Quest for

Dynamical Origin of Mass

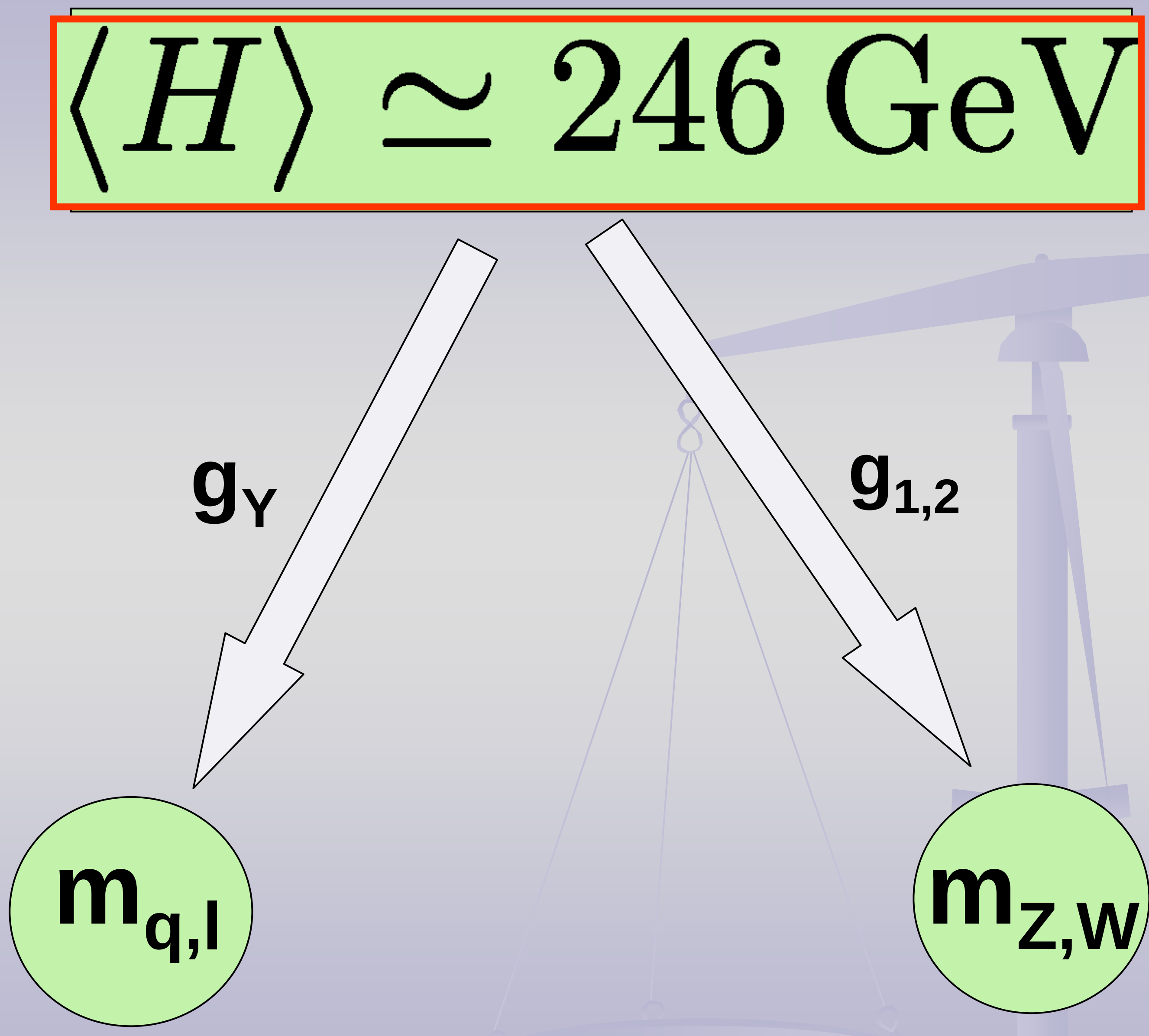
K. Yamawaki (Nagoya)

Jan.26,2009

@ YKIS 08

Dynamical Symmetry Breaking
with Large Anomalous Dimension

ORIGIN
of
MASS ?



$\langle H \rangle \simeq 246\,\mathrm{GeV}$
g_Y
$g_{1,2}$
$m_{q,l}$
$m_{Z,W}$

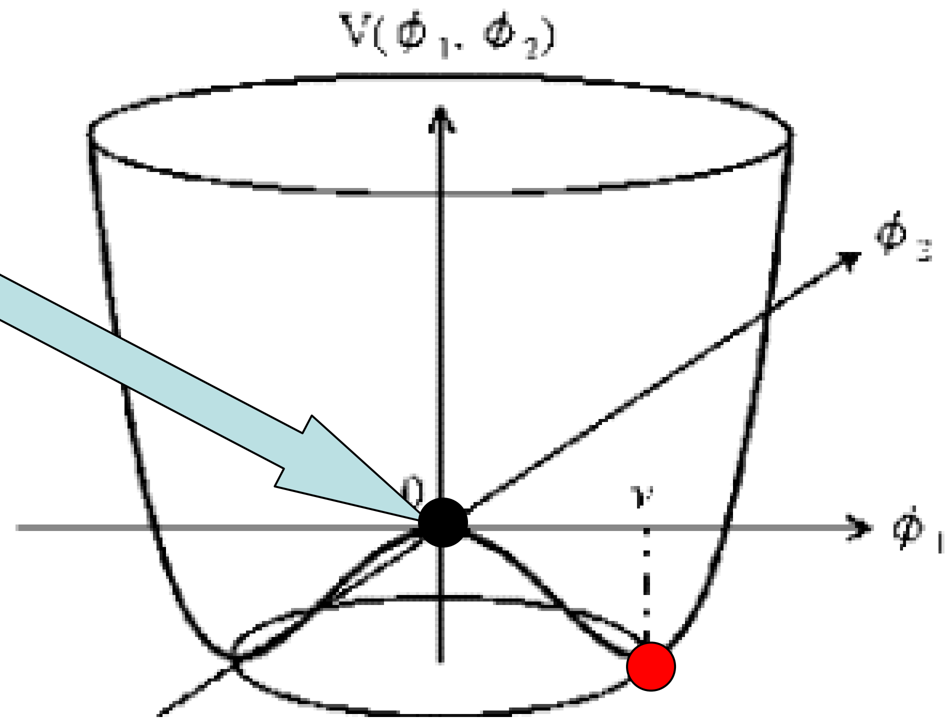
$$\mathcal{L}_{\rm Higgs} = |\partial_\mu \phi|^2 - \mu^2 |\phi|^2 - \lambda |\phi|^4$$

$$v = \langle Re\, \phi^0 \rangle = \langle H \rangle = 246\,\text{GeV}$$

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

$\mu^2 < 0$

Tachyon ! ?



$\phi_2 = Im\, \phi^0$

$\phi_1 = Re\, \phi^0$

$$\mathcal{L}_{\text{Higgs}} = |\partial_\mu \phi|^2 - \lambda \left(|\phi|^2 - \frac{1}{2} v^2 \right)^2$$

$$v = \langle H \rangle = 246\,\text{GeV}$$

$$\mu^2 = -\lambda v^2 \,(< 0)$$

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} i\pi_1 + \pi_2 \\ \sigma - i\pi_3 \end{pmatrix}$$

$$H \leftrightarrow \sigma$$

$$\mathcal{L}_{\text{GL}} = \frac{1}{2} \left((\partial_\mu \sigma)^2 + (\partial_\mu \pi^a)^2 \right) - \frac{\lambda}{4} \left((\sigma)^2 + (\pi^a)^2 - v^2 \right)^2$$

Gell-Mann-Levy (1960)

$$v = \langle \sigma \rangle = 93\,\text{MeV} = f_\pi$$

The Nobel Prize in Physics 2008

"for the discovery of the mechanism of
spontaneous broken symmetry
in subatomic physics"

Spontaneous Symmetry Breaking

was born as

Dynamical Symmetry Breaking

BCS Analogue

Spontaneous Symmetry Breaking

Macroscopic

Microscopic

Superconductor

Ginzburg-Landau

Bardeen-Cooper-Schriefer

$$\mu^2 < 0$$

Attractive Force (BCS Instability)

$$p \gg \langle \phi \rangle$$

$< \phi >$ → $< \psi^{\uparrow} \psi^{\downarrow} >$

Order parameter

Cooper Pair

Energy gap

Nobel Prize Winning Paper

PHYSICAL REVIEW VOLUME 122, NUMBER 1 APRIL 1, 1961

Dynamical Model of Elementary Particles Based on an Analogy with Superconductivity. I*

Y. NAMBU AND G. JONA-LASINIO†
The Enrico Fermi Institute for Nuclear Studies and the Department of Physics, The University of Chicago, Chicago, Illinois

(Received October 27, 1960)

It is suggested that the nucleon mass arises largely as a self-energy of some primary fermion field through the same mechanism as the appearance of energy gap in the theory of superconductivity. The idea can be put into a mathematical formulation utilizing a generalized Hartree-Fock approximation which regards real nucleons as quasi-particle excitations. We consider a simplified model of nonlinear four-fermion interaction which allows a γ_5-gauge group. An interesting consequence of the symmetry is that there arise automatically pseudoscalar zero-mass bound states of nucleon-antinucleon pair which may be regarded as an idealized pion. In addition, massive bound states of nucleon number zero and two are predicted in a simple approximation.

The theory contains two parameters which can be explicitly related to observed nucleon mass and the pion-nucleon coupling constant. Some paradoxical aspects of the theory in connection with the γ_5 transformation are discussed in detail.

[2] J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Phys. Rev. **106**, 162 (1957).

$$\frac{G}{2}(\bar{N}N)(\bar{N}N)$$

m_N $\quad$ m_N

$$= -G\langle \bar{N}N \rangle$$

Gap eq.

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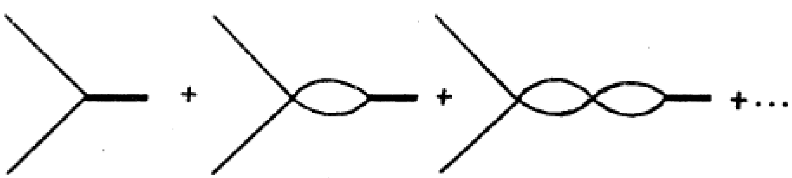
$$\pi \sim \bar{N}N$$

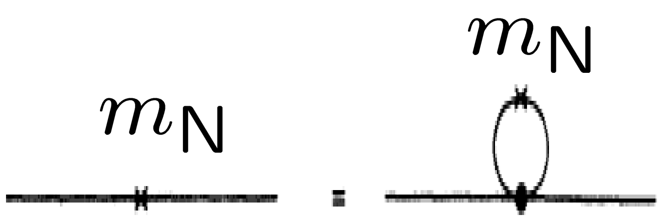


Fermi-Yang Model (1948)

p, n ⇒ p, n, Λ

Sakata Model (1956)

$$\frac{G}{2}(\bar{N}N)(\bar{N}N)$$



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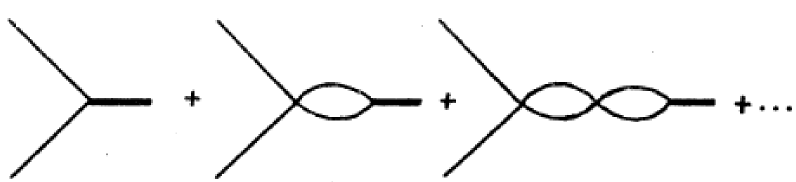
σ (``Higgs'')

$$\pi \sim \bar{N}N$$



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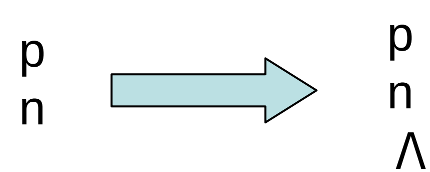


Sakata Model (1956)

Dynamical Generation of

Nucleon Mass M_{N} $\Longleftrightarrow$ $\langle \bar{N} N \rangle$

Masssless Pion f_π

Yukawa Coupling $G_{\mathrm{NN}\pi}$

Cooper Pair Condensate

(Spontaneously Broken) Chiral Symmetry !

- Goldberger-Treiman Relation

$$M_{\mathrm{N}}\, g_{\mathrm{A}} = G_{\mathrm{NN}\pi}\, f_\pi$$

Essentially the same in QCD

- Nucleon

$$\langle \bar{N} N \rangle$$

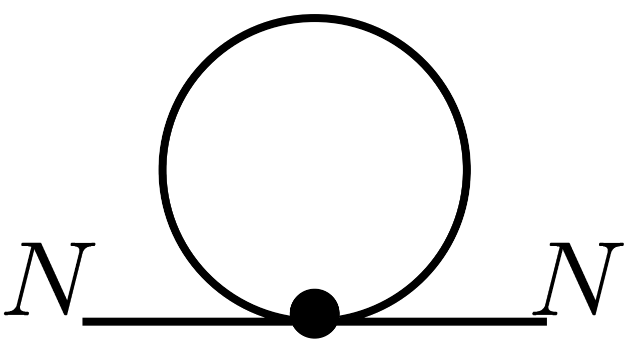
$$M_{\mathrm{N}}$$

Quark

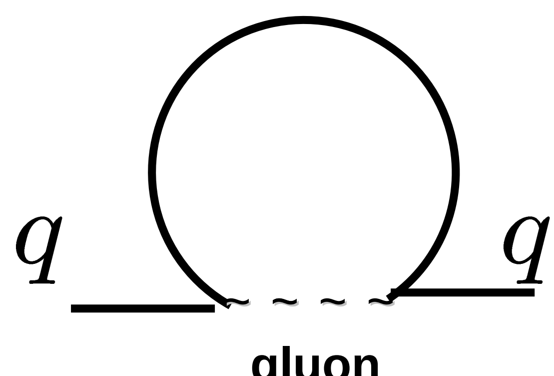
$$\langle \bar{q} q \rangle$$

$$m_{\mathrm{q}}^{*} \approx \frac{M_{\mathrm{N}}}{3}$$

- 4-fermion int.



