Black hole and shock wave collisions in higher dimensions

Carlos A. R. Herdeiro
Departamento de Física da Universidade de Aveiro
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Part I with V. Cardoso, L. Gualtieri, A. Nerozzi, U. Sperhake, H. Witek, M. Zilhão
PRD 81 (2010) 084052; 82 (2010) 104014; 83 (2011) 044017

Part II with Flávio Coelho, Carmen Rebelo and Marco Sampaio
JHEP 07 (2011) 121; PRL 108 (2012) 181112; arXiv: 1206:5839 [hep-th]

Gr@v
gravitation.web.ua.pt

Kyoto, Japan, 17 January 2013

Ultra-relativistic particle collision:

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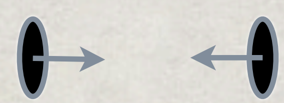

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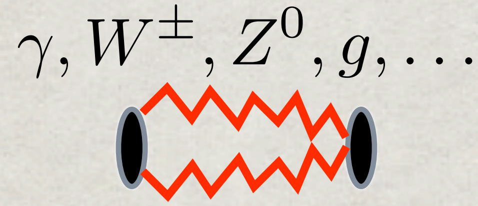

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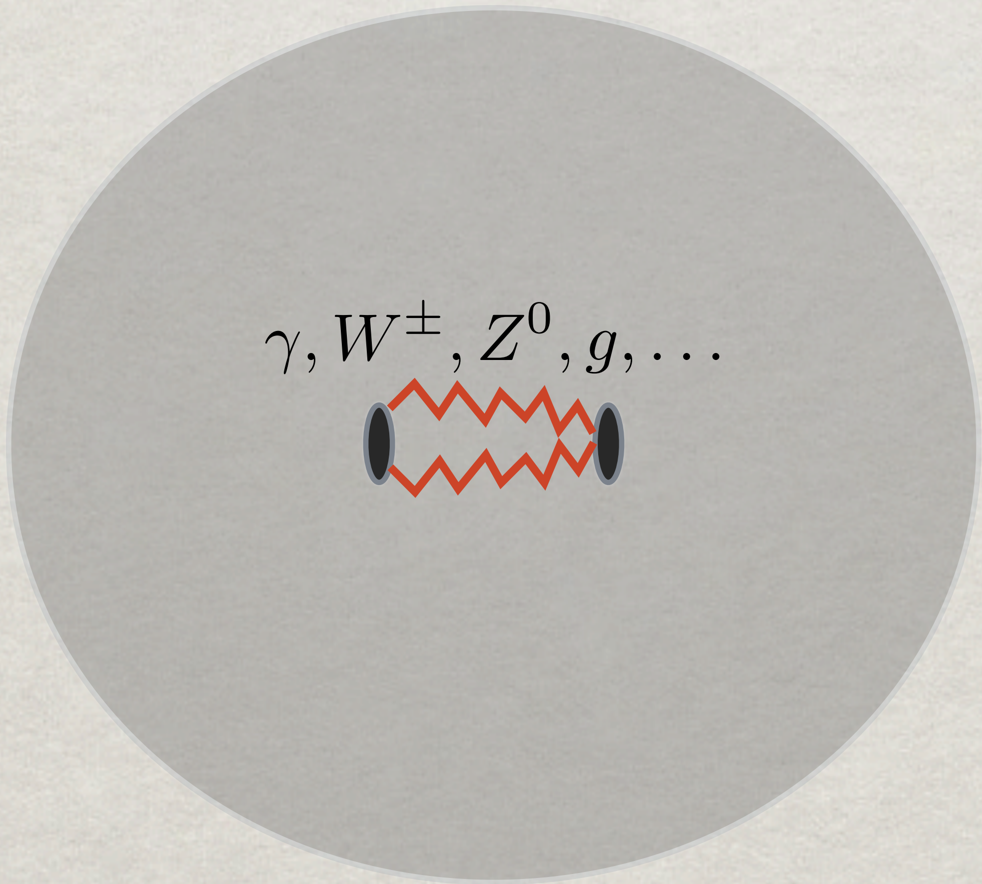
At sufficiently high energies:



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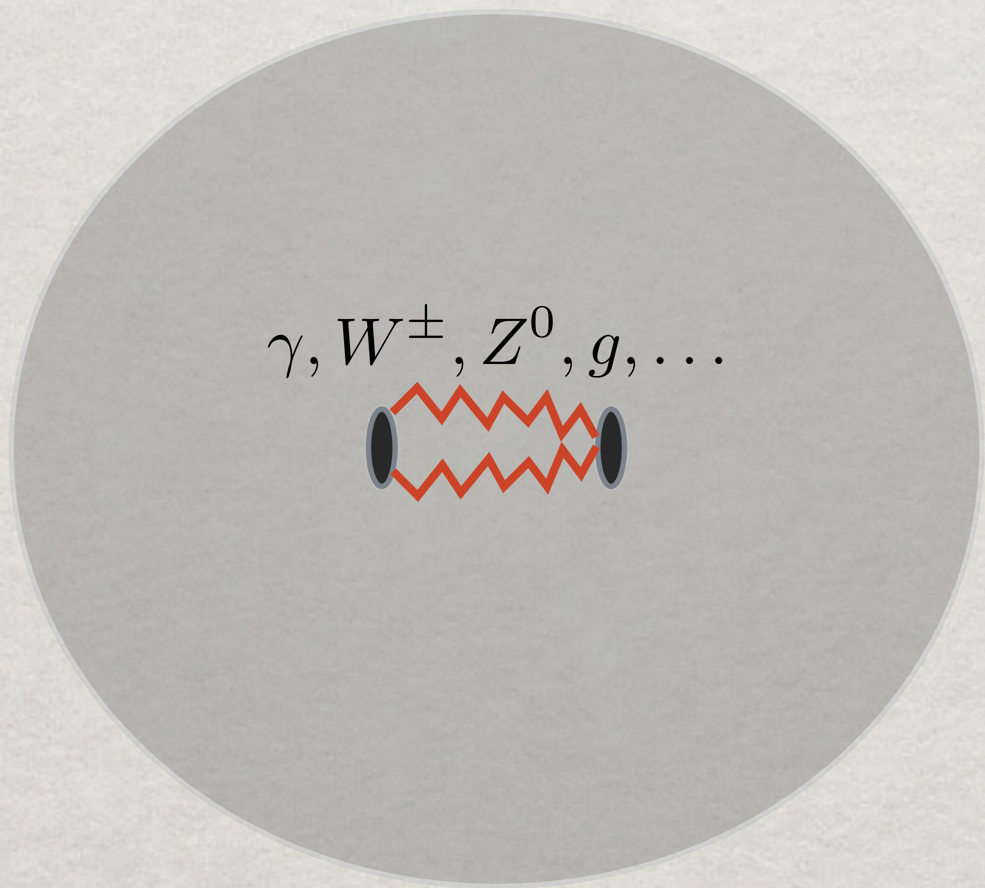

At energies well above the Planck energy, black hole production sets in, accompanied by the coherent emission of real gravitons (gravitational waves) 't Hooft '87;

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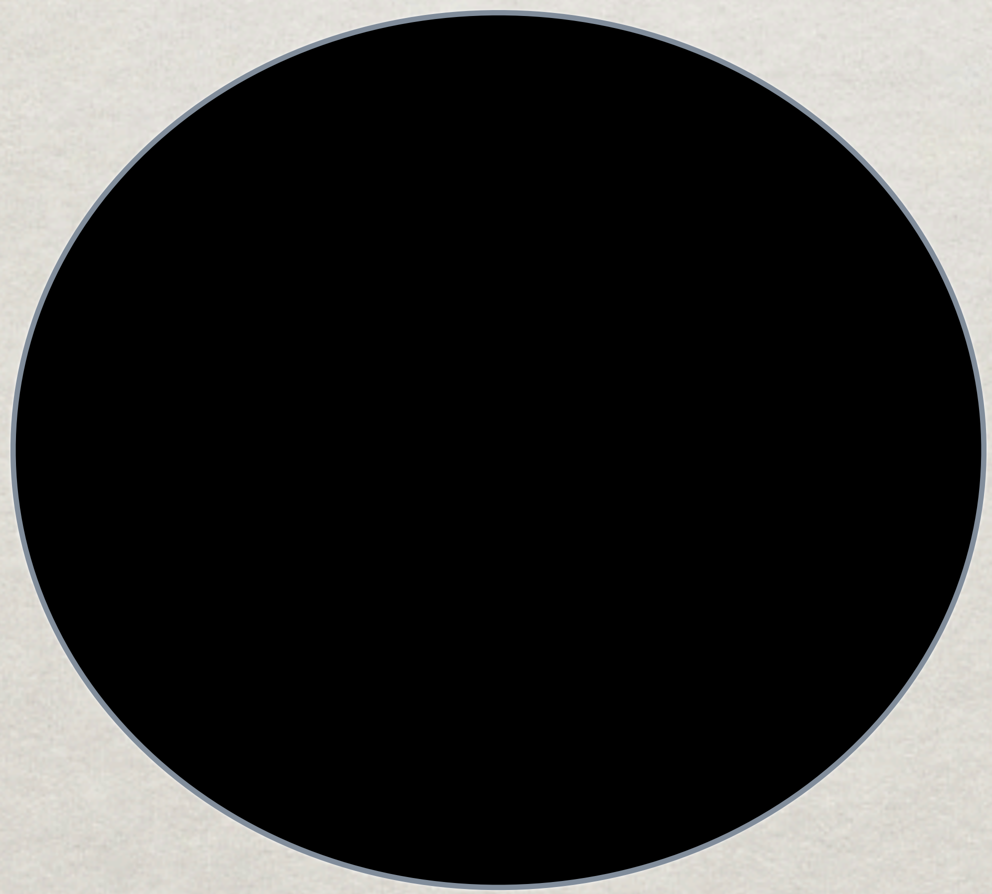

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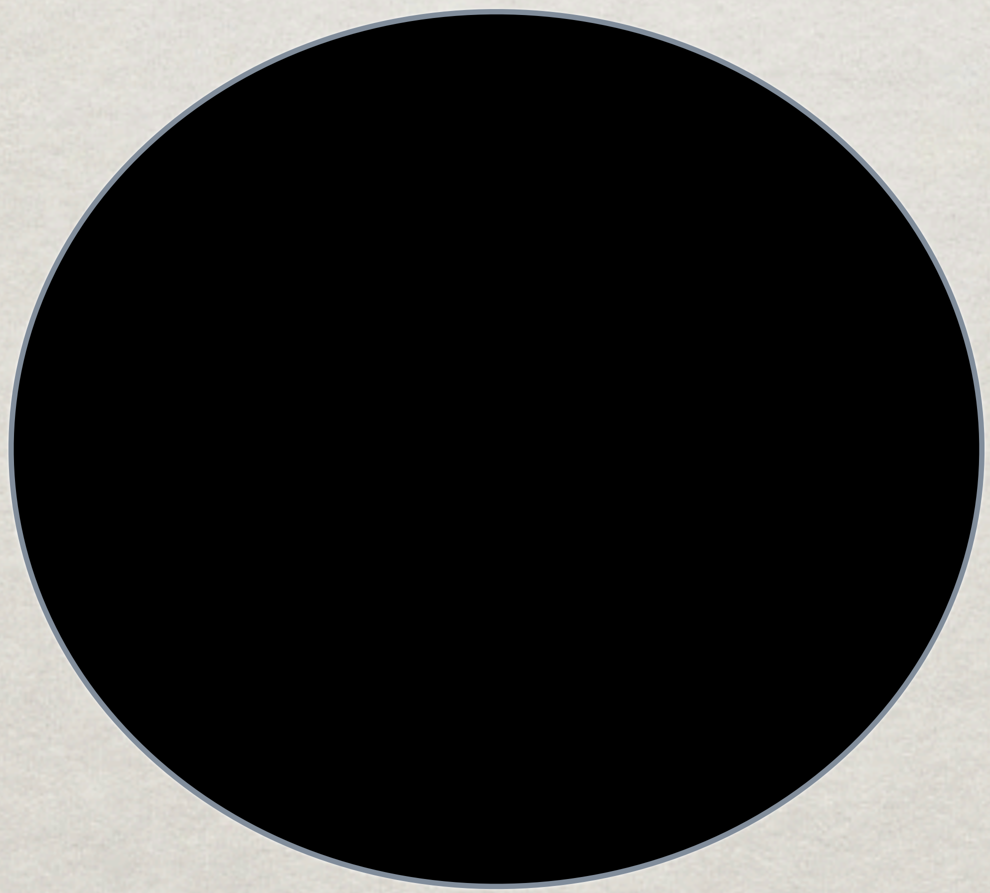

$$Riemann\Big|_{Horizon} \sim \frac{1}{s}$$

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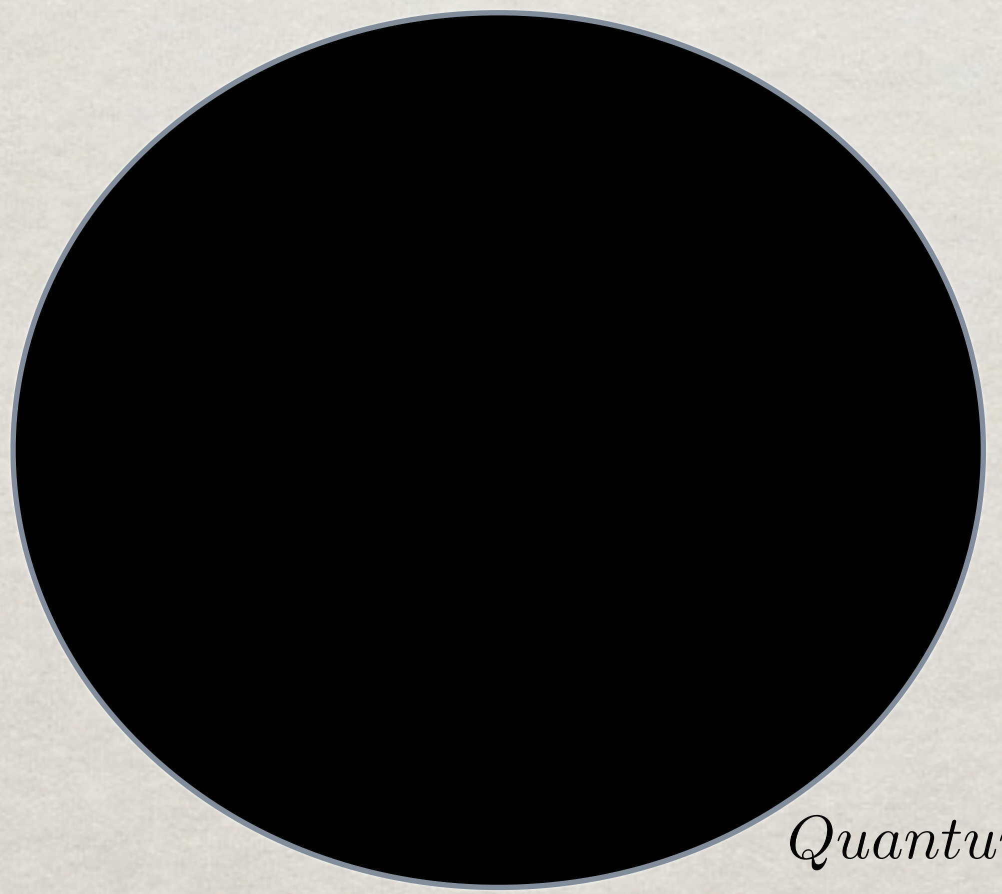

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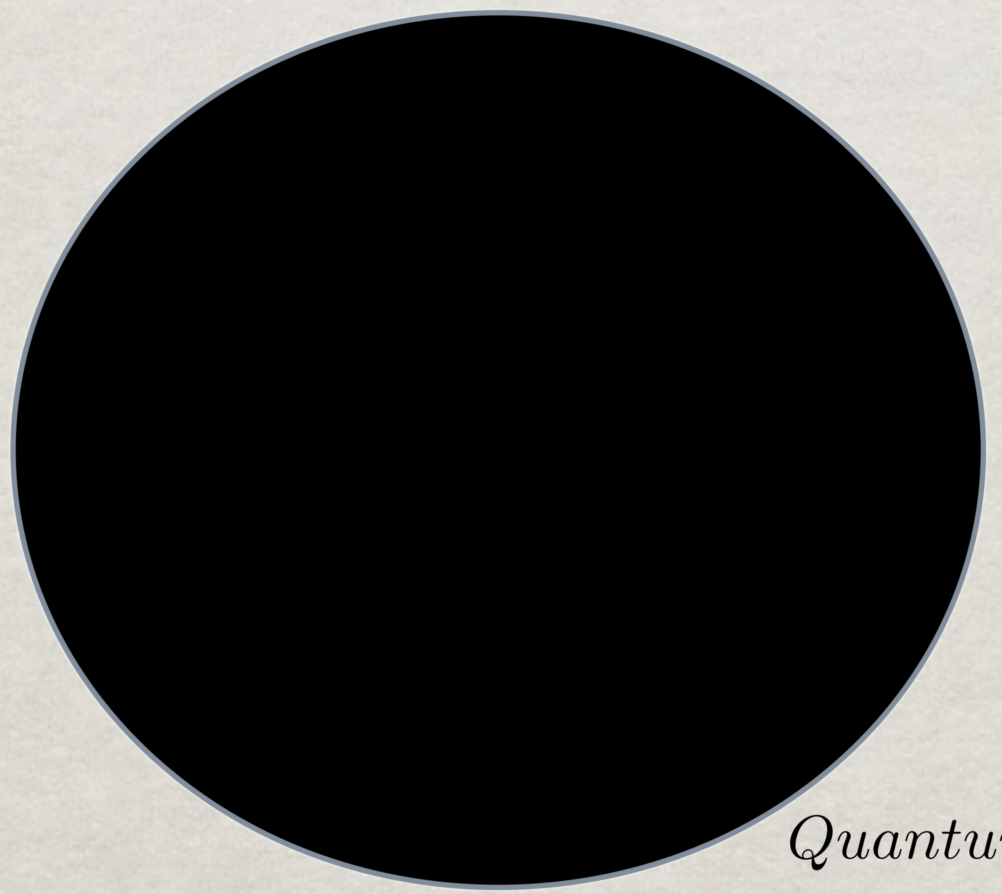

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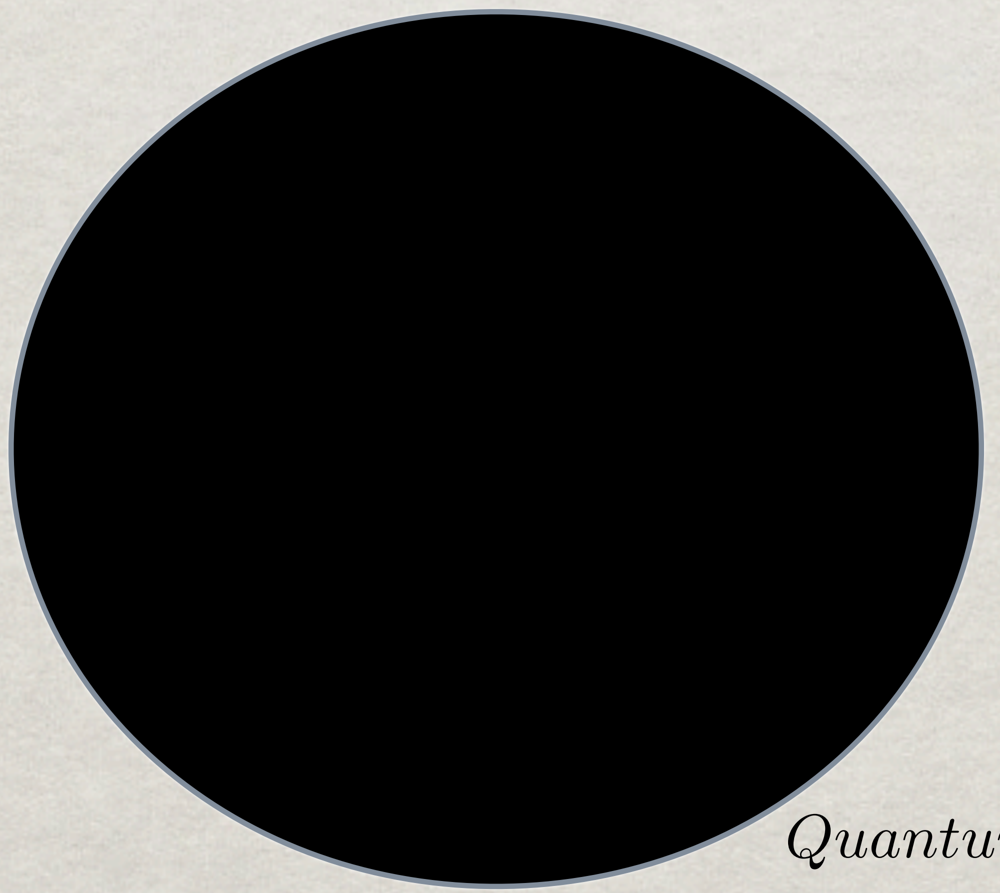

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Graviton dominance in ultra-high-energy scattering

Formation of an Apparent Horizon (AH) from a high energy collision of boson stars

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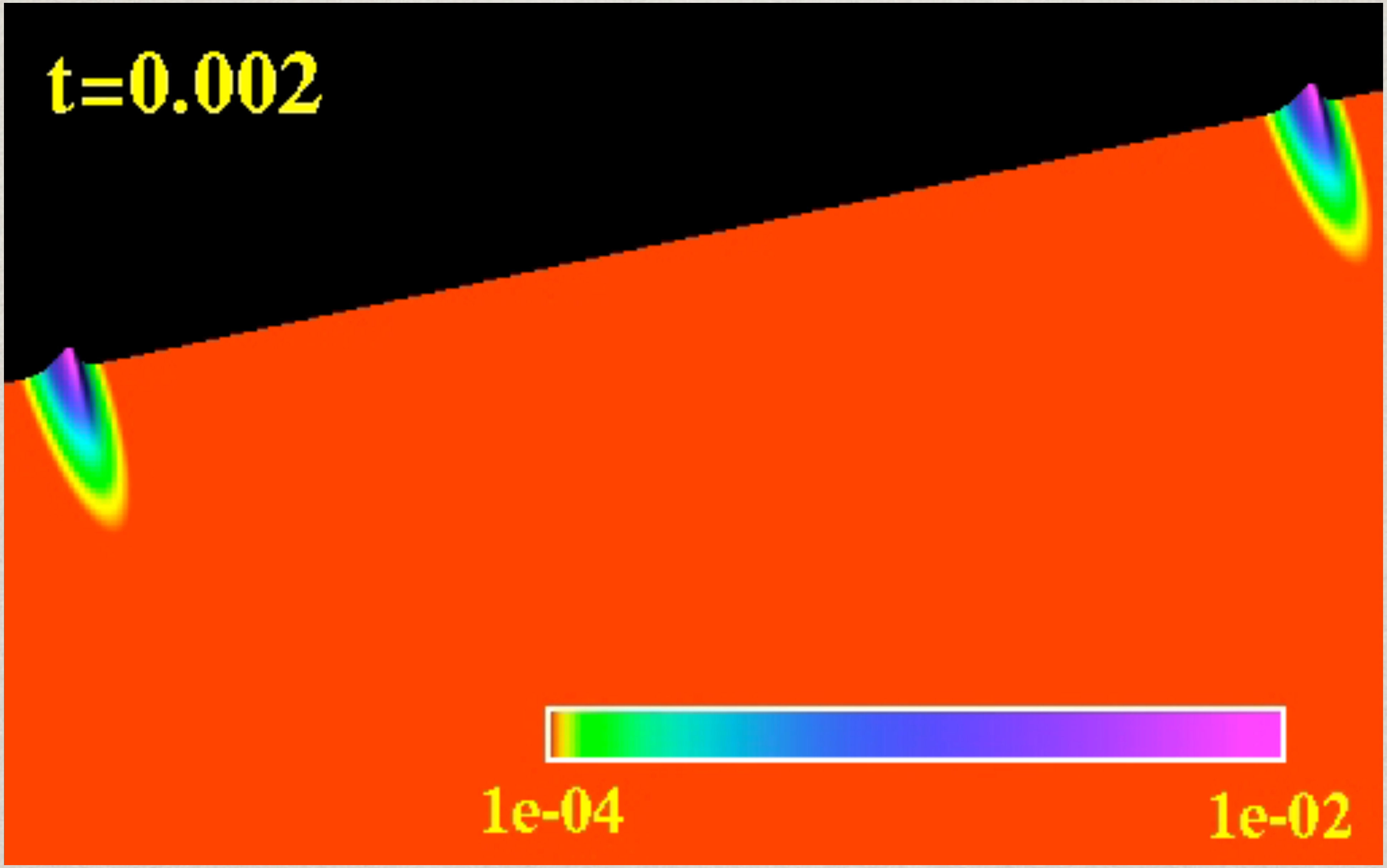


Depicted is the magnitude of the scalar field. Copyright Frans Pretorius; http://physics.princeton.edu/~fpretori/

What is the fraction of the total energy radiated away (inelasticity) in a head-on collision of two particles at (almost) the speed of light computed in classical gravity?

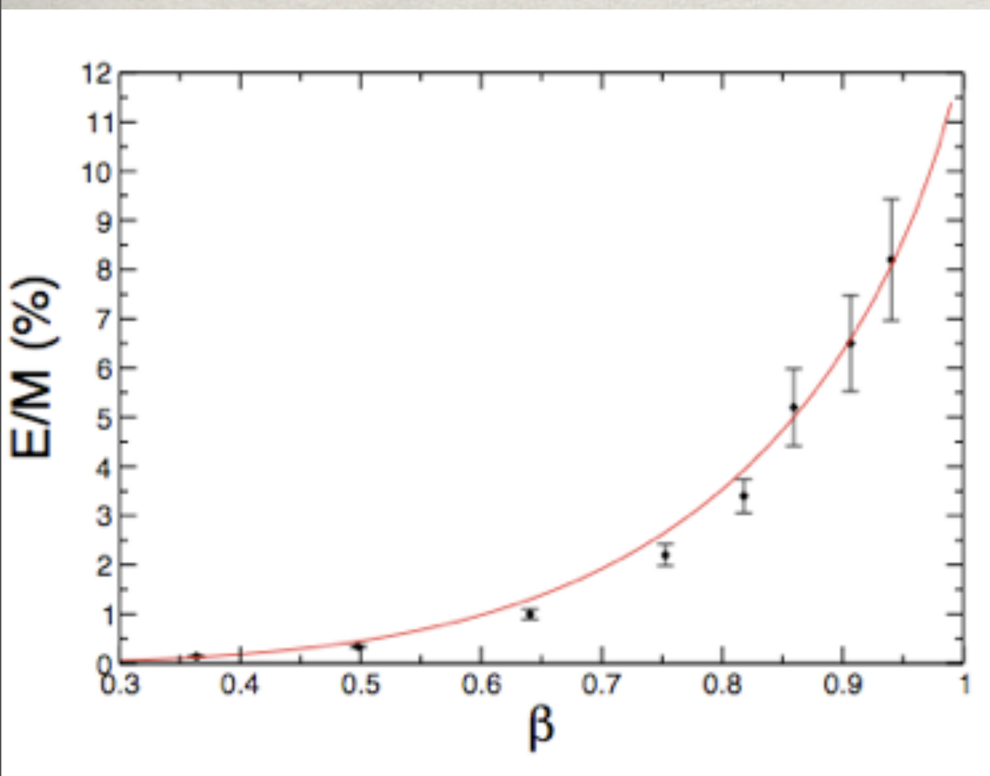
Part I:

Black hole collisions using numerical relativity:

- in higher-dimensions

Study black holes head-on collisions (e.g. test cosmic censorship)

Sperhake, Cardoso, Pretorius, Berti, Gonzales, '08

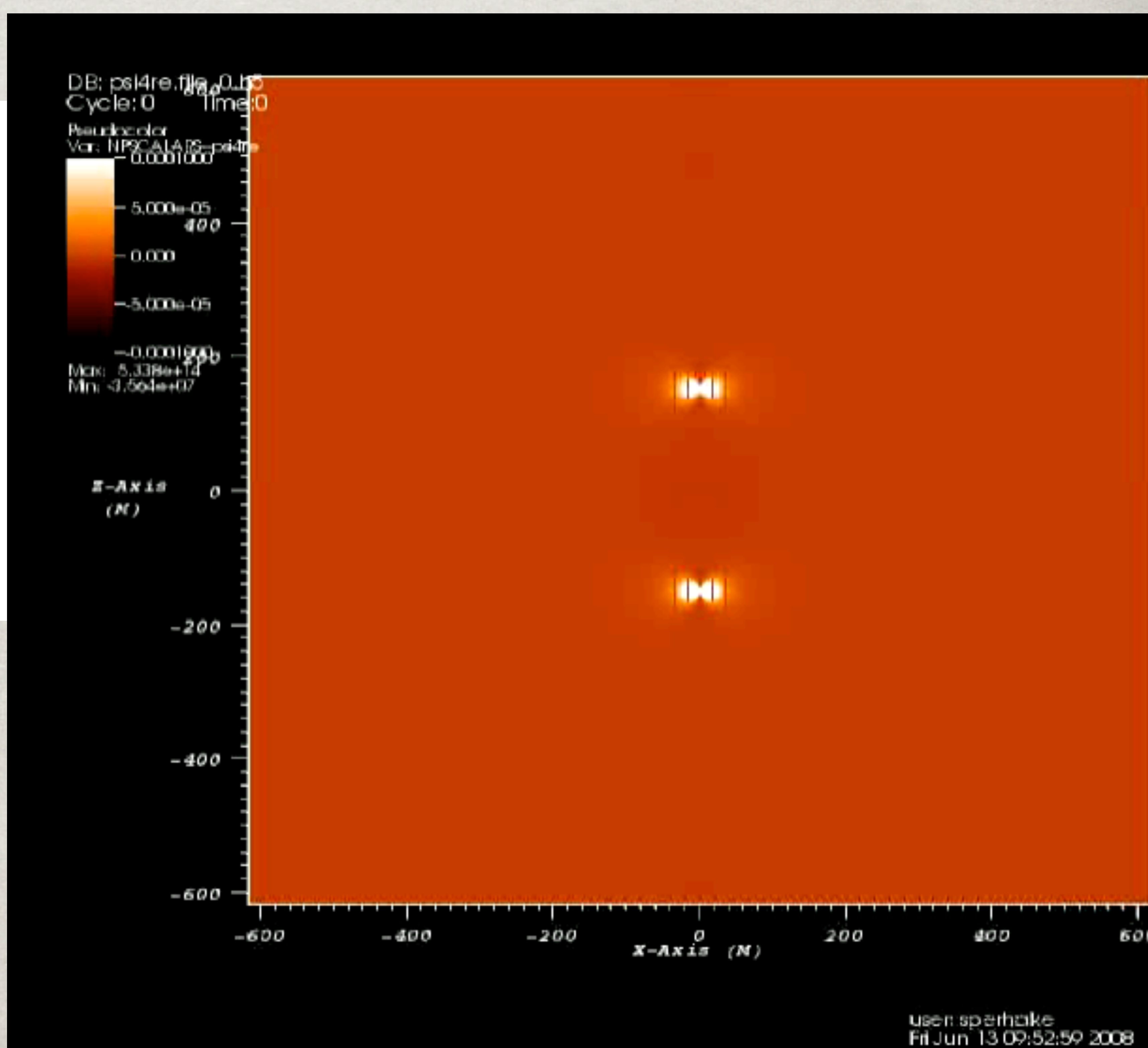


Ultra-relativistic regime

$$\frac{E}{M} \simeq 14 \pm 3\%$$

Luminosity

$$10^{-2}\frac{c^5}{G}$$



Head on collision of equal mass black holes (no spin). Copyright Ulrich Sperhake.

