FRW/CFT duality: Holographic formulation of eternal inflation and its applications

Yasuhiro Sekino

（Okayama Institute for Quantum Physics)

Collaboration w/ B. Freivogel (UC Berkeley),

L. Susskind (Stanford), C.-P. Yeh (Natl. Taiwan U);

hep-th/0606204, 0908.3844[hep-th], work in progress

Outline

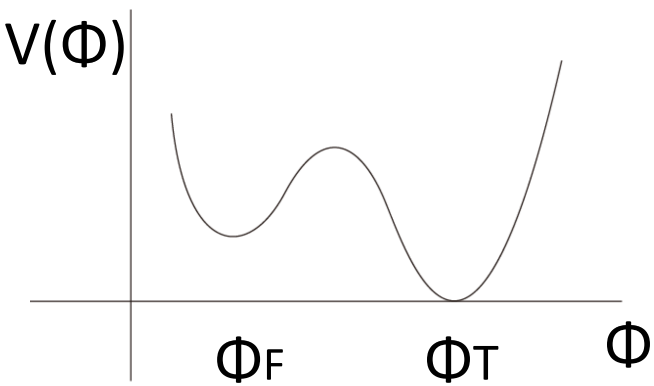
- Holographic dual description for eternal inflation
  (for an universe created by bubble nucleation)
- Observational consequences of bubble nucleation

Plan of the talk

- Bubble nucleation in eternal inflation
- The conjecture of holographic duality
- Correlation functions
- Interpretation in the dual theory
- General features of the CMB spectrum

A simplified model for the landscape

- A scalar field coupled with gravity

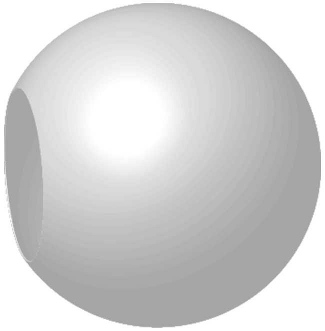


False ("ancestor") vacuum:
positive c.c. (de Sitter)
Hubble parameter H_A
True vacuum: zero c.c.
(3+1) D spacetime

- Holographic dual formulation will give support for the existence of string landscape.
  - We can give a meaning to the meta-stable vacua beyond the low-energy approximation.

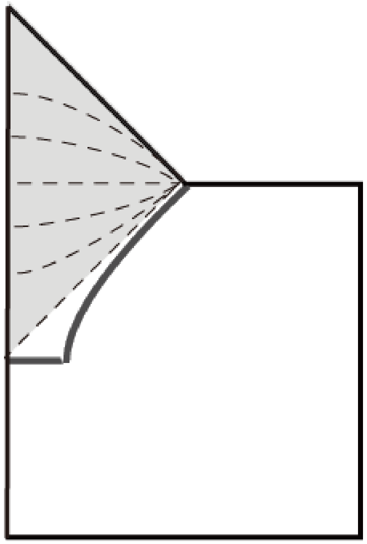
Decay of false vacuum (CDL instanton)

- False vacuum decays through bubble nucleation.
- Described by Coleman-De Luccia (CDL) instanton
  - Euclidean ("bounce") solution with SO(4) symmetry.
  - Euclidean Geometry: $(-\infty \leq X \leq \infty)$



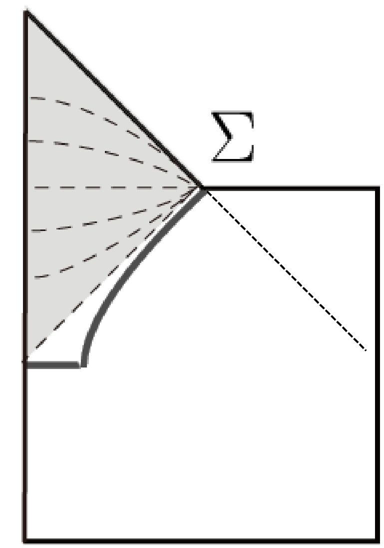
$$ds_E^2 = a^2(X)\left(dX^2 + d\theta^2 + \sin^2\theta d\Omega_2^2\right)$$

$$a(X) = \tilde{H}_A^{-1} e^X \quad \text{(flat)}, \qquad a(X) = \frac{H_A^{-1}}{\cosh X} \quad \text{(de Sitter)}$$



- Evolution after nucleation:
  - The spacetime has SO(3,1) sym.