Gauge int.



$$\pi,\ \sigma \sim \bar{N}N \qquad\qquad \pi,\ \sigma \sim \bar{q}q$$

$$m_\pi = 0 \qquad\qquad m_\pi = 0$$

$$m_\sigma \simeq 2M_\mathsf{N} \quad \Longrightarrow \quad m_\sigma \simeq 2m^*_\mathsf{q}\ ?$$

$$M_\mathsf{N}\, g_\mathsf{A} = G_{\mathsf{NN}\pi}\, f_\pi \qquad\qquad m^*_\mathsf{q} = G_{\mathsf{qq}\pi} f_\pi$$

$$m_\rho \simeq 2m^*_\mathsf{q}$$

Origin of Nucleon Mass

Linear Sigma Model (Gell-Mann-Levy)

$\mu^2 < 0$ tachyon

$\langle \sigma \rangle = f_\pi = 93\mathrm{MeV}$

σ

π

$M_\mathrm{N} = G_{\mathrm{NN}\pi} \langle \sigma \rangle$

$p \gg f_\pi$

QCD

Attractive force (BCS instability)

$\langle \bar{q} q \rangle \sim f_\pi^3$

$\bar{q} q$

$\bar{q} i \gamma_5 q$

$m_N \approx 3 m_q^*$

Nambu Theory is established in QCD

Contents

- Introduction

  (Nambu Theory realized in QCD)
- Walking/Conformal Technicolor
- Effective Theory (Hidden Local Symmetry and Holographic View)
- Top Quark Condensate
- Probing Composite Higgs in LHC

$$v = \langle H \rangle = 246\,\text{GeV}$$

$$\mathcal{L}_{\text{Higgs}} = |\partial_\mu \phi|^2 - \lambda \left(|\phi|^2 - \frac{1}{2} v^2 \right)^2$$

h

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} i\pi_1 + \pi_2 \\ \sigma - i\pi_3 \end{pmatrix}$$

$$H \leftrightarrow \sigma$$

$$\mathcal{L}_\sigma = \frac{1}{2} \left((\partial_\mu \sigma)^2 + (\partial_\mu \pi^a)^2 \right) - \frac{\lambda}{4} \left((\sigma)^2 + (\pi^a)^2 - v^2 \right)^2$$

$$v = \langle \sigma \rangle = 93\,\text{MeV} = f_\pi$$

Underlying QCD

Technicolor: a Scale-Up of QCD

S. Weinberg (1976)
L. Susskind (1979)

$H \sim \bar{F}F \qquad F_\pi = 246\ \text{GeV}$

$\langle \bar{F}F \rangle \sim (700\ \text{GeV})^3$

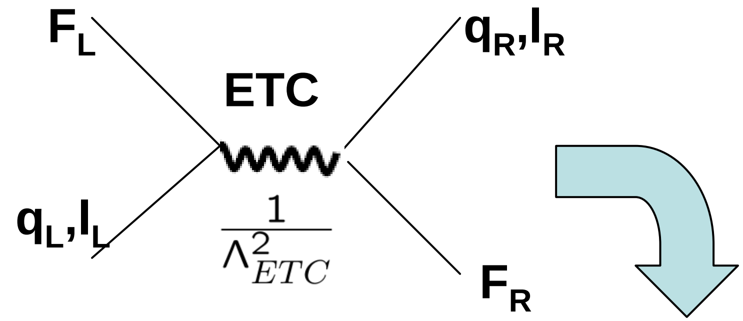
$\frac{N_{TC}}{N_C}$ $\qquad \sqrt{\frac{N_C}{N_{TC} N_D}}$

X 2600

$\sigma \sim \bar{q}q \qquad f_\pi = 93\ \text{MeV}$

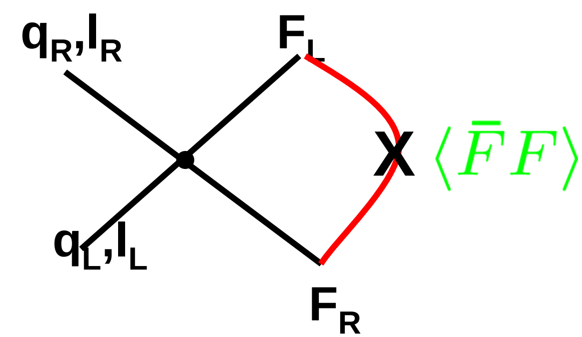
$\langle \bar{q}q \rangle \sim (250\ \text{MeV})^3$

Problems:



Mass of Quarks/Leptons

$$m_{q,l} \sim \frac{1}{\Lambda^2_{ETC}} \langle \bar{F} F \rangle$$



FCNC

$$\frac{1}{\Lambda^2_{ETC}} \bar{s} d \bar{s} d \quad < 10^{-6} \quad \mathrm{TeV}^{-2}$$

$$m_s < 10^{-6}\ \mathrm{TeV}^{-2} \times (0.7\ \mathrm{TeV})^3 \sim 10^{-1}\ \mathrm{MeV}$$

Needs 10^3 enhancement $\times N_D^{-3/2}$

Anomalous Scaling

Holdom (1981)

$$\langle\bar{\psi}\psi\rangle_{\Lambda} = Z_m^{-1}\langle\bar{\psi}\psi\rangle_{\mu}$$

QCD

$$Z_m^{-1} = \left(\frac{\Lambda}{\mu}\right)^{\gamma_m} \Longleftrightarrow \left(\frac{\ln(\Lambda/\Lambda_{QCD})}{\ln(\mu/\Lambda_{QCD})}\right)^{\frac{A}{2}}$$

$$\left(A = \frac{18 C_2(F)}{11 N_c - 2 N_f}\right)$$

$$m_{q,l} \sim \frac{1}{\Lambda_{ETC}^2}\langle\bar{F}F\rangle_{\Lambda_{ETC}}$$

$$\frac{\langle\bar{F}F\rangle_{\Lambda_{ETC}}}{\langle\bar{F}F\rangle_{\Lambda_{TC}}} \simeq \left(\frac{\Lambda_{ETC}}{\Lambda_{TC}}\right)^{\gamma_m} \sim \left(10^3\right)^{\gamma_m} > 10^3$$

$$\Longrightarrow \gamma_m > 1$$

Explicit Dynamics?

Walking/Conformal Technicolor

・K.Y., Bando, Matumoto (1986)

・Akiba, Yanagida (1986)

・Appelquist, Karabali, Wijewardhane (1986)

(Holdom (1985))

$$\gamma_m = 1$$

K.Y., Bando, Matumoto (1986)

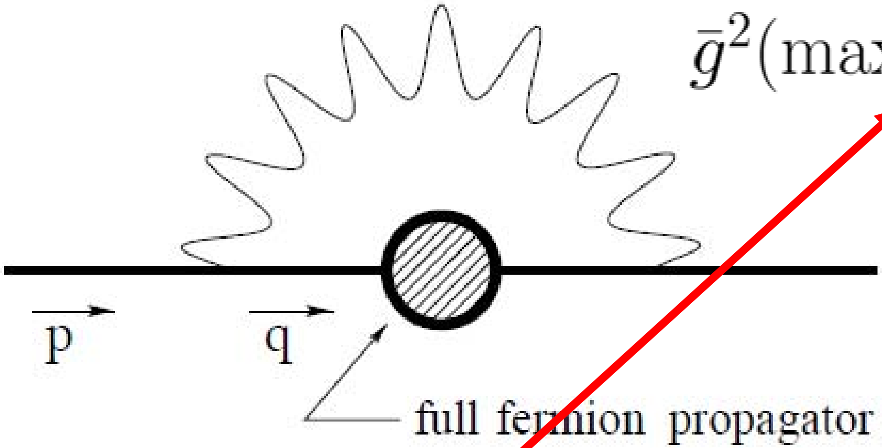
Ladder Schwinger-Dyson Equation with Λ^2

Maskawa-Nakajima (1974)

Schwinger-Dyson Gap Equation

$$iS_F^{-1}(p) - \not{p} \;=$$

$\Sigma(q)$



$\bar{g}^2(\max(P^2, Q^2))$

$\Sigma(q)$

$$iS_F^{-1}(p) - \not{p} = C_2 \int \frac{d^4q}{i(2\pi)^4} \bar{g}^2(p,q) \frac{1}{(p-q)^2} \times \left(g_{\mu\nu} - \frac{(p-q)_\mu (p-q)_\nu}{(p-q)^2} \right) \gamma^\mu i S_F(q) \gamma^\nu,$$

$$\Sigma(x) = C_2 \frac{3}{16\pi^2} \int^{\Lambda^2} dy \, \frac{y\,\Sigma(y)}{y+\Sigma^2(y)} \left(\frac{\bar{g}^2(x)}{x}\theta(x-y) + \frac{\bar{g}^2(y)}{y}\theta(y-x) \right)$$

$$x,\, y \equiv P^2,\, Q^2$$

$$\bar{\alpha}(Q^2) \sim \text{const.} = \alpha$$

$$\Rightarrow \quad (x\,\Sigma(x))'' + \frac{3C_2}{4\pi}\alpha \frac{\Sigma(x)}{x+\Sigma(x)^2} = 0$$

Scale-inv. form

+ UVBC & IRBC

$$\left(\Sigma + x\Sigma'\right)\Big|_{x=\Lambda^2} = m_0(\Lambda) = 0$$

$$\Sigma(x) \sim x^{\left(-1\pm\sqrt{1-\frac{3C_2}{\pi}\alpha}\right)/2}$$

$$\alpha > \frac{\pi}{3C_2} = \alpha_{\rm cr} \quad \Rightarrow$$

Oscillating Sol= SSB Solution

$$\sim x^{-1/2}$$

$\alpha \sim$ const ($> \alpha_{cr}$)

≈ Fixed point

Quasi-conformal

$$\Sigma(Q^2) \sim \frac{1}{Q}$$

DSB solution

$$\Longleftrightarrow \quad \Sigma(Q^2) \overset{\text{OPE}}{\sim} \frac{1}{Q^{2-\gamma_m}}$$

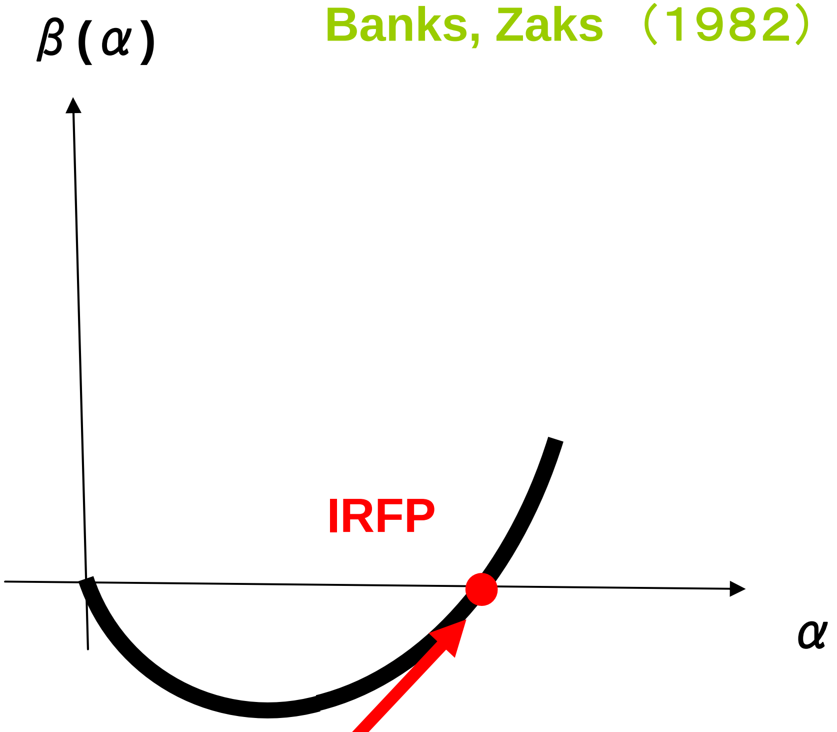
$$\longrightarrow \quad \gamma_m \sim 1$$

Realistic Dynamics for

$\alpha(Q^2) \sim \text{Constant}$ **?**

Large N_f QCD

Banks, Zaks （1982）



Walking/Conformal Coupling

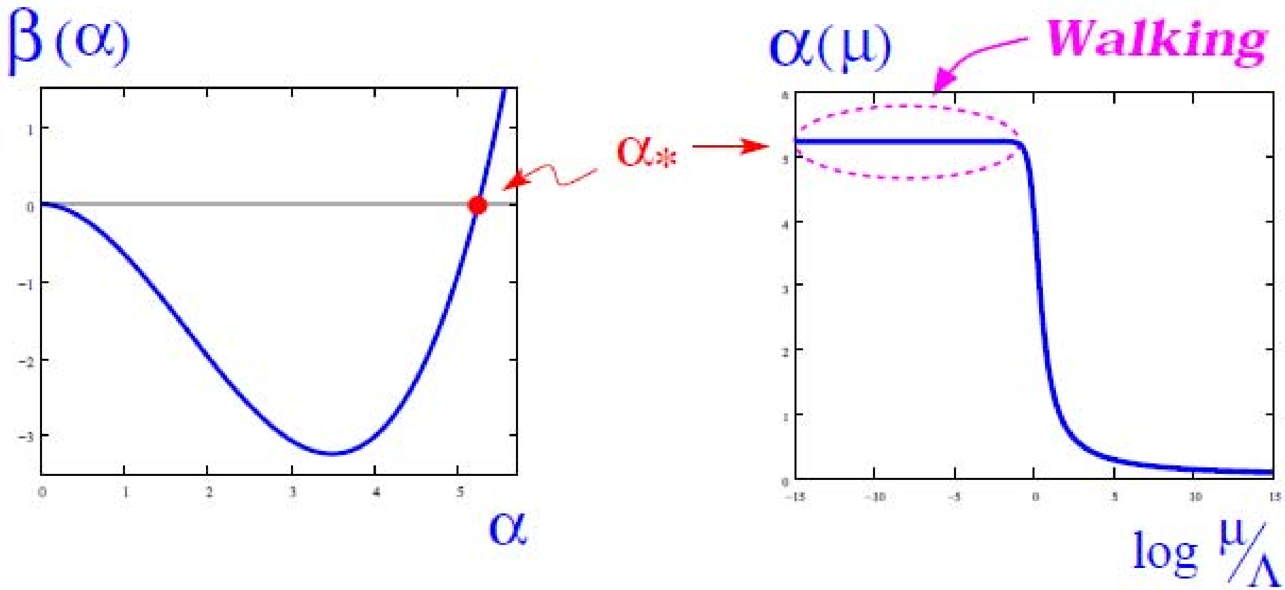
α (Q) ≈ Const. ≈ α_* $\xrightarrow[N_f \uparrow]{}$ 0