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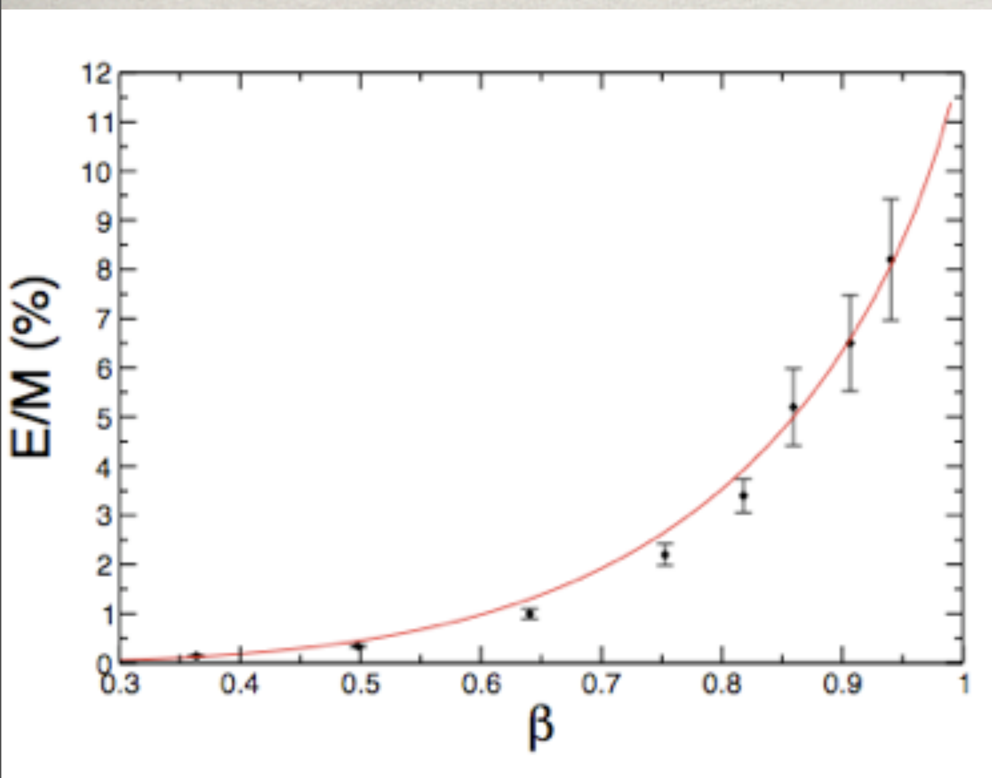

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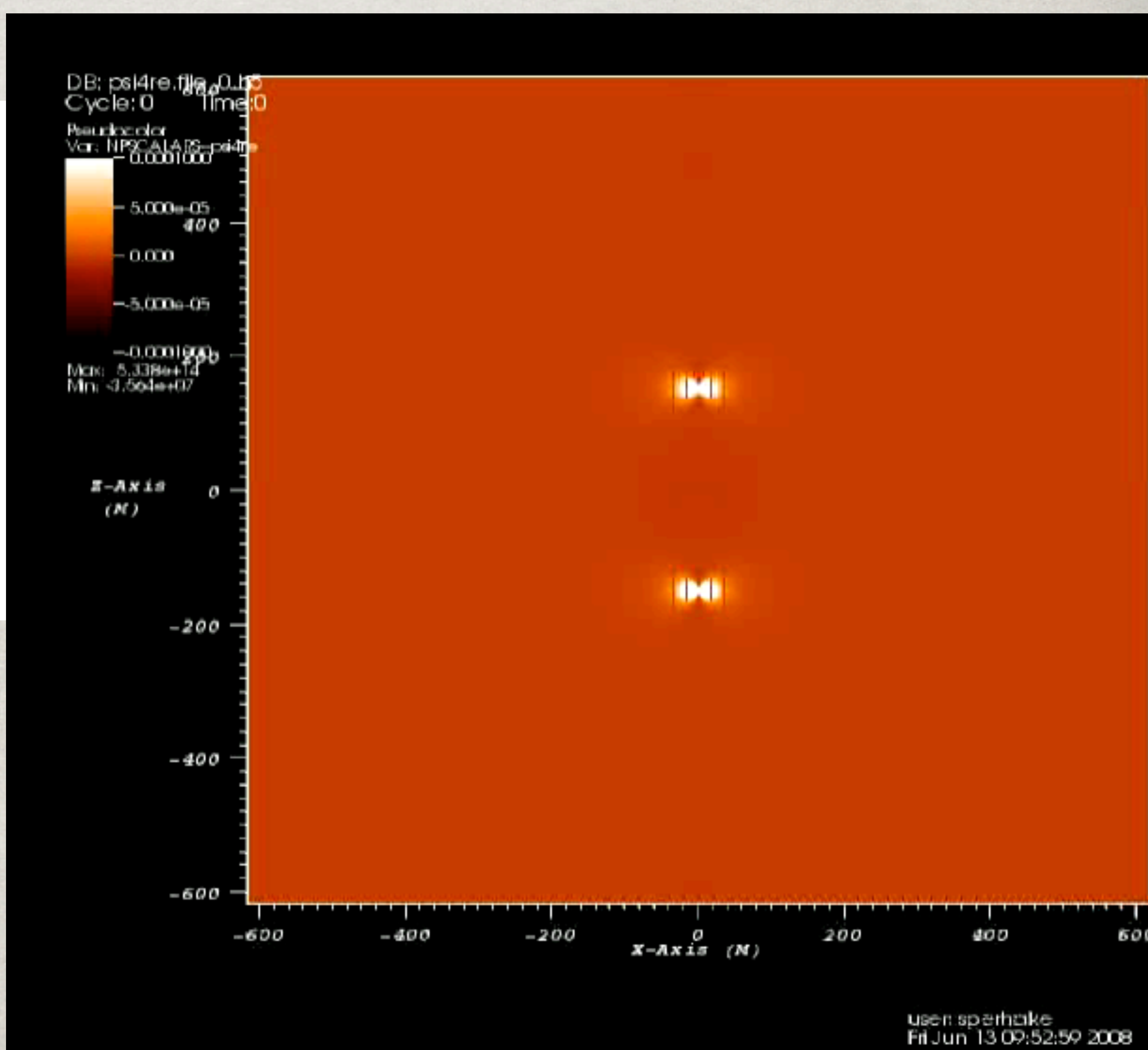


Head on collision of equal mass black holes (no spin). Copyright Ulrich Sperhake.

Study black hole scattering problem
(e.g. analyse energy conversion and orbital motion)

Sperhake, Cardoso, Pretorius, Berti, Hinderer, Yunes '09

Ultra-relativistic regime

$$\frac{E}{M} \simeq 35 \pm 5\%$$

Luminosity

$$0.1 \frac{c^5}{G}$$

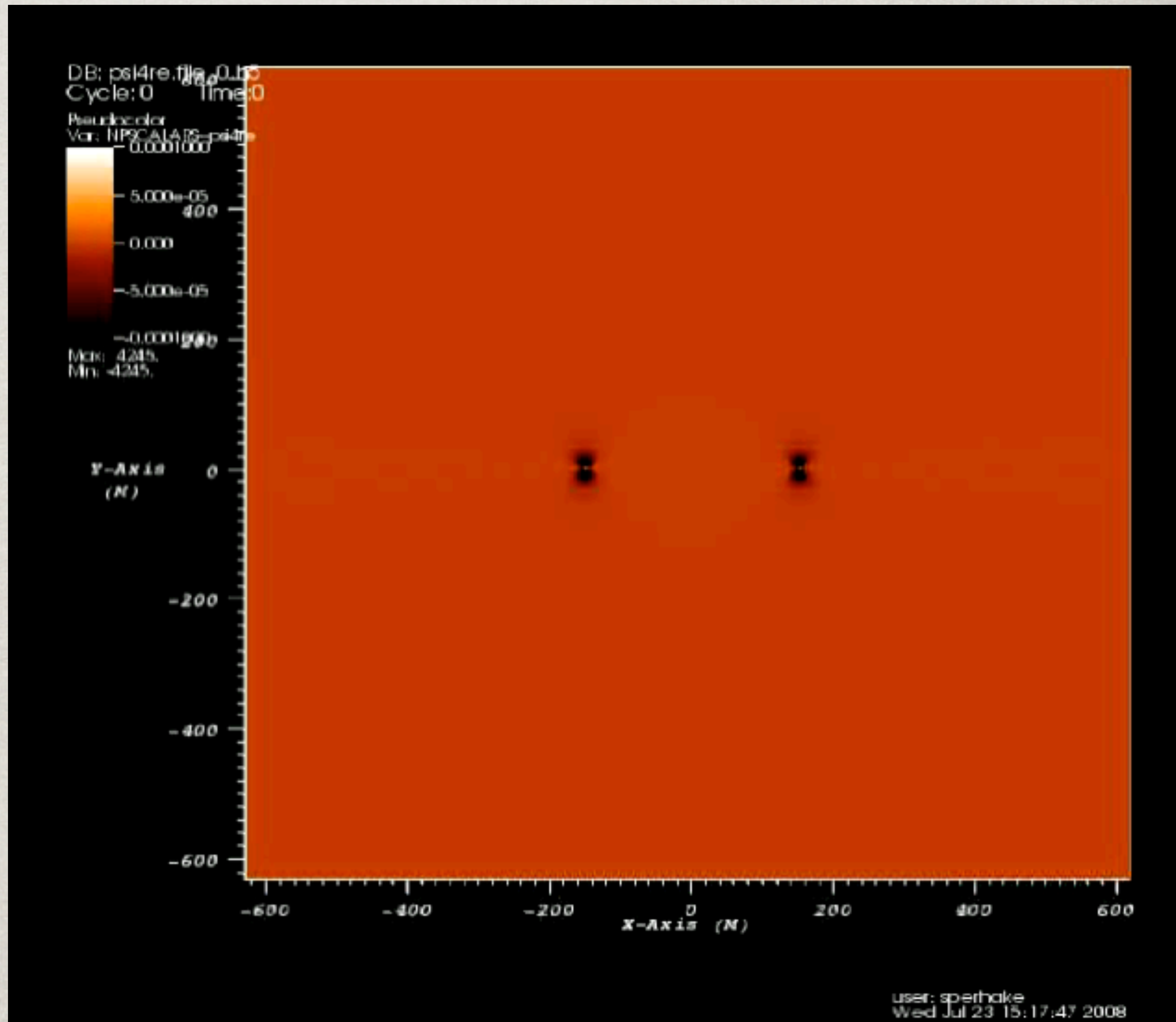
Remnant black hole very close to extremal

$$a \gtrsim 0.95 \ M$$

Two special values for impact parameter:

-scattering threshold

-threshold for immediate merger



Non-head on collision of equal mass black holes (no spin). Copyright Ulrich Sperhake.

Thursday, January 17, 2013

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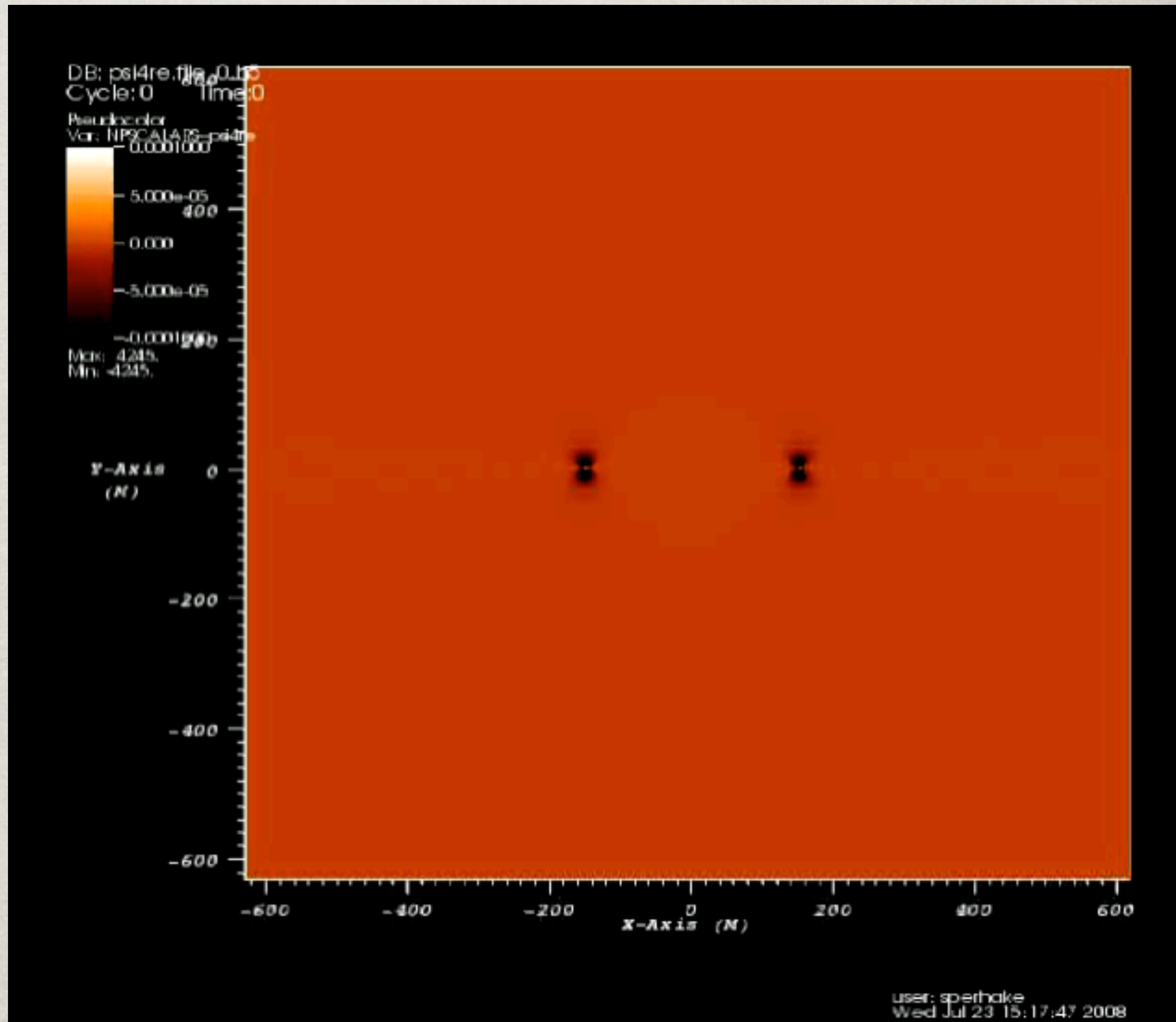

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Black hole collisions:
i) Higher dimensional black holes

PHYSICAL REVIEW D **81,** 084052 (2010)

Numerical relativity for *D* dimensional axially symmetric space-times: Formalism and code tests

Miguel Zilhão,[1,*] Helvi Witek,[2,†] Ulrich Sperhake,[3,‡] Vitor Cardoso,[2,4,§] Leonardo Gualtieri,[5,∥]
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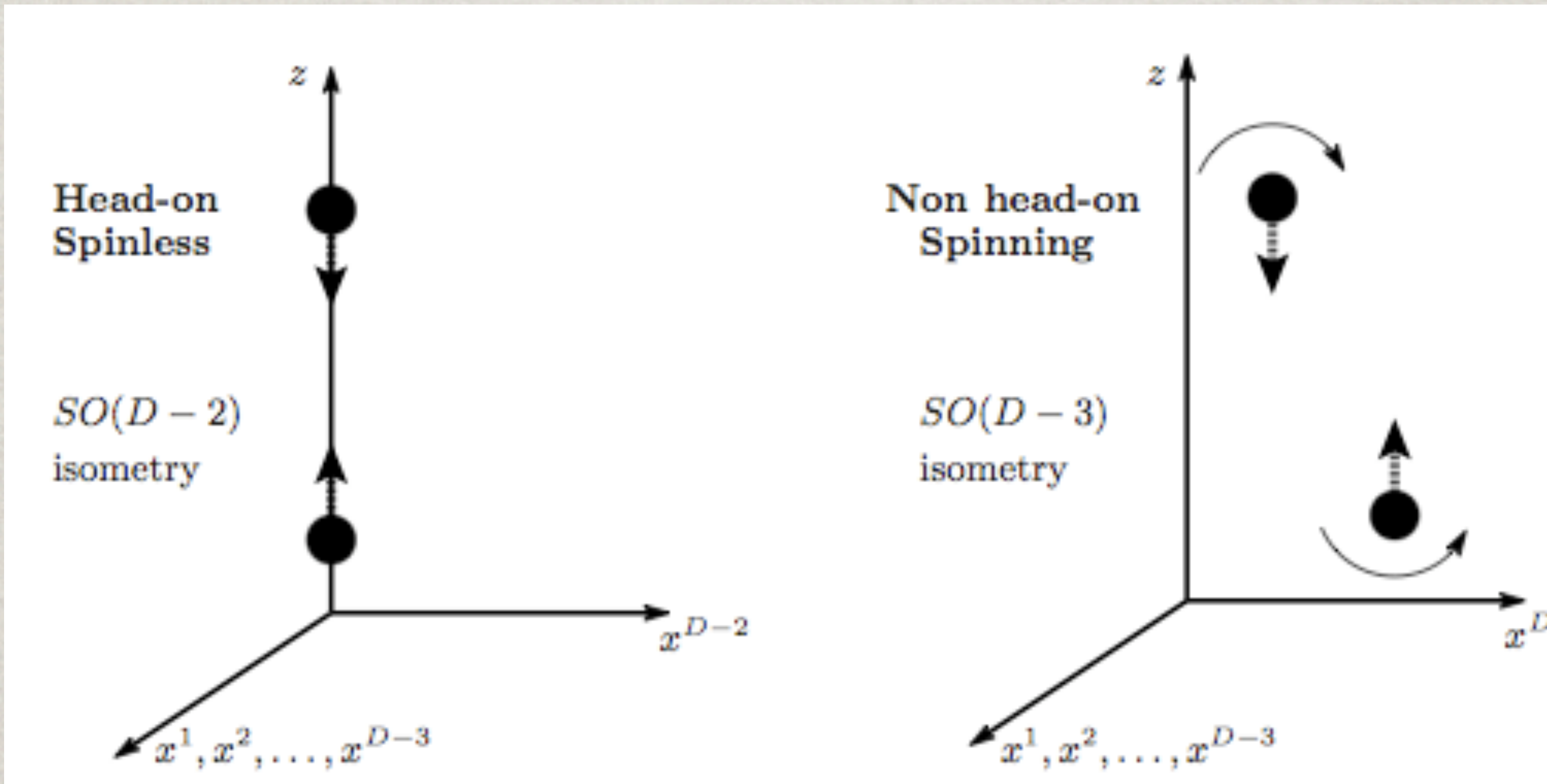
[5]*Dipartimento di Fisica, Università di Roma "Sapienza" & Sezione, INFN Roma1, P.A. Moro 5, 00185, Roma, Italy, USA*
(Received 18 January 2010; published 29 April 2010)

The numerical evolution of Einstein's field equations in a generic background has the potential to answer a variety of important questions in physics: from applications to the gauge-gravity duality, to modeling black hole production in TeV gravity scenarios, to analysis of the stability of exact solutions, and to tests of cosmic censorship. In order to investigate these questions, we extend numerical relativity to more general space-times than those investigated hitherto, by developing a framework to study the numerical evolution of D dimensional vacuum space-times with an $SO(D-2)$ isometry group for $D \geq 5$, or $SO(D-3)$ for $D \geq 6$. Performing a dimensional reduction on a $(D-4)$ sphere, the D dimensional vacuum Einstein equations are rewritten as a $3+1$ dimensional system with source terms, and presented in the Baumgarte, Shapiro, Shibata, and Nakamura formulation. This allows the use of existing $3+1$ dimensional numerical codes with small adaptations. Brill-Lindquist initial data are constructed in D dimensions and a procedure to match them to our $3+1$ dimensional evolution equations is given. We have implemented our framework by adapting the LEAN code and perform a variety of simulations of nonspinning black hole space-times. Specifically, we present a modified *moving puncture* gauge, which facilitates long-term stable simulations in $D = 5$. We further demonstrate the internal consistency of the code by studying convergence and comparing numerical versus analytic results in the case of geodesic slicing for $D = 5, 6$.

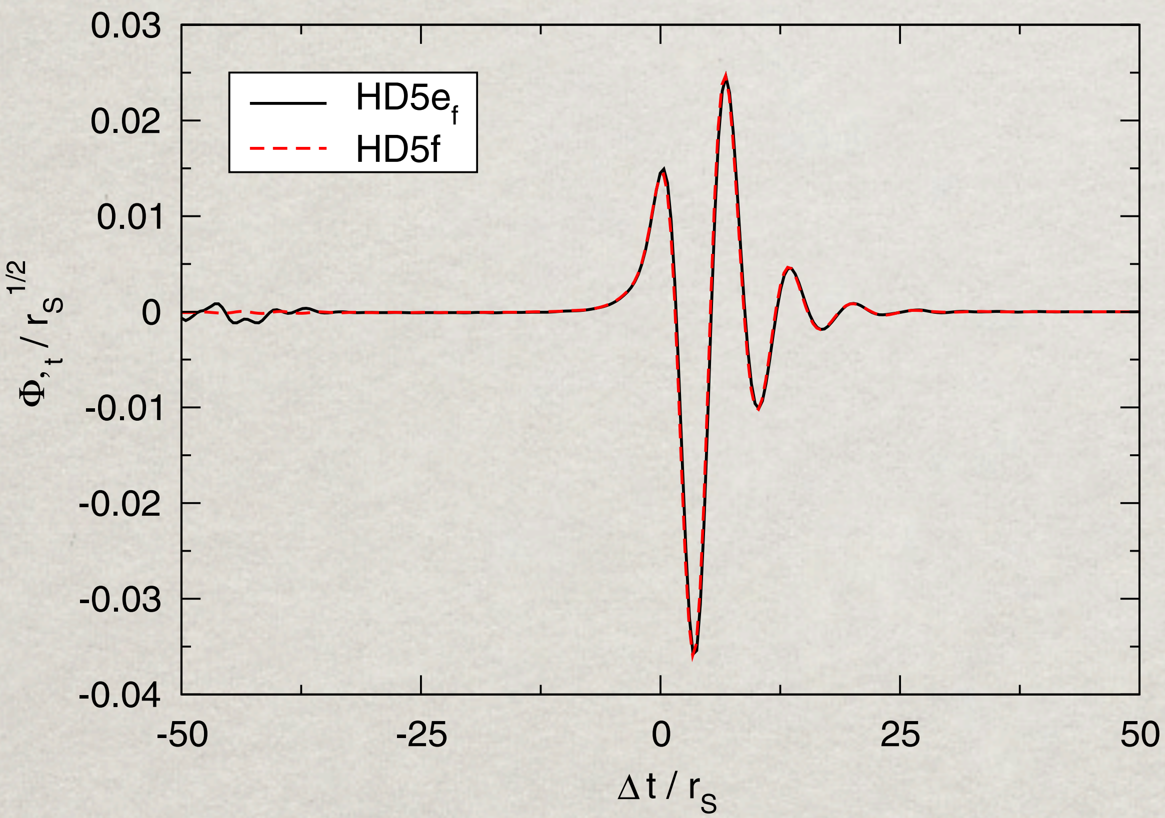
In higher dimensions: Dimensional reduction by isometry and quasi-matter

Impose "axial" symmetry SO(D-3); make dimensional reduction on (D-4)-sphere



D-dimensional vacuum Einstein equations yield 4-dimensional Einstein equations with "quasi-matter" (scalar field)

Black hole and shock wave collisions:
i) Higher dimensional black holes



D	$r_S\omega(l=2)$	$E^{\rm rad}/M(\%)$	$E^{\rm area}/M(\%)$	$E^{\rm rad}_{l=4}/E^{\rm rad}_{l=2}$
4	0.7473 − i0.1779	0.055	29.3	$<10^{-3}$
5	0.9477 − i0.2561	0.089	20.6	$<10^{-4}$

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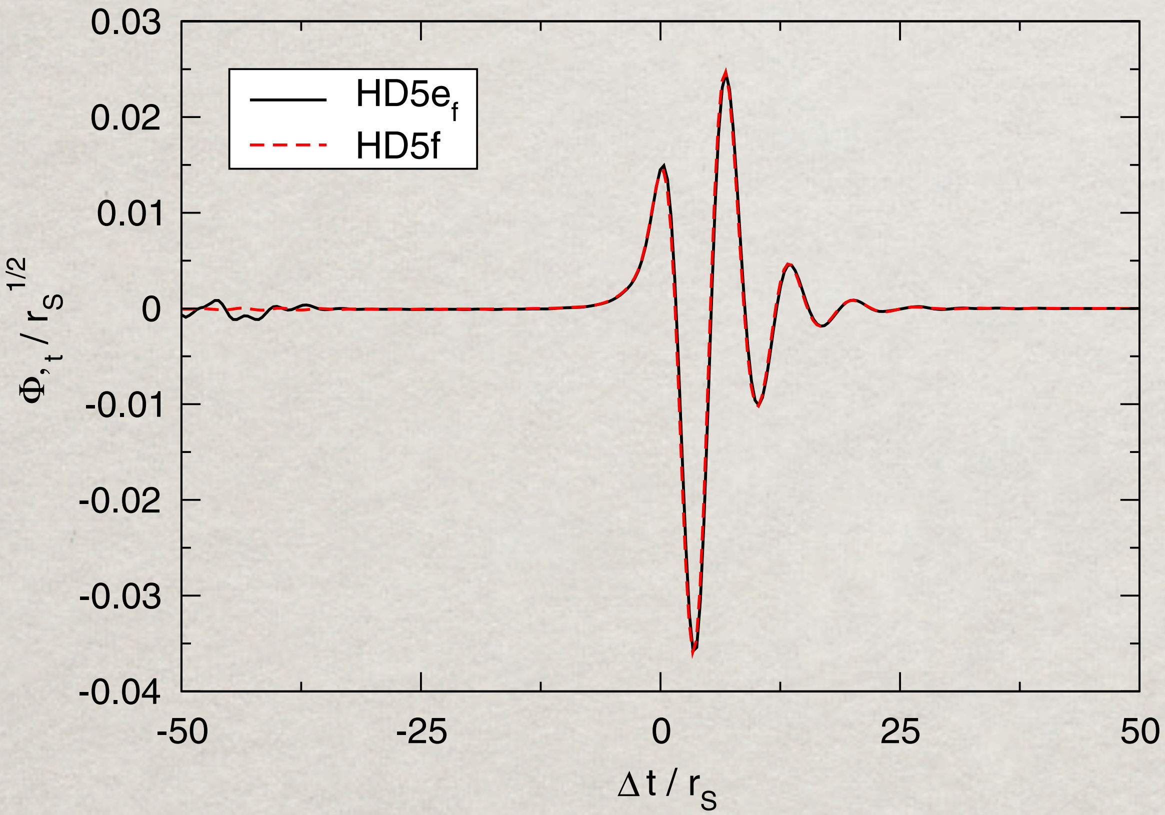

Achieved:
- First higher D BH collisions
- For unequal masses there is good agreement with point particle results
- Difficult to get to (highly) boosted collisions **PRD 84 (2011) 084039**

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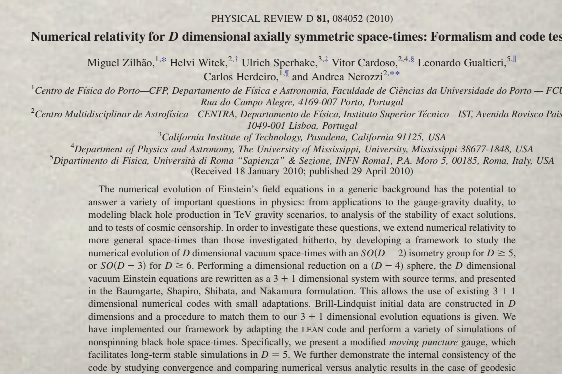
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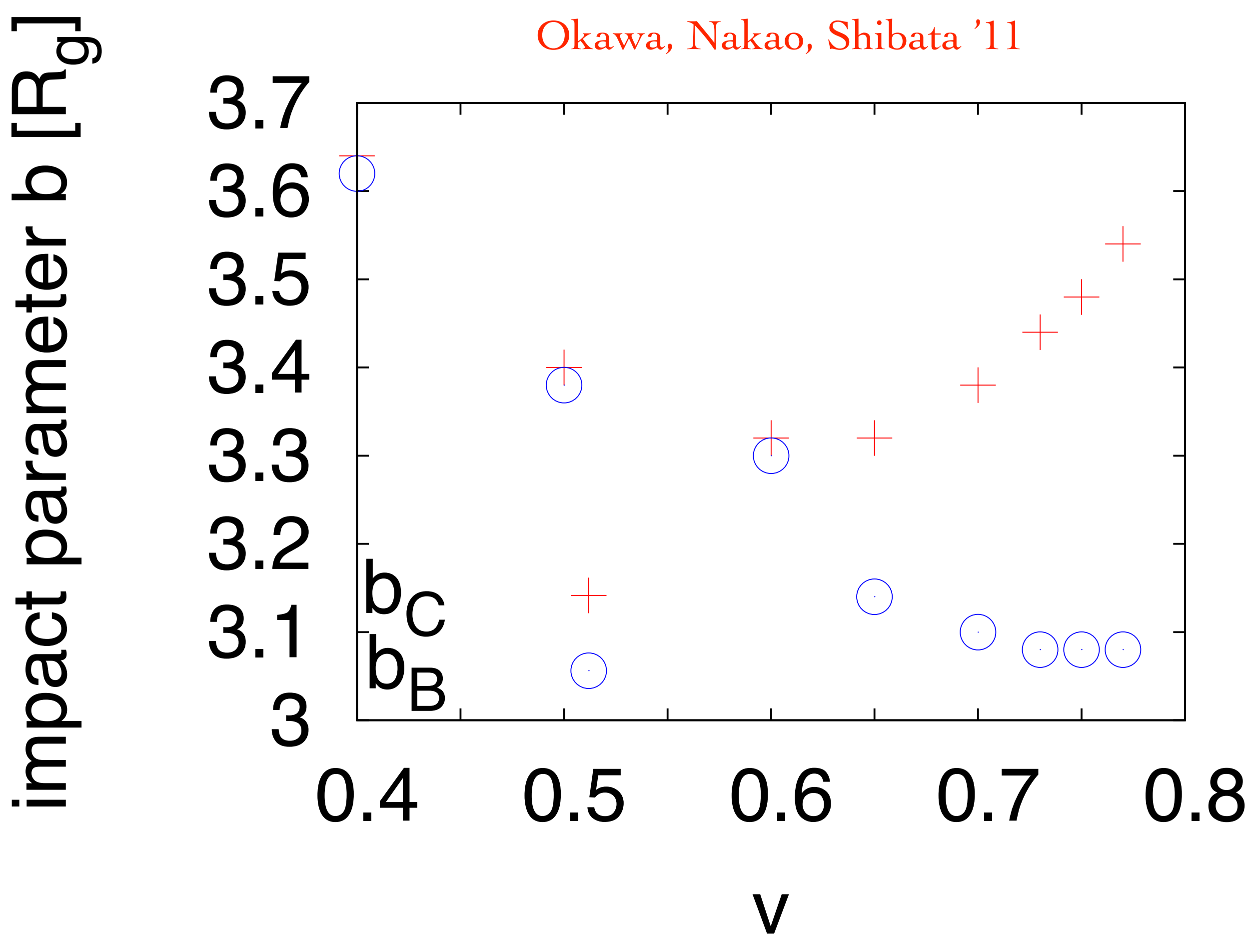
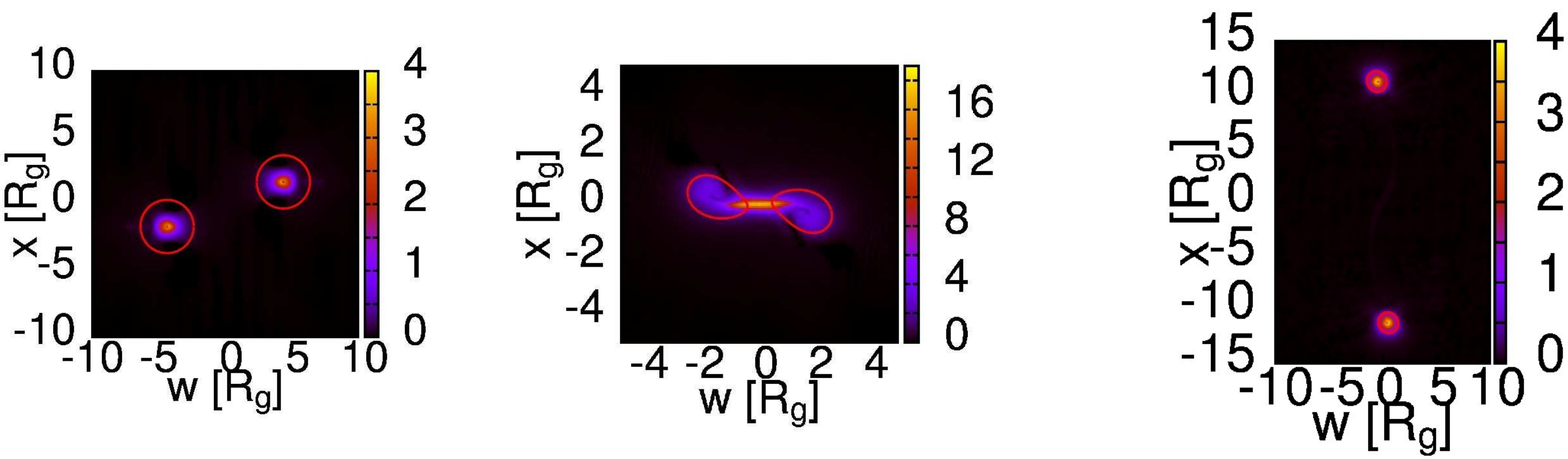
Other results in higher D:

Scattering in D=5

Okawa, Nakao, Shibata '11





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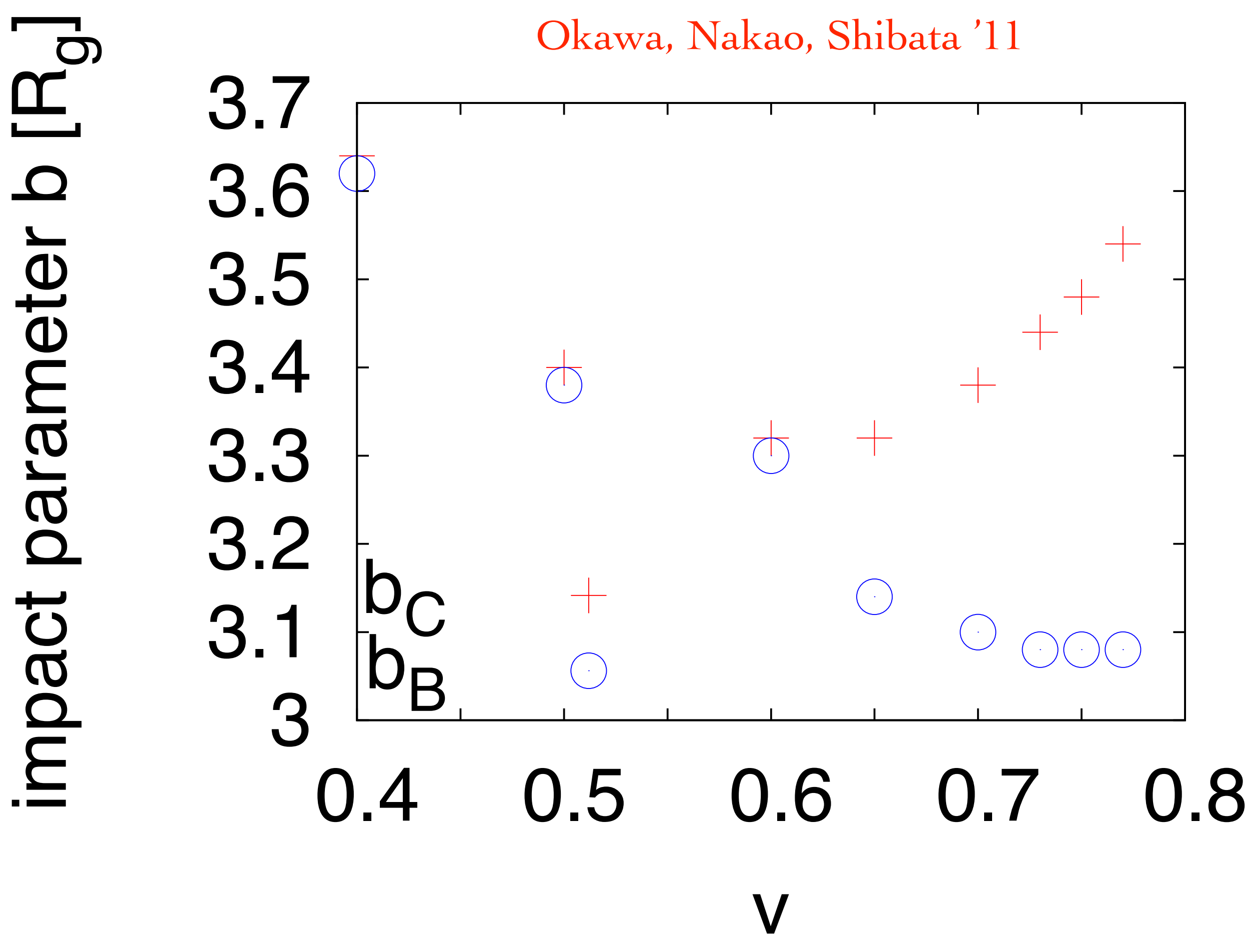
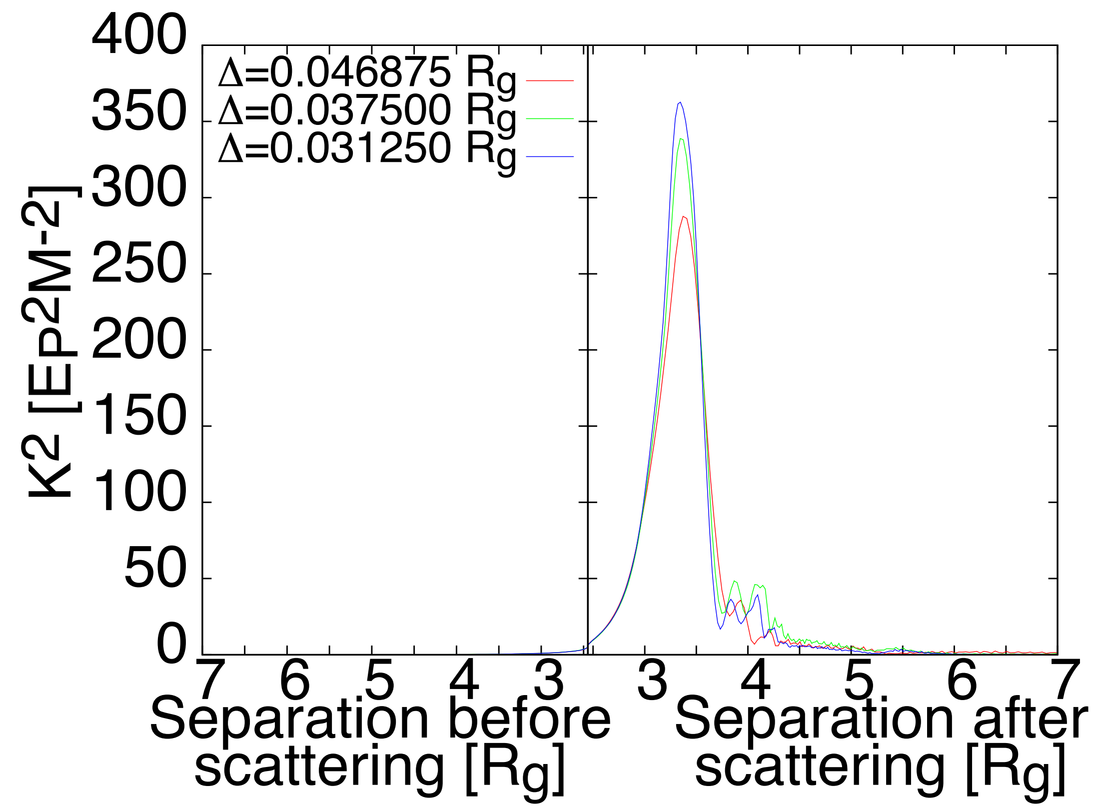
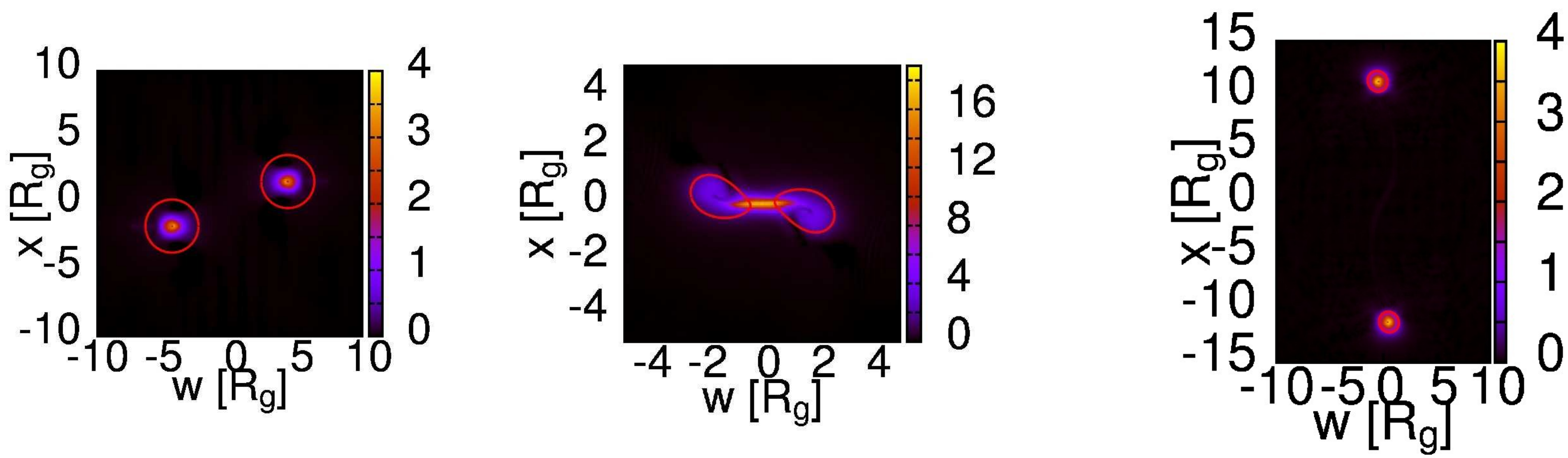




$$\mathcal{K}_{\max} \simeq 19 \left(\frac{E_P}{M}\right).$$



Part II:

Shock wave collisions in D-dimensions

(semi-analytical technique)

Gravitational field of a point particle at rest:

$$ds^2 = -\left(1 - \frac{16\pi G_D M}{(D-2)\Omega_{D-2}} \frac{1}{r^{D-3}}\right) dt^2 + \left(1 - \frac{16\pi G_D M}{(D-2)\Omega_{D-2}} \frac{1}{r^{D-3}}\right)^{-1} dr^2 + r^2 d\Omega_{D-2}^2\,,$$

Curved

Curved • Curved

Curved

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Curved

Curved • Curved

Curved

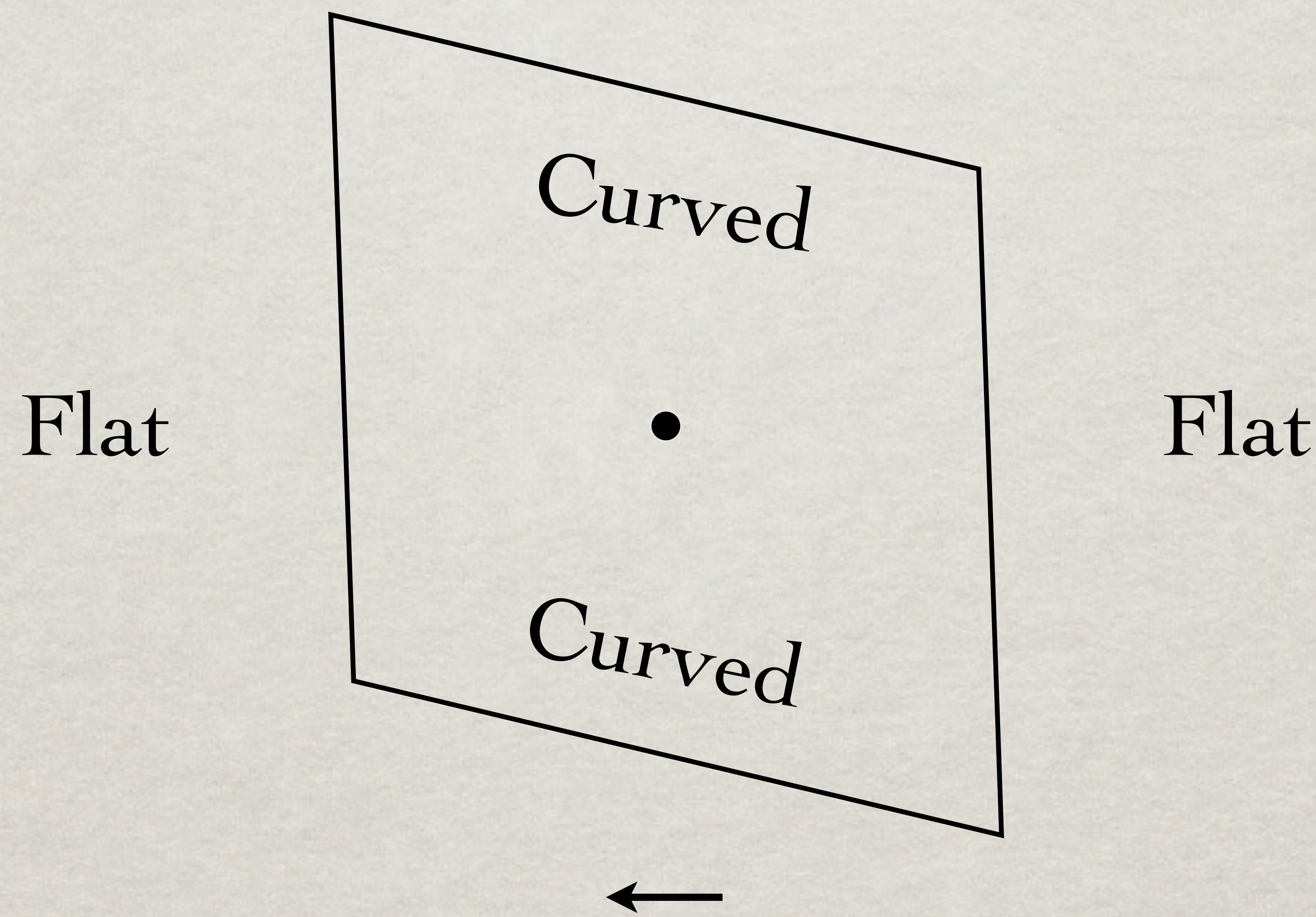
←

Boost particle and take a limit of fixed energy

Gravitational field of a point particle moving at the speed c:

$$ds^2 = -dudv + d\rho^2 + \rho^2 d\Omega_{D-3}^2 + \kappa\Phi(\rho)\delta(u)du^2 \,,$$

Aichelburg-Sexl shock wave



$$\Phi(\rho) = \begin{cases} -2\ln(\rho)\,, & D=4 \\ \dfrac{2}{(D-4)\rho^{D-4}}\,, & D>4 \end{cases}$$

Gravitational field of a point particle moving at the speed c:

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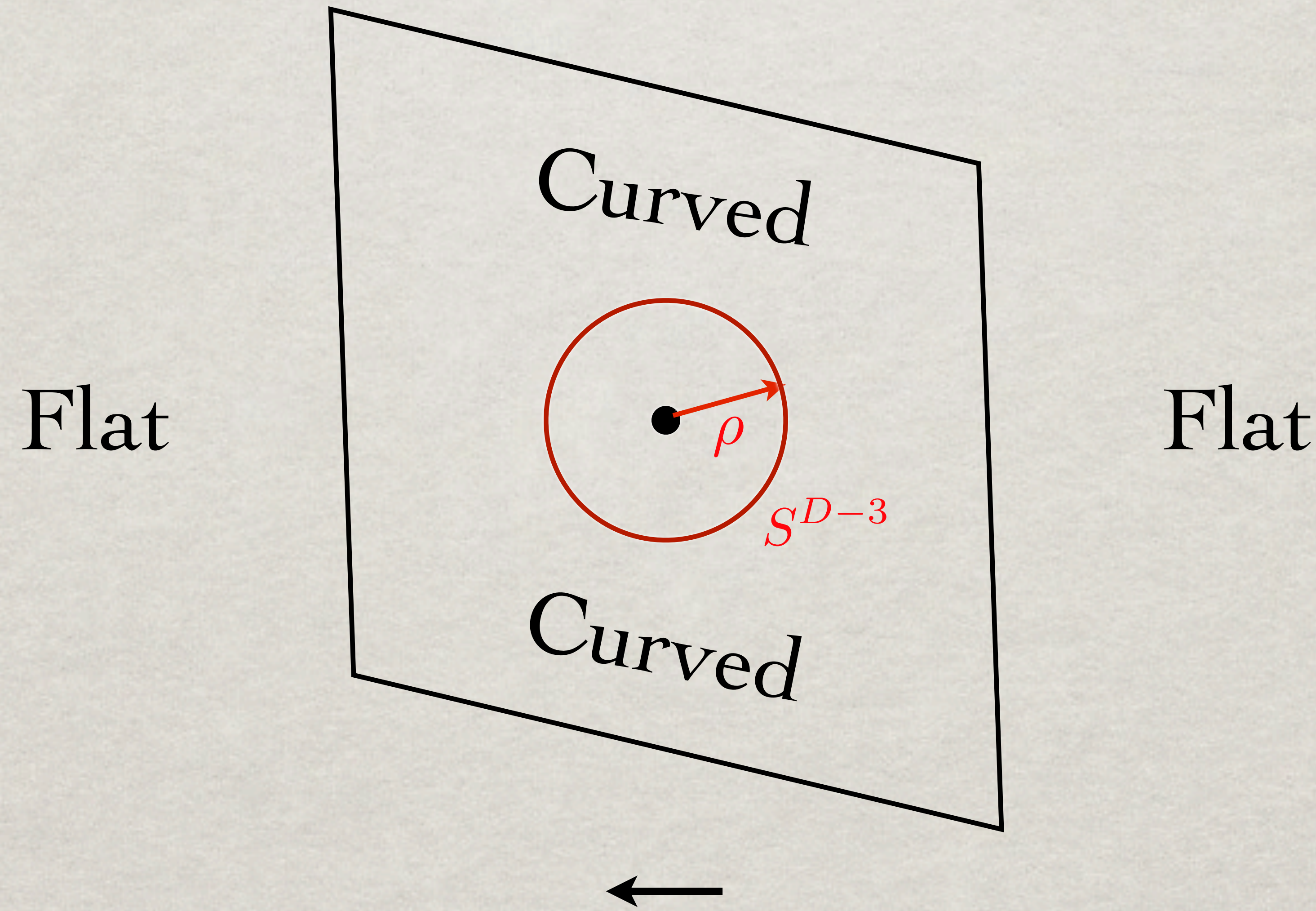

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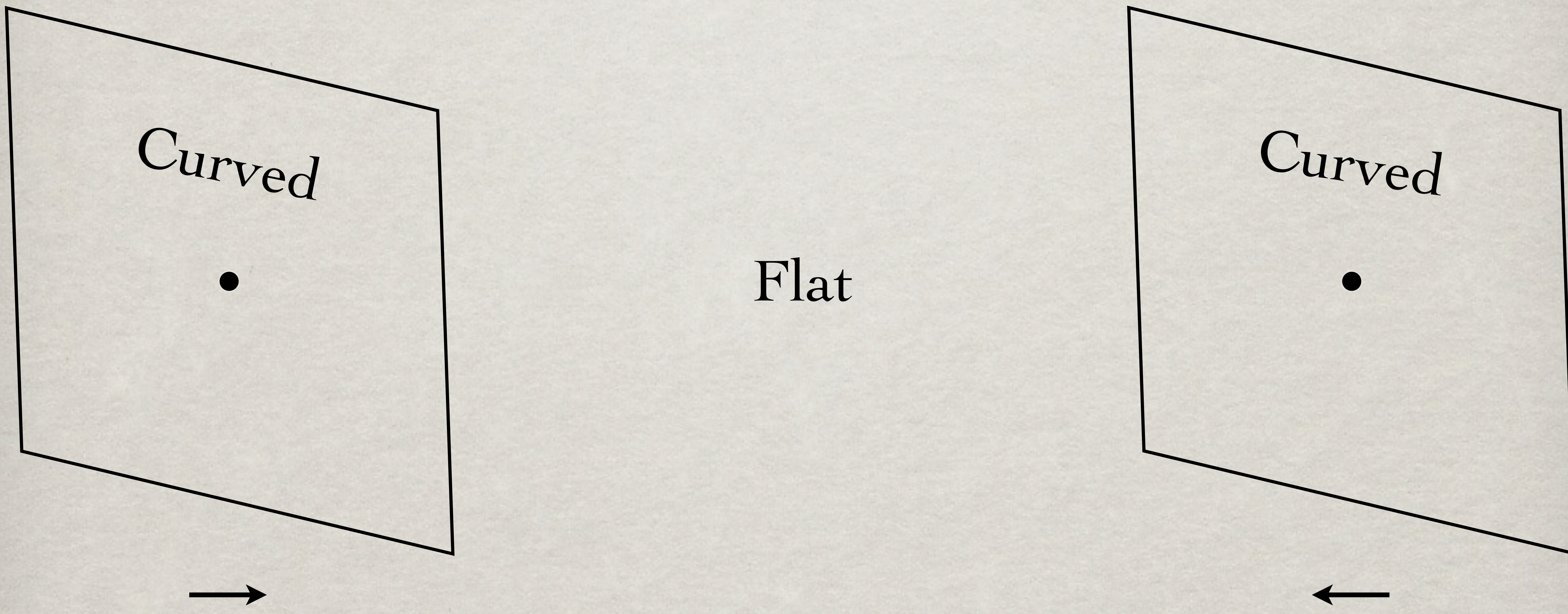
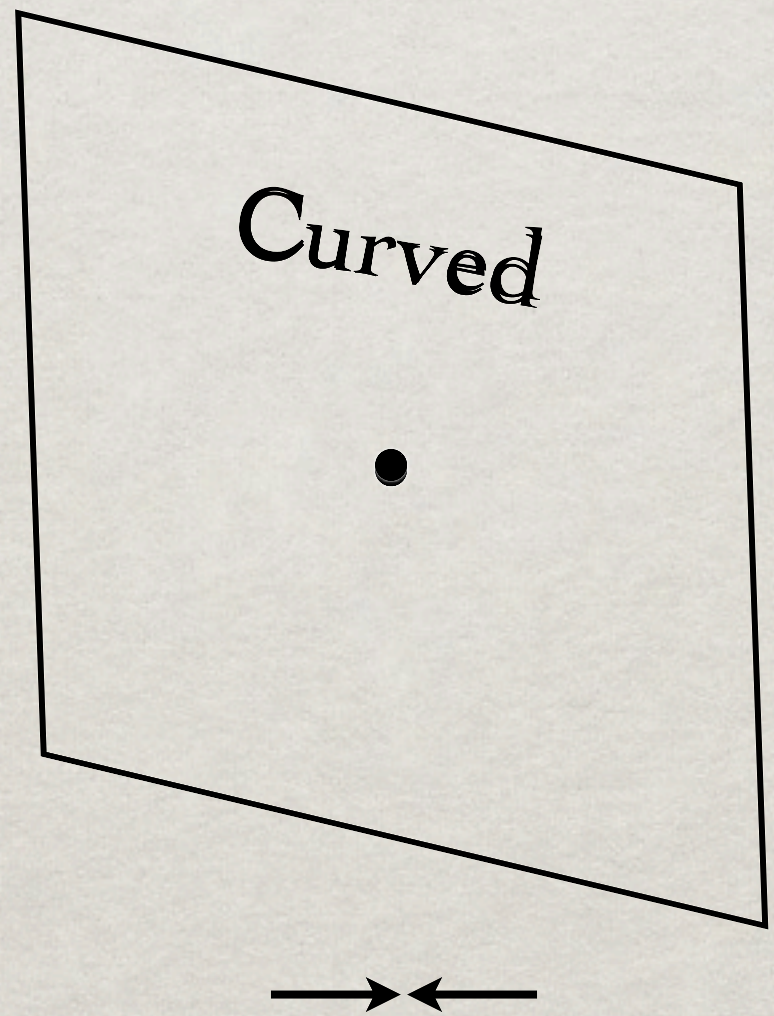
Gravitational field of two colliding shock waves:



Metric is the superposition of the two shock waves (until collision)

Gravitational field of two colliding shock waves:



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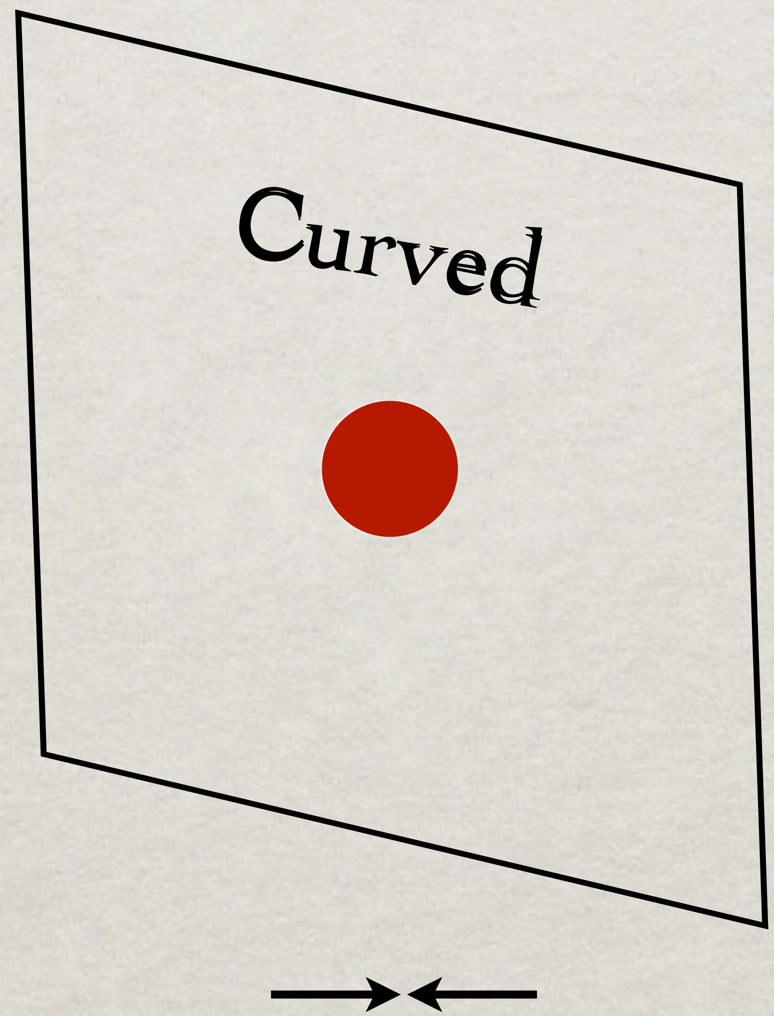
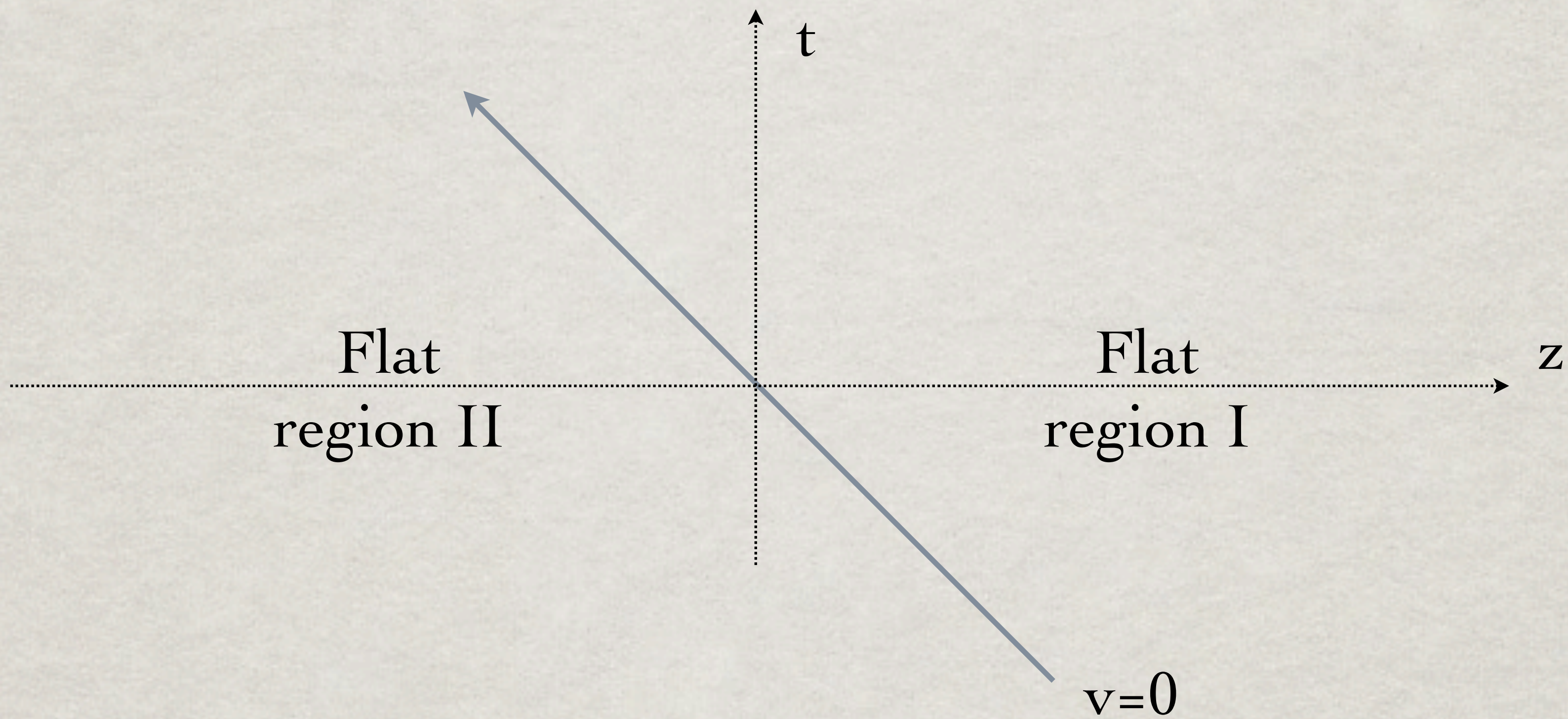


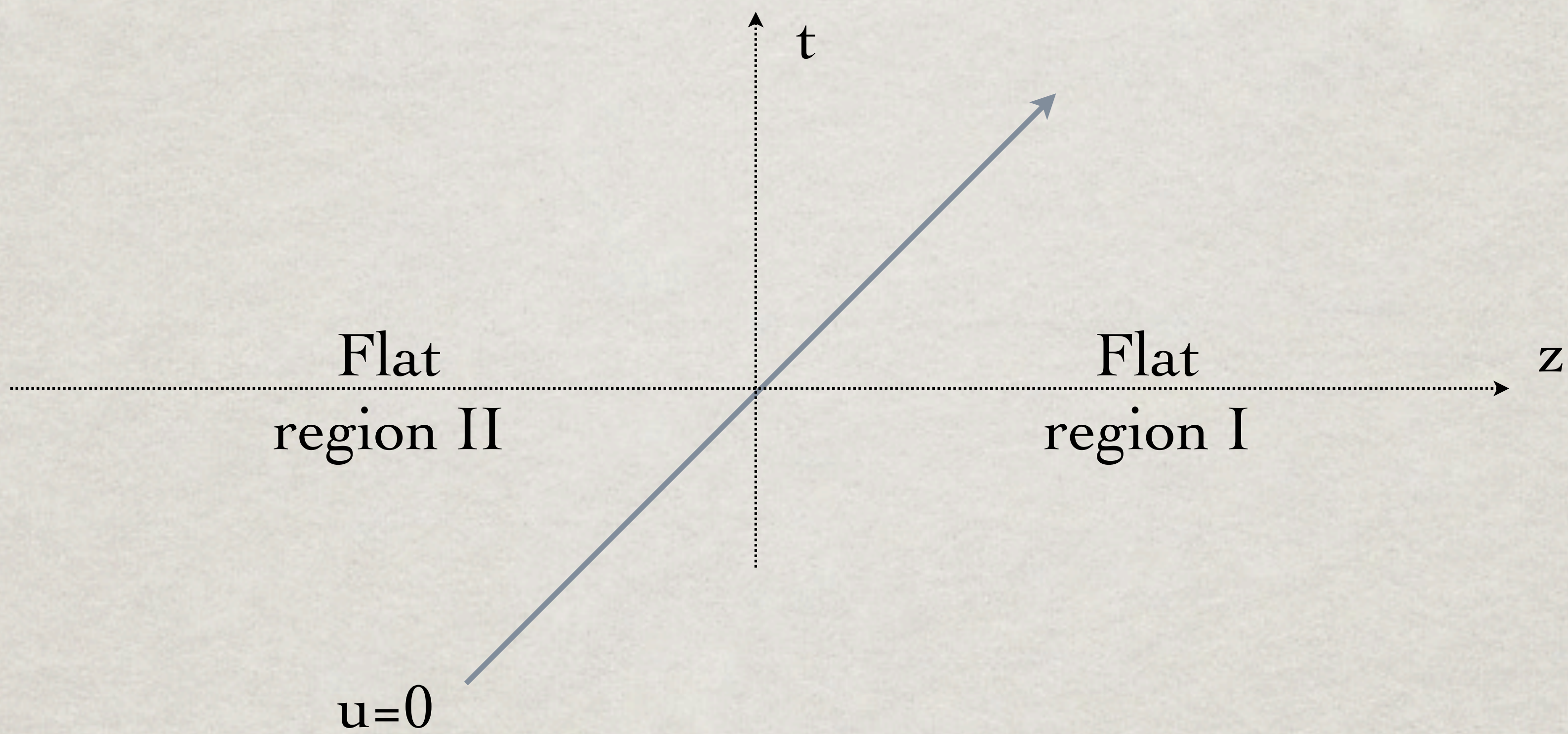
In a head on collision we expect a black hole to form
(smoking gun: find an apparent horizon)