Open FRW universe in the bubble



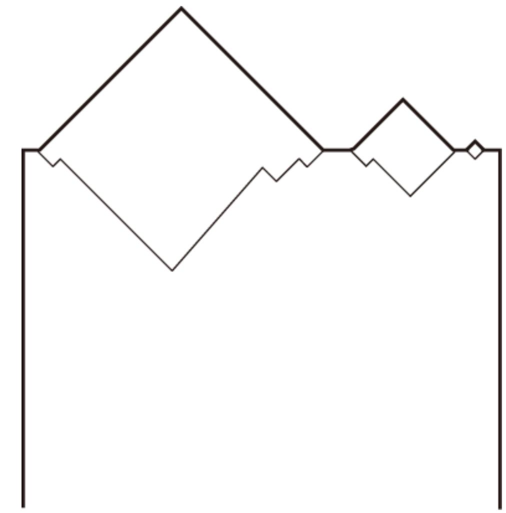
- FRW universe: shaded region

$$ds^2 = a^2(T)(-dT^2 + dR^2 + \sinh^2 R d\Omega_2^2)$$

  - Constant time slices (dashed lines): H^3 (isometry: SO(3,1))
  - Boundary (S^2 at $R \to \infty$): Σ
  - The beginning of the FRW time $T \to -\infty$ is smooth (locally flat) : $a(T) = H_A^{-1} e^T, \qquad (a(t) \sim t)$

- A single observer ("census taker") can see the whole FRW universe.
  - We expect consistent quantum theory exists.

Eternal Inflation

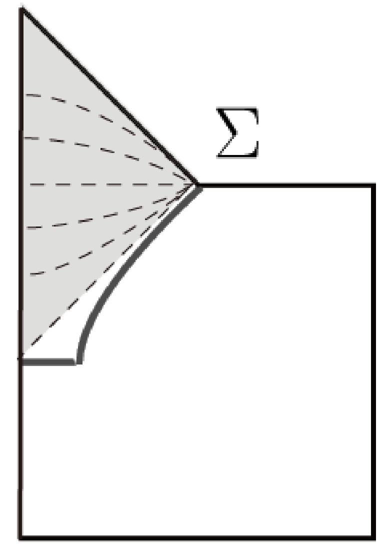


- If nucleation rate is small $\Gamma \lesssim H_A^4$, true vacuum does not "percolate". Physical volume continues to be dominated by false vacuum.

- Infinite number of bubble collisions occur eventually. (Guth-Weinberg '83)

- Interior geometry after the collision depends on the type of the bubbles. E.g. collision with a bubble of the same vacuum: space is smoothed out (one time-like infinity).

Holographic duality



<u>The conjecture:</u>

The dual theory is a CFT on S^2

(at the boundary Σ)

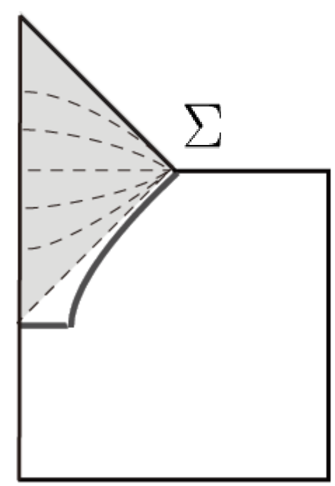
- SO(3,1): conformal sym in 2D.
  (as in AdS/CFT)
- <u>The dual has 2 less dimensions than the bulk</u>.
- The dual theory contains gravity (Liouville field).
  - Gravity is not decoupled at the boundary
    (due to the compactness of the Euclidean space)

Dual theory is 2D gravity coupled to matter with c>25

Holographic duality

- In the semi-classical regime, we can identify Liouville field as time (as in the Wheeler-DeWitt theory).
- One field in the bulk corresponds to an infinite tower of the CFT operators.

- Different from dS/CFT correspondence
  - Defined on the region accessible to a single observer.
  - In dS/CFT, it is not clear how to think about terminal vacua.