Two-loop running coupling in the large N_f QCD

Banks, Zaks （1982）

RGE $\mu \frac{d}{d\mu} \alpha(\mu) = -b\, \alpha^2(\mu) - c\, \alpha^3(\mu)$

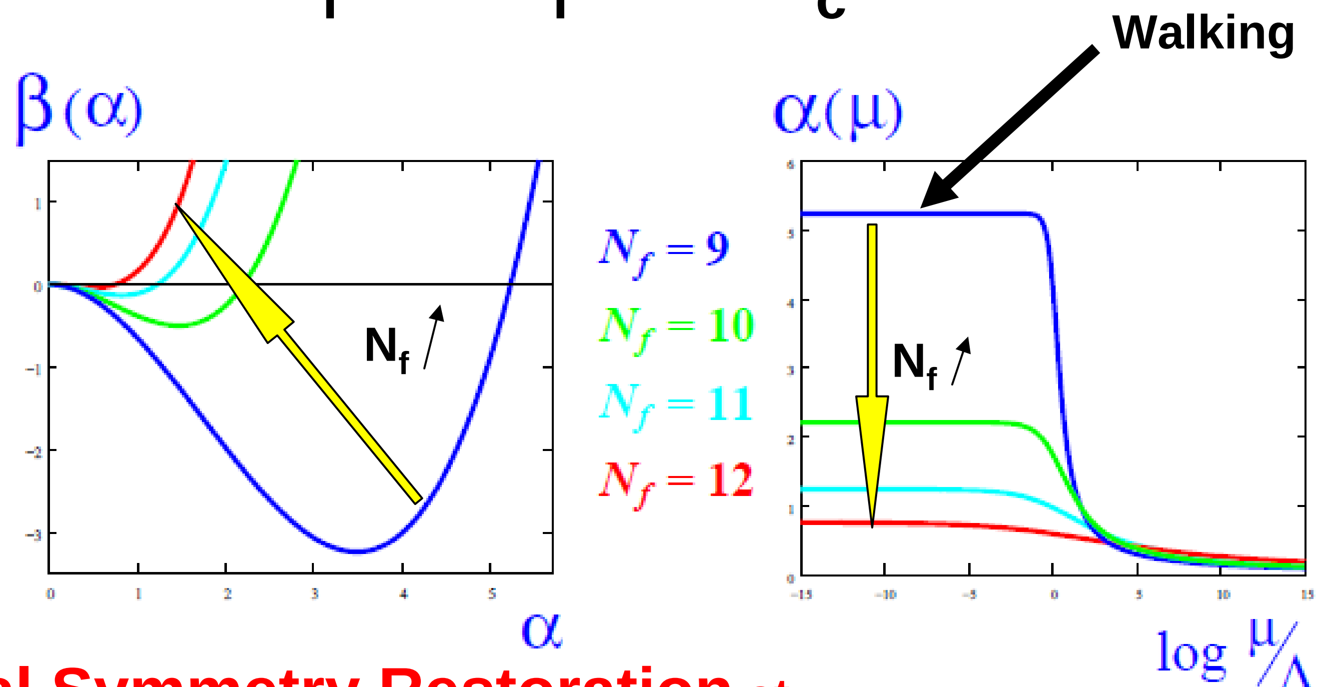
$(N_c = 3)$	$N_f < 8.05$	$8.05 < N_f < 16.5$	$16.5 < N_f$
$b = \frac{1}{6\pi}(33 - 2N_f)$	+	+	−
$c = \frac{1}{12\pi^2}(153 - 19N_f)$	+	−	−



$(\alpha_* = -c/b)$ **IR Fixed Point**

``Conformal Window''

$$N_f^{cr} < N_f < 11N_c/2$$



Chiral Symmetry Restoration at

$$\alpha_* = \alpha_*(N_f, N_c) < \alpha_{cr} = \frac{\pi}{4}$$ ⇐ **SD equation**

$$N_f^{cr} \simeq 4N_c = 12$$

Appelquist,Terning,Wijewardhana (1996)

Conformal Phase Transition

Miransky & K.Y. (1997)

$$\Sigma(m^2) \approx \Lambda \exp\left(- \frac{\pi}{\sqrt{\frac{\alpha_*}{\alpha_{cr}} - 1}} \right) \qquad \sqrt{\frac{\alpha_*}{\alpha_{cr}} - 1} \equiv \omega$$

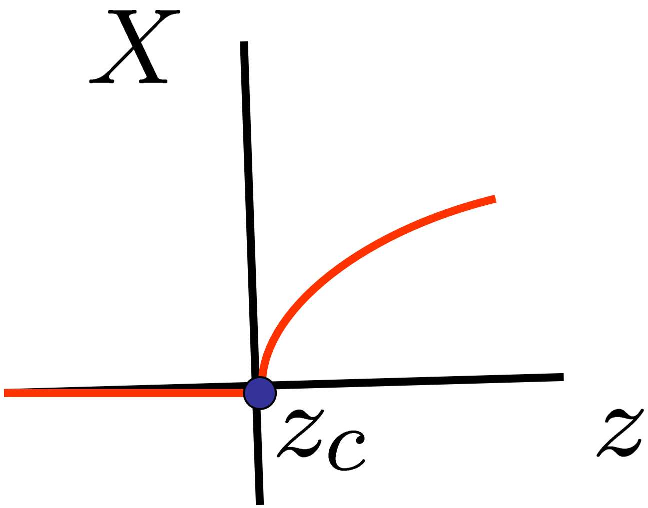
$$\Sigma \sim \frac{m}{\sqrt{1-\frac{\alpha^*}{\alpha_{cr}}}} \left(\frac{x}{m^2}\right)^{\left(-1\pm\sqrt{1-\frac{\alpha^*}{\alpha_{cr}}}\right)/2} \xrightarrow[\alpha^* > \alpha_{cr}]{} \frac{1}{\omega} x^{-1/2} \sin\left(\frac{\omega}{2}\ln(x/m^2) + \delta\right)$$

$$(\Sigma + x\Sigma')\Big|_{x=\Lambda^2} = 0 \implies \omega \ln(\Lambda/m) + \delta' = n\pi$$

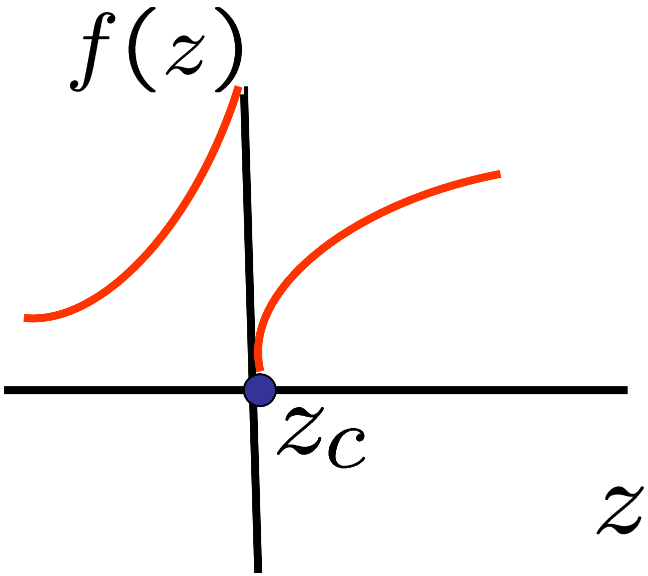
$\delta' \to 0$

Order parameter

$$X = \Lambda f(z) \quad (z > z_c)$$
$$= 0 \quad (z < z_c)$$



$f(z)$: essential singularity at $z = z_c$



1. No light spectrum for $z < z_c$

"unparticle"

2. No parameter s.t.

$$\left.\frac{d^nV}{dX^n}\right|_{X=0} (n = 2, 4, \cdots) > 0\,(z < z_c), \;\; < 0(z > z_c)$$

~~Ginzburg-Landau~~

- usual QCD

$\alpha > 0$ $$m \sim \Lambda \cdot \underline{e^{-\frac{1}{b\alpha(\Lambda)}}}$$

$\alpha < 0$ No bound states (unphysical)

- Gross-Neveu Model

Conformal sym. breaking

$g > 0$ $$m = \bar{\varphi} \sim \Lambda \cdot \underline{e^{-\frac{1}{2g(\Lambda)}}}$$

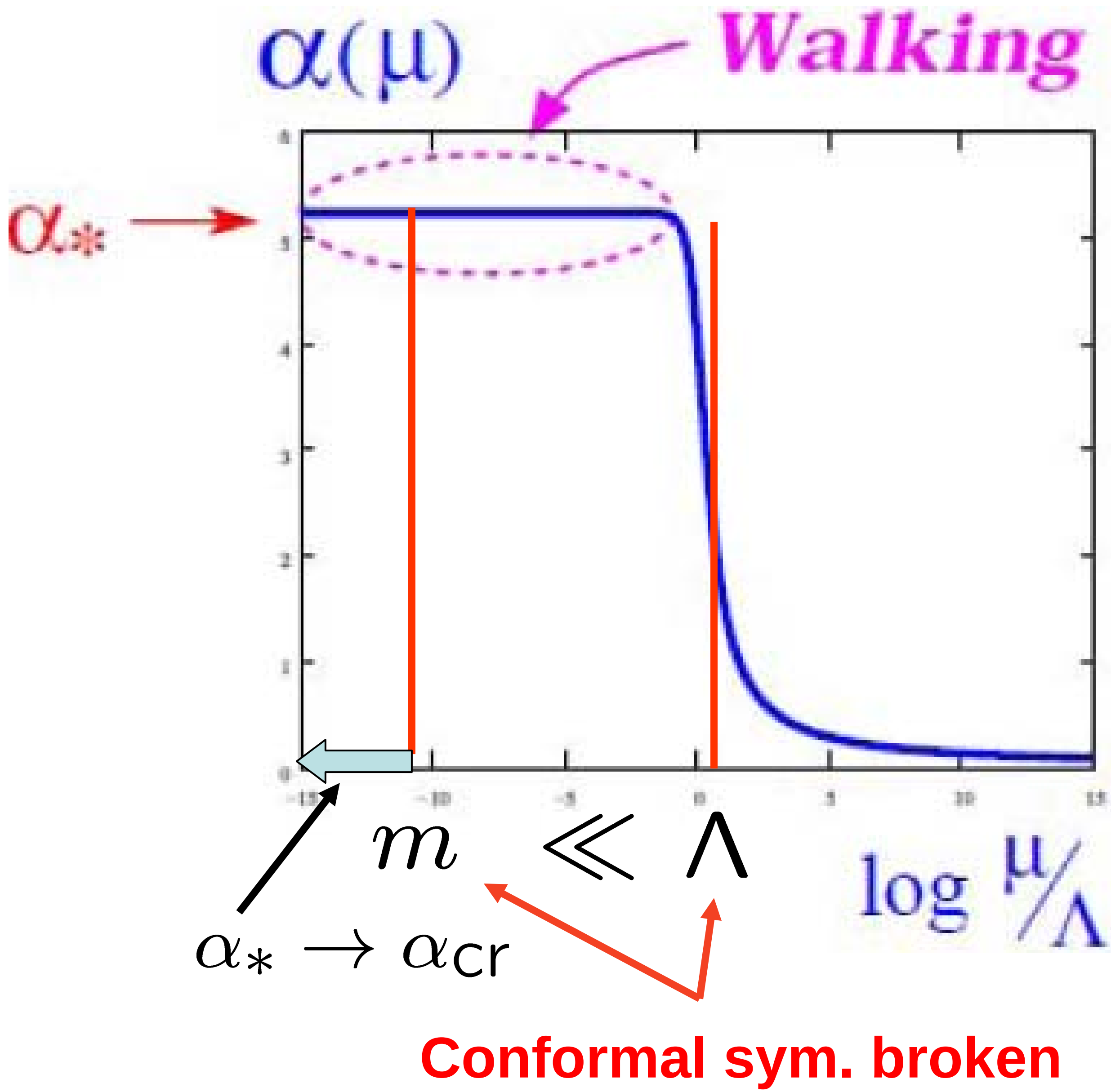
: scalar bound state (``Dilaton'') $M_\sigma = 2m$

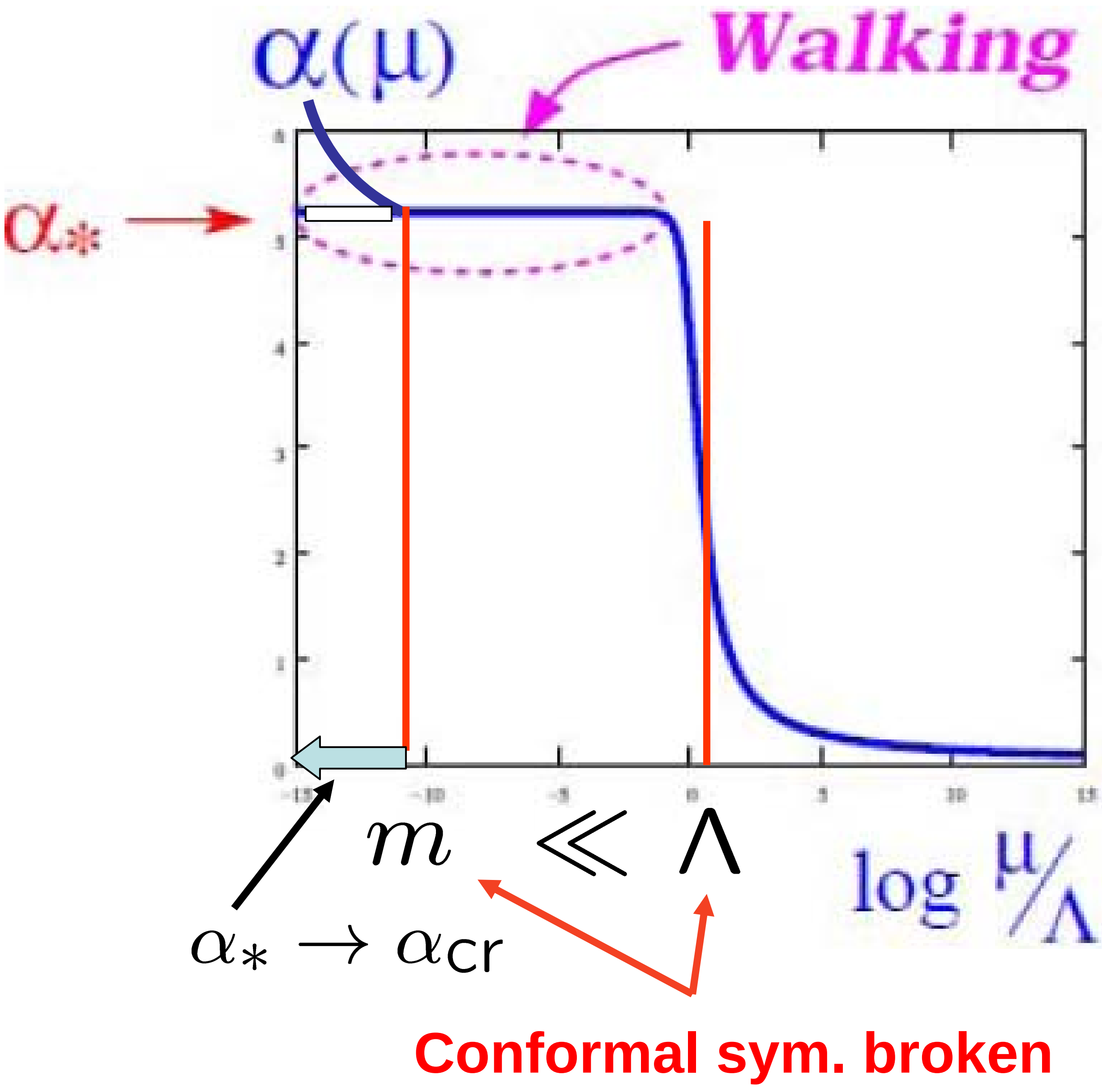
would-be NG boson "$M_\pi = 0$"

$g < 0$: Repulsive four-fermion int.

