chi_7,\chi_8) \otimes (\chi_9',\chi_{10}',\chi_{11}') \\
&+ \eta_{10}(\chi_2',\chi_3') \otimes (\chi_9,\chi_{10},\chi_{11}) \otimes (\chi_{12},\chi_{13},\chi_{14}) \\
&+ \eta_{11}(\chi_4,\chi_5)^3_2 + \eta_{12}(\chi_4,\chi_5)_2 \otimes (\chi_9,\chi_{10},\chi_{11})^2_{3_1} + \eta_{13}(\chi_4,\chi_5)_2 \otimes (\chi_9',\chi_{10}',\chi_{11}')^2_{3_1} \\
&+ \eta_{14}(\chi_4,\chi_5)_2 \otimes (\chi_6,\chi_7,\chi_8)_{3_1} \otimes (\chi_{12},\chi_{13},\chi_{14})_{3_1} + \eta_{15}(\chi_6,\chi_7,\chi_8)^2_{3_1} \otimes (\chi_9',\chi_{10}',\chi_{11}')_{3_1} \\
&+ \eta_{16}(\chi_9,\chi_{10},\chi_{11})^3_{3_1} + \eta_{17}(\chi_6,\chi_7,\chi_8)_{3_1} \otimes (\chi_9,\chi_{10},\chi_{11})_{3_1} \otimes (\chi_{12},\chi_{13},\chi_{14})_{3_1} \\
&+ \eta_{18}(\chi_9,\chi_{10},\chi_{11})_{3_1} \otimes (\chi_9',\chi_{10}',\chi_{11}')^2_{3_1} + \eta_{19}(\chi_9',\chi_{10}',\chi_{11}')_{3_1} \otimes (\chi_{12},\chi_{13},\chi_{14})^2_{3_1}.
\end{aligned}
$$

Consider only large VEVs of $(\langle\chi_6\rangle, \langle\chi_7\rangle, \langle\chi_8\rangle) = \langle\chi_6\rangle(1,1,1)$, $(\langle\chi'_2\rangle, \langle\chi'_3\rangle) = \langle\chi'_2\rangle(1,1)$, and $(\langle\chi_{12}\rangle, \langle\chi_{13}\rangle, \langle\chi_{14}\rangle) = \langle\chi_{14}\rangle(0,0,1)$.

Since we have a solution

$$\eta_2(\chi_6^2 + \chi_7^2 + \chi_8^2) + \eta_3\chi_{14}^2 = 0,\ \eta_6(\chi_7^2 - \chi_8^2) = 0,\ \frac{1}{\sqrt{6}}\eta_6(-2\chi_6^2 + \chi_7^2 + \chi_8^2) = 0,$$

$$\eta_8 = \eta_9 = \eta_{10} = \eta_{14} = \eta_{15} = \eta_{17} = \mu_6 = 0,$$

vacuum alignments $\langle\chi_6\rangle = \langle\chi_7\rangle = \langle\chi_8\rangle$, $\langle\chi_{12}\rangle = \langle\chi_{13}\rangle = 0$ are possible solutions. On the other hand, $\langle\chi'_2\rangle = \langle\chi'_3\rangle$ is not guaranteed;

$$\eta_9\left(-\frac{1}{\sqrt{2}}\chi'_2 + \frac{1}{\sqrt{6}}\chi'_3\right) + \eta_{15}\chi_6\chi_7 = 0,\quad \eta_{10}\left(-\frac{1}{\sqrt{2}}\chi'_2 + \frac{1}{\sqrt{6}}\chi'_3\right)\chi_{14} = 0.$$

We may need another mechanism to realize the vacuum alignment of $(\langle\chi'_2\rangle, \langle\chi'_3\rangle)$.

- Flavor Symmetry Breaking and Vacuum Alignment on Orbifolds
  (T. Kobayashi, Y. Omura, and K. Yoshioka, arXiv:0809.3064. @Kyoto)
  Vacuum alignments are realized by boundary conditions in extra dimension.

4. Summary and Future work

Summary

- We have presented a successful flavor model with S_4 symmetry to unify quarks and leptons in the framework of SU(5) SUSY GUT.
- Vacuum alignments of scalars are also required to realize the tri-bimaximal mixing of neutrino flavors.
- Our model predicts the quark mixing as well as the tri-bimaximal mixing of leptons. The Cabbibo angle is predicted to be around 15°.

Future work

- We will study the origin of the vacuum alignments.
- We will study CP violation in lepton and quark sectors.
- We will apply to SUSY sector; slepton and squark spectra, FCNC

- Multiplication of $2 \times 3_1$ is

$$(a_1, a_2)_2 \times (b_1, b_2, b_3)_{3_1} = \begin{pmatrix} a_2 b_1 \\ -\frac{1}{2}(\sqrt{3} a_1 b_2 + a_2 b_2) \\ \frac{1}{2}(\sqrt{3} a_1 b_3 - a_2 b_3) \end{pmatrix}_{3_1} + \begin{pmatrix} a_1 b_1 \\ \frac{1}{2}(\sqrt{3} a_2 b_2 - a_1 b_2) \\ -\frac{1}{2}(\sqrt{3} a_2 b_3 - a_1 b_3) \end{pmatrix}_{3_2}$$

- Multiplication of $3_1 \times 3_2$ is

$$(a_1, a_2, a_3)_{3_1} \times (b_1, b_2, b_3)_{3_2} = \left(\sum_{j=1}^{3} a_j b_j \right)_{1_2} + \begin{pmatrix} \frac{1}{\sqrt{6}}(2a_1 b_1 - a_2 b_2 - a_3 b_3) \\ \frac{1}{\sqrt{2}}(a_2 b_2 - a_3 b_3) \end{pmatrix}_2 + \begin{pmatrix} a_3 b_2 - b_2 a_3 \\ a_1 b_3 - a_3 b_1 \\ a_2 b_1 - a_1 b_2 \end{pmatrix}_{3_1} + \begin{pmatrix} a_2 b_3 + b_3 a_2 \\ a_1 b_3 + a_3 b_1 \\ a_1 b_2 + a_2 b_1 \end{pmatrix}_{3_2}$$

- A_4

$$(l_e,\ l_\mu,\ l_\tau) : 3$$
$$e^c : 1, \quad \mu^c : 1'', \quad \tau^c : 1'$$
$$(N_e^c,\ N_\mu^c,\ N_\tau^c) : 3$$

$$M_D \propto I, \quad M_l \propto \mathrm{diag}(m_e, m_\mu, m_\tau)$$

- S_4

$$(l_e,\ l_\mu,\ l_\tau) : 3_1$$
$$(e^c,\ \mu^c) : 2, \quad \tau^c : 1_1$$
$$(N_e^c,\ N_\mu^c) : 2, \quad N_\tau^c : 1_1$$