Correlation functions in open FRW

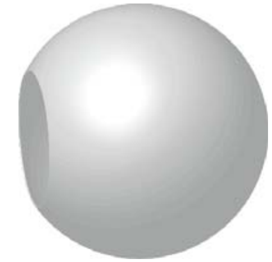
- Basic fact about open universe:
  harmonics on H^3 :

$$\nabla^2_H q^{(k)}(R) = -(k^2+1) q^{(k)}(R), \qquad q^{(k)}(R) = \frac{\sin kR}{\sinh R}$$

  - Normalizable modes: decay exponentially at R>>1
  - Non-normalizable (supercurvature) mode: imaginary k
- Initial condition (vacuum): determined by Euclidean
  - Compute the correlator on the Euclidean CDL, and analytically continue it to open FRW.
- Correlator can be expressed as a discrete sum.

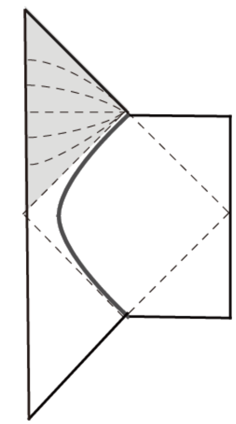
- Earlier work: Garriga, Montes, Sasaki, Tanaka, ‘99,
  Also, Hertog, Gratton, Turok

Calculation of the Euclidean correlator



- e.o.m. for a minimally coupled scalar:

$$\left[-\partial_X^2 + \frac{a''}{a} - \nabla_S^2 + m^2 a^2\right](a\phi) = 0$$

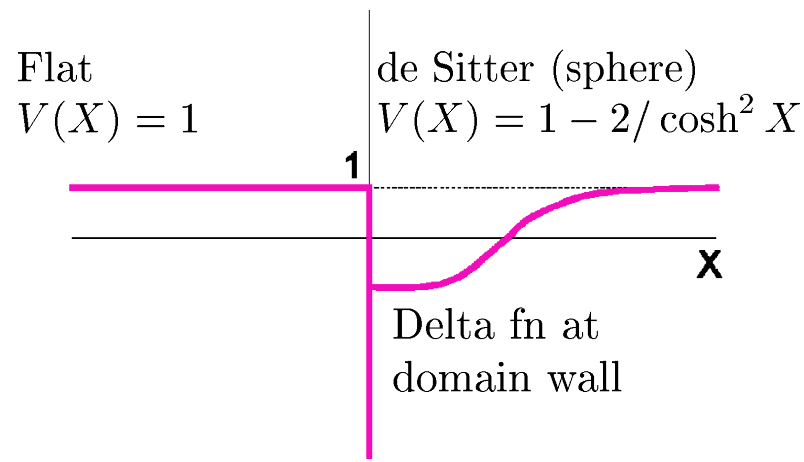


- Calculation of the correlator is essentially a 1-dimensional scattering problem:

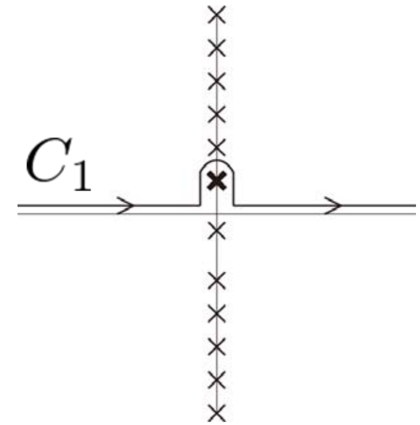
$$\left[-\partial_X^2 + \frac{a''}{a} + m^2 a^2\right] u_k(X) = (k^2+1) u_k(X)$$

- Massless case:
  bound state at k=i
- Bound state exists when mass is small compared to H_A
- There is at most one bound state.

$V(X) = a''/a$ (in the thin-wall limit)



Euclidean correlator



- On the flat side, $(X, X' \to -\infty)$

$$\langle \phi(X,\theta)\phi(X',0)\rangle = \tilde{H}_A^2 e^{-(X+X')} \int_{C_1} dk \Big(e^{ik(X-X')} + \mathcal{R}(k) e^{-ik(X+X')} \Big) \frac{\sinh k(\pi-\theta)}{\sinh k\pi \sin\theta}$$

  – The last factor: Green's fn. on S^3 with mass (k^2+1) It has poles at $k = ni$.