(no bound states)

conformal


α(μ)
Walking
α*
m ≪ Λ
α* → αcr
log μ/Λ
Conformal sym. broken



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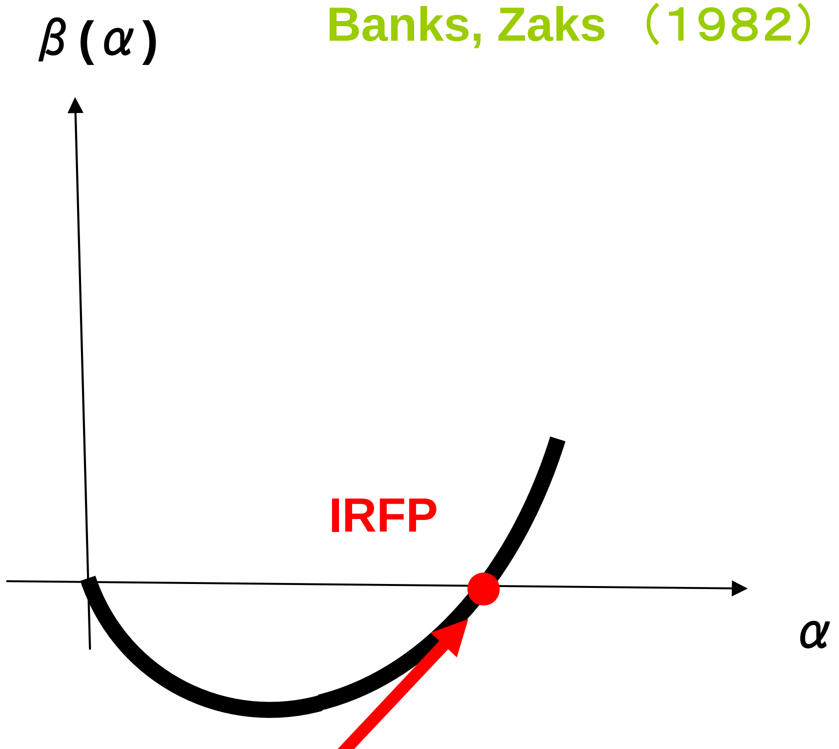
Large N_f QCD

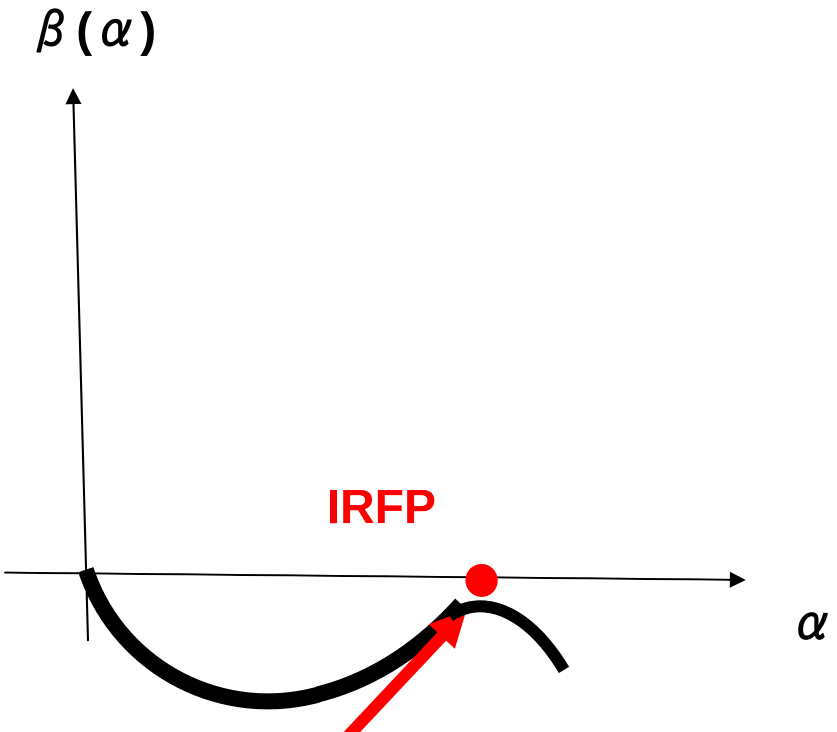
Banks, Zaks （1982）

Walking/Conformal Coupling



$\alpha\ (Q) \approx$ **Const.** $\approx \alpha_* \xrightarrow[N_f \uparrow]{} 0$

Walking/Conformal Technicolor



α (Q) ≈ Const. ≈ α_*

Various Issues

1. Existence of IR fixed point

2. Determination of N_f^{cr}

3. Light spectrum

$$M_\sigma, M_\rho, M_{a_1} \quad : vs. F_\pi$$

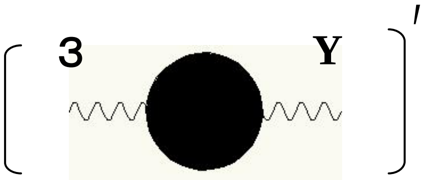
dilaton

$$\frac{F_\pi^2}{M_\rho^2}$$

4. S Parameter

⋮

Electroweak Constraints



L R

$$W^3_{\mu\nu} B^{\mu\nu}$$

$$S = 16\pi \left[\frac{\partial}{\partial q^2} \Pi_{33}\Big|_{q^2=0} - \frac{\partial}{\partial q^2} \Pi_{3Q}\Big|_{q^2=0} \right]$$

$$T = \frac{4\pi}{\sin^2\theta \cos^2\theta M_Z^2} \left[\Pi_{WW}\Big|_{q^2=0} - \Pi_{33}\Big|_{q^2=0} \right]$$

$$U = 16\pi \left[\frac{\partial}{\partial q^2} \Pi_{WW}\Big|_{q^2=0} - \frac{\partial}{\partial q^2} \Pi_{33}\Big|_{q^2=0} \right]$$

$$F.T. < 0|T \tilde{A}^A_\mu(0) \tilde{A}^B_\mu(x)|0 >= g_{\mu\nu} \Pi_{AB} - q_\mu q_\nu \Pi^T_{AB}$$

$S^{(exp)} = -16\pi L_{10} = 0.32 \pm 0.04$ (QCD)

$S^{(pert)} = N_D N_c / (6\pi)$

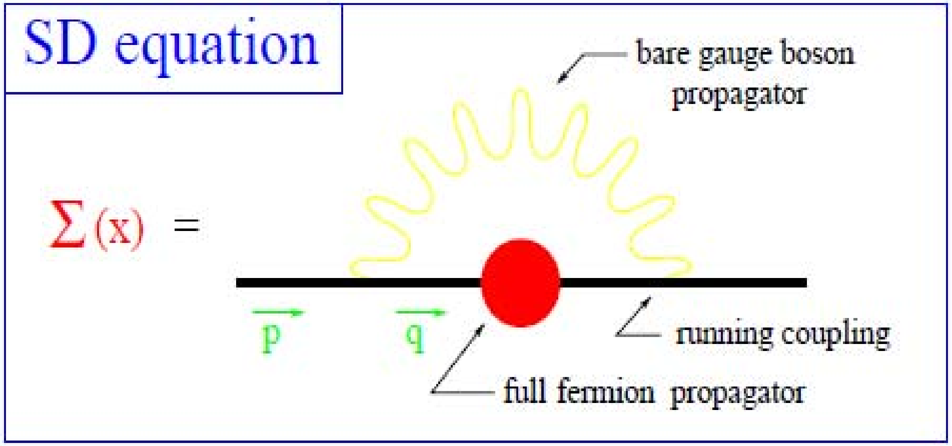
→ 0.16 (QCD)

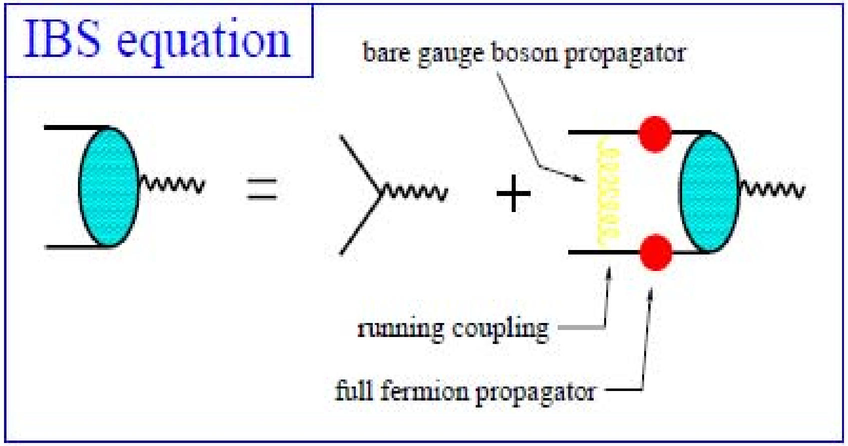
?

$S^{(exp)} < 0.1$

Straightforward Calculation

(Improved) Ladder SD & BS Equations





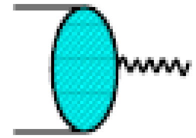
Works in Real-life QCD

Harada, Kurachi, KY (2006)

Procedure

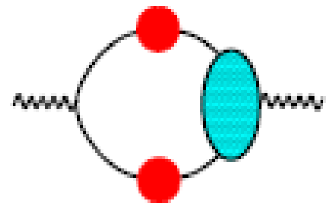
Harada, Kurachi, K.Y. (2004)

IBS & SD eqs. →



$\chi_{\alpha\beta}^{(J)}$

→



$\Pi_{JJ}(q_E^2)$

→ $\Pi_{VV}(q_E^2) - \Pi_{AA}(q_E^2)$

- Current-current correlator Π_{JJ} :

$$\delta^{ab}\left(\frac{q_\mu q_\nu}{q^2} - g_{\mu\nu}\right)\Pi_{JJ}(q^2) = i\int d^4x\, e^{iqx}\, \langle 0|T J_\mu^a(x) J_\nu^b(0)|0\rangle$$

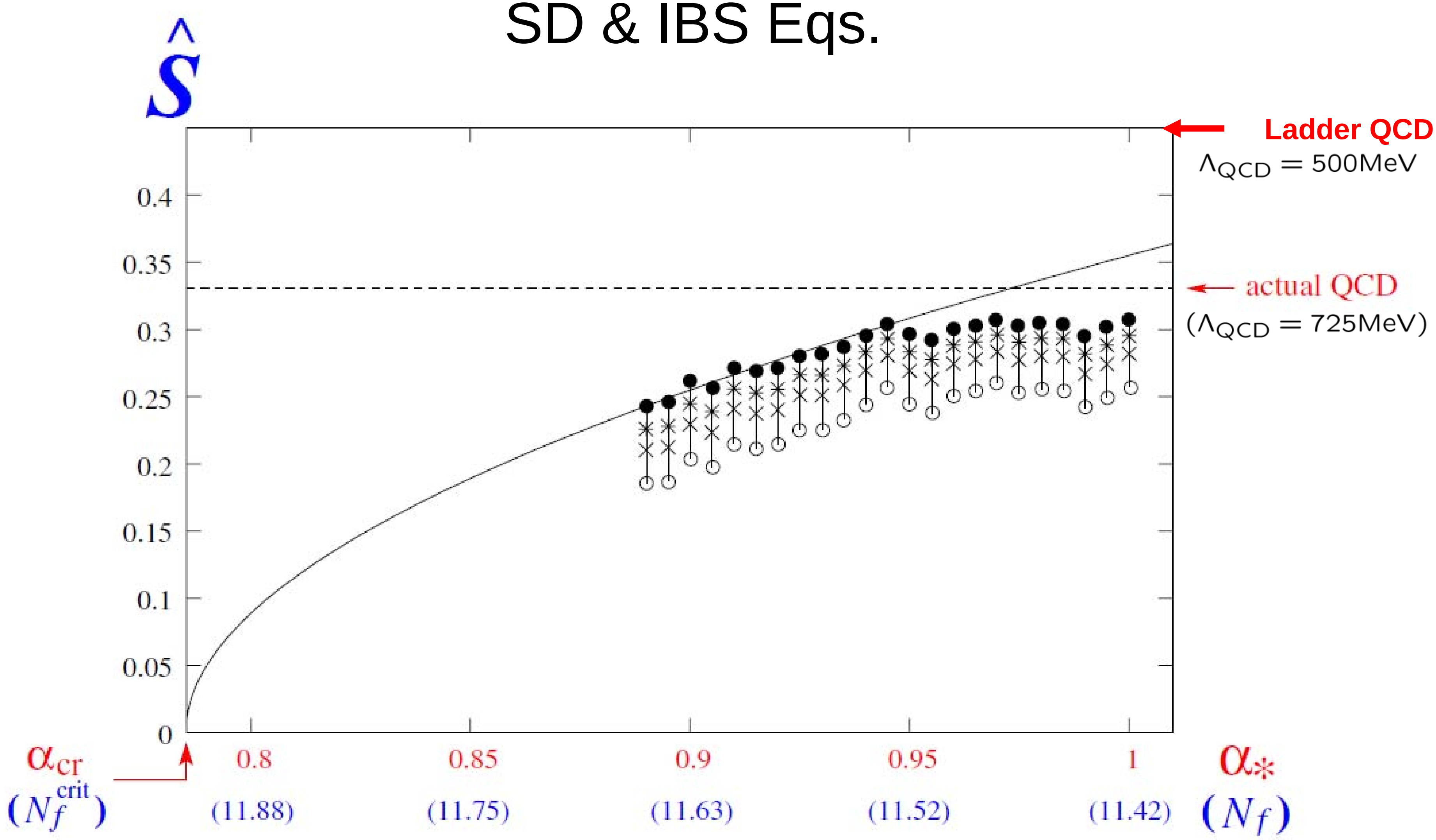
$$J_\mu^a(x) \ : \ \begin{cases} V_\mu^a(x) = \bar\psi(x)\,\frac{\lambda^a}{2}\,\gamma_\mu\psi(x), \\ A_\mu^a(x) = \bar\psi(x)\,\frac{\lambda^a}{2}\,\gamma_\mu\gamma_5\psi(x), \end{cases}$$

$$f_\pi^2 = \Pi_{VV}(0) - \Pi_{AA}(0)$$

$$\hat{S} = -4\pi \frac{d}{dQ^2}\left[\Pi_{VV}(Q^2) - \Pi_{AA}(Q^2)\right]\Big|_{Q^2=0}$$

$$\hat{S} = \frac{S}{N_f/2}$$

SD & IBS Eqs.