Shock Wave Collisions



$$ds^2_{AS}(E, v = 0)$$

Shock Wave Collisions



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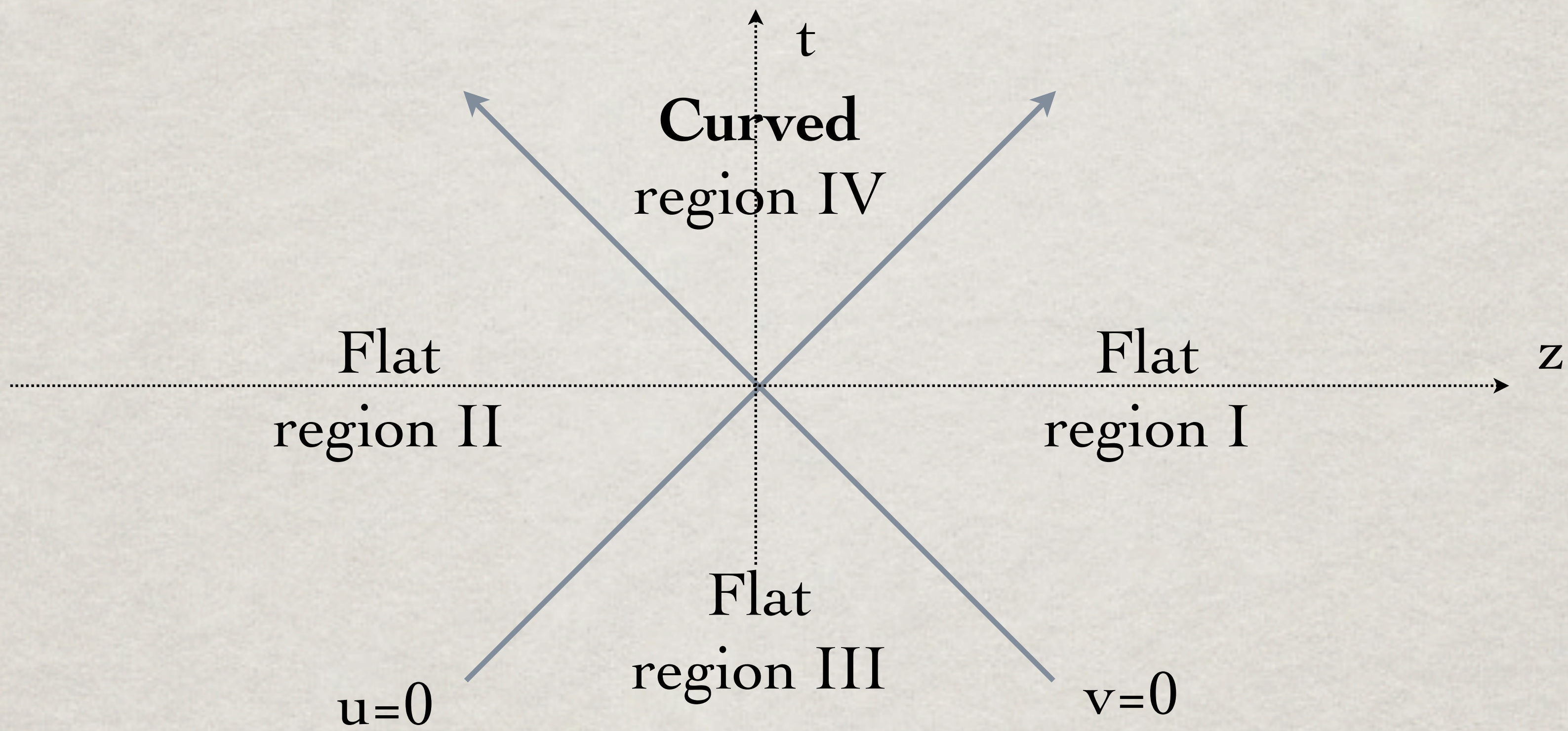
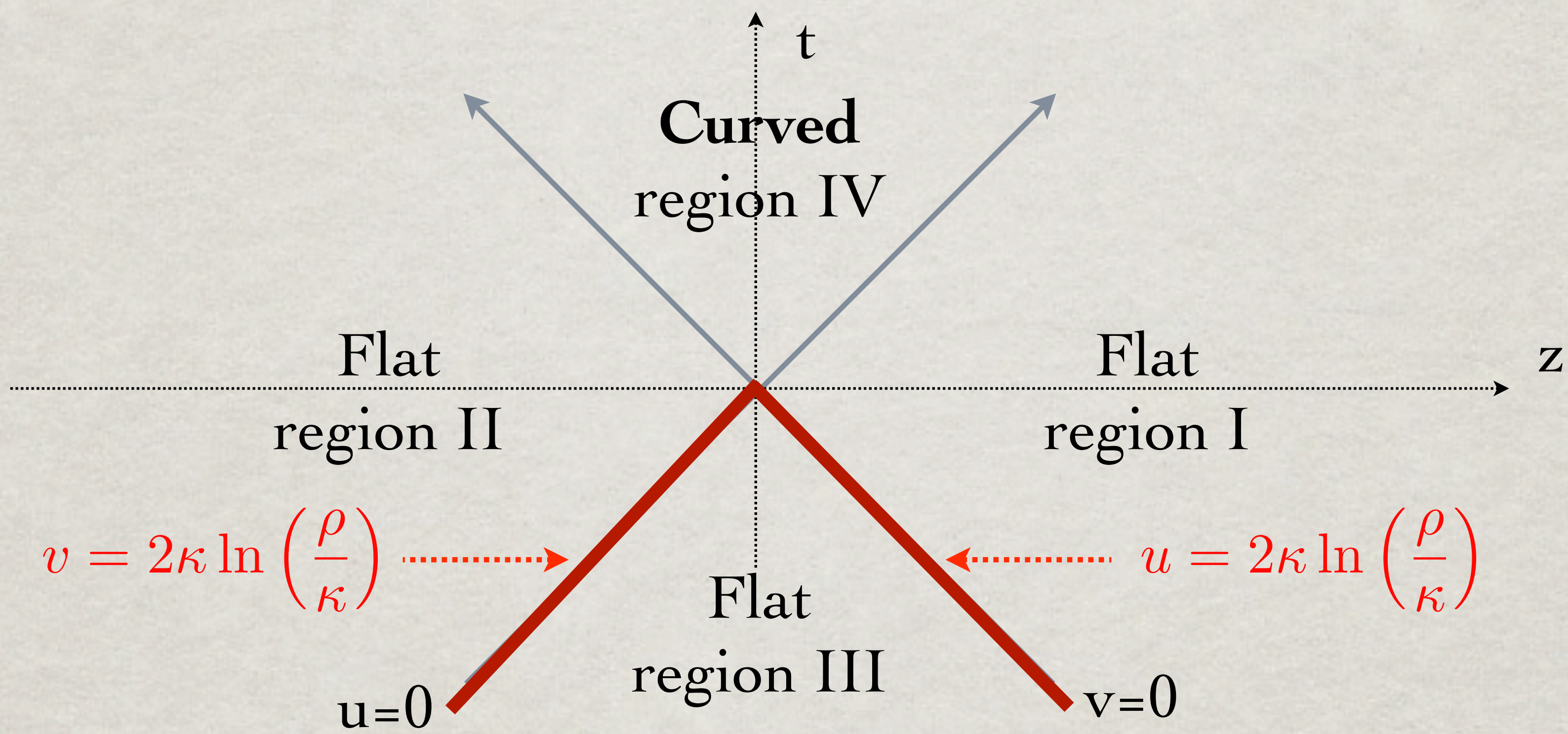
Shock Wave Collisions



regions I, II and III $\quad ds^2_{AS}(E, u=0) \quad + \quad ds^2_{AS}(E, v=0) \quad$ region IV ?

Shock Wave Collisions

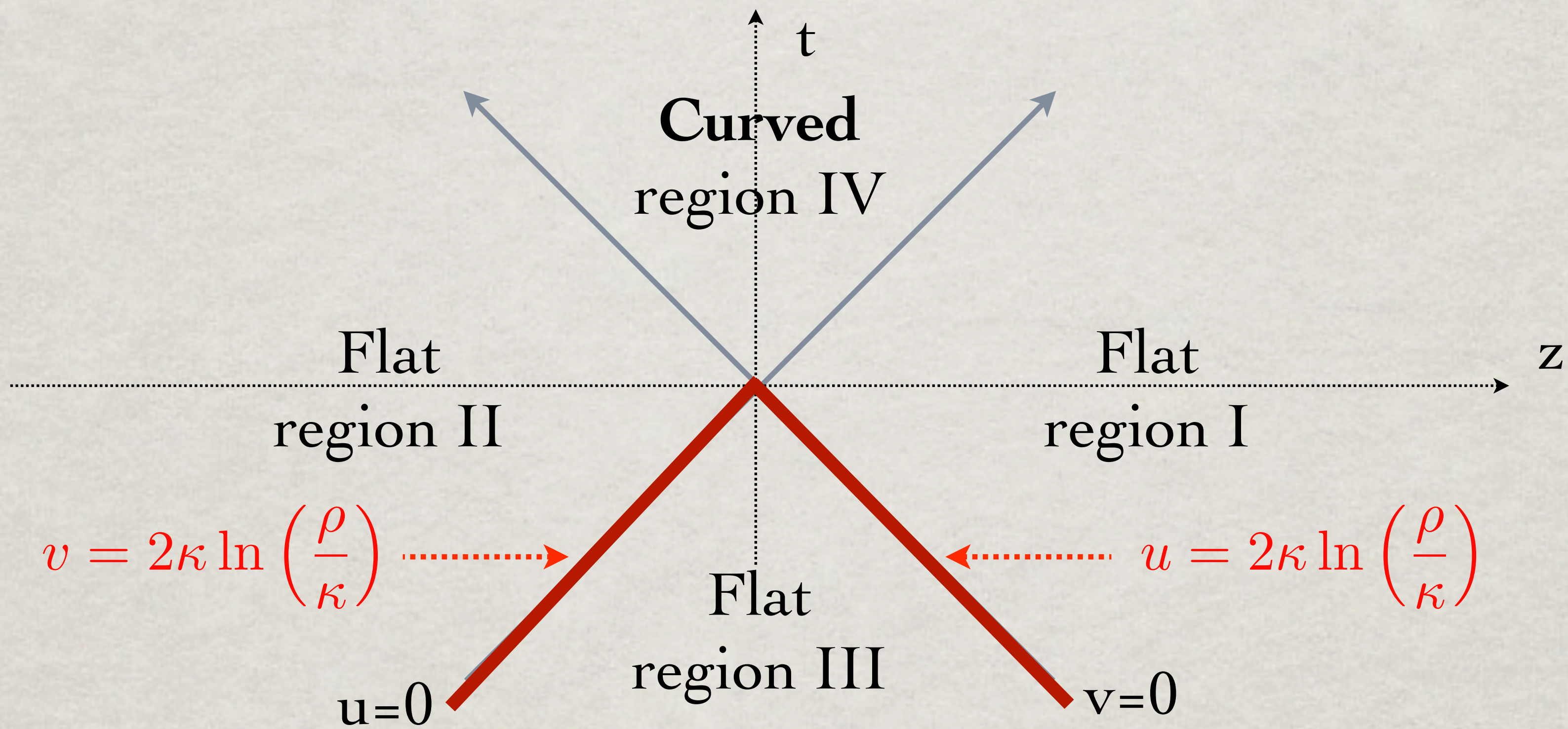


regions I, II and III $\quad ds^2_{AS}(E, u=0) \quad + \quad ds^2_{AS}(E, v=0) \quad$ region IV ?

Penrose Penrose '74 found an apparent horizon with area

$$\text{Area} = 32\pi E^2 \;\Rightarrow\; E_{rad} \leq \left(1 - \frac{1}{\sqrt{2}}\right) 2E$$

Shock Wave Collisions



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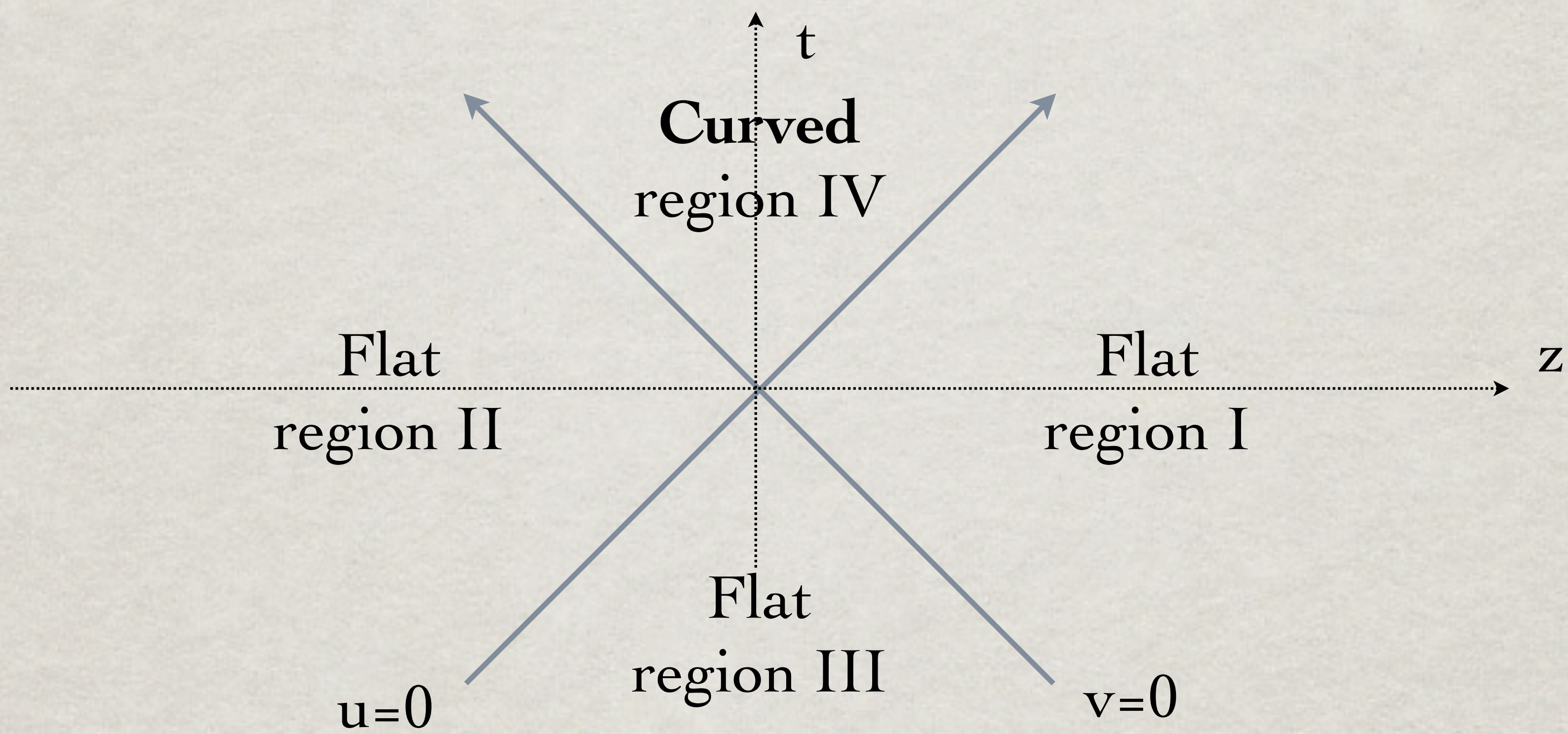
Estimate that no more than 29.3% of the initial energy
in the shock waves can be transformed
into gravitational radiation.

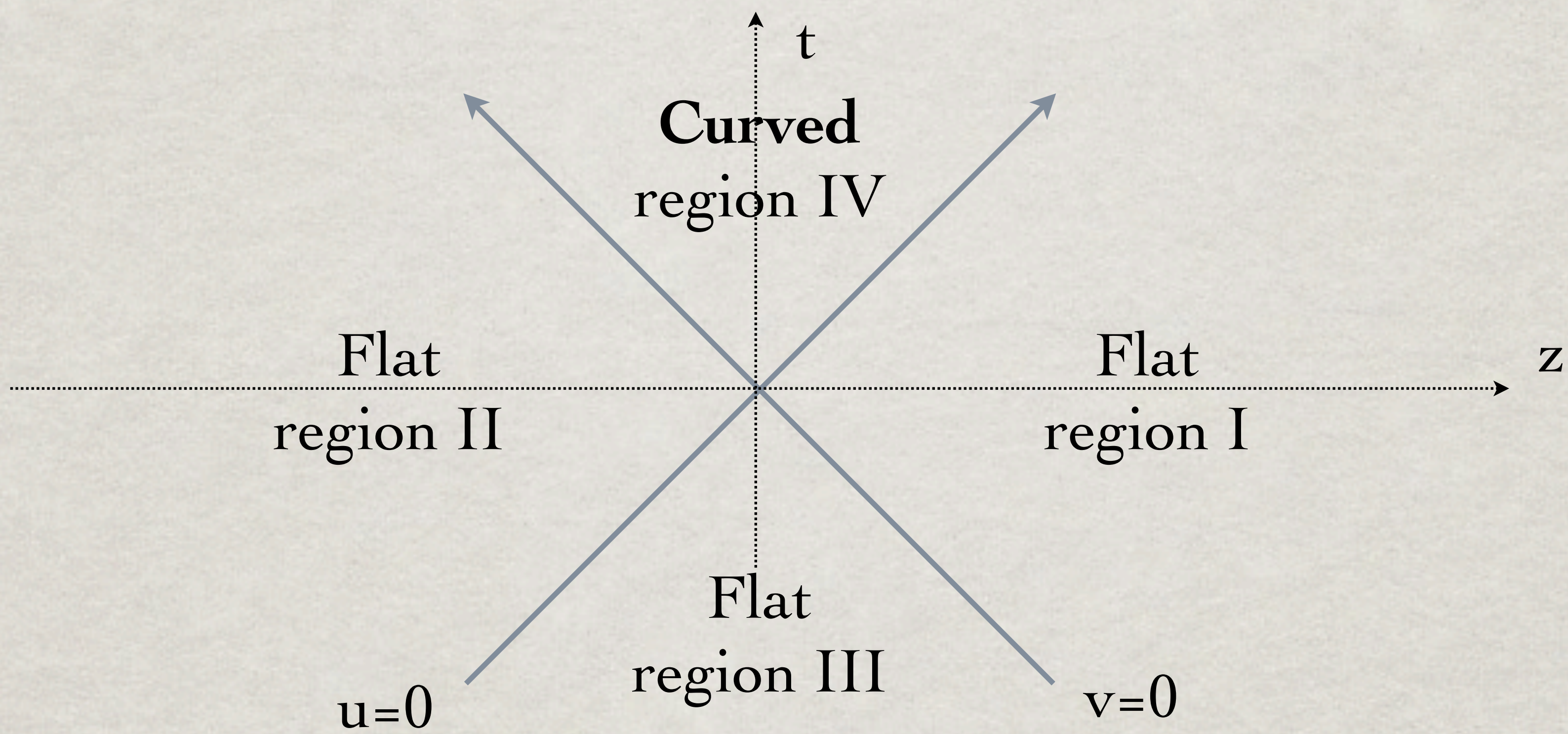
Shock Wave Collisions



regions I, II and III $ds^2_{AS}(E, u=0)$ $+$ $ds^2_{AS}(E, v=0)$ region IV ?

Has the intrinsic geometry of two flat disks at u=0, v=0:

Shock Wave Collisions



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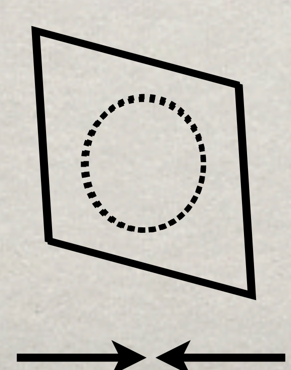
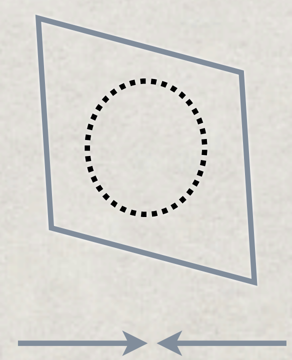
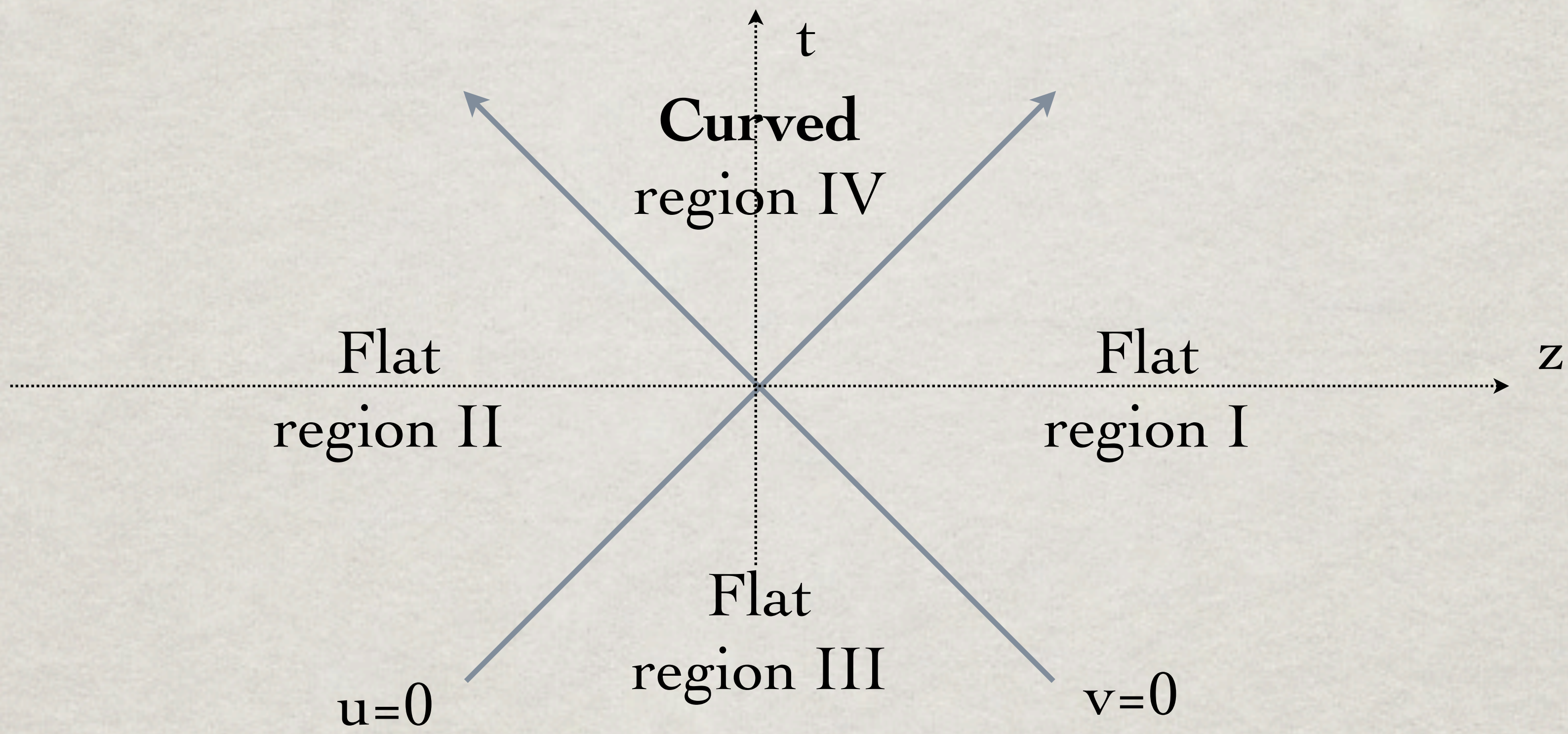


Very similar in D dimensions; at u=0, v=0: Eardley and Giddings '02



D	4	5	6	7	8	9	10	11
AH bound	29.3	33.5	36.1	37.9	39.3	40.4	41.2	41.9

Shock Wave Collisions



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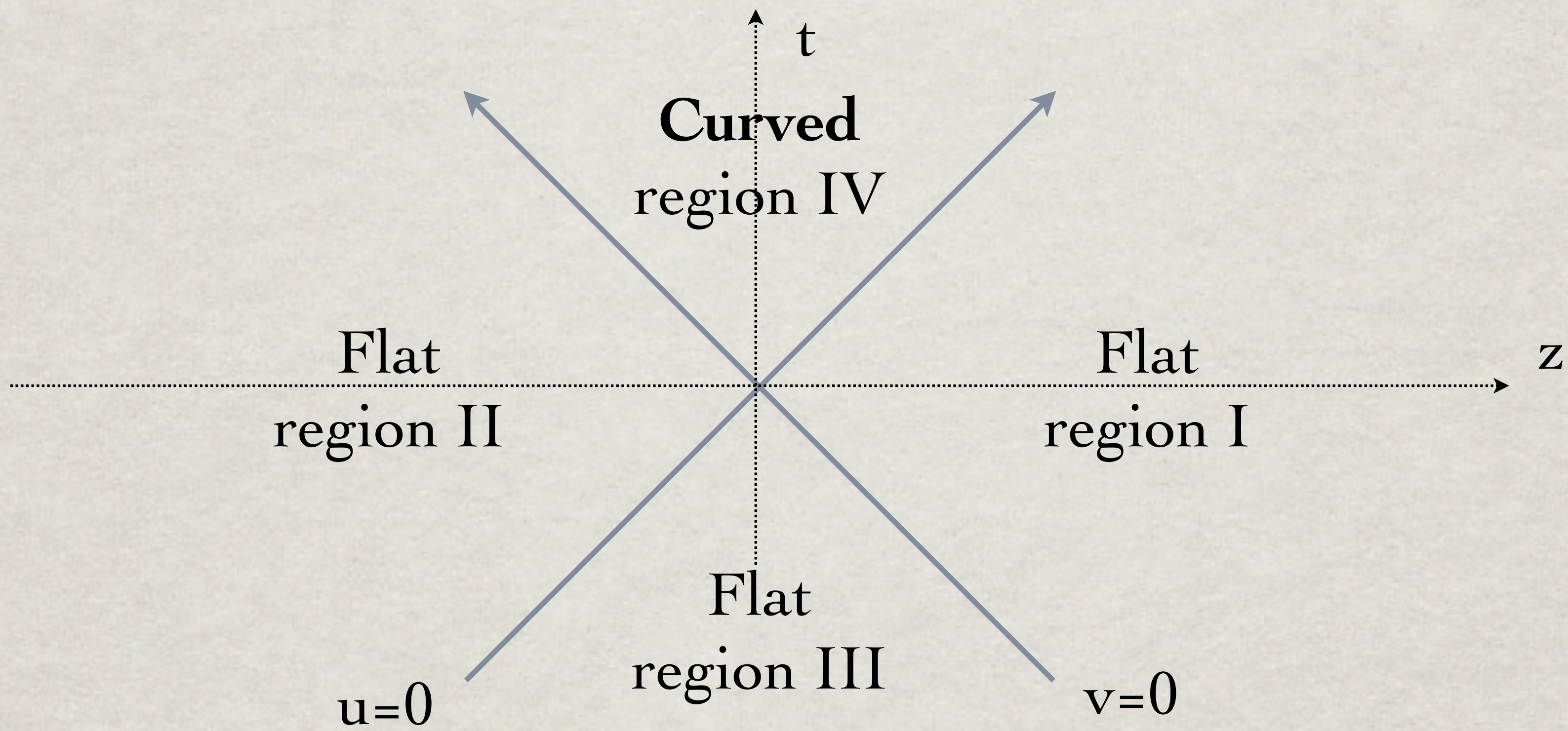
D'Eath and Payne '92 gave a more precise estimate of the energy radiated:

Shock Wave Collisions



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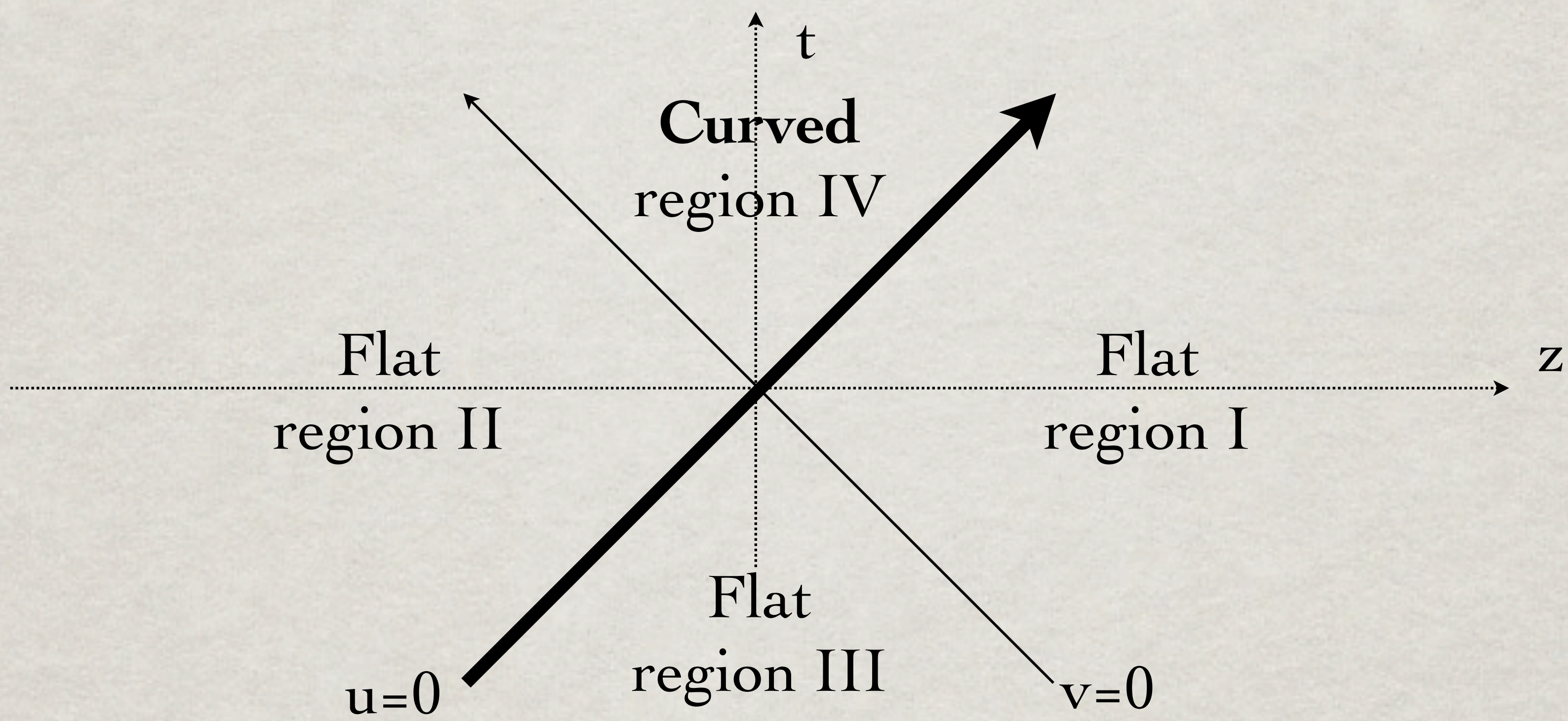

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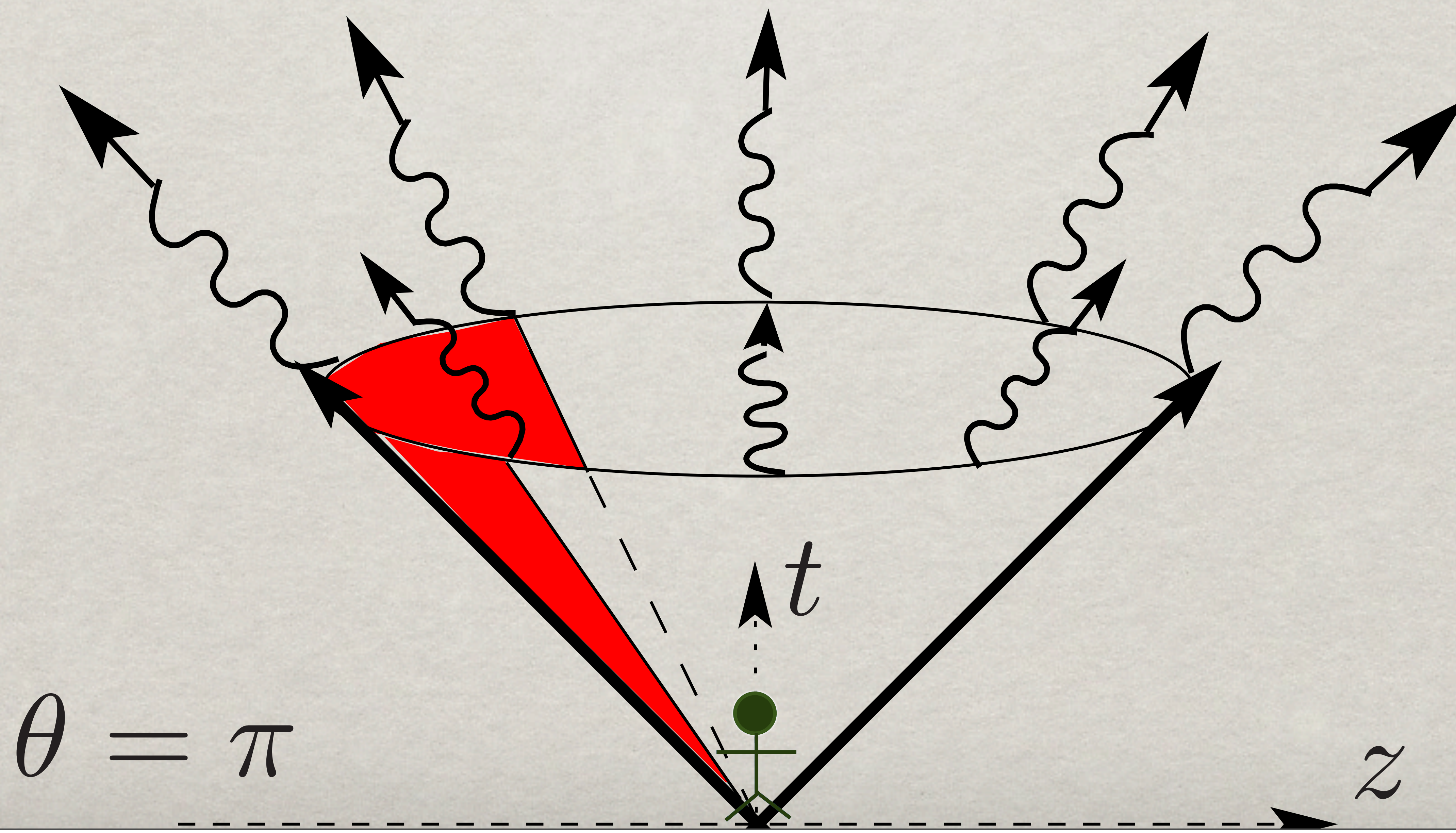
Centre of Mass Frame



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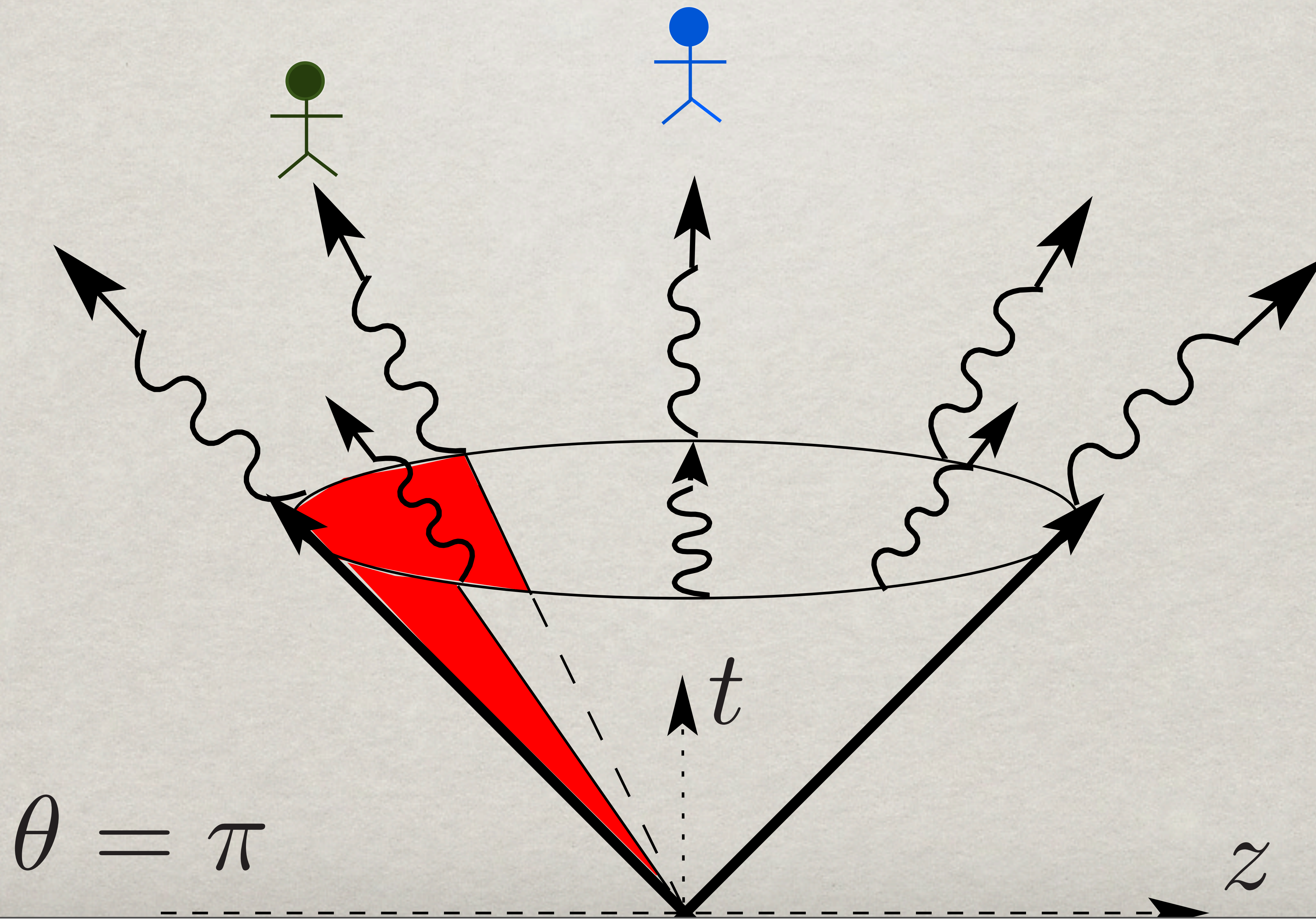
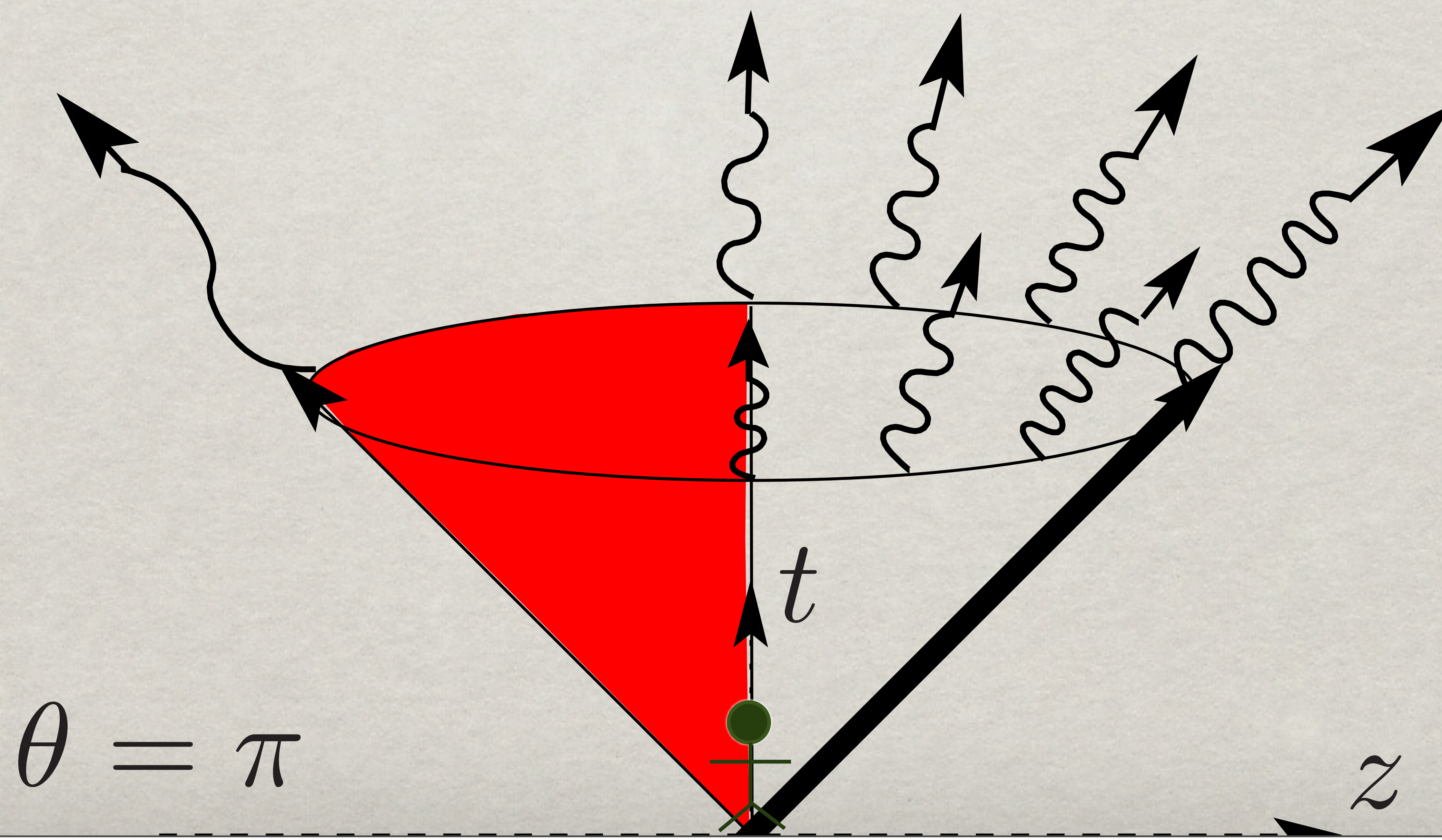


Boosted Frame



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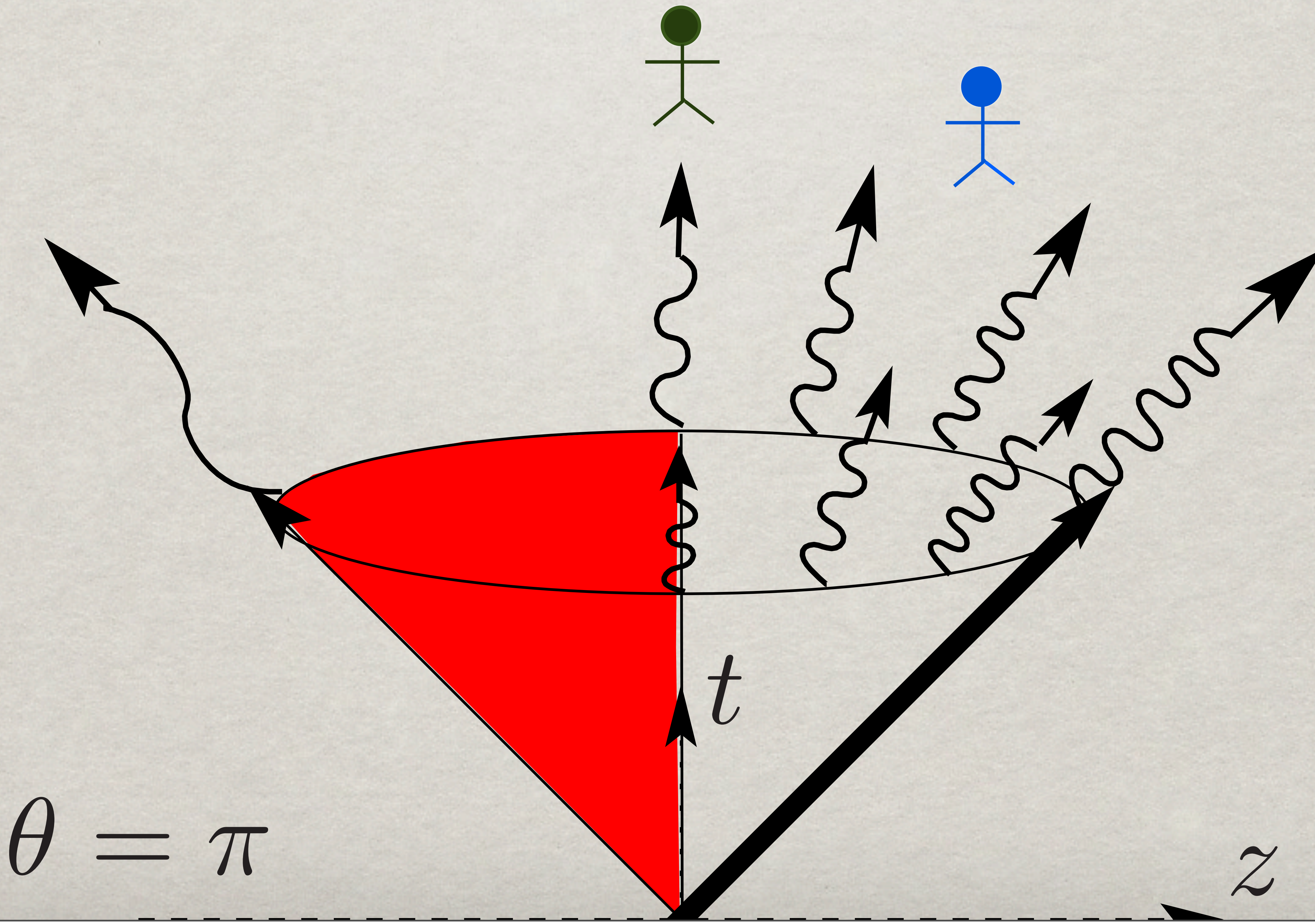

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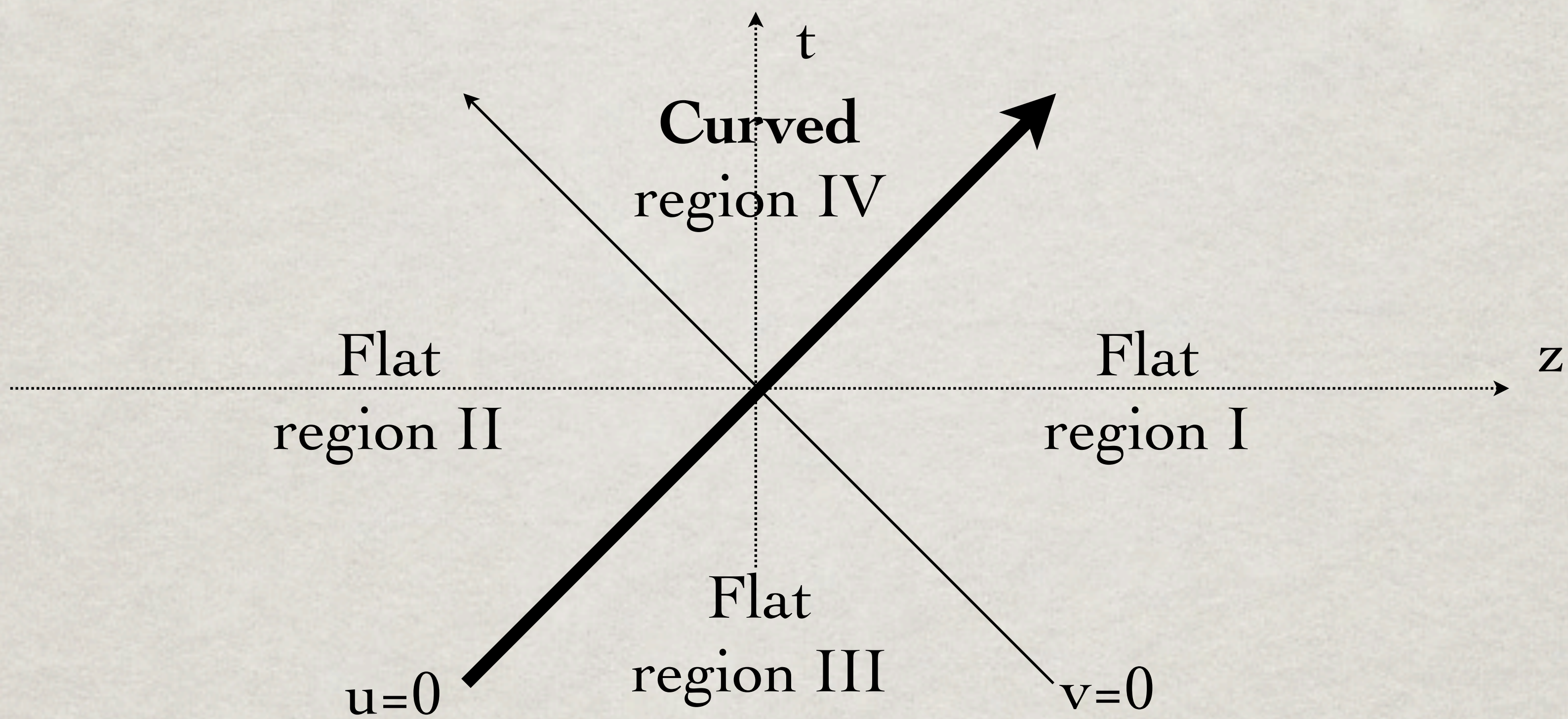

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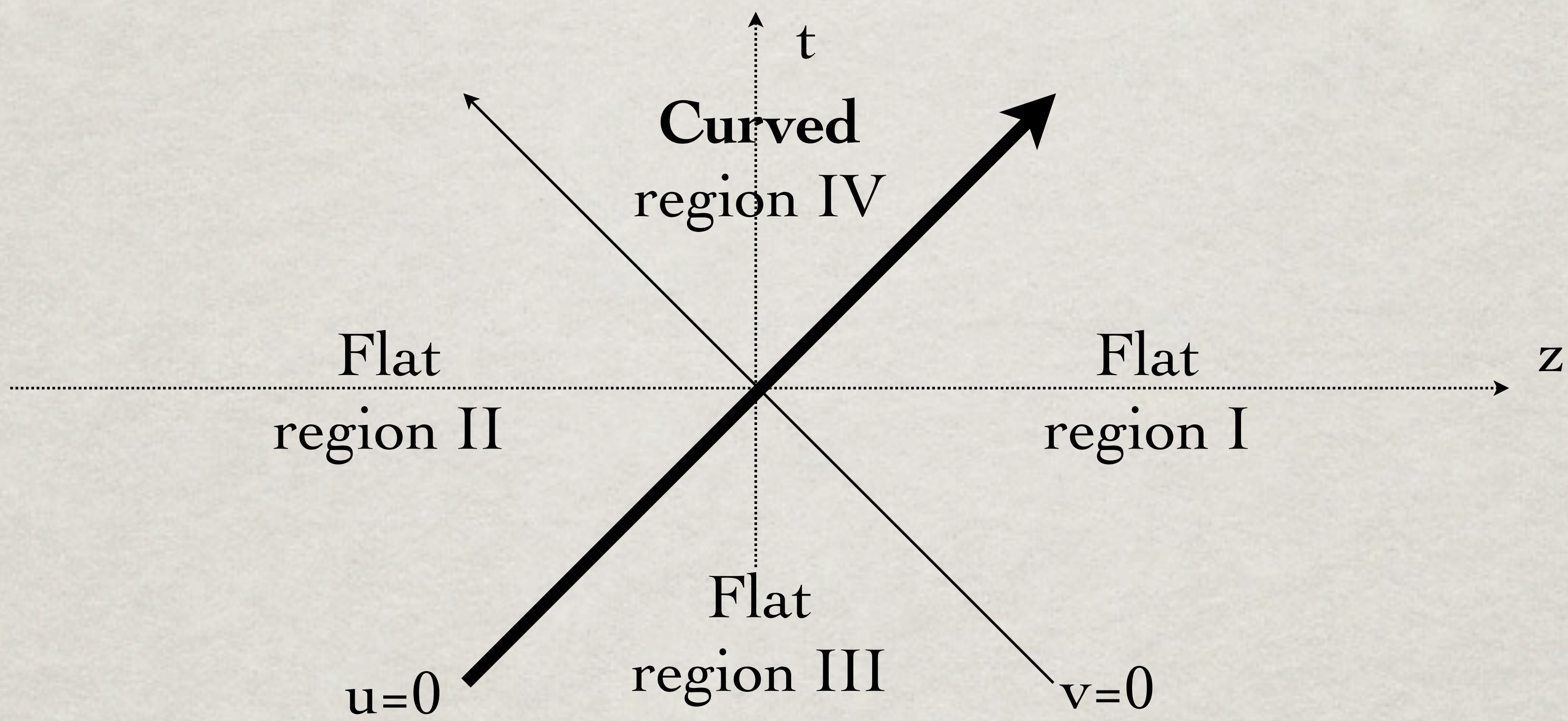

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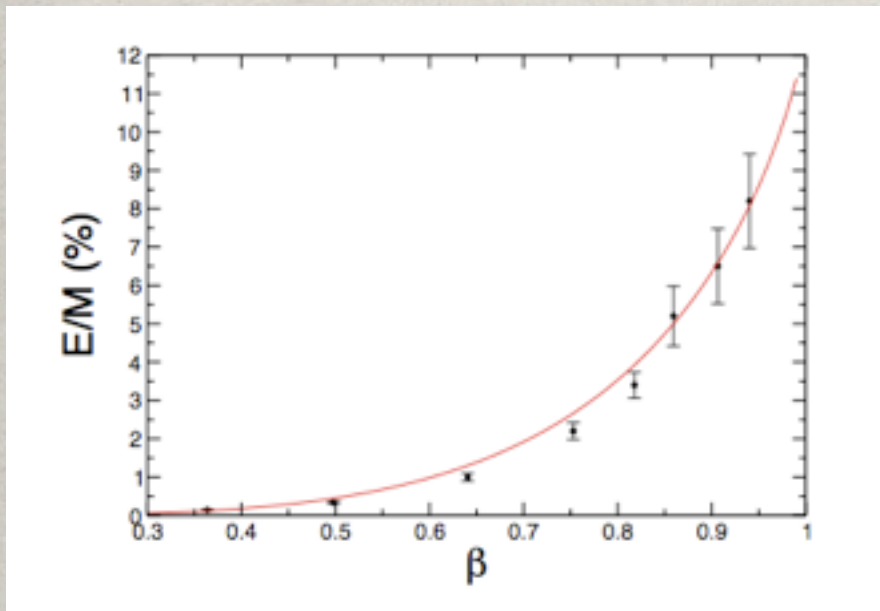
3) computed the "news" function and obtained that 16,3% of the initial shock waves energy is radiated into gravitational radiation

Suggestive agreement with Numerical Relativity I:

Sperhake, Cardoso, Pretorius, Berti, Gonzales, '08



Ultra-relativistic regime

$$\frac{E}{M} \simeq 14 \pm 3\%$$

Luminosity

$$10^{-2}\frac{c^5}{G}$$

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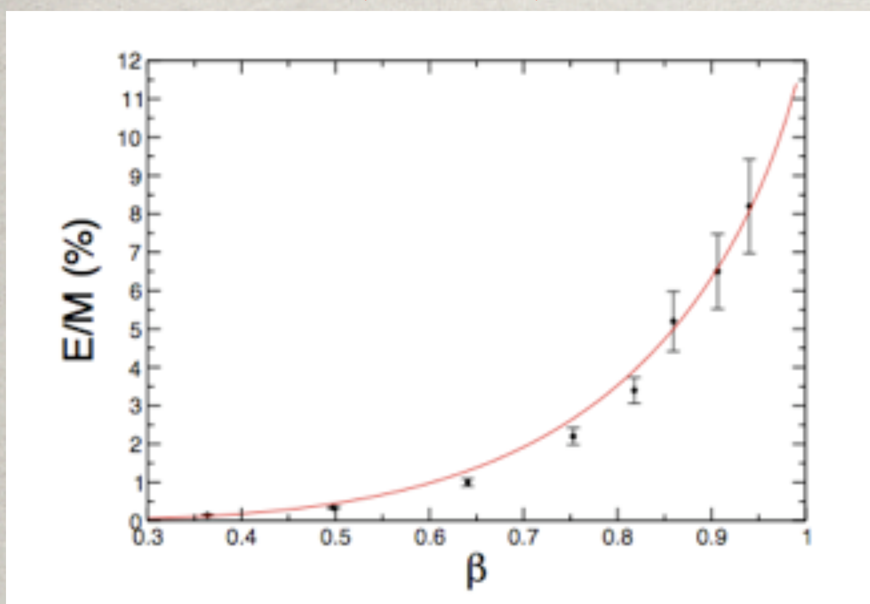

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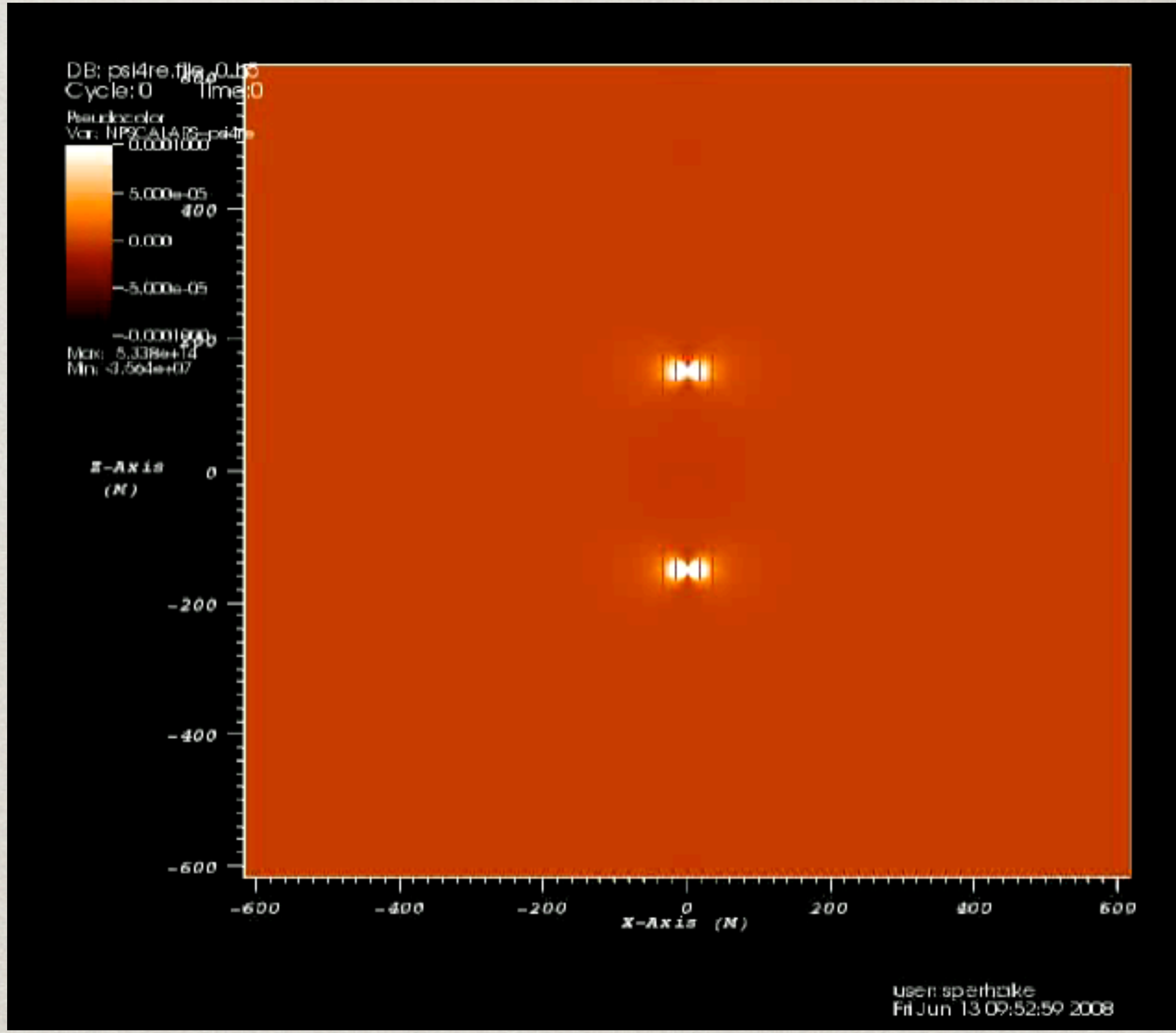
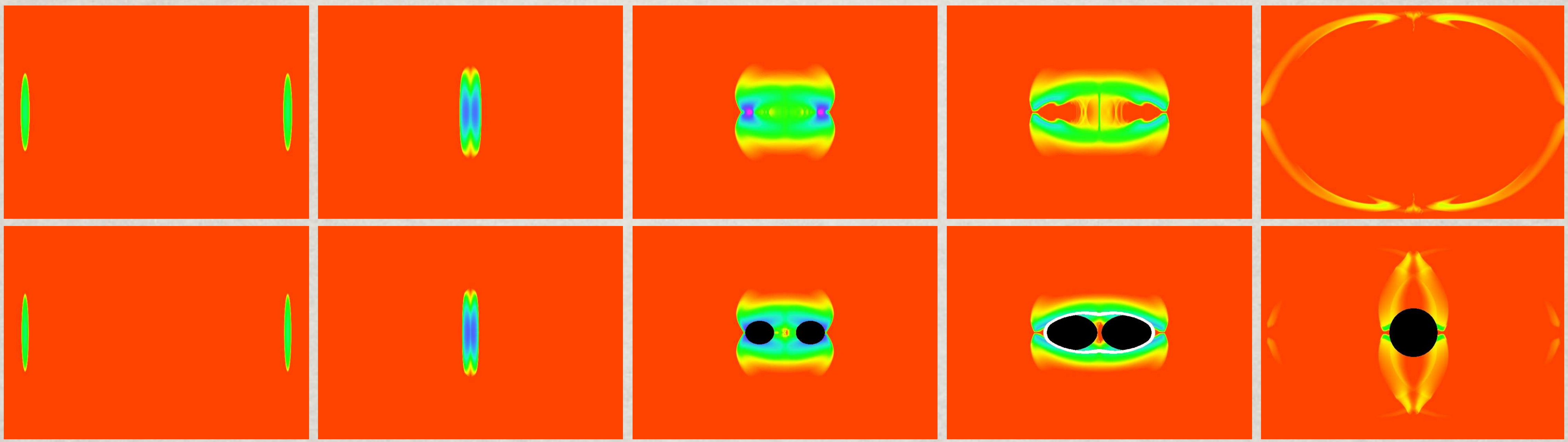


Suggestive agreement with Numerical Relativity II:

Head on collision of fluid particles East, Pretorius '12



Energy radiated: $\frac{E}{M} = 16 \pm 2\% \quad (\gamma = 10)$ $\quad \frac{v}{c} = 0.995$

Luminosity: $1.4 \times 10^{-2} \frac{c^5}{G}$

New qualitative feature: two individual apparent horizons form, before the common one.

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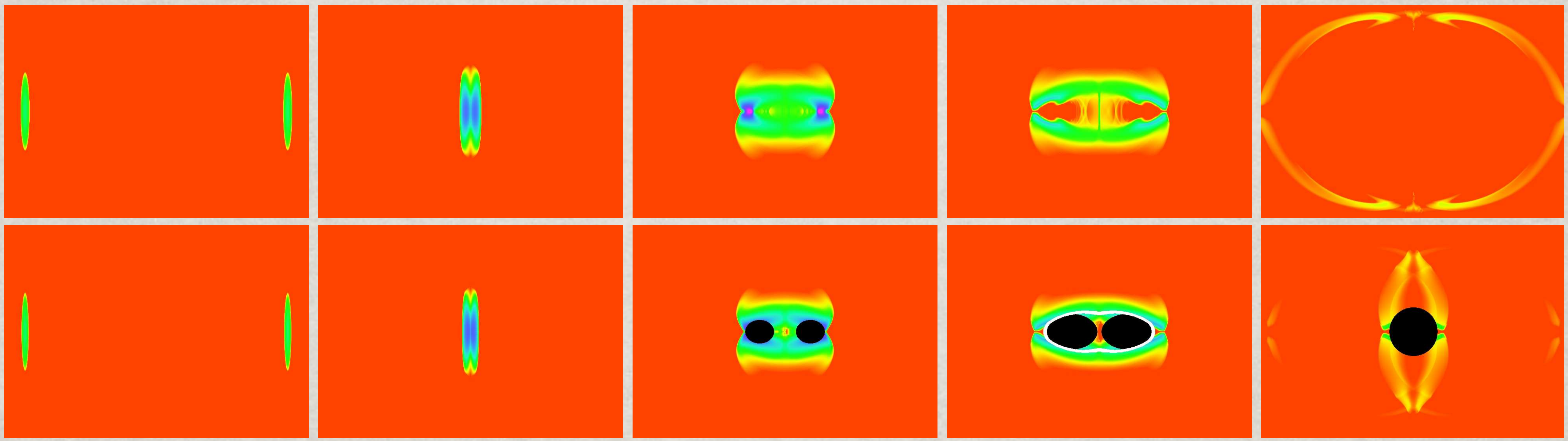

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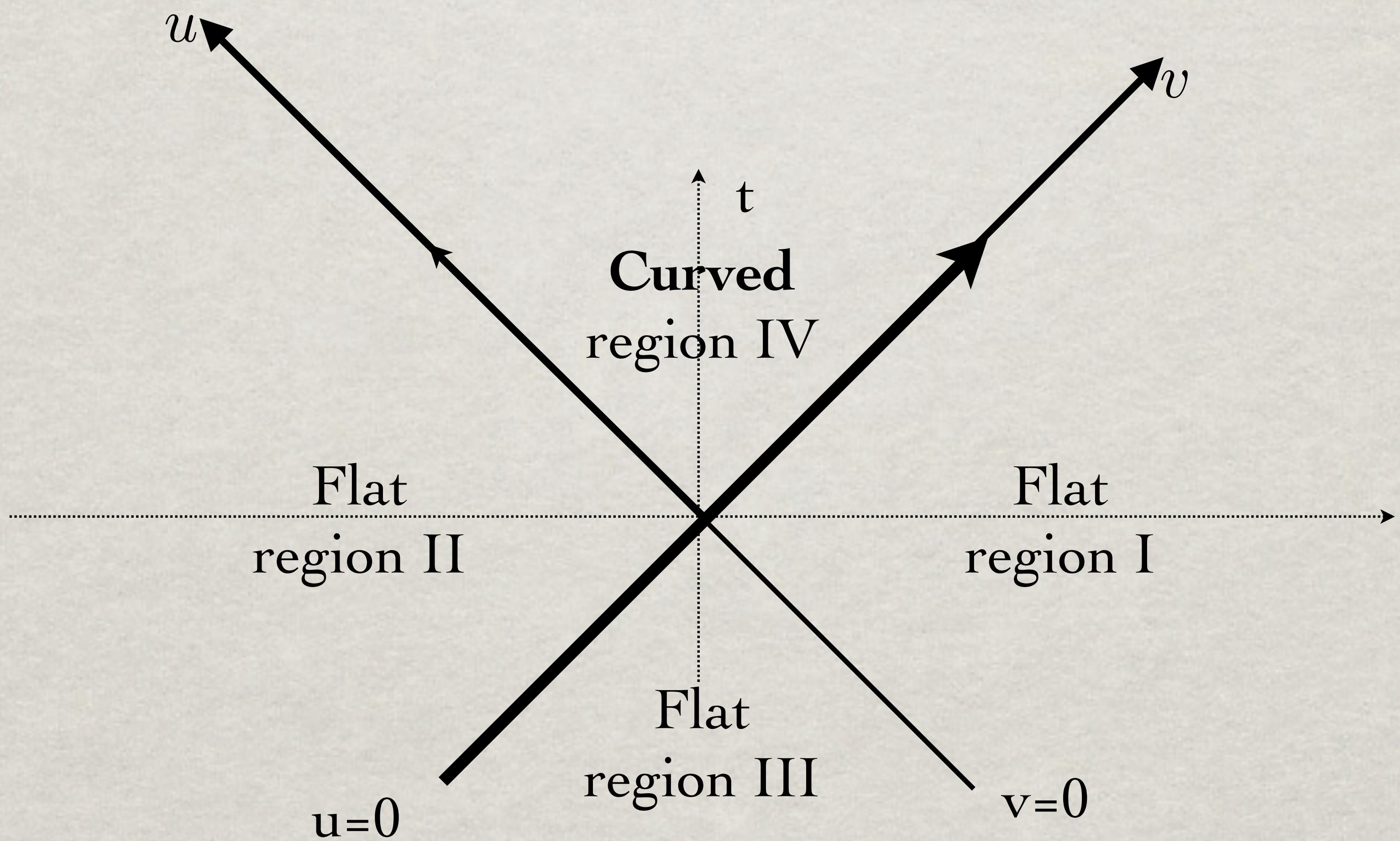
Evidence for "Matter does not Matter"

Compute the fraction of the total energy radiated away (inelasticity) in a head-on collision of two D-dimensional shock waves.

Shock waves approach:

1st order perturbation theory



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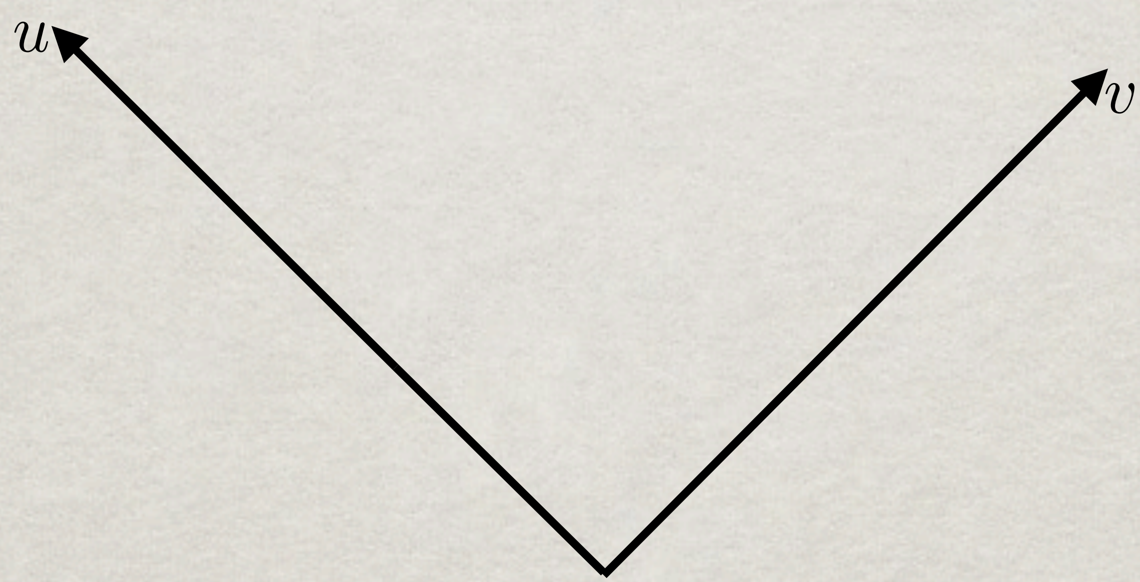

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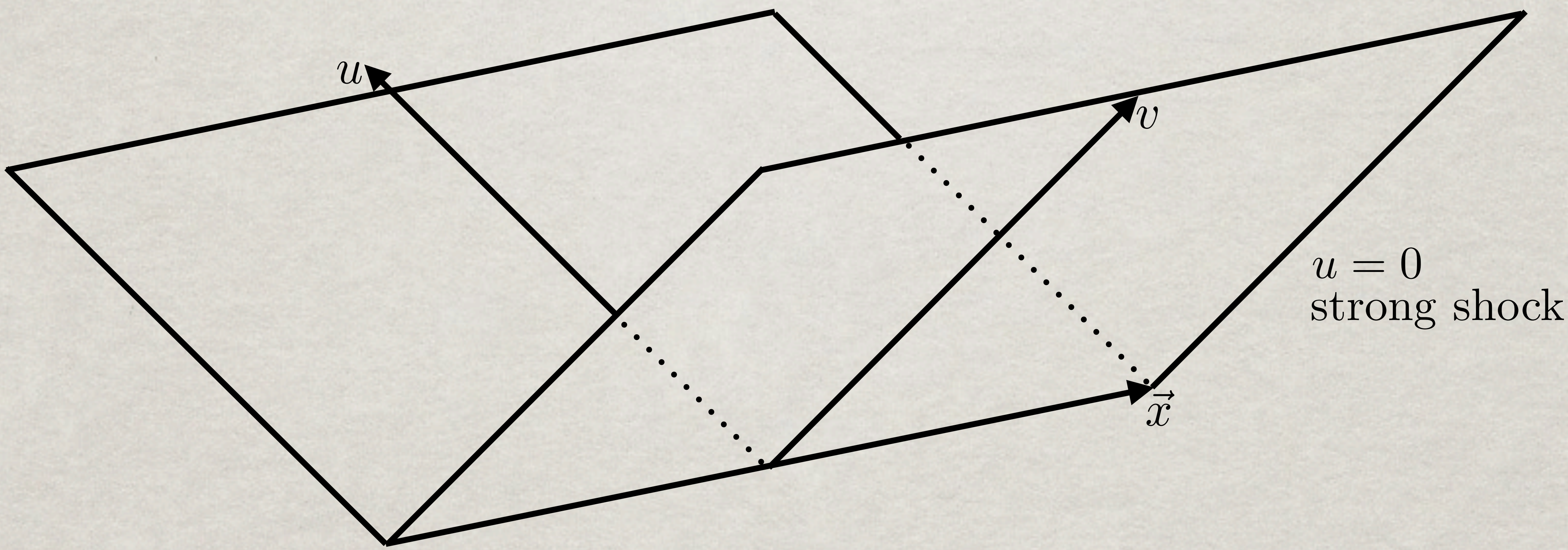

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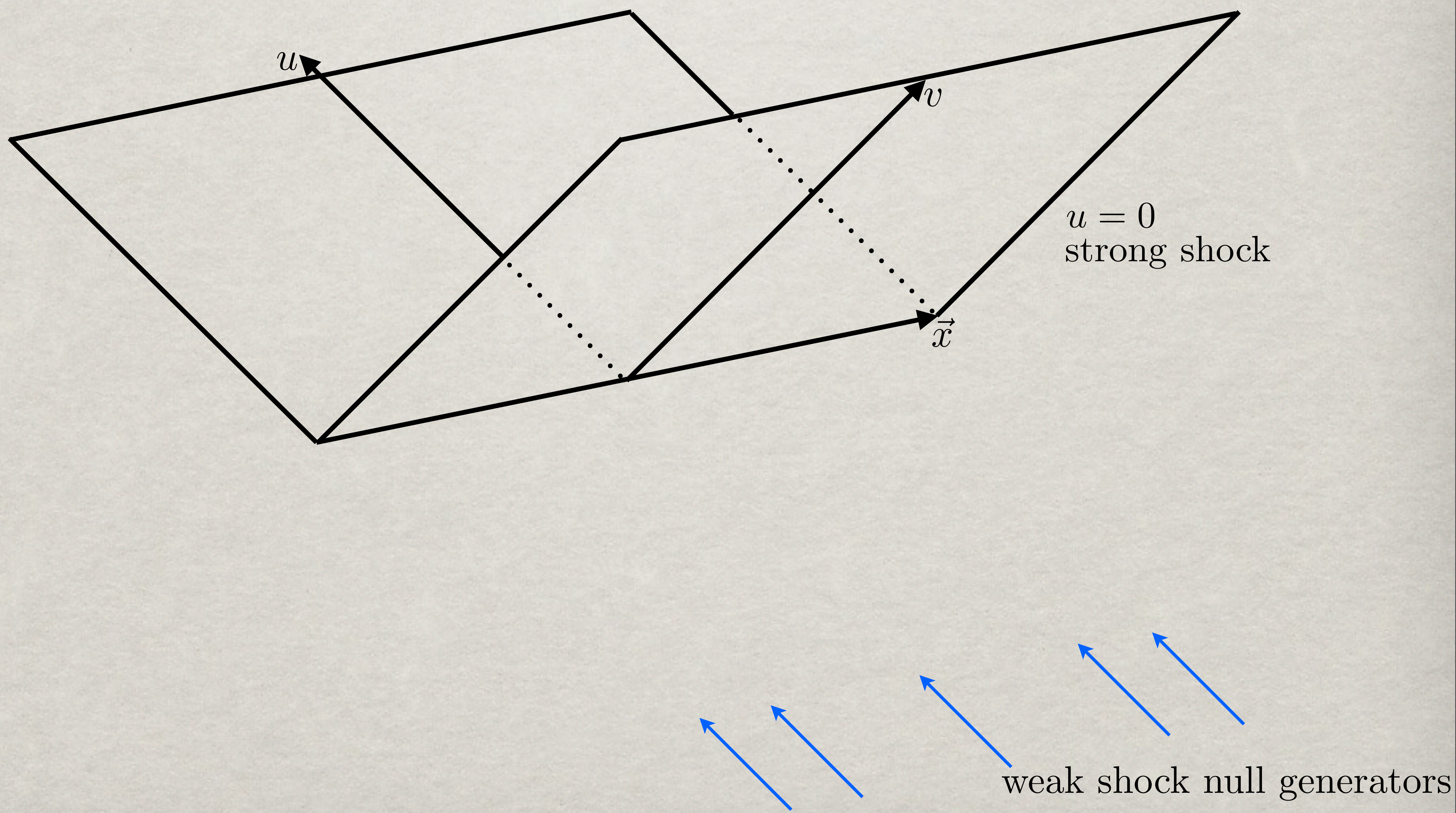

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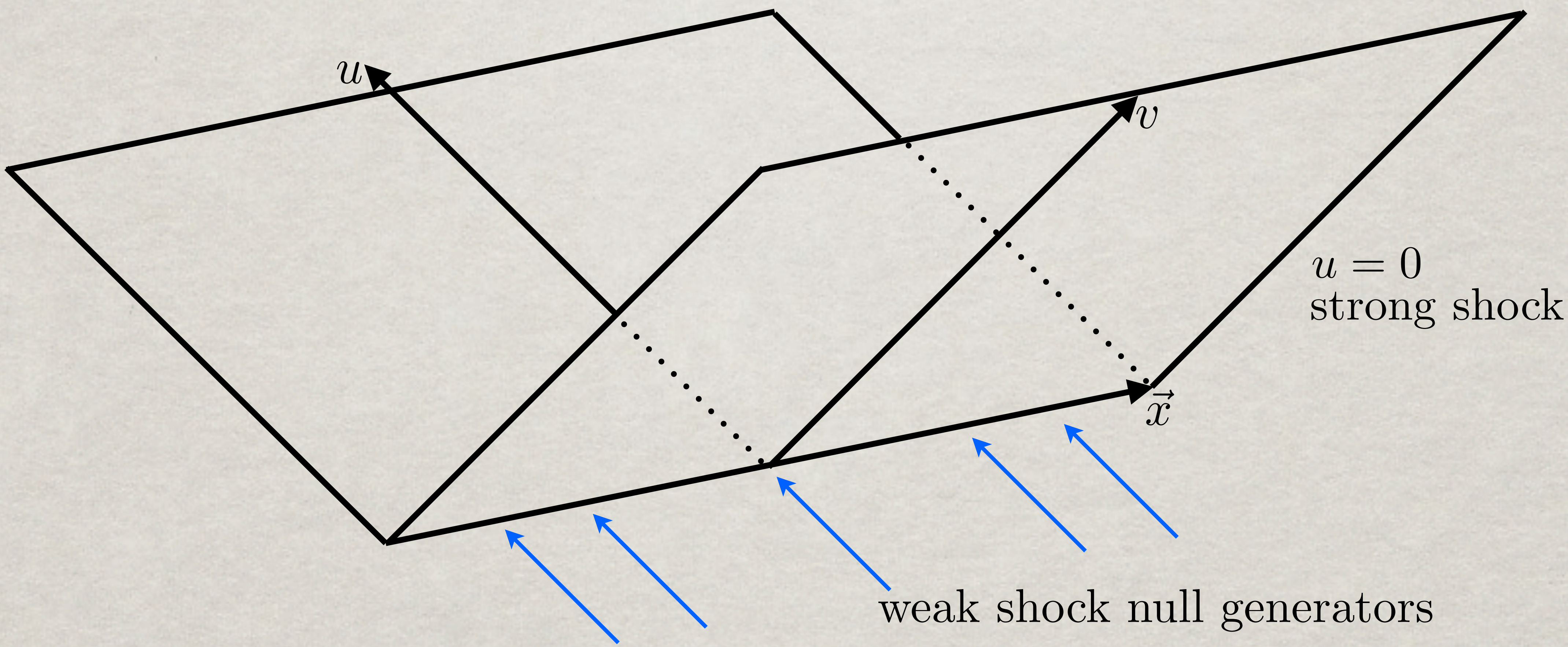

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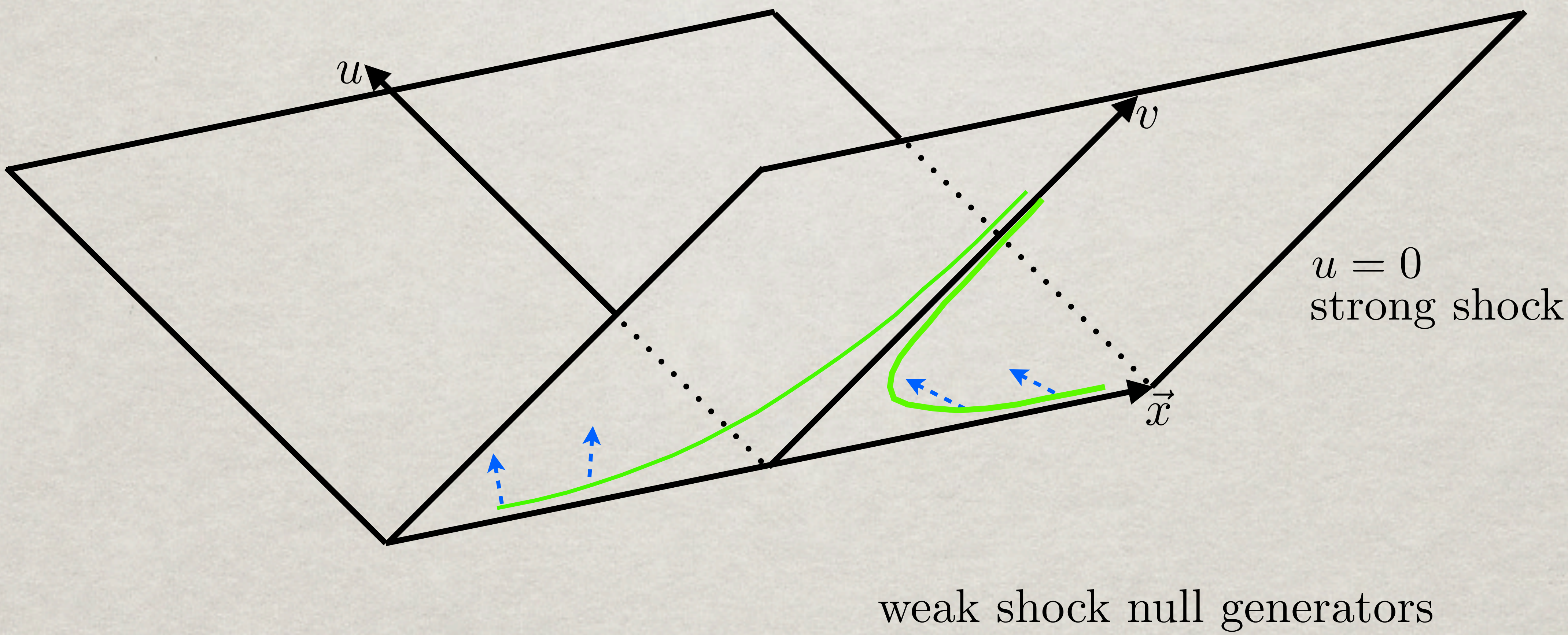

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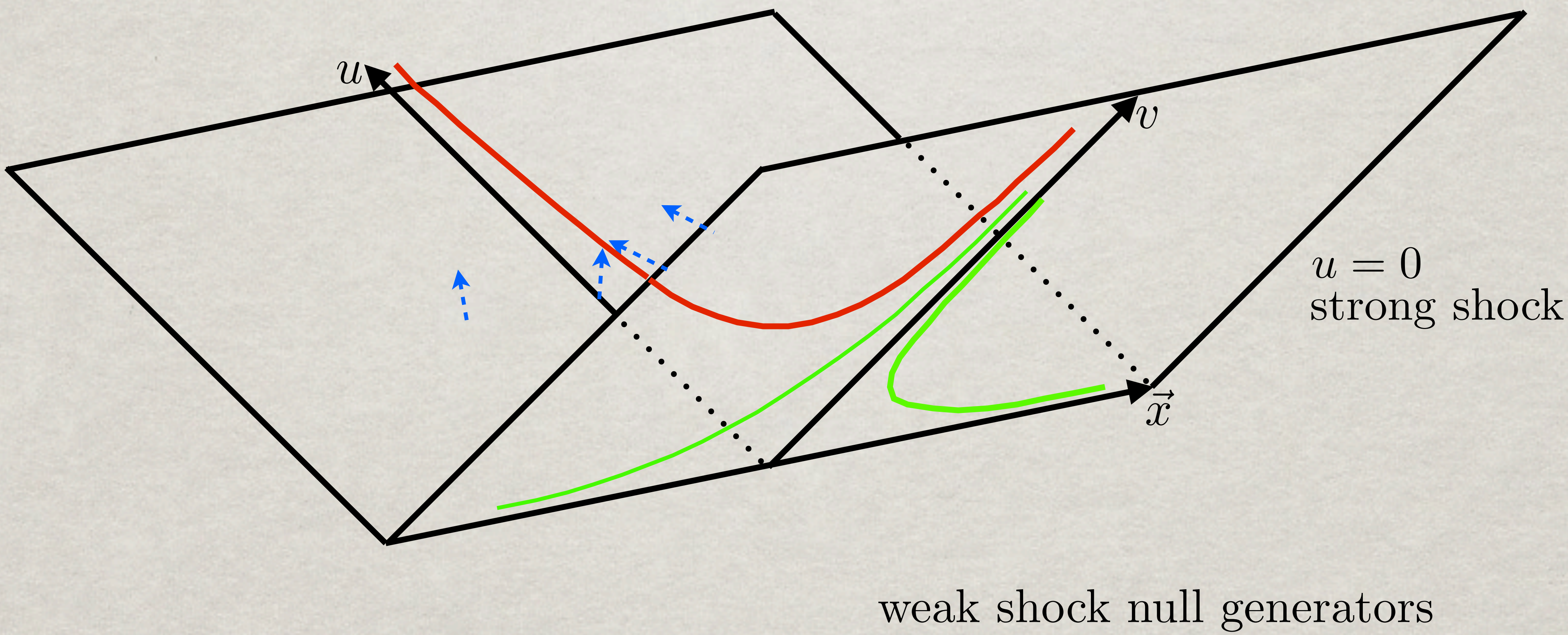

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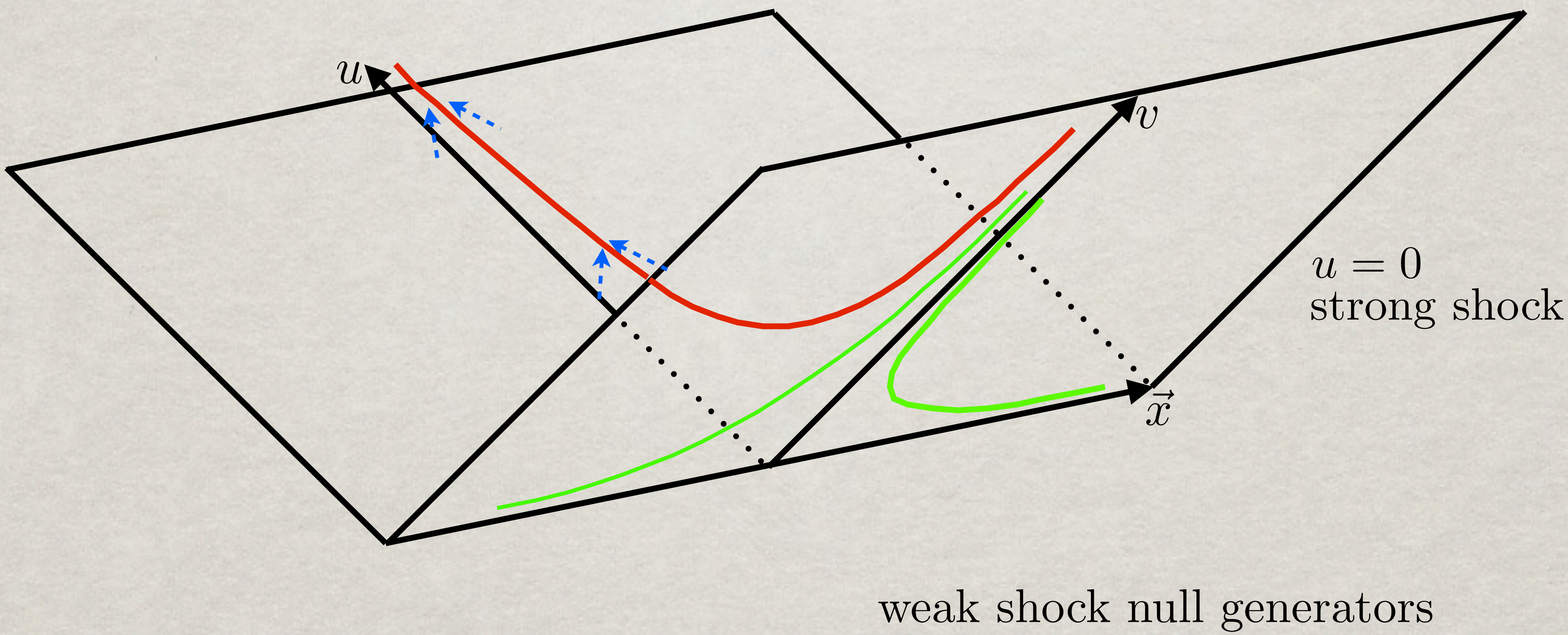

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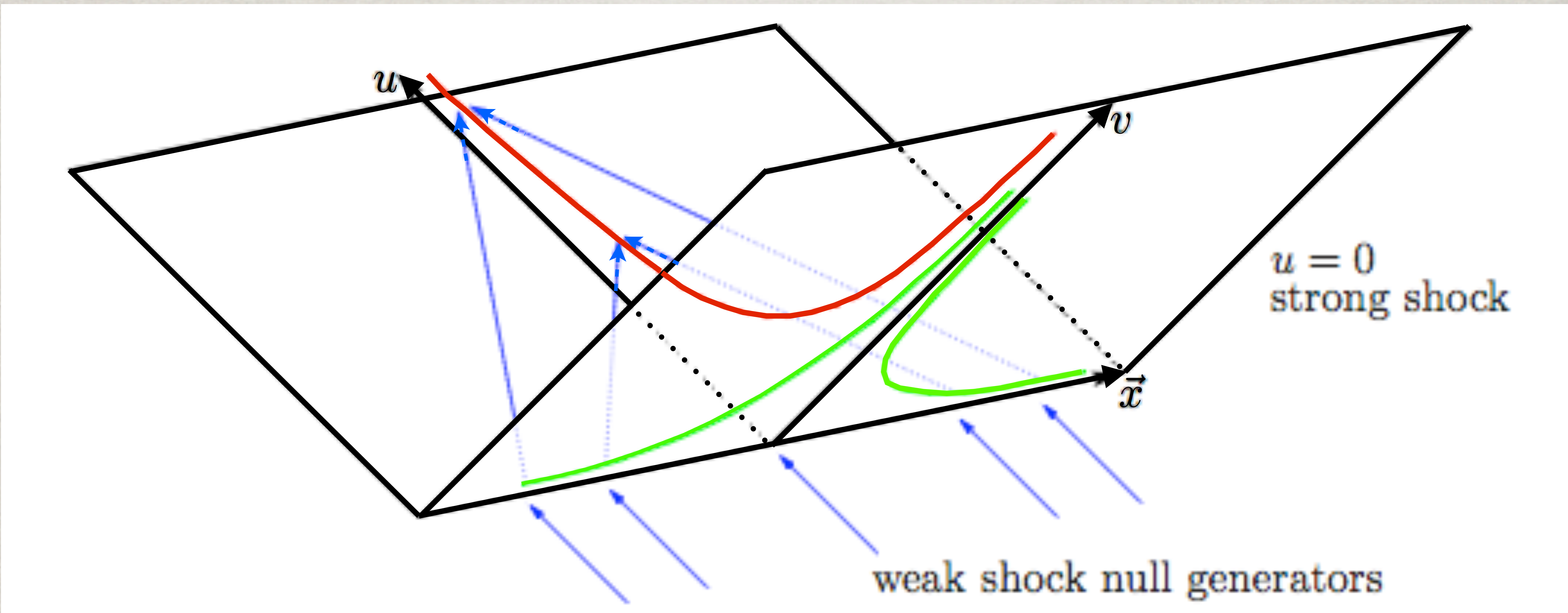

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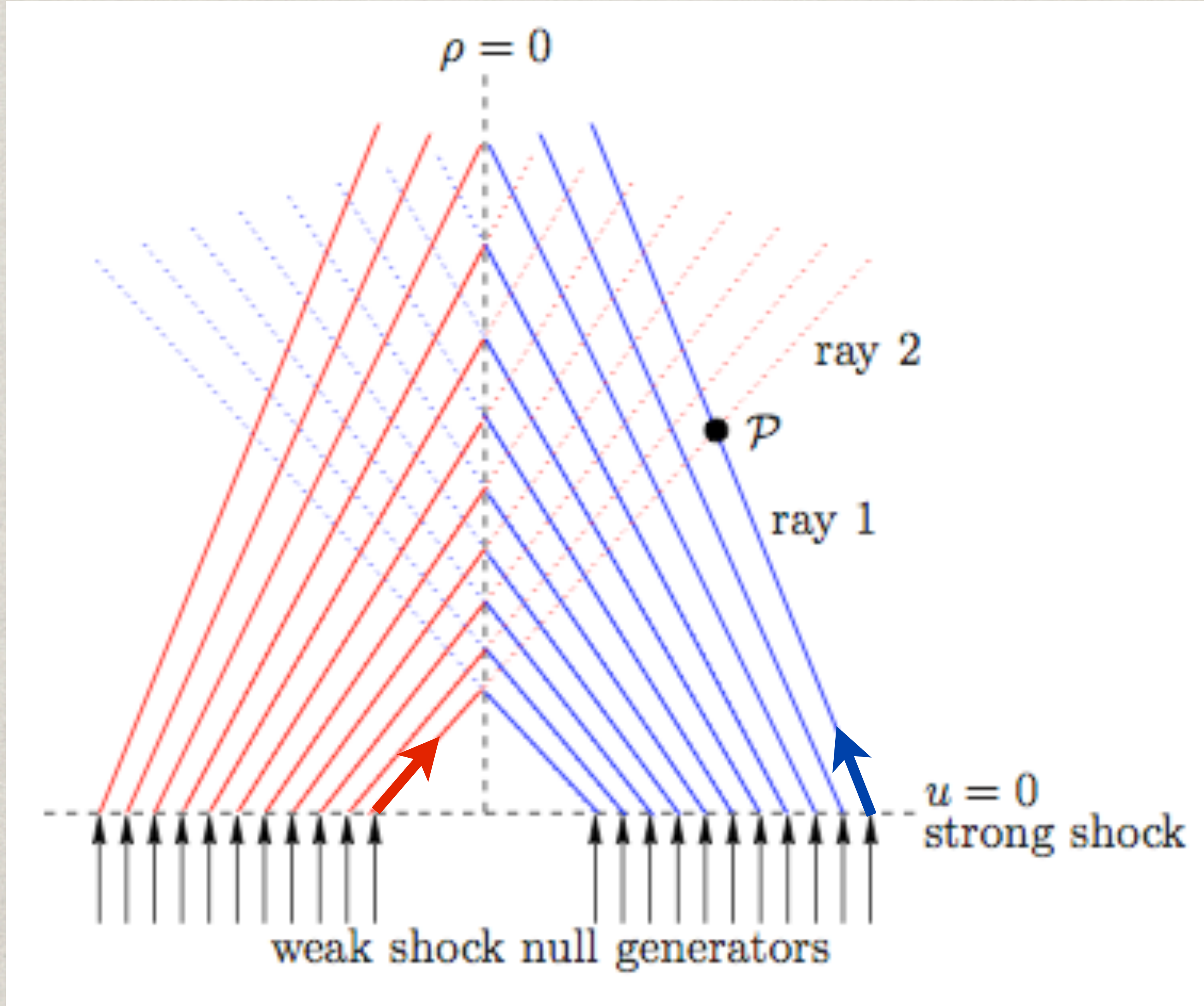
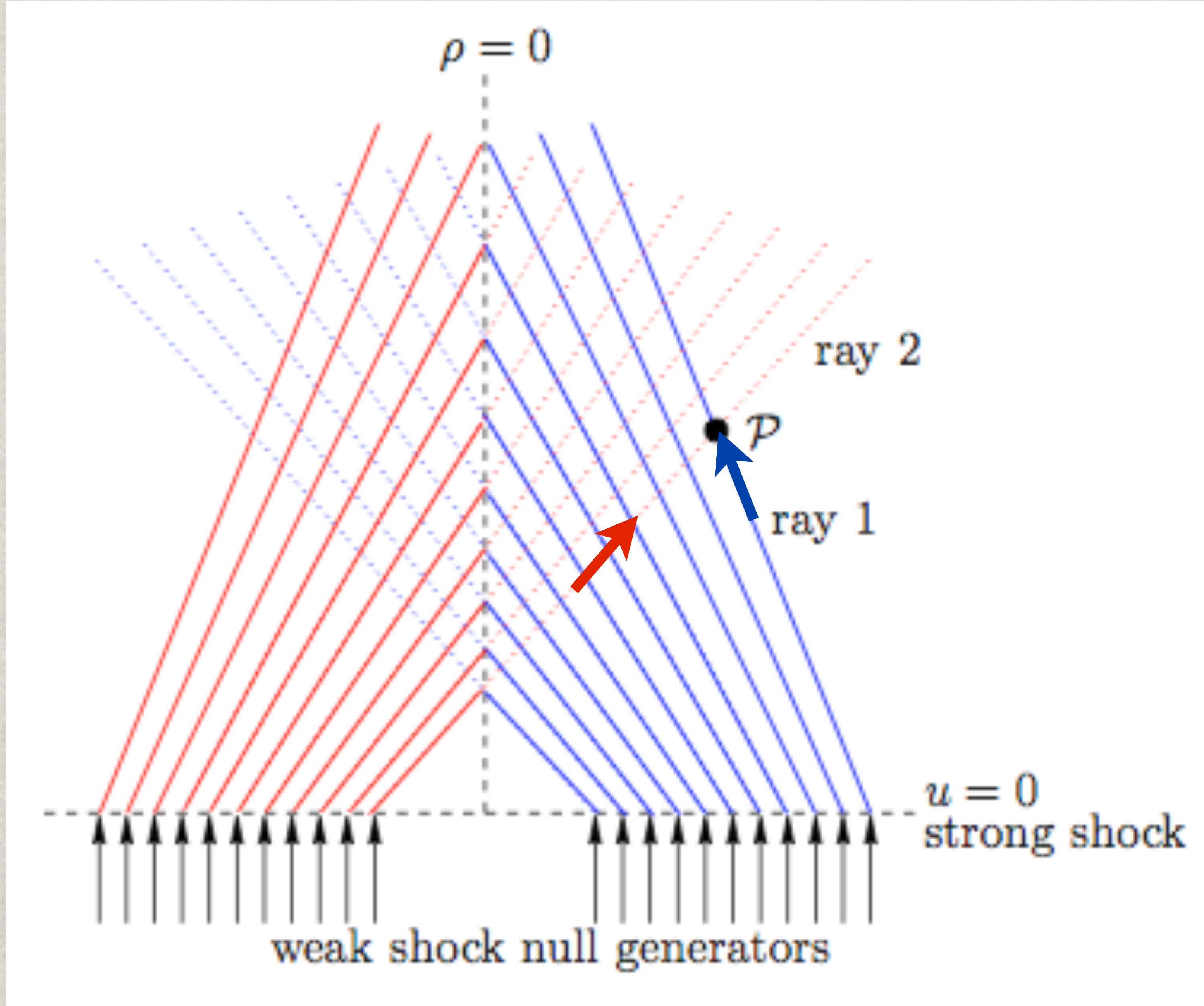


Spatial Projection

Shock waves approach:

1st order perturbation theory

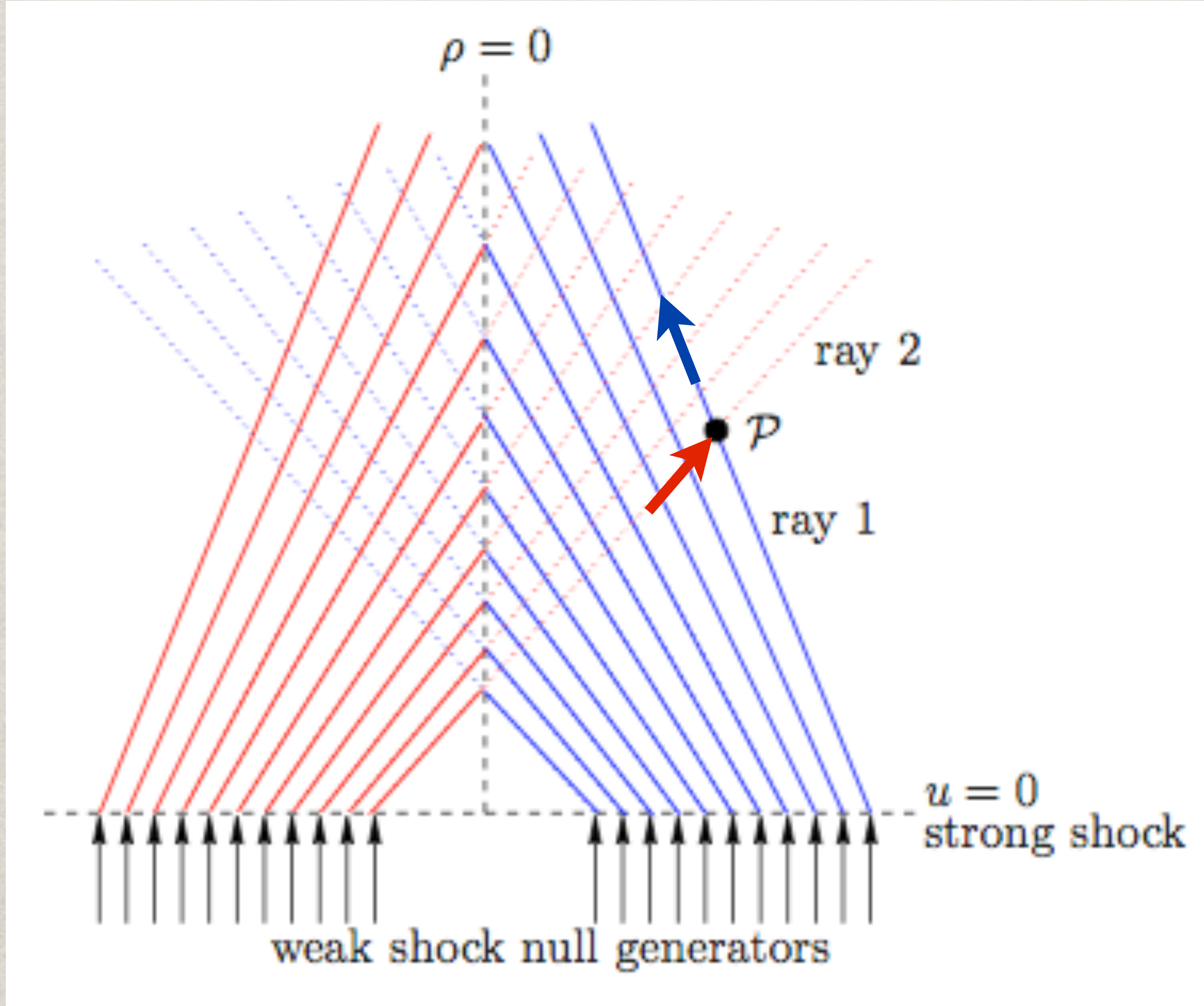


Occurs at τ_1

Spatial Projection

Shock waves approach:

1st order perturbation theory



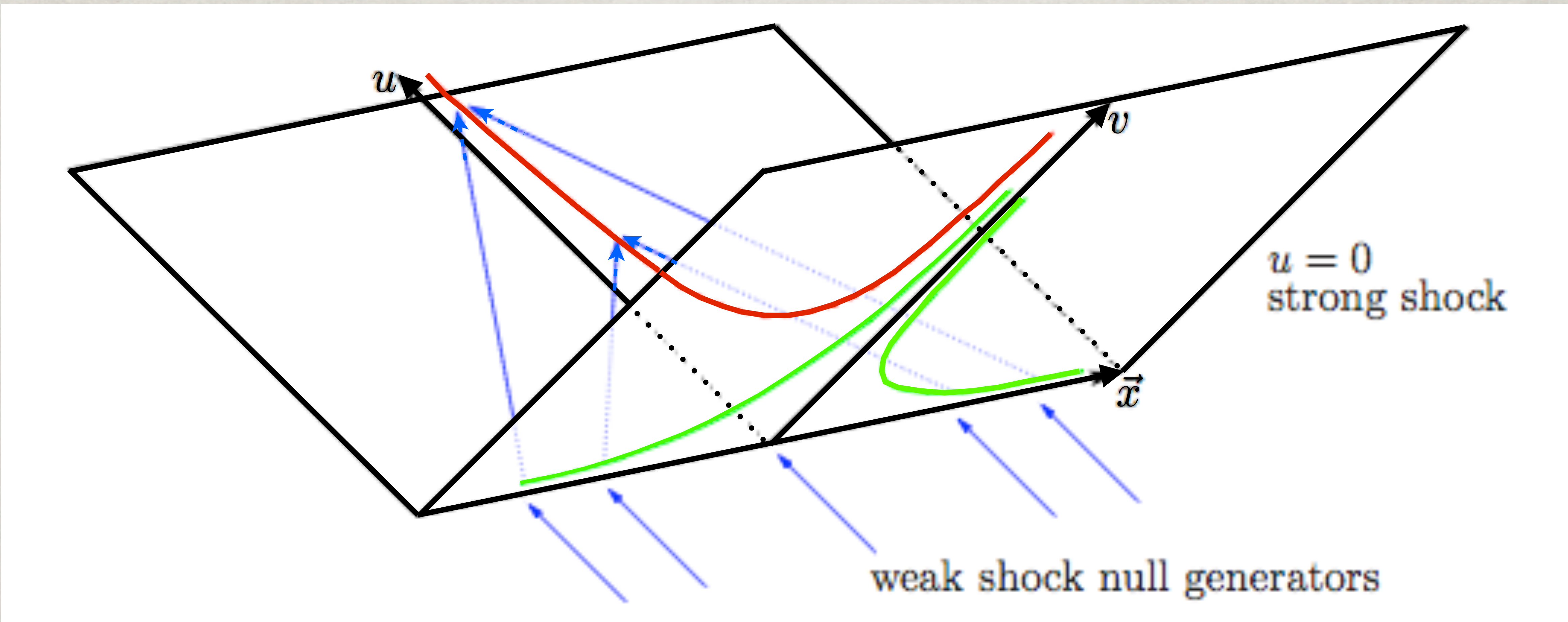
Occurs at τ_2

Spatial Projection

For u<0 and u=0:

$$g_{\mu\nu} = \nu^{\frac{2}{D-3}} \left[\eta_{\mu\nu} + \frac{\lambda}{\nu} h^{(1)}_{\mu\nu} + \left(\frac{\lambda}{\nu}\right)^2 h^{(2)}_{\mu\nu} \right] ,$$

$$\eta_{\mu\nu} dx^\mu dx^\nu = -2dudv + \delta_{ij} dx^i dx^j$$



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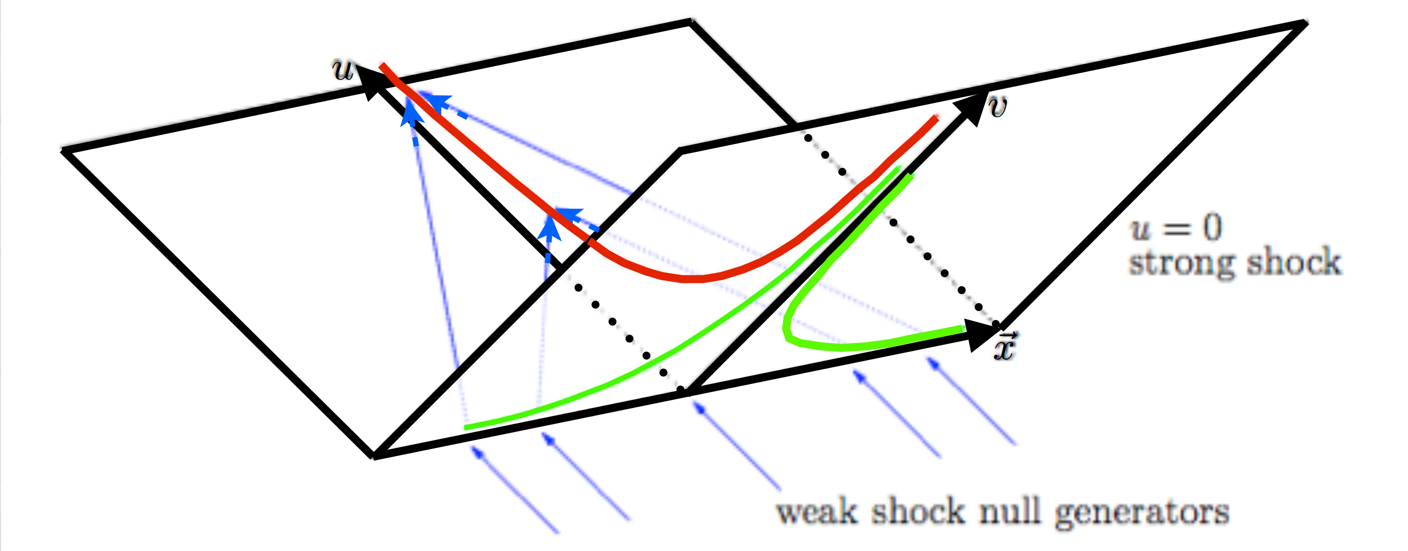


where:

$$h^{(1)}_{uu} = (D-3)\Phi'^2 h(v,\rho) , \qquad h^{(1)}_{ui} = -\frac{x_i}{\rho}\sqrt{2}(D-3)\Phi' h(v,\rho) ,$$

$$h^{(1)}_{ij} = 2\left(-\delta_{ij} + (D-2)\frac{x_i x_j}{\rho^2}\right) h(v,\rho) ,$$

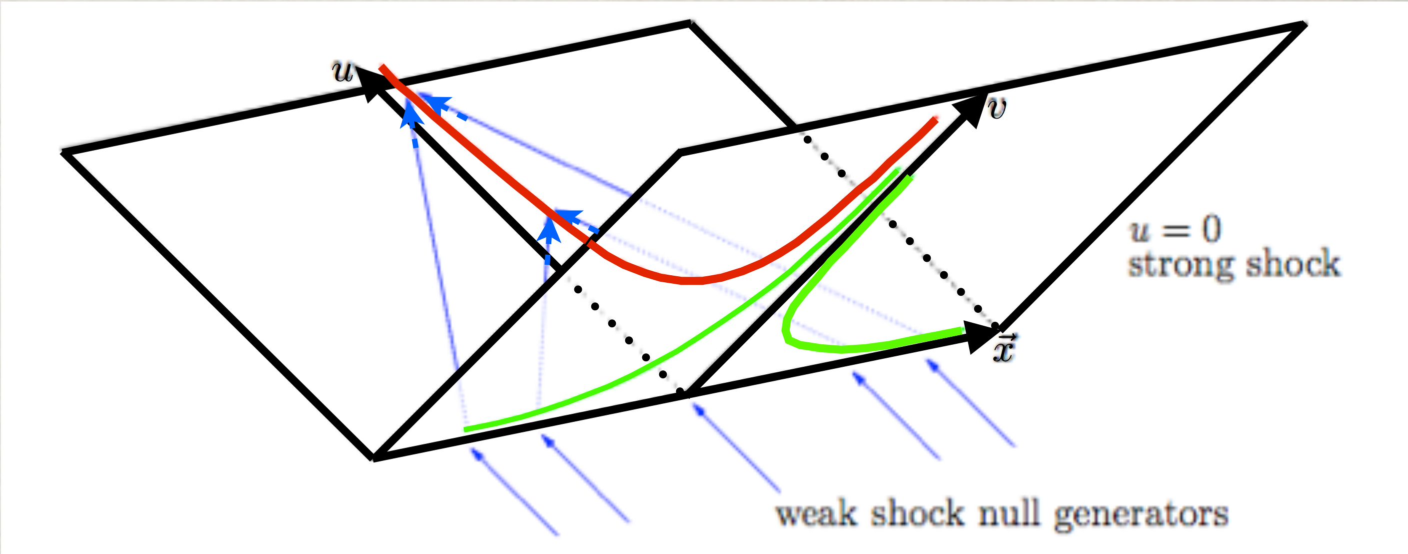
$$h^{(2)}_{uu} = \frac{(D-3)^2}{2}\Phi'^2 h(v,\rho)^2 , \qquad h^{(2)}_{ui} = -\frac{x_i}{\rho}\frac{\sqrt{2}(D-3)^2}{2}\Phi' h(v,\rho)^2 ,$$

$$h^{(2)}_{ij} = \left(\delta_{ij} + (D-2)(D-4)\frac{x_i x_j}{\rho^2}\right) h(v,\rho)^2 ,$$

$$h(v,\rho) \equiv -\frac{\Phi'}{2\rho}\left(\sqrt{2}v - \Phi\right)\theta\left(\sqrt{2}v - \Phi\right) .$$

For u>0:

$$g_{\mu\nu} = \nu^{\frac{2}{D-3}} \left[\eta_{\mu\nu} + \sum_{i=1}^{\infty} \left(\frac{\lambda}{\nu} \right)^i h_{\mu\nu}^{(i)} \right] ,$$



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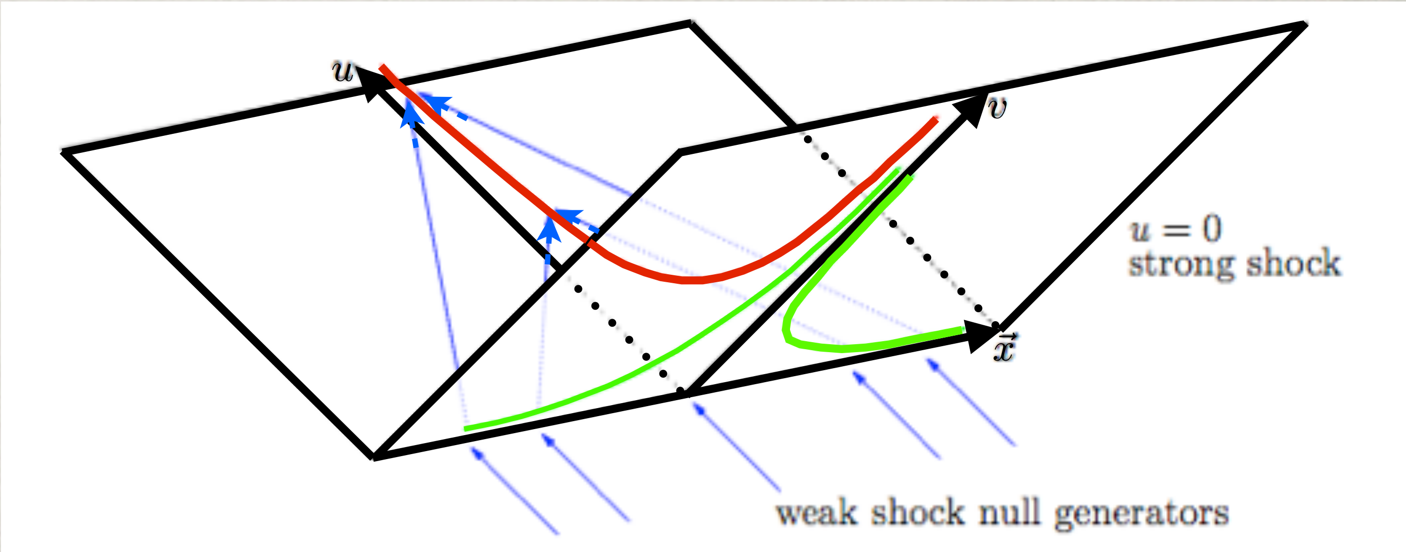

Einstein equations are solved order by order.
To linear order, in de Donder gauge (to decouple perturbations), one solves a sourceless wave equation in Minkowski space:

$$\Box \bar{h}^N_{\mu\nu} = 0 \quad \Leftrightarrow \quad \left(-2\partial_u \partial_v + \partial_i^2 \right) \bar{h}^N_{\mu\nu} = 0 \ .$$

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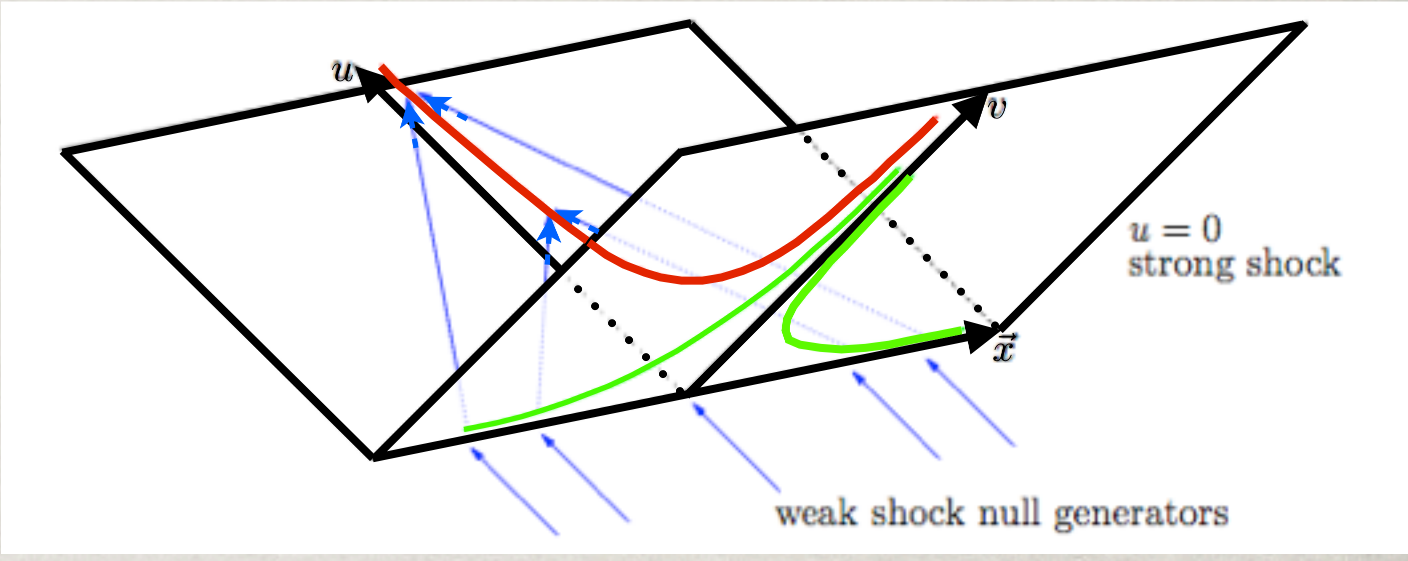

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Integral solution for the wave equation: $\Box F = 0$

$$F(u,v,x_i) = \frac{1}{(2\pi u)^{\frac{D-2}{2}}} \int d^{D-2}x' \partial_{v'}^{\frac{D-2}{2}} F(0,v',x_i') \ ,$$

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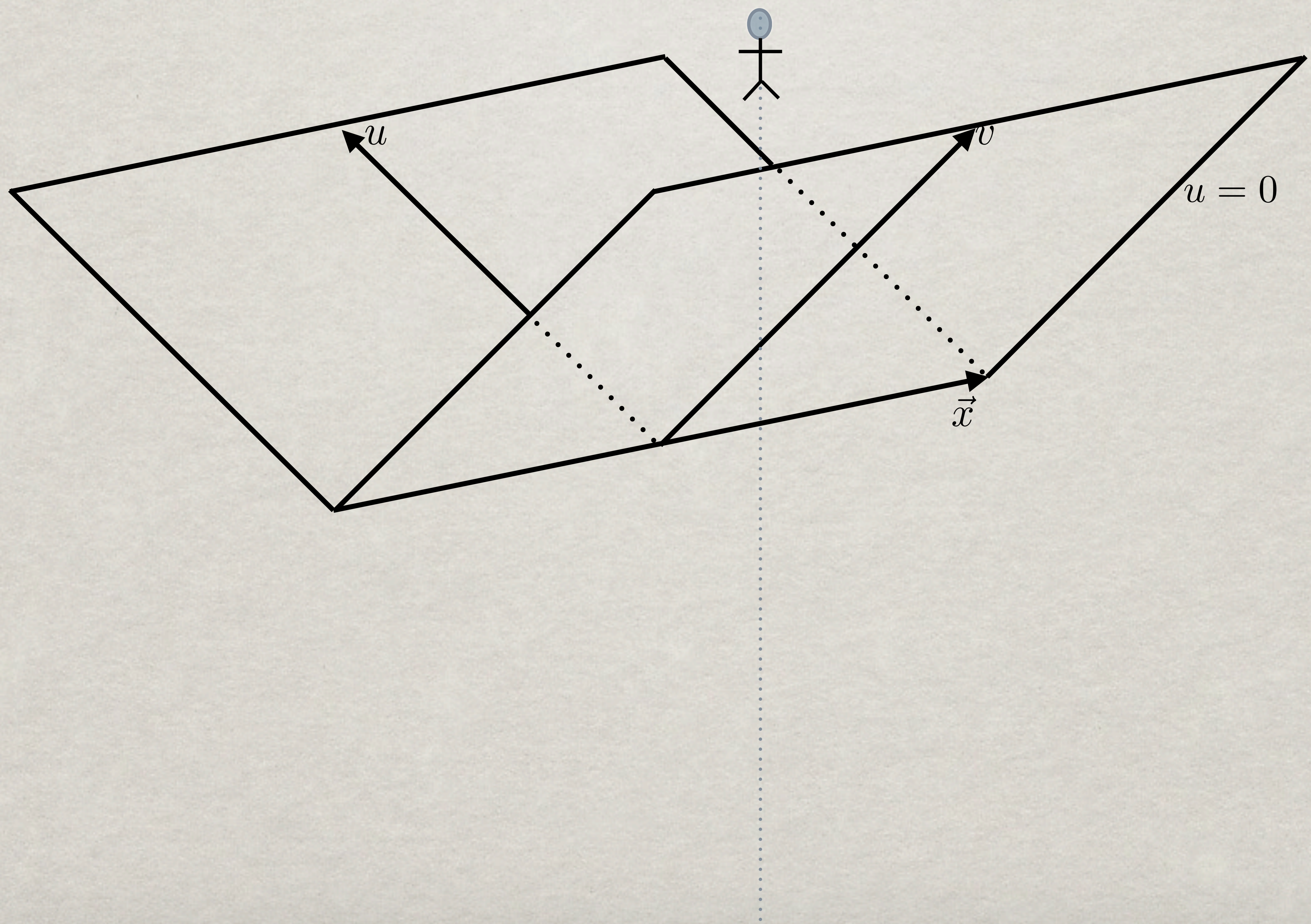
1st order perturbation theory



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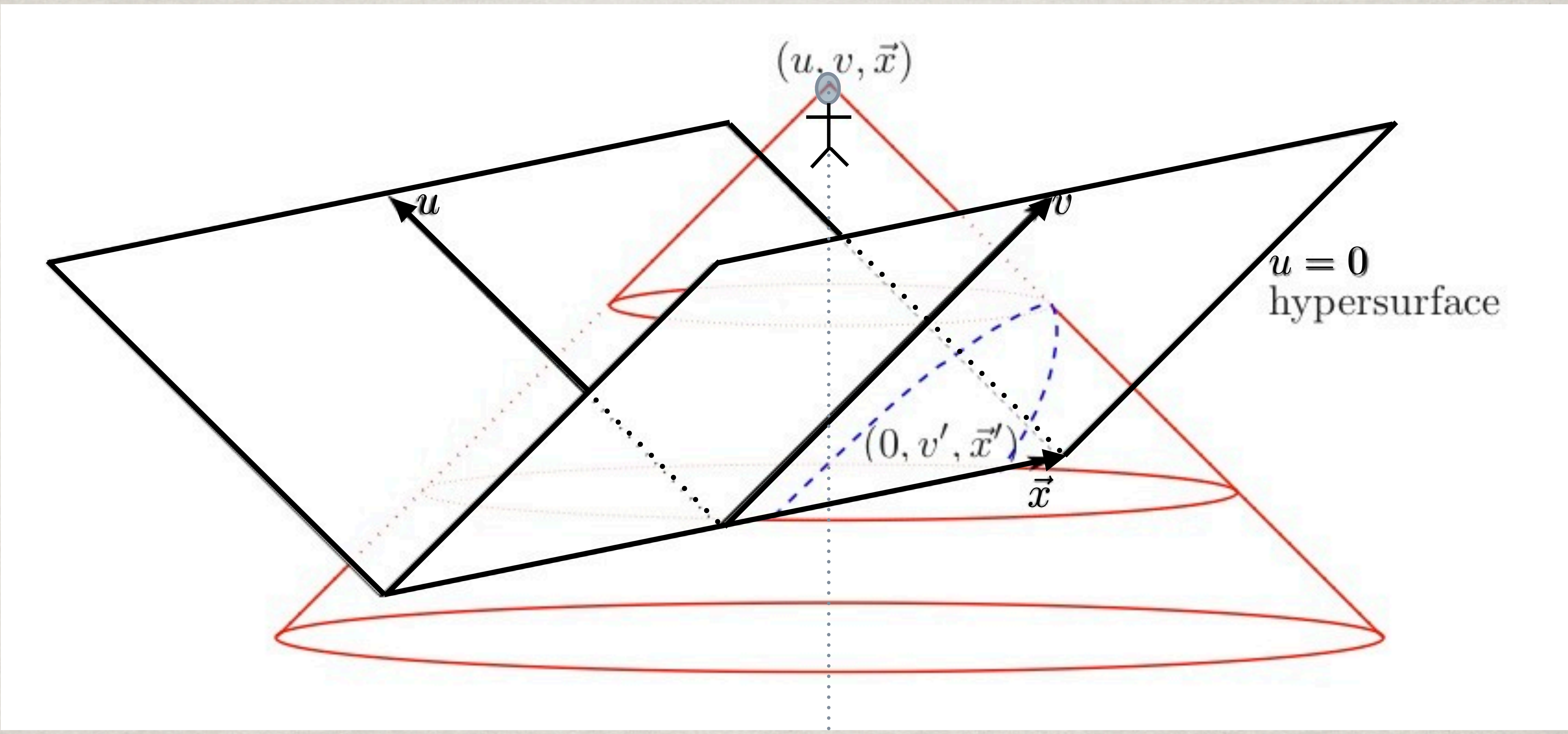

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Wave extraction: use Landau-Lifshitz pseudo-tensor:

$$16\pi G_D t^{\mu\nu}_{LL} = h^{\mu\nu}{}_{,\alpha} h^{\alpha\beta}{}_{,\beta} - h^{\mu\alpha}{}_{,\alpha} h^{\nu\beta}{}_{,\beta} + \frac{1}{2}\eta^{\mu\nu}\left(h^{\alpha\beta}{}_{,\sigma} h^{\sigma}{}_{\alpha,\beta} - \frac{1}{2} h^{\beta\sigma,\alpha} h_{\beta\sigma,\alpha} \right)$$
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Yields a well defined energy:

$$E_{\text{radiated}} = \int t_{LL}^{0i} n_i dS dt \,,$$

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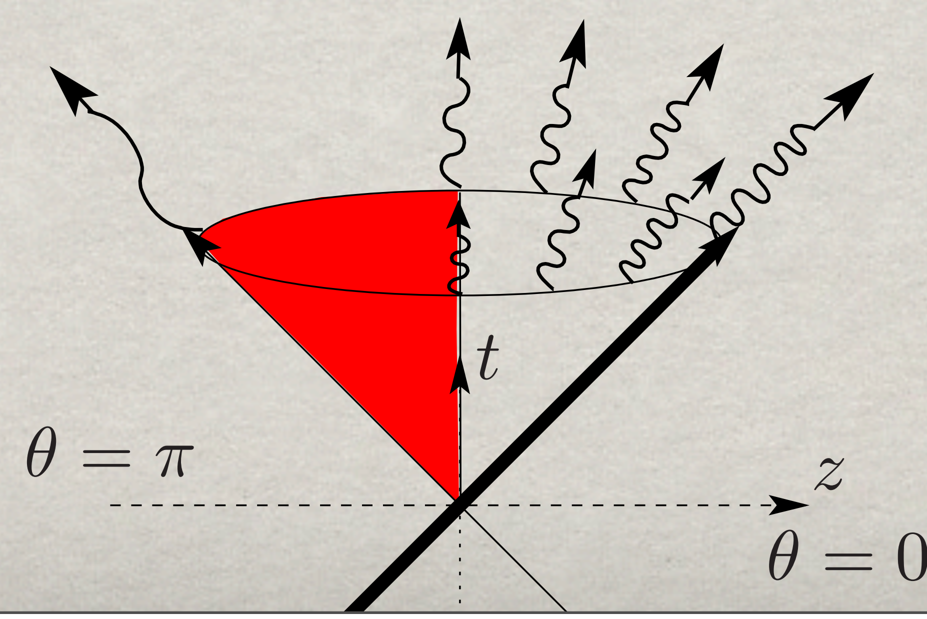

$\hat{\theta} = \pi - \theta$

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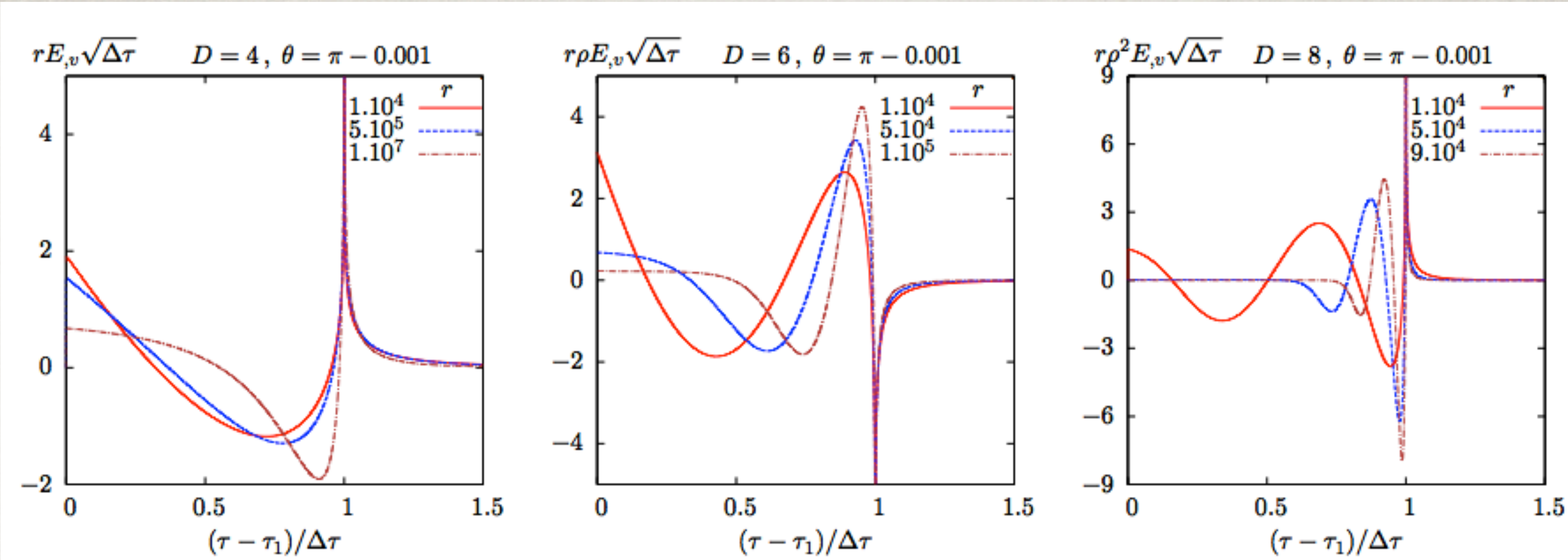
where the wave amplitude is:

$$E_{,v} = -\frac{\sqrt{8}\Omega_{D-4}}{(2\pi u)^{\frac{D-2}{2}}} \int_0^{+\infty} \frac{d\rho'}{\rho'} \int_{-1}^{1} dx \, \frac{d}{dx}\left[x(1-x^2)^{\frac{D-3}{2}}\right] \delta^{\left(\frac{D-4}{2}\right)}\left(v'_1 - v'_2\right) \,,$$

where

$$v'_1 \equiv v - \frac{\rho^2 - 2\rho\rho' x + \rho'^2}{2u} \,, \qquad v'_2 \equiv \frac{\Phi(\rho')}{\sqrt{2}} \,.$$

Wave forms from first order computation (even D)



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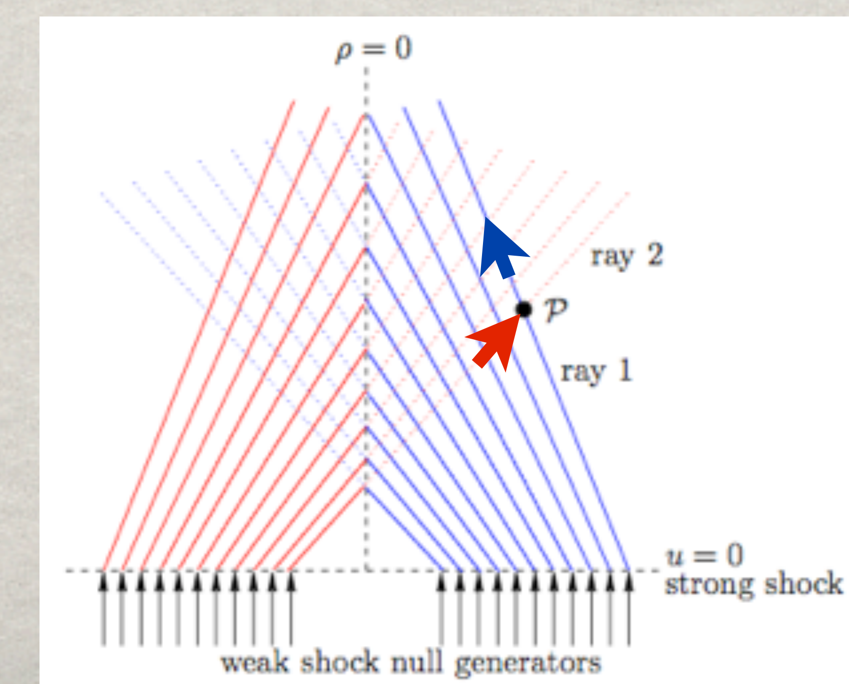
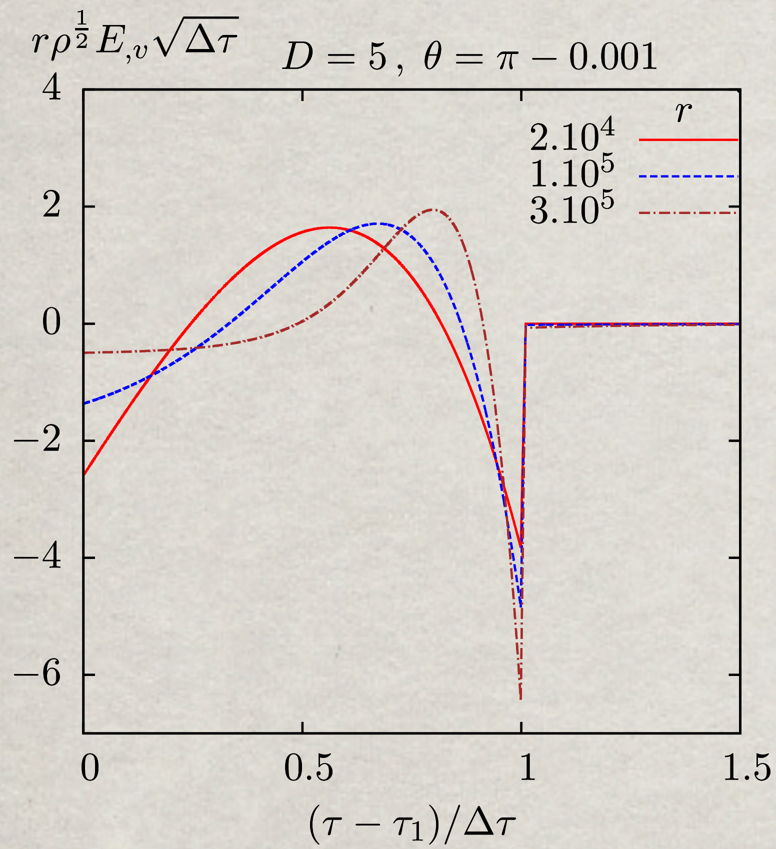
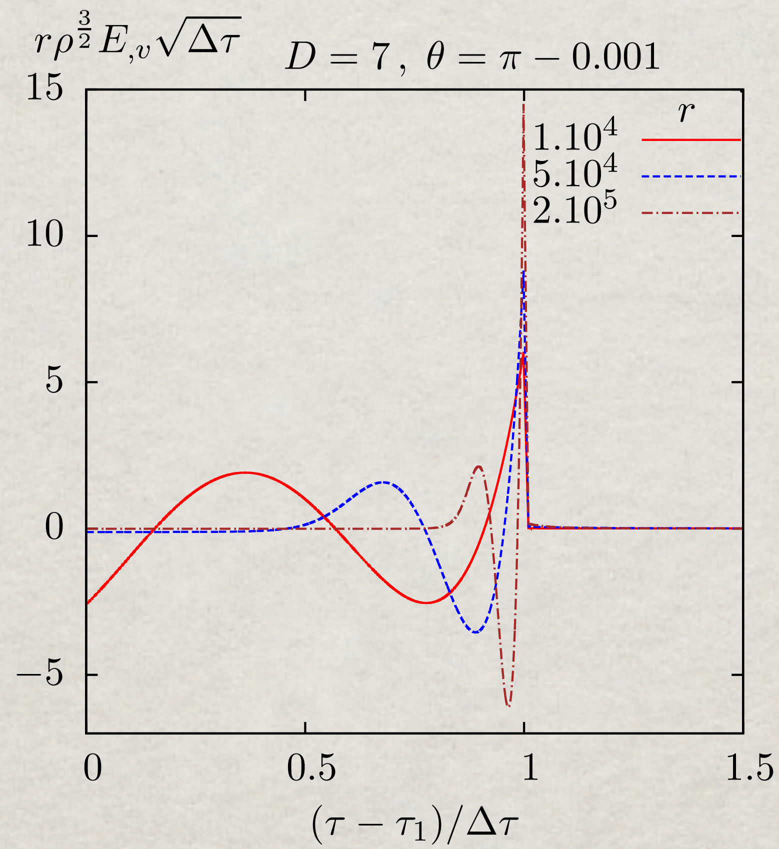


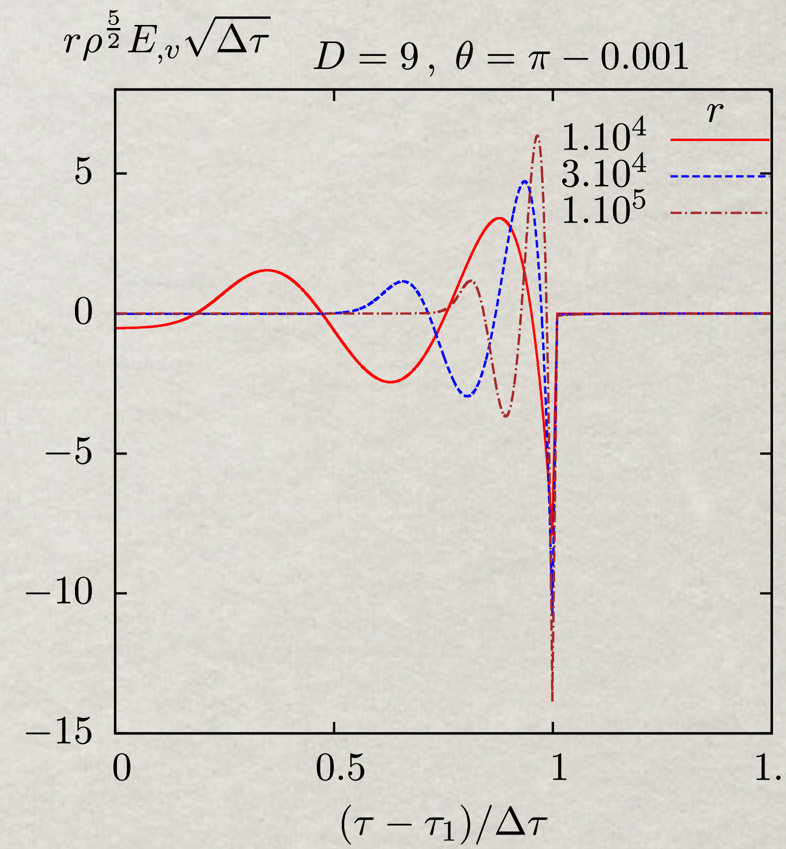
Occurs at τ_1
Occurs at τ_2

$$\Delta\tau=\tau_2-\tau_1$$

Wave forms from first order computation (odd D)







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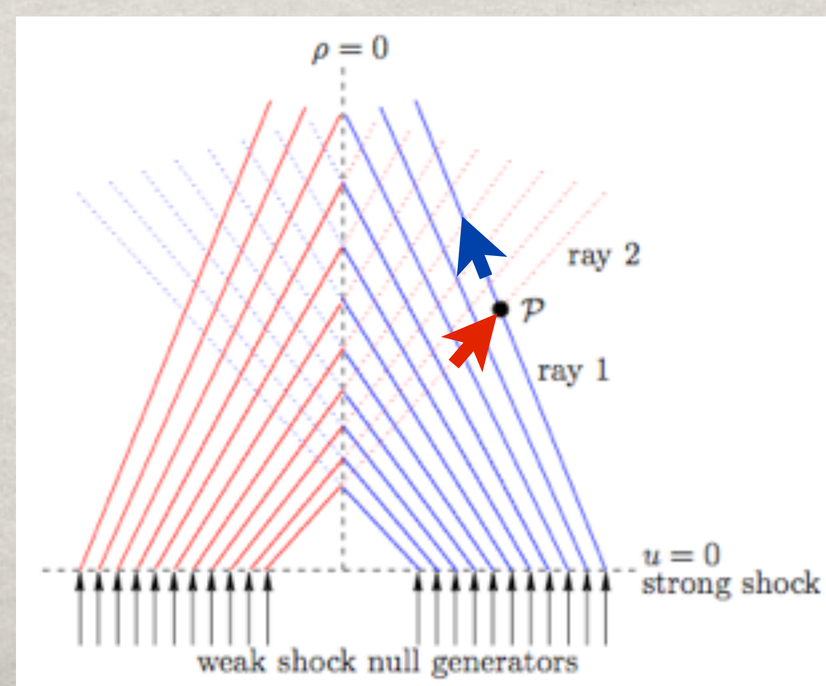

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Shock Wave Collisions

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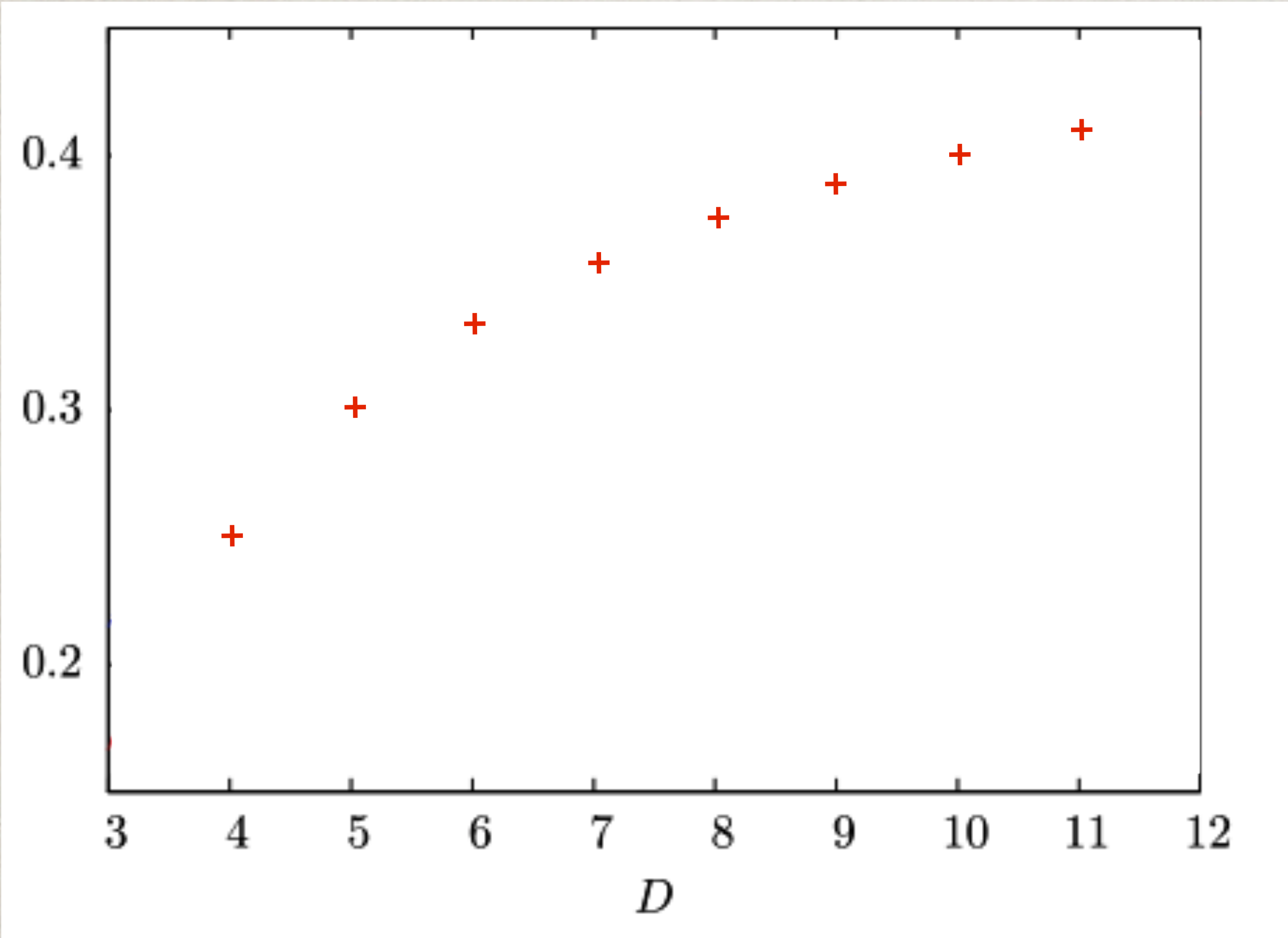

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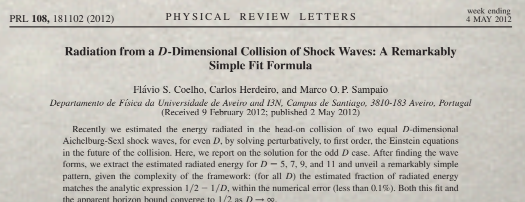
PRL **108**, 181102 (2012) PHYSICAL REVIEW LETTERS week ending 4 MAY 2012

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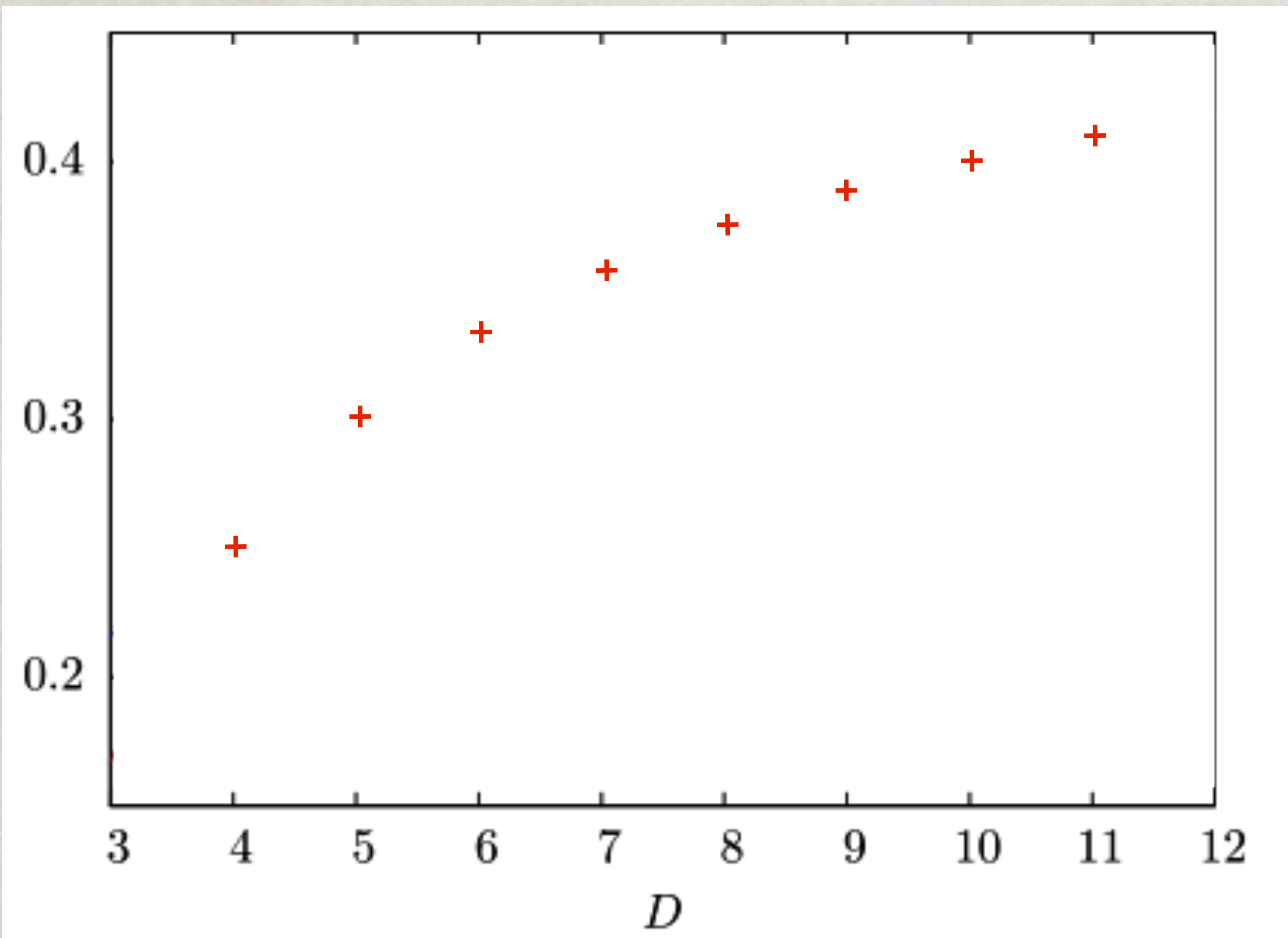

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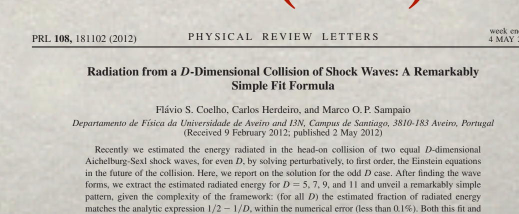
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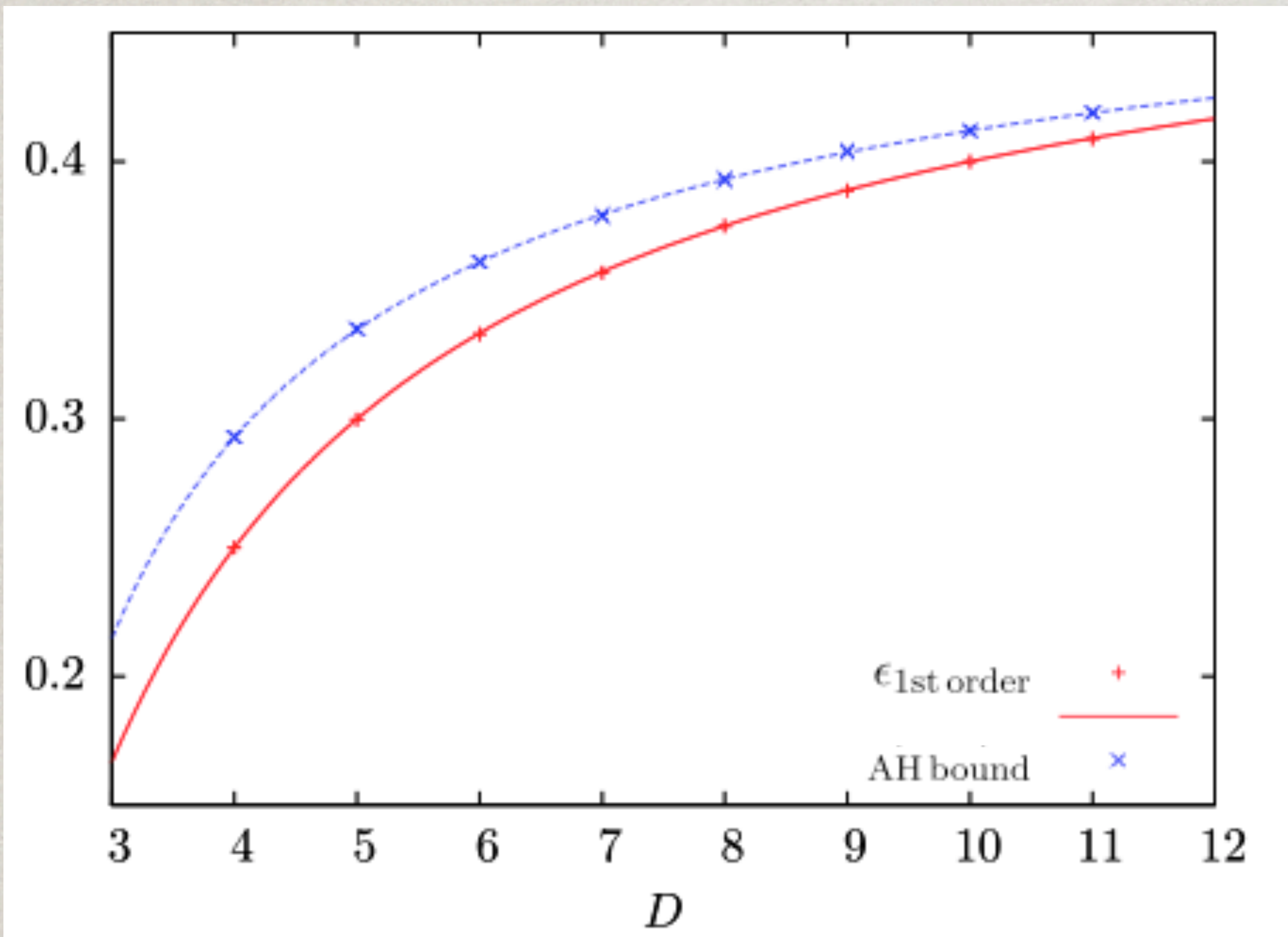

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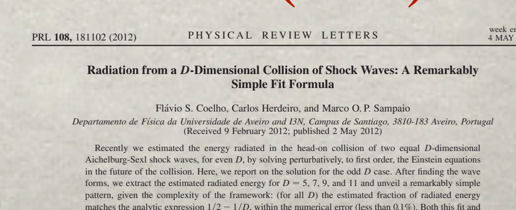
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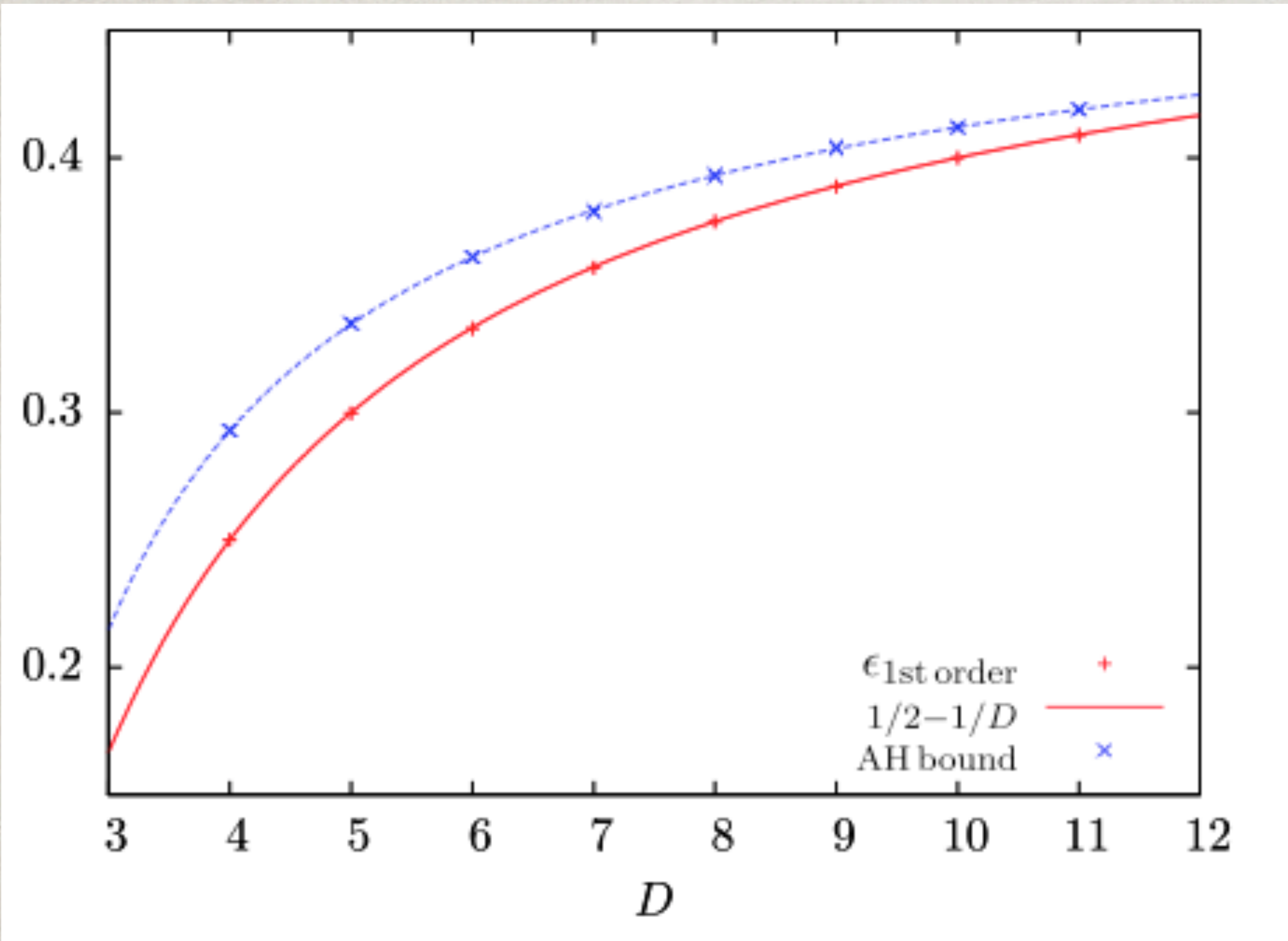

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Remarks:

1) Example of how D dimensional gravity can be used as a tool to understand four dimensional one!

2) Results have been checked by using a method with a Bondi mass loss formula in D dimensions.

3) Method stops being legitimate for charged shocks.

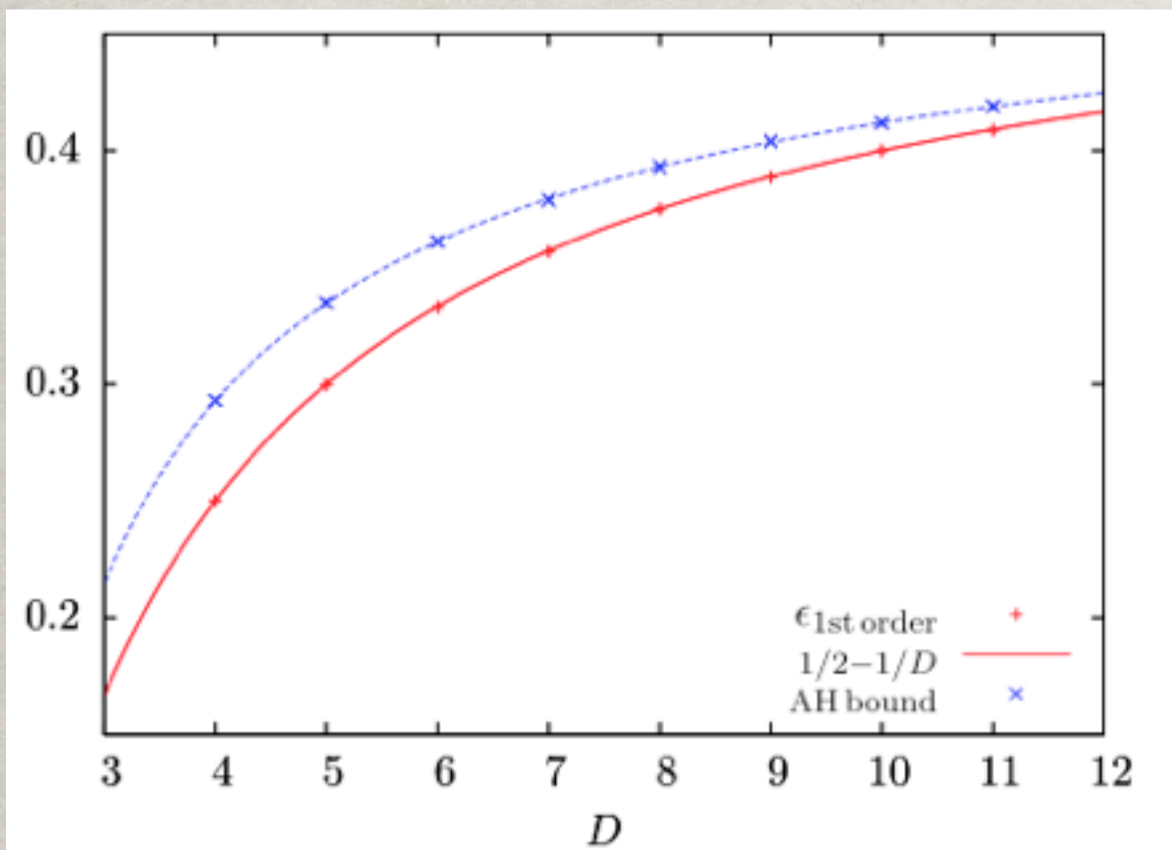
Questions:

1) Is this fit formula exact in first order perturbation theory? Can one derive it analytically?

2) Is there an analogous simple formula in second/higher order theory?

3) Isotropy emission assumption - how good is it?

THANK YOU FOR YOUR ATTENTION !



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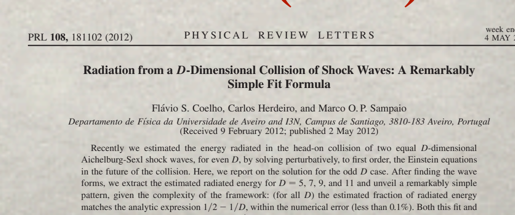

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