  – A bound state corresponds to a pole of $\mathcal{R}(k)$ at $k = ib \quad (0 < b \leq 1)$. If there is a bound state, we have to deform the contour and include this pole.

- Analytic continuation to FRW:

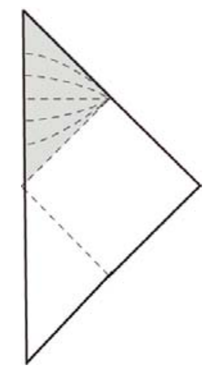
$$X \to T + \frac{\pi}{2} i, \quad \theta \to iR$$

Correlator in the FRW

- In the thin-wall limit (or at early FRW time),

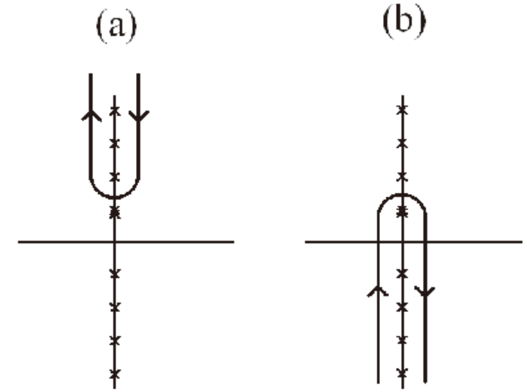
$$\langle \phi(T,R)\phi(T',0)\rangle = \tilde{H}_A^2 e^{-(T+T')}\int_{C_1} dk\Big(e^{ik(T-T')}\cosh k\pi + \mathcal{R}(k)e^{-ik(T+T')}\Big)\frac{\sin kR}{\sinh k\pi \sinh R}$$

(R: geodesic distance on H^3)



- <u>The first term: "flat space piece"</u>:
  Minkowski correlator written in open slicing
- <u>The second term: "ancestor piece":</u>
  May contain a non-normalizable mode.
  Modifies the spectrum of normalizable mode.

Near-boundary limit



- By contour deformation (T+R>0, T-R<0):

$$\langle\phi(T,R)\phi(T',0)\rangle = \sum_{\Delta} G^{(1)}_{\Delta} e^{-\Delta(R_1+R_2)} e^{-\Delta(T_1+T_2)} (1-\cos\Omega)^{-\Delta}$$
$$+\sum_{\Delta'=2}^{\infty} G^{(2)}_{\Delta'} e^{-\Delta'(R_1+R_2)} e^{(\Delta'-2)(T_1+T_2)} (1-\cos\Omega)^{-\Delta'}$$

where we have used $R \sim R_1 + R_2 + \log(1-\cos\Omega) \quad (\text{when } R\to\infty)$

- Sum over terms with definite dimensions Δ (w.r.t. the spatial dependence: $(1-\cos\Omega)^{-\Delta}$)

  1st sum: poles at $k = i(1-\Delta) \quad (\Delta \geq 0)$

  2nd sum: poles at $k = i(\Delta - 1) \quad (\Delta \geq 2)$

  All integer dimensions from 2 appear.

The scales in the dual theory

- UV cutoff : $a \leftrightarrow e^{-(T+R)}$

  (Late time in the bulk: UV cutoff becomes finer)
- Reference scale: $\delta \leftrightarrow e^{-R}$

- Interpretation of the prefactors: wave fn. renormalization
  - Operators in the 1st sum (which go like $e^{-\Delta(T+R)}$): "RG-invariant" operators (defined at the UV scale)
  - Operators in the 2nd sum (which go like $k = i(1-\Delta)$): "RG-covariant" operators (defined at reference scale, e.g. effective action or energy momentum tensor)

Graviton correlator

- The transverse-traceless mode (on H^3) is essentially equivalent to the massless scalar.
- There is a non-normalizable mode (Δ=0).
  - This is pure gauge in the bulk, but has physical effects on the boundary. Boundary (2D) curvature correlator:

    $$\langle R^{(2)} R^{(2)} \rangle = \frac{1}{(1 - \cos\Omega)^2}$$

    Gravity is not decoupled at the boundary.
- Dimension 2 piece of graviton is transverse (conserved)-traceless in 2D.
  - Identified as energy-momentum tensor of the 2D CFT