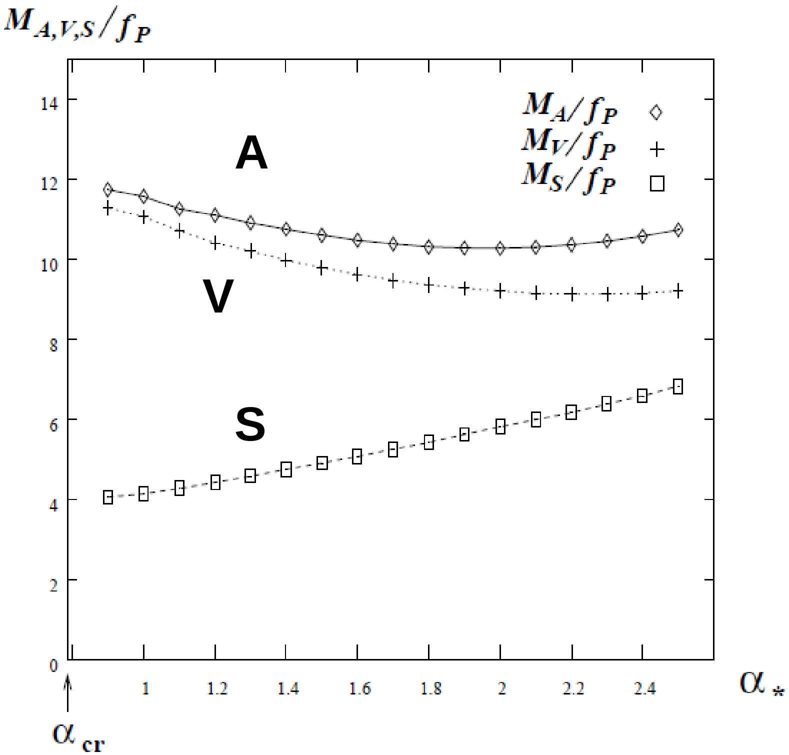
Ŝ
Ladder QCD
$\Lambda_{QCD} = 500$MeV
actual QCD
$(\Lambda_{QCD} = 725$MeV$)$
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
α_{cr}
(N_f^{crit})
0.8
0.85
0.9
0.95
1
(11.88)
(11.75)
(11.63)
(11.52)
(11.42)
α_*
(N_f)

Light Spectra (SD+HBS)

Harada-Kurachi-KY (2003)

$$f_\pi^2 : M_S^2 : M_V^2 : M_A^2 \simeq 1 : 17 : 121 : 132$$

$$m^2 : M_S^2 : M_V^2 : M_A^2 \simeq 1 : 2.4 : 17 : 18.5$$



$$\frac{(M_V/f_P)_{\rm WL}}{(M_\rho/f_\pi)} \simeq 1.3$$

$$\frac{(M_A/f_P)_{\rm WL}}{(M_{a_1}/f_\pi)} \simeq 0.86\,,$$

$$\frac{(M_S/f_P)_{\rm WL}}{(M_{a_0}/f_\pi)} \simeq 0.38\ .$$

Kurachi-Shrock (2006)

Ladder $M_{dilaton} \sim \sqrt{2}m$

Effective Field Theory

- **Light Composite Spectra**

SSB phase: π , ρ, a_1 , σ …

Bando-Kugo-Uehara-KY-Yanagida (1985)

Hidden Local Symmetry

$$M_\rho \sim 2\, m_q \longrightarrow 0$$

$$N_f \rightarrow N_f^{cr}$$

Effective Field Theory

- **Light Composite Spectra**

SSB phase: π, ρ, a_1, σ …

Bando-Kugo-Uehara-KY-Yanagida (1985)

Generalized Hidden Local Symmetry

Bando-Kugo-KY (1985)

$$M_\rho \sim 2\ m_q \xrightarrow[N_f \to N_f^{cr}]{} 0$$

Loop Effects (π , ρ) : proportional to N_f

M. Harada & KY (1999, 2001)

$f_\pi^{\ 2} \longrightarrow 0$

Quadratic div

Matched with OPE
proportional to Nc

$N_f \longrightarrow N_f^{cr} \approx 5\ (N_c/3)$

$N_f^{cr} \approx 12\ (N_c/3)$

Ladder SD

$$M_\rho^2 = a g^2 f_\pi^2 \to 0$$

$g \to 0$, $f_\pi^2 \to 0$

Vector Manifestation
Of Chiral Sym.

π , ρ: chiral partner

Dual ?

SU(N_c) QCD ⟺ SU(N_f) HLS

Completely Confined

Completely Higgsed

$$\hat{S} = 4\pi \left(\frac{F_\rho}{M_\rho}\right)^2 = 4\pi a \left(\frac{f_\pi}{M_\rho}\right)^2 = \frac{4\pi}{g^2}$$

$$a \simeq 2 \quad \text{(QCD)}$$

$$\longrightarrow \infty$$

$$g \sim \langle \bar{q} q \rangle \to 0$$

$$\hat{S} = 4\pi \left[\left(\frac{F_\rho}{M_\rho}\right)^2 - \left(\frac{F_{a_1}}{M_{a_1}}\right)^2 \right]$$

$$= \frac{4\pi}{g^2} \left(1 - \left(\frac{b}{b+c}\right)^2 \right) \xrightarrow[c/b \to 0?]{g \to 0} \textbf{?}$$

$$b \simeq c \simeq 2 \text{ (QCD)}$$

Higher resonances ?

Hidden Local Symmetry

Reviews: **M. Bando, T.Kugo, K.Y., Phys. Rep. 164('88) 217** **(tree)**
M. Harada, K.Y., Phys. Rep. 381('03) 1 **(loop)**

Bando-Kugo-Uehara-KY-Yanagida (1985)

$$G/H \underset{m_\rho \nearrow \infty}{\approx} G_{global} \times H_{local} \longleftarrow \rho$$

$\pi \nearrow G/H$

$$G_{global} \supset H_{global}$$

$$\approx G_{global} \times G_{local} \longleftarrow \rho,\ a_1$$

Bando – Kugo –KY (1986)

Bando – Fujiwara –KY (1988)

$$\approx G_{global} \times G_{local} \times H_{local} \longleftarrow \rho,\ a_1,\ \rho'$$

Bando – Kugo –KY (1988)

$$\approx G_{global} \times G_{local} \times G_{local} \times \cdots$$

Hidden Local Symmetry

Reviews: **M. Bando, T.Kugo, K.Y., Phys. Rep. 164('88) 217 (tree)**
M. Harada, K.Y., Phys. Rep. 381('03) 1 (loop)

Bando-Kugo-Uehara-KY-Yanagida (1985)

$$G/H \overset{m_\rho \uparrow \infty}{\approx} G_{global} \times H_{local} \longleftarrow \rho$$

π

$$\cup$$

$$H_{global} \rightarrow H_{global+local}$$

$$\approx G_{global} \times G_{local} \longleftarrow \rho\ ,\ a_1$$

Bando – Kugo –KY (1986)

Bando – Fujiwara –KY (1988)

$$\approx G_{global} \times G_{local} \times H_{local} \longleftarrow \rho\ ,\ a_1\ ,\ \rho'$$

Bando – Kugo –KY (1988)

$$\approx G_{global} \times G_{local} \times G_{local} \times \cdots$$

Hidden Local Symmetry

Reviews: **M. Bando, T.Kugo, K.Y., Phys. Rep. 164('88) 217 (tree)**
M. Harada, K.Y., Phys. Rep. 381('03) 1 (loop)

Bando-Kugo-Uehara-KY-Yanagida (1985)

$m_\rho \to \infty$

$G/H \simeq G_{global} \times H_{local} \leftarrow \rho$

π

$\cup$

$H_{global} \to H_{global+local}$

$\simeq G_{global} \times G_{local} \leftarrow \rho, a_1$ Bando – Kugo –KY (1986)

Bando – Fujiwara –KY (1988)

$\simeq G_{global} \times G_{local} \times H_{local} \leftarrow \rho, a_1, \rho'$

Bando – Kugo –KY (1988)

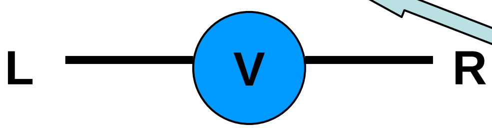
$\simeq G_{global} \times G_{local} \times G_{local} \times \cdots$

Heavy HLS Bosons Integrated out

$$U(x) = e^{i\frac{2\pi(x)}{f_\pi}} \quad \rightarrow g_L U(x) g_R^\dagger$$

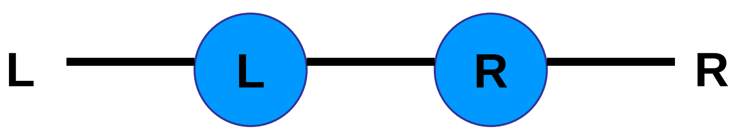
$$= \xi_L^\dagger(x) \cdot \xi_R(x)$$

$$\xi_{L,R}(x) \rightarrow \xi'_{L,R}(x) = h(x)\xi_{L,R}(x) g^\dagger_{L,R}$$

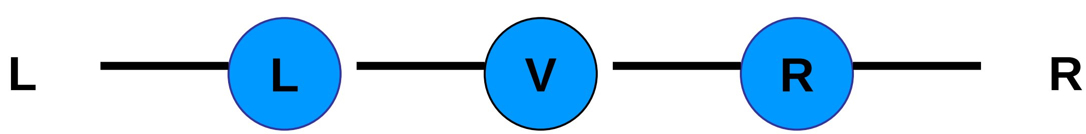


arbitrariness=gauge symmetry

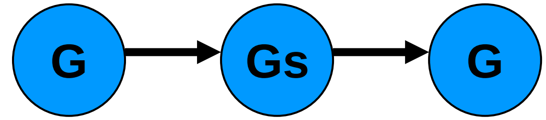
$$= \xi_L^\dagger(x) \cdot \xi_M(x) \cdot \xi_R(x)$$



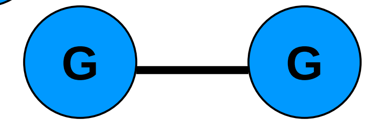
$$= \xi_{L_1}^\dagger(x) \cdot \xi_{L_2}^\dagger(x) \cdot \xi_{R_2}(x) \cdot \xi_{R_1}(x)$$

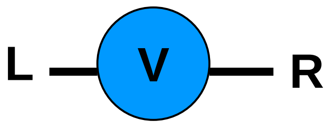


Moose (Georgi 1986)

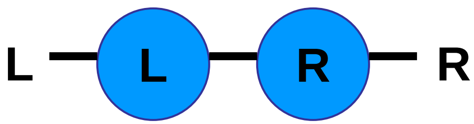


Condensed Moose (Arkani-Hamed-Cohen-Georgi 2001)





G_{global} x H_{local} "3-site model"



G_{global} x G_{local} "5-site model"

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Deconstructed/Latticized 5-dimensional Gauge Field ⟺ **HLS**

↑

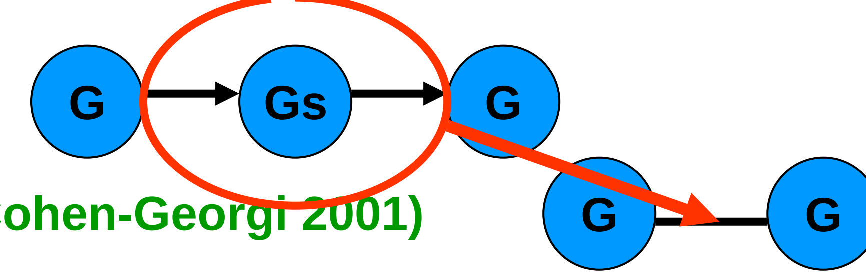
Arkani-Hamed-Cohen-Georgi(2001)

Cheng-Hill-Pokorski-Wang (2001)

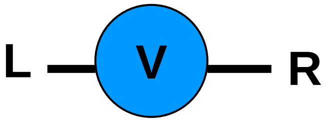
AdS/QCD, Holographic QCD

Higgsless Model, Little Higgs

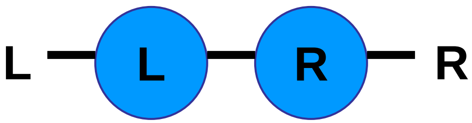
Moose (Georgi 1986)



Condensed Moose (Arkani-Hamed-Cohen-Georgi 2001)



G_{global} x H_{local} "3-site model"



G_{global} x G_{local} "5-site model"

.
.
.

Deconstructed/Latticized 5-dimensional Gauge Field ⇔ **HLS**

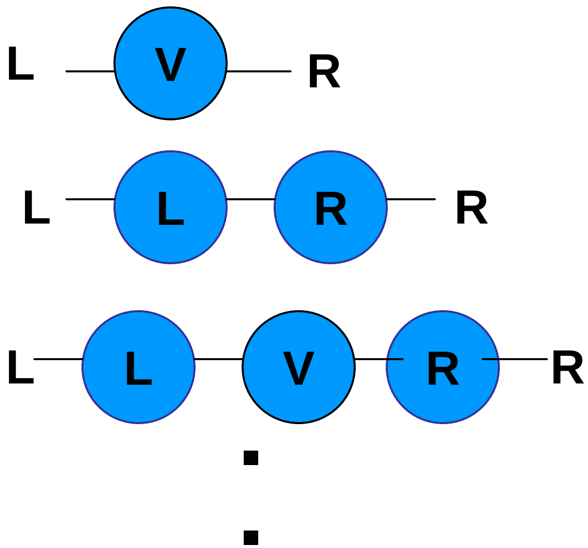
Arkani-Hamed-Cohen-Georgi(2001)

Cheng-Hill-Pokorski-Wang (2001)

AdS/QCD, Holographic QCD
Higgsless Model, Little Higgs

Infinite Sequence of Linear Moose

Son-Stephanov(2004)



G_{global} x H_{local} "3-site model" ρ

G_{global} x G_{local} "4-site model" ρ, a_1

G_{global} x G_{local} x H_{local} "5-site model"

ρ, a_1, ρ'

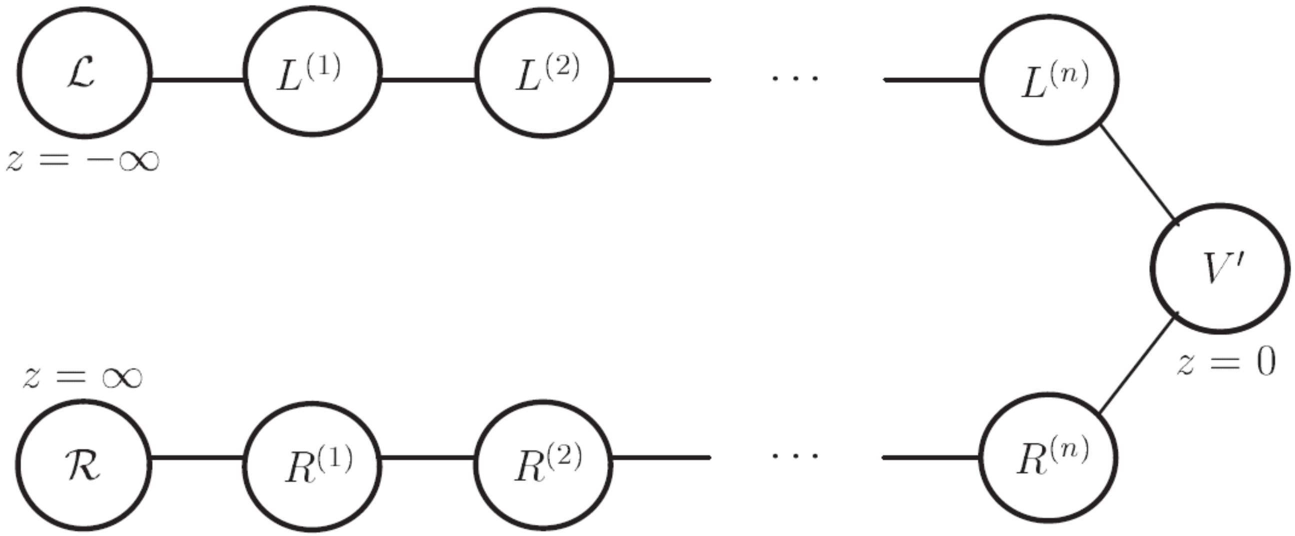
Deconstructed/Latticized
5-dimensional Flavor Gauge Field

Bottom-up Approach

AdS/QCD, Holographic QCD

Infinite Sequence of Linear Moose

Son-Stephanov(2004)
Sakai-Sugimoto(2005)

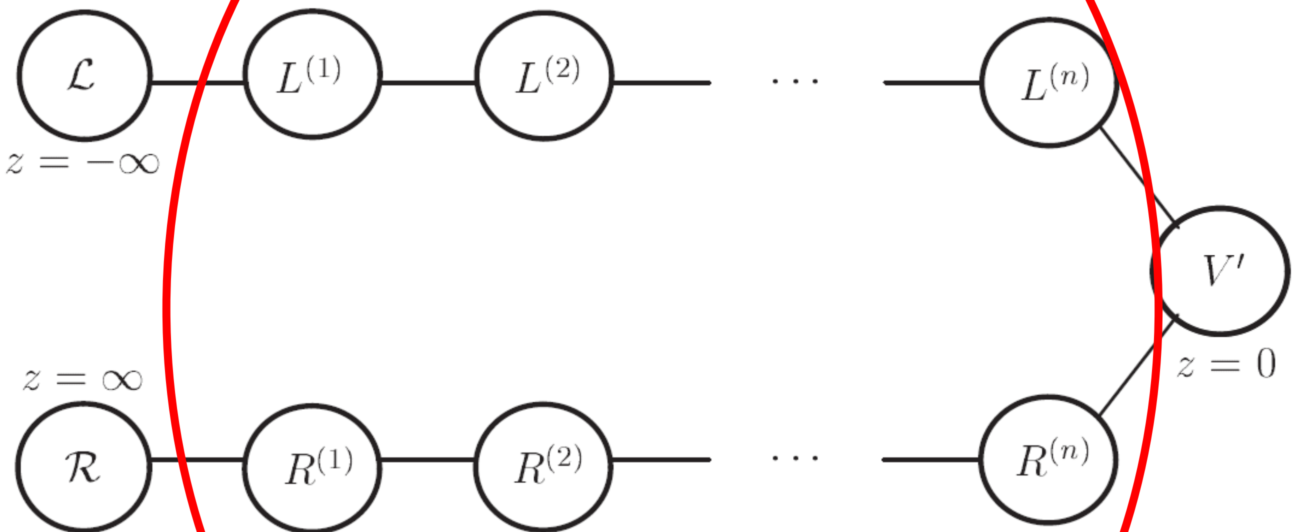


$$L^{(i)} = V^{(i)} - A^{(i)},\ R^{(i)} = V^{(i)} + A^{(i)}$$

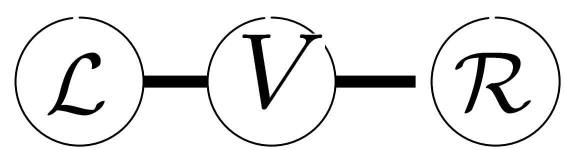
5D Gauge Theory for Flavor Symmetry

Infinite Sequence of Linear Moose

Son-Stephanov(2004)
Sakai-Sugimoto(2005)



$$L^{(i)} = V^{(i)} - A^{(i)},\ R^{(i)} = V^{(i)} + A^{(i)}$$



$$+\mathcal{O}(p^4)$$

Integrating out via Eq. Mot.

Harada-Matsuzaki-KY (2006)

Holographic Walking/Conformal TC

Holographic correspondence

J. M. Maldacena, (1998)

4D strongly coupled theory in large Nc limit ⟷ **5D weakly coupled theory at tree level**

correspondence

= QCD,TC

Arkani-Hamed, et al (2001)

= {HLS}

- All 5D bulk fields $\chi(x,z)$ couple to 4D operator $\mathcal{O}(x)$ at UV boundary:

$$\text{Operator: } \mathcal{O}(x) \longleftrightarrow \text{Field: } \chi(x,z)$$

- 5D effective action corresponds to generating functional of $\mathcal{O}(x)$
  (which obtained from integrating out 5-th direction “z” after substituting $\chi^{cl}(x,z)$ into 5D action. ($\chi^{cl}(x,z)$: classical solution of χ))

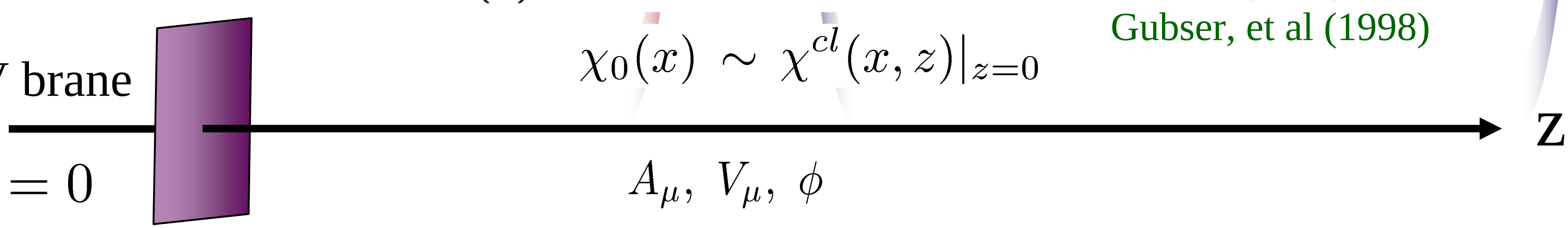
$$\langle e^{\int d^4x \chi_0(x)\mathcal{O}(x)} \rangle = e^{S_{\rm eff}[\chi^{cl}(x,z=0)]}$$

Then UV boundary value of $\chi^{cl}(x,z)$ corresponds to source of $\mathcal{O}(x)$.

Witten, (1998), Gubser, et al (1998)

$$\chi_0(x) \sim \chi^{cl}(x,z)|_{z=0}$$

UV brane



➢ Bottom-Up Holographic QCD

Erlich, Katz, Son and Stephanov (2005), Da Rold and Pomarol, (2005)

Works within 30%

4D Strongly Coupled (Color) Theory

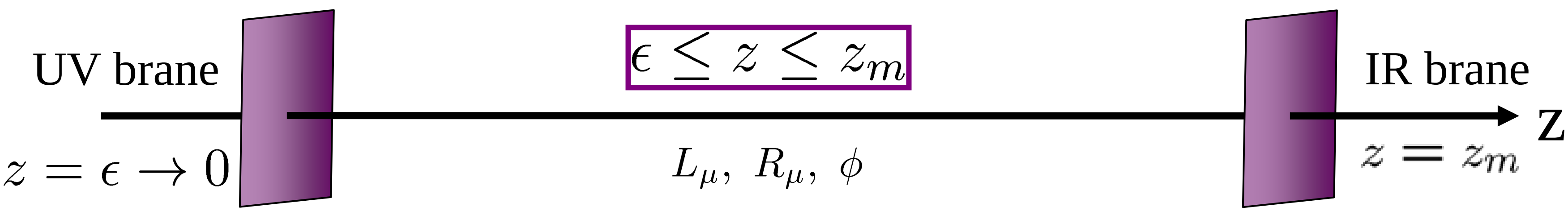
- $SU(N_f)_L \times SU(N_f)_R$ global symmetry
- 4D operators
  L-, R-current j^L_μ, j^R_μ, chiral operator $\bar{q}q$

5D Weakly Coupled (Flavor) Theory

- Choose AdS5 metric :

$$ds^2 = a^2(z)\left(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2\right), \quad a(z) = \frac{L}{z}$$

- Compactify 5th direction "z" from ε to z_m:



- $SU(N_f)_L \times SU(N_f)_R$ gauge symmetry
- Bulk fields coupling to 4D operators, respectively :
  L-, R-gauge field L_μ, R_μ , scalar field ϕ

$$\begin{pmatrix} L_\mu \\ R_\mu \\ \phi \end{pmatrix} \Leftrightarrow \begin{pmatrix} j^L_\mu \\ j^R_\mu \\ \bar{q}q \end{pmatrix}$$

$$m_5^2 = \frac{\Delta(\Delta-4)}{L^2}$$

$$\Delta = 3 - \gamma_m$$

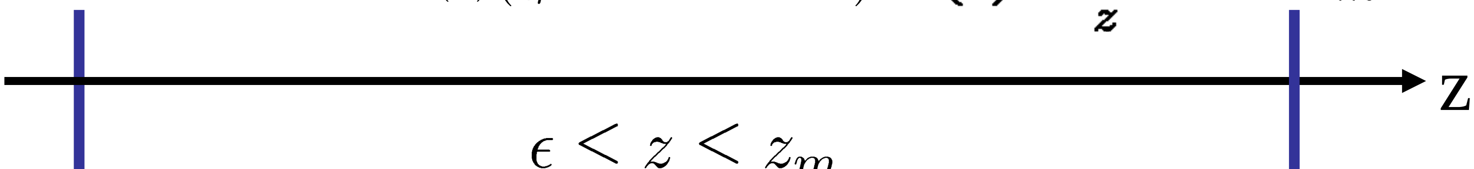
•Calculation method

$$S_5 = \int d^5x\sqrt{g}\,\frac{1}{g_5^2}Tr\left[-\frac{1}{4}F_{L,R}^2 + \frac{1}{2}|D_M\Phi|^2 - \frac{1}{2}m_5^2|\Phi|^2\right]$$

UV brane $z = \epsilon \to 0$

$ds^2 = a^2(z)\left(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2\right)$ $\quad a(z) = \frac{L}{z}$

IR brane $z = z_m$



$$\epsilon \leq z \leq z_m$$

UVBC

$$A_\mu\Big|_\epsilon \equiv a_\mu$$

$$V_\mu\Big|_\epsilon \equiv v_\mu$$

$$\left(\frac{L}{\epsilon}\right)^{1+\gamma_m} S_0\Big|_\epsilon \equiv M$$

✓Solve EOM in 5D theory at tree level.

✓Substitute the solution into 5D action.

AdS/CFT

$$\langle e^{\int d^4x\,\chi_0(x)\mathcal{O}(x)}\rangle_{\text{conn}} = S_{\text{eff}}[\chi_0^{cl}]$$

$$\chi_0(x) \sim \chi^{cl}(x,\epsilon)|_{\epsilon\to 0}$$

IRBC

$$\partial_z A_\mu\Big|_{z_m} = 0$$

$$\partial_z V_\mu\Big|_{z_m} = 0$$

$$LS_0\Big|_{z_m} \equiv \xi$$

$$\longrightarrow \quad S_{eff}[V^{cl}, A^{cl}, S_0^{cl}] \sim \int_x \text{Tr}\left[\,v^\mu \Pi_V v_\mu + a^\mu \Pi_A a_\mu + M\bar{q}q\,\right]$$

➢ Holographic W/C TC

Hong, and Yee (2006)   M.Piai, (2006)

- The action is given :

$$S_5 = \int d^5 x \sqrt{g}\, \frac{1}{g_5^2} Tr\left[-\frac{1}{4} F_{L,R}^2 + \frac{1}{2} |D_M \Phi|^2 - \frac{1}{2} m_5^2 |\Phi|^2 \right]$$

$$D_M \Phi = \partial_M \Phi + i L_M \Phi - i \Phi R_M\,, \quad \mathrm{Tr}[T^a T^b] = \delta_{ab},$$

- Bulk scalar mass m_5 is related to γ_m:

Conformal dimension of $\langle \bar{T} T \rangle$: $\boxed{\Delta = 3 - \gamma_m}$

$$m_5^2 = \frac{\Delta(\Delta - 4)}{L^2} \xrightarrow[\gamma_m \sim 1]{\text{W/C TC}} \sim -\frac{2}{L^2}$$

$\gamma_m \simeq 0$ for QCD

• **S parameter**

$$f_\pi^2 = -\Pi_A(0)$$

$$\widehat{S} = -4\pi\left(\Pi'_V(0) - \Pi'_A(0)\right)$$

From HWTC, we find

$$\Pi'_V(0) \sim \frac{L}{g_5^2}\log\left(\frac{z_m}{\epsilon}\right) \quad \Pi'_A(0) = \frac{L}{g_5^2}\int_\epsilon^{z_m}\frac{dz'}{z'}\left(\frac{z'}{\epsilon}K_{1/\Delta}\left(\frac{\sqrt{2}\xi}{3z_m^2}z^2\right)\right)^2$$

$$\xrightarrow[g_5^2/L\,=\,12\pi^2/N_c]{\epsilon\to 0} \quad \hat{S}_{HWTC} = \hat{S}(z_m,\xi) \qquad \text{c.f. } S \equiv \frac{N_c}{3}\cdot\frac{N_f}{2}\cdot\hat{S}$$

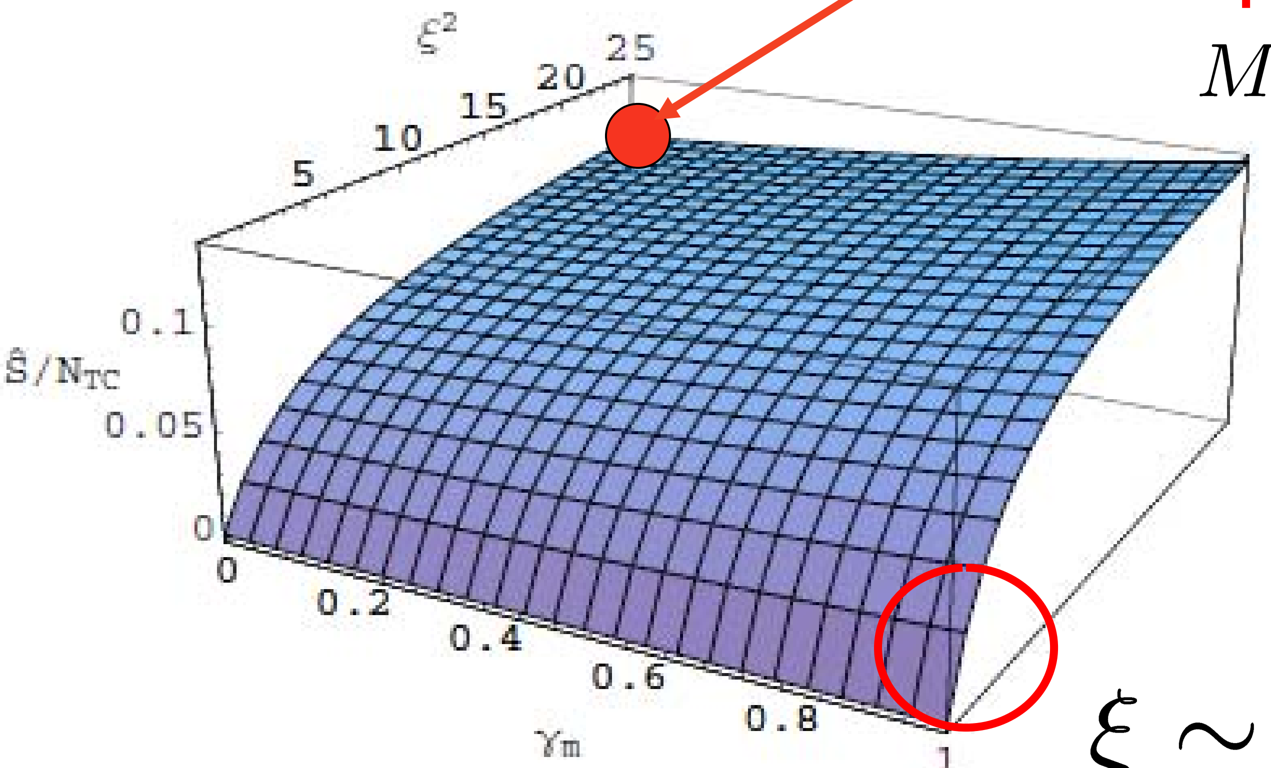
2 input parameter: $z_m, \;\; \xi$

- z_m: IR cutoff ⟶ fit by $M_\rho \simeq 2.4/z_m$
- $\xi \equiv LS_0\Big|_{z_m}$ ⟶ (AdS/CFT) $\xi = \dfrac{2}{(3-\gamma_m)}\dfrac{L}{g_5^2}z_m^3\langle\bar{q}q\rangle_{z_m^{-1}}$ ← $\dfrac{f_\pi}{M_\rho}$

K. Haba, S. Matsuzaki , K.Y. (2008)

QCD $(\xi^{(\mathrm{exp})} \simeq 4.8)$

M_ρ, f_π

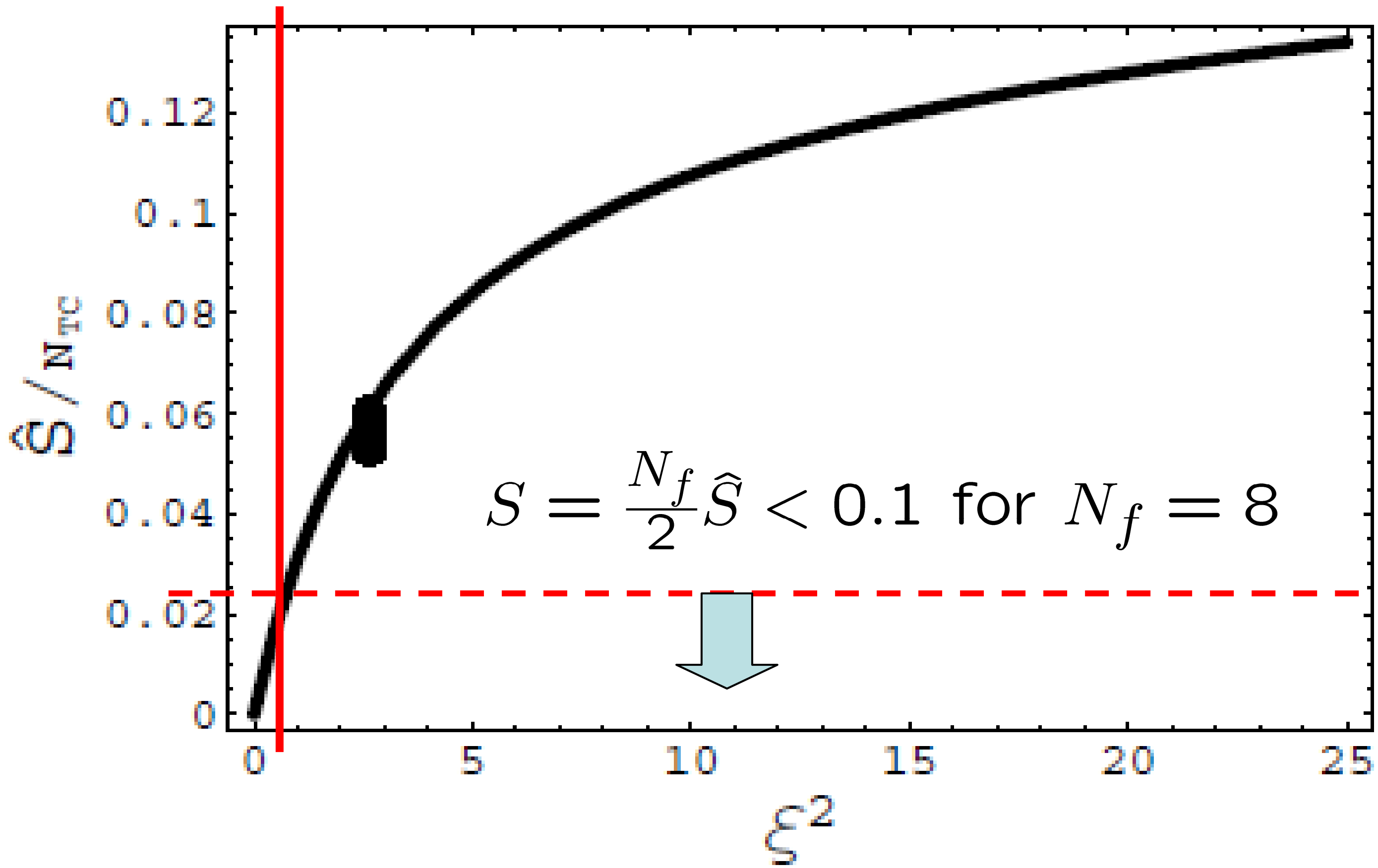


$$\xi \sim \frac{f_\pi}{M_\rho}$$

Insensitive to γ_m

Free parameter !

$\boxed{\gamma_m \simeq 1}$

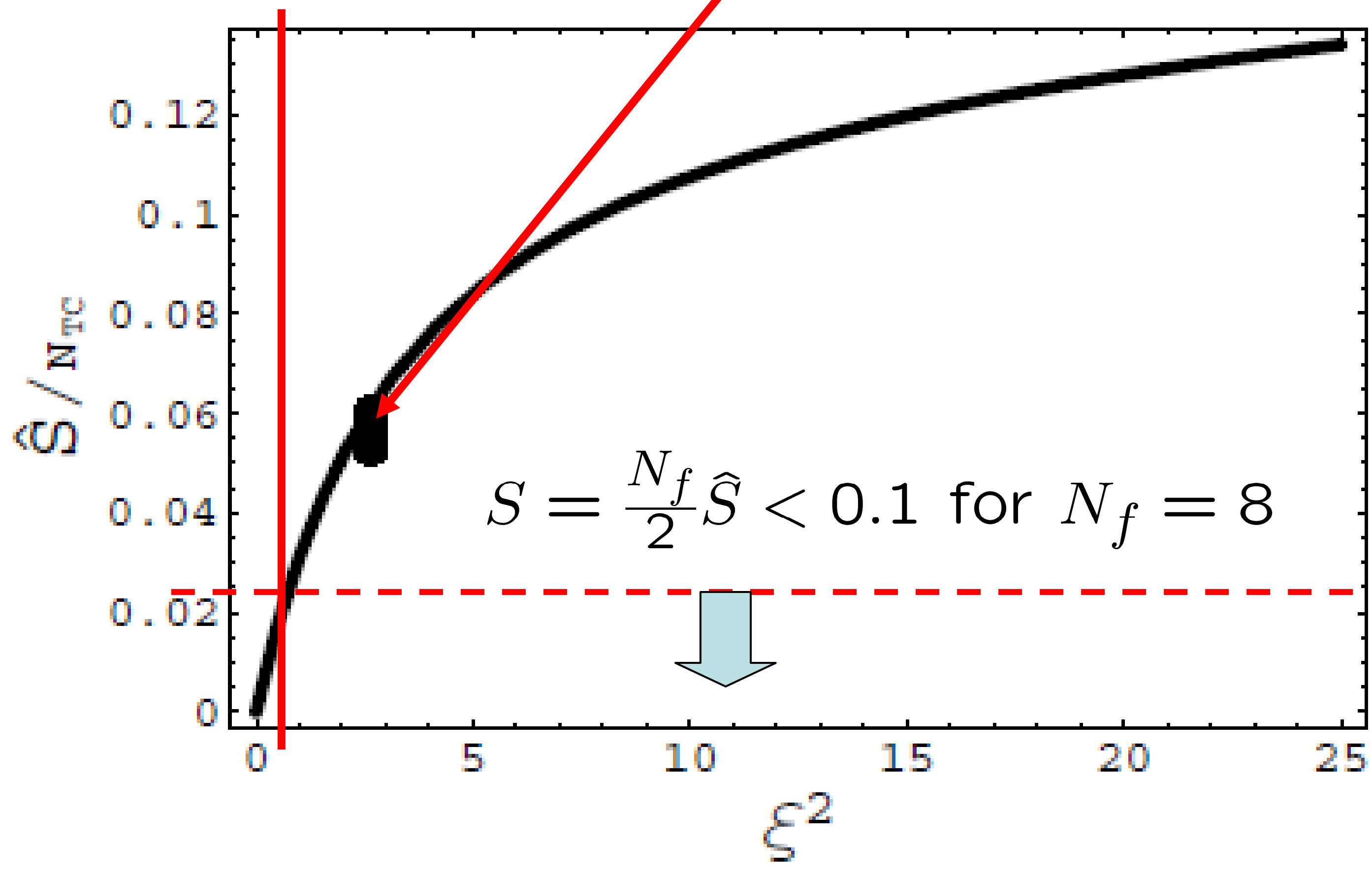


$$\xi < 0.59 \quad \left(\frac{f_\pi}{M_\rho} < 0.032,\ M_\rho > 3.9\,\text{TeV}\right)$$

$\gamma_m \simeq 1$

Ladder & SD HBS (M_ρ, f_π) & SD+IBS (S)

Harada, Kurachi, K.Y. (2003, 2006)



$$\xi < 0.59 \quad \left(\frac{f_\pi}{M_\rho} < 0.032,\ M_\rho > 3.9\,\text{TeV}\right)$$

Holographic W/C TC

- S ~ (f_π**/M**$_\rho$**)**$^{\mathbf{2}}$ fairly independent of γ_m even including infinite tower of vector and axialvector mesons
- Holography alone does not predict $\mathbf{f}_\pi$**/M**$_\rho$. S Needs more explicit dynamics

Various Issues

1. Existence of IR fixed point

2. Determination of N_f^{cr}

3. Light spectrum

$$M_\sigma\,, M_\rho, M_{a_1} \quad : vs. F_\pi$$

dilaton

4. S Parameter $\frac{F_\pi}{M_\rho}$

Various Issues

1. Existence of IR fixed point

2. Determination of N_f^{cr}

3. Light spectrum

$M_\sigma, M_\rho, M_{a_1}$ $: vs. F_\pi$

dilaton

4. S Parameter $\frac{F_\pi}{M_\rho}$

Lattice

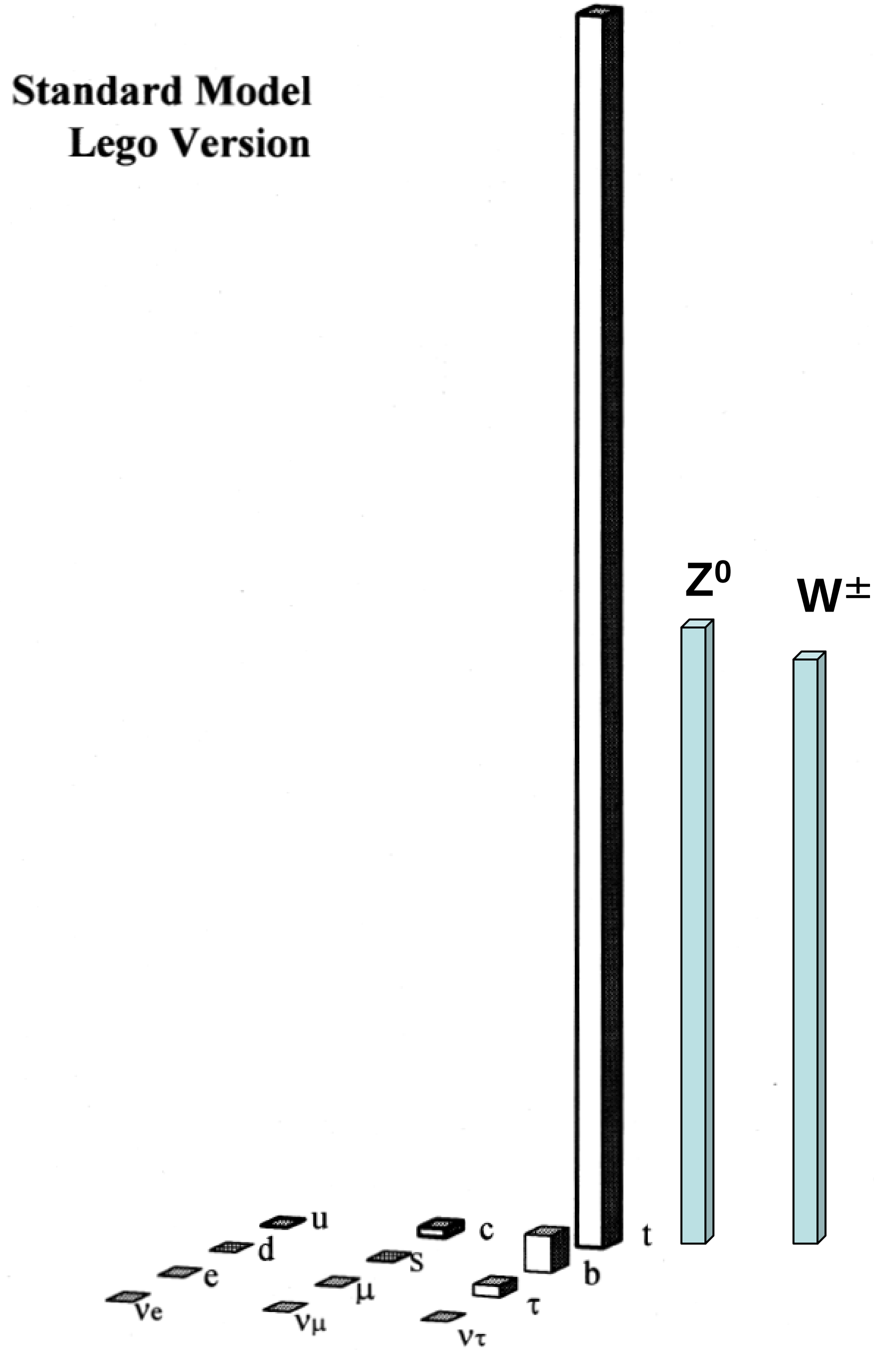
Appelquist et al, Sannino et al, Lombardo et al, Onogi et al (Hayakawa et al), ……

5. Top quark mass ?

$m_t \gg m_b$ ⟺ (?) $\langle \bar{U}U \rangle \simeq \langle \bar{D}D \rangle$

T parameter

Top quark is special



Standard Model
Lego Version
Z0
W±
u
d
e
νe
c
s
μ
νμ
b
τ
ντ
t

Top Quark Condensate
(Top Mode Standard Model)

Miransky,Tanabashi and K.Y. (1989)
Nambu(1989)
Bardeen, Hill and Lindner (1990)

$$H \sim \bar{t}t$$

☆ Explicit Model (gauged NJL model)

Miransky,Tanabashi & K.Y. (1989)
Bardeen, Hill & Lindner (1990)

$$\mathcal{L}_{\rm TMSM} = \mathcal{L}_{\rm SM} \quad -\mathcal{L}_{\rm Higgs} \quad +\mathcal{L}_{4F}$$

$$\begin{aligned}\mathcal{L}_{4F} &= G_t(\bar{\psi}_L^i t_R)(\bar{t}_R \psi_{Li}) + G_b(\bar{\psi}_L^i b_R)(\bar{b}_R \psi_{Li}) \\ &+ G_{tb}\left(\epsilon^{ik}\epsilon_{jl}\bar{\psi}_L^i \psi_{Rj}\bar{\psi}_L^k \psi_{Rl}\right) + h.c.\end{aligned}$$

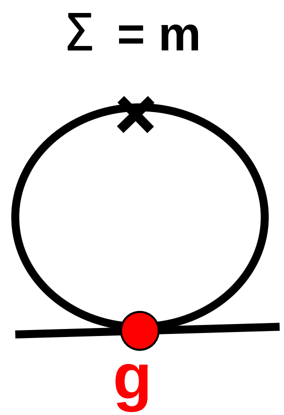
G_{tb}: t ⟷ b ~~$U(1)_A$~~ ⟹ m_b

$$\mathcal{L}_{\mathsf{NJL}} = \bar\psi i\partial\!\!\!/\psi + \tfrac{1}{2}G\left[\left(\bar\psi\psi\right)^2 + \left(\bar\psi i\gamma_5\tau^a\psi\right)^2\right]$$

$$G\langle\bar\psi\psi\rangle\bar\psi\psi$$

$$-m$$

$$\mathbf{m} = \Sigma\,(p\!\!\!/) =$$



$$G \equiv g\,\frac{4\pi^2}{N_c\Lambda^2}$$

$$m = -G\,\langle\bar\psi\psi\rangle = G\,TrS(p) = 4G\,N_c\int\frac{d^4p}{(2\pi)^4 i}\frac{m}{m^2-p^2}$$

$$= m\cdot\frac{G\,N_c}{4\pi^2}\left(\Lambda^2 - m^2\ln\frac{\Lambda^2}{m^2}\right) = m\cdot g\left(1-\left(\frac{m}{\Lambda}\right)^2\ln\frac{\Lambda^2}{m^2}\right)$$

g

Gap Eq.

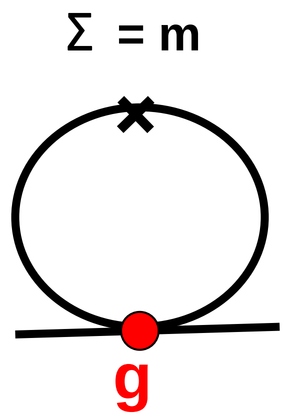
$$\mathcal{L}_{\mathrm{NJL}} = \bar{\psi} i \partial\!\!\!/ \psi + \frac{1}{2} G \left[\left(\bar{\psi}\psi \right)^2 + \left(\bar{\psi} i \gamma_5 \tau^a \psi \right)^2 \right]$$

$$G \langle \bar{\psi}\psi \rangle \bar{\psi}\psi$$

$$-m$$

$$G \equiv g \, \frac{4\pi^2}{N_c \Lambda^2}$$

$$\mathbf{m} = \Sigma \, (\not{p}) =$$



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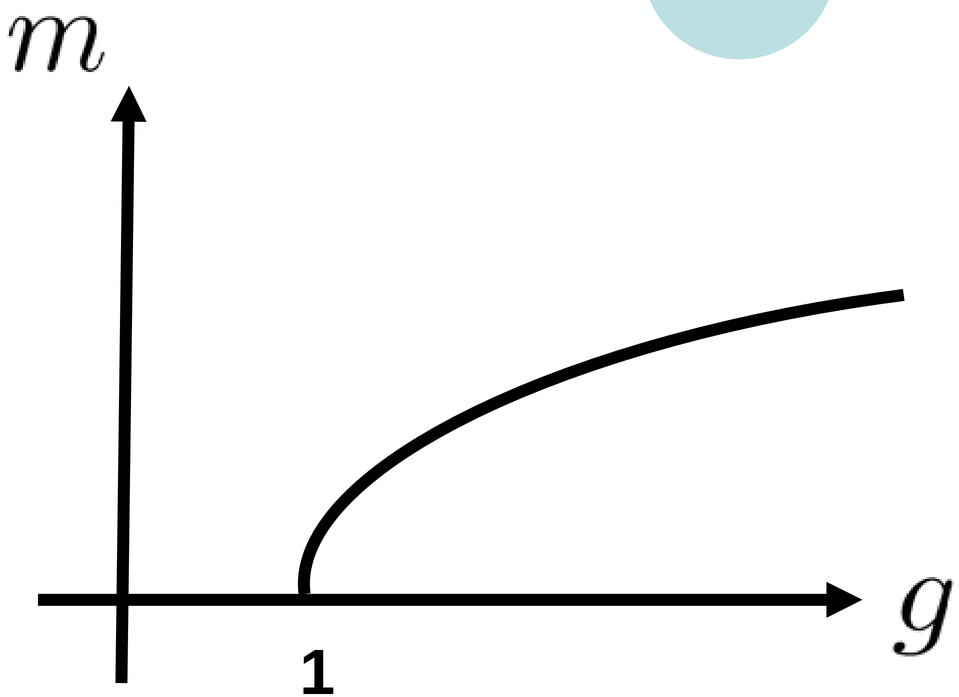
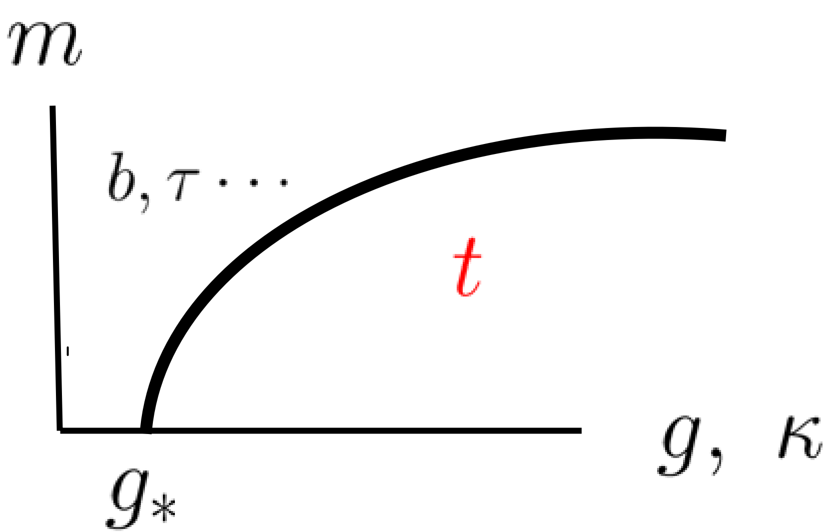
g **Gap Eq.** **m≠0** solution

$$\ln\frac{\Lambda^2}{m^2}\cdot\left(\frac{m}{\Lambda}\right)^2 \simeq \frac{1}{g^*}-\frac{1}{g}\,,$$

1



$g^* = g_{\mathrm{cr}} = 1$



given

- $m_t \gg m_b, m_\tau, \cdots$
- $m_t = m_t(\Lambda, g, \kappa) \sim \mathcal{O}(\langle H \rangle)$

$$\langle H \rangle = F_\pi = F_\pi(m_t, \Lambda, g, \kappa)$$

Decay const. of composite NG boson

(Pagels-Stokar formula)

$$\phi^i \sim G \cdot \bar{t}_R \psi_L^i$$

$$F_\pi^2 = \frac{N_c}{8\pi^2} m_t^2 \ln \frac{\Lambda^2}{m_t^2}$$

Pagels-Stokar formula

$$m_t = 172\,\text{GeV} \Longleftrightarrow \Lambda \sim 10^{13}\text{GeV}$$

$$(m_t \sim 600\,\text{GeV} \Longleftrightarrow \Lambda \sim \mathcal{O}(\text{TeV}))$$

1/Nc leading (QCD corrections) **Miransky,Tanabashi and K.Y. (1989)**

$$m_t > 250\,\text{GeV} \Longleftrightarrow \Lambda < 10^{19}\,\text{GeV}$$

Incl. subleading (QCD,electroweak gauge + composite higgs loop)

$$m_t > 220\,\text{GeV}$$

Bardeen, Hill & Lindner (1990)

Various Issues

- **Too Large m_t**

  Strong ETC TC (With / Without Top Condensate)

  Topcolor-Assisted TC (TC&Top Condensates)

  Top Seesaw (Extra Vector-Like Top)

  SM with Extra Dimension (Top KK modes)

- **Origin of Four-fermion Coupling**

  Topcolor

  SM with Extra Dimension (Gluon KK modes)

  Strong Yukawa

- **Other q / l Masses**

  From Top Condensate

  From TC Condensate

Probing Composite Higgs in LHC

- Walking/Conformal TC : Techni-dilaton

  $\sqrt{2}M_{\mathsf{F}}(500 - 600\mathrm{GeV})$

- Top Quark Condensate : Top-sigma

  $< 2m_t(200 - 300\mathrm{GeV})$

- Topcolor-Assisted TC (TC2):

  Top-pion/Top-sigma

- Top Seesaw: Top-sigma $\mathcal{O}(\mathrm{TeV})$
- Top-mode SM with Extra Dim. (D=6,8)

  Bulk Composite Scalar Zero Mode

  $\sim 180\,\mathrm{GeV}$

Search for Higgs

- **Present Lower Limit （LEP)**

$$m_H > 114 \ \mathrm{GeV/c^2}$$

- **LHC （2008~ ）:**

Could be searched for $m_H < 1 \ \mathrm{TeV/c^2}$

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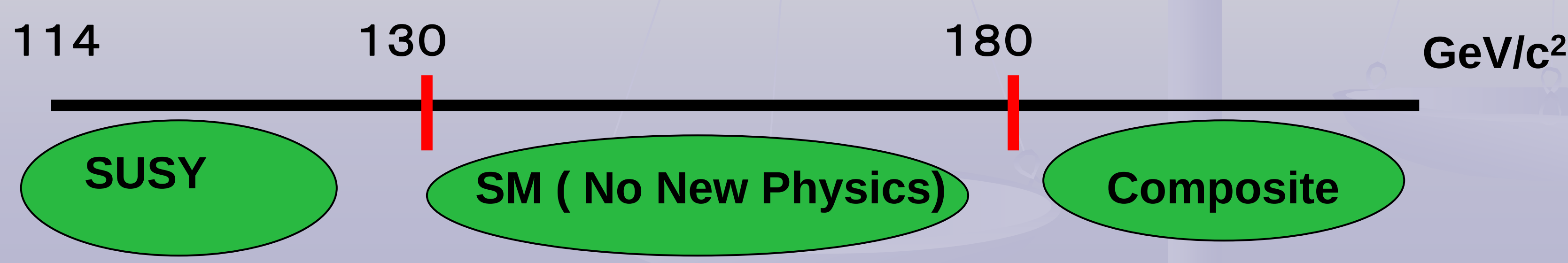
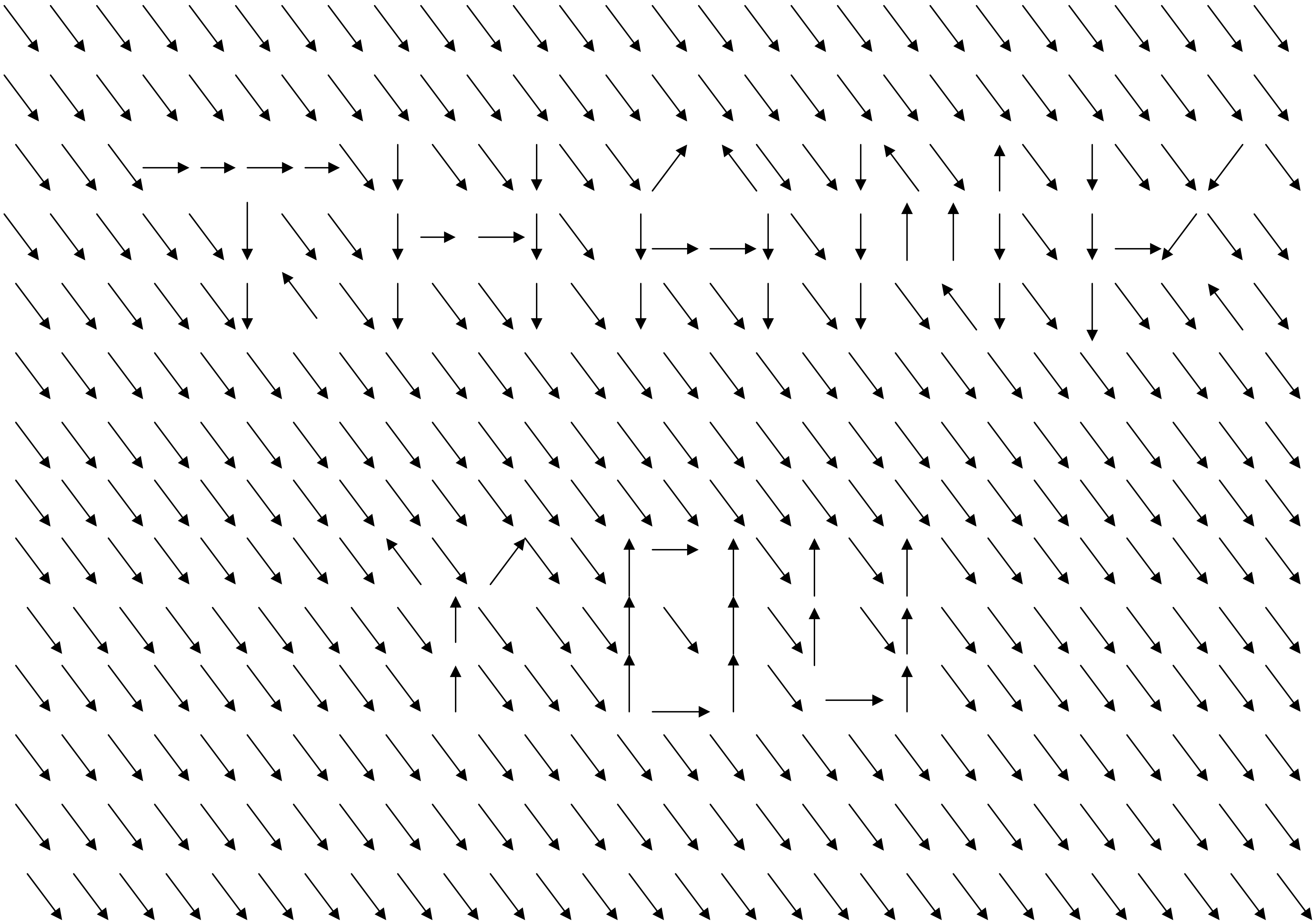


Come Back again?



Prof. Nambu’s reply to my congraturations