Three-point functions

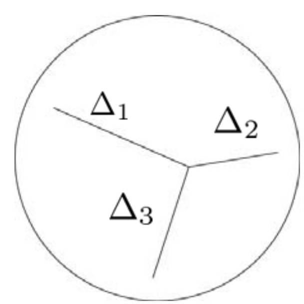
(work in progress /w Daniel Park)

- Graviton-scalar-scalar 3-point function will tell us:
  Central charge (~ de Sitter entropy, from dim. analysis)
  Operator algebra:

$$T(w)\mathcal{O}(0) \sim \frac{\Delta \mathcal{O}(0)}{w^2} + \frac{\partial \mathcal{O}(0)}{w} + \dots$$

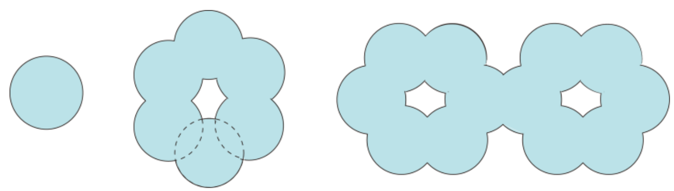
- Computation: Analytic continuation from Euclidean.
  Final expression: Sum of AdS 3-point functions, with the dimensions of the external lines summed over.

$$\sum_{\Delta_1 \Delta_2 \Delta_3} \int dH_3$$



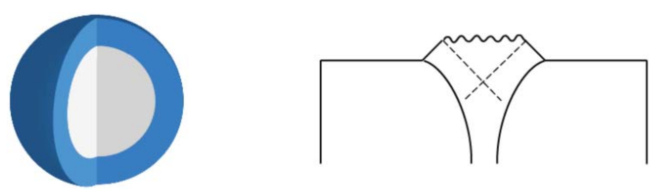
Comments on bubble collisions

- Universe (true vacuum region) with a boundary with arbitrary genus (connected to a single observer) can arise. (Bousso, Freivogel, YS, Shenker, Susskind, Yang, Yeh, '08)



We have to sum over topology of the base space of CFT.

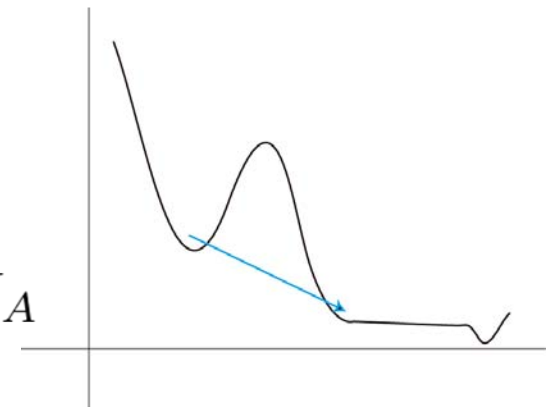
- Multiple boundaries have to be separated by singularity. (Kodama, Maeda, Sasaki, Sato, '82)



- Characterization of the "phases" of eternal inflation: work in progress (w/ Shenker and Susskind)

Observational consequences

(work in progress with Freivogel and Susskind)



- Assume:

  Slow-roll inflation after tunneling $H_I \ll H_A$

  Minimal # of e-foldings

- Assume negative curvature $\Omega_0 \sim 0.98$

  Then, radius of the last scattering surface (Freivogel et al '05):

$$R_{\rm l.s.} = \int_{z=1100}^{z=0} \frac{dt}{a} \sim 0.5 R_{\rm curv}$$

- We might be able to see the effect of the curvature or the ancestor vacuum in the CMB spectrum.

Basic intuition

- Early FRW time: curvature dominated (until $t \sim H_I^{-1}, \quad (T \sim \log(H_A/H_I))$)
- E.o.m. during the curvature domination:

$$\ddot{\phi}_k + \frac{3}{t}\dot{\phi}_k - (k^2+1)\phi_k = 0, \qquad \phi_k \sim t^{-1 \pm ik}$$

Amplitude basically decays as 1/t.

- We evaluate the correlator by contour deformation
  - How large the effect of the ancestor could be compared to the fluctuations generated during the slow-roll inflation?

Amplitude at the end of curvature domination

- Correlator during the curvature domination:

$$\langle\phi(T,R)\phi(T',0)\rangle = \tilde{H}_A^2 e^{-(T+T')}\int_{C_1} dk\Big(e^{ik(T-T')}\cosh k\pi + \mathcal{R}(k)e^{-ik(T+T')}\Big)\frac{\sin kR}{\sinh k\pi \sinh R}$$

- 1st term (usual initial condition for the SR): $H_A^2 e^{-(T+T')} \sim H_I^2$
- 2nd term (effect of the ancestor):
  - Contribution from a pole at $k = i(1-\Delta)$:

$$H_I^2 e^{2(1-\Delta)(T+T')} = H_I^2\left(\frac{H_A}{H_I}\right)^{2(1-\Delta)}$$

  - The lowest dimension gives the leading contribution. If there is NNM ($0<\Delta<1$), larger than H_I^2

The spectrum

- We evolve forward the initial condition into slow-roll regime using the e.o.m.
- The first term (fluctuations generated after tunneling):

$$\begin{aligned} \langle \phi(T,R)\phi(T',0)\rangle &= H_I^2 \int_{-\infty}^{\infty} dk \frac{1}{k^2+1} \frac{\cosh k\pi}{\sinh k\pi} \frac{\sin kR}{\sinh R} \\ &= -H_I^2 \left\{ \log(\cosh R - 1) - R\frac{\cosh R}{\sinh R} \right\} \end{aligned}$$

  – In terms of angular harmonics (for $R_{\mathrm{l.s.}} \sim 0.5$), the spectrum is almost scale invariant: $C_l^{(\mathrm{scale\ inv.})} = \frac{H_I^2}{l(l+1)}$ (1% suppression at $l=2$,due to the curvature)

The effect of the ancestor vacuum

- Characteristic behavior: exponential decrease in l .

$$C_l^{(\text{ancestor})} \sim \left(\frac{R}{2}\right)^{2l} \quad (\text{ when } R < 1)$$

  - Because the radial function for the H^3 harmonics satisfies

$$\left[\frac{1}{\sinh^2 R}\partial_R(\sinh^2 R)\partial_R + \frac{l(l+1)}{\sinh^2 R} + (k^2+1)\right] q^{(kl)}(R) = 0$$

    When R<1, $q^{(kl)}(R) \sim R^l$ (suppression near R=0 due to centrifugal barrier).

- If there is a non-normalizable mode, we will see this type of enhancement at low-l.

When is the effect of the ancestor large?

Scalar mode (curvature perturbation)

- Small effect:
  - There will be no non-normalizable mode, since the tunneling field should have large mass in the ancestor. (Otherwise CDL instanton does not exist).

For extra fields ("isocurvature" perturbations)

- The effect could be large.
  - There could be fields with small mass in the ancestor, but we should understand how they contribute to observable quantities.

How large is the effect of the ancestor?

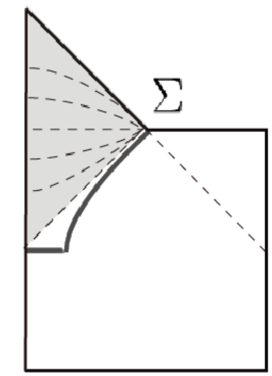
Tensor modes

- Effect on the temperature fluctuations

$$\frac{\delta \mathrm{T}(\Omega)}{\mathrm{T}} = \int dT \partial_T h_{RR}(T, R(T), \Omega)$$

  - There is no NNM (it is pure gauge), and the lowest dimension is $\Delta = 1 + \alpha$ where $\alpha \sim (r_c H_A)^2$
  - The ancestor effect is small: $H_I^2 \left(\frac{H_I}{H_A}\right)^{2\alpha}$
  - When $\alpha \ll 1$ (the bubble is very small), the effect could be large: H_I^2/α

Open questions



On holographic duality

- Description of the de Sitter side
  (and application to the measure problem)
- Problems with non-zero final c.c.?
- Matter content of the dual theory

  (possibly related to D-brane world volume theory)

On observational consequence

- Curvature perturbations in two-field models (when the tunneling field is different from the slow-roll field) could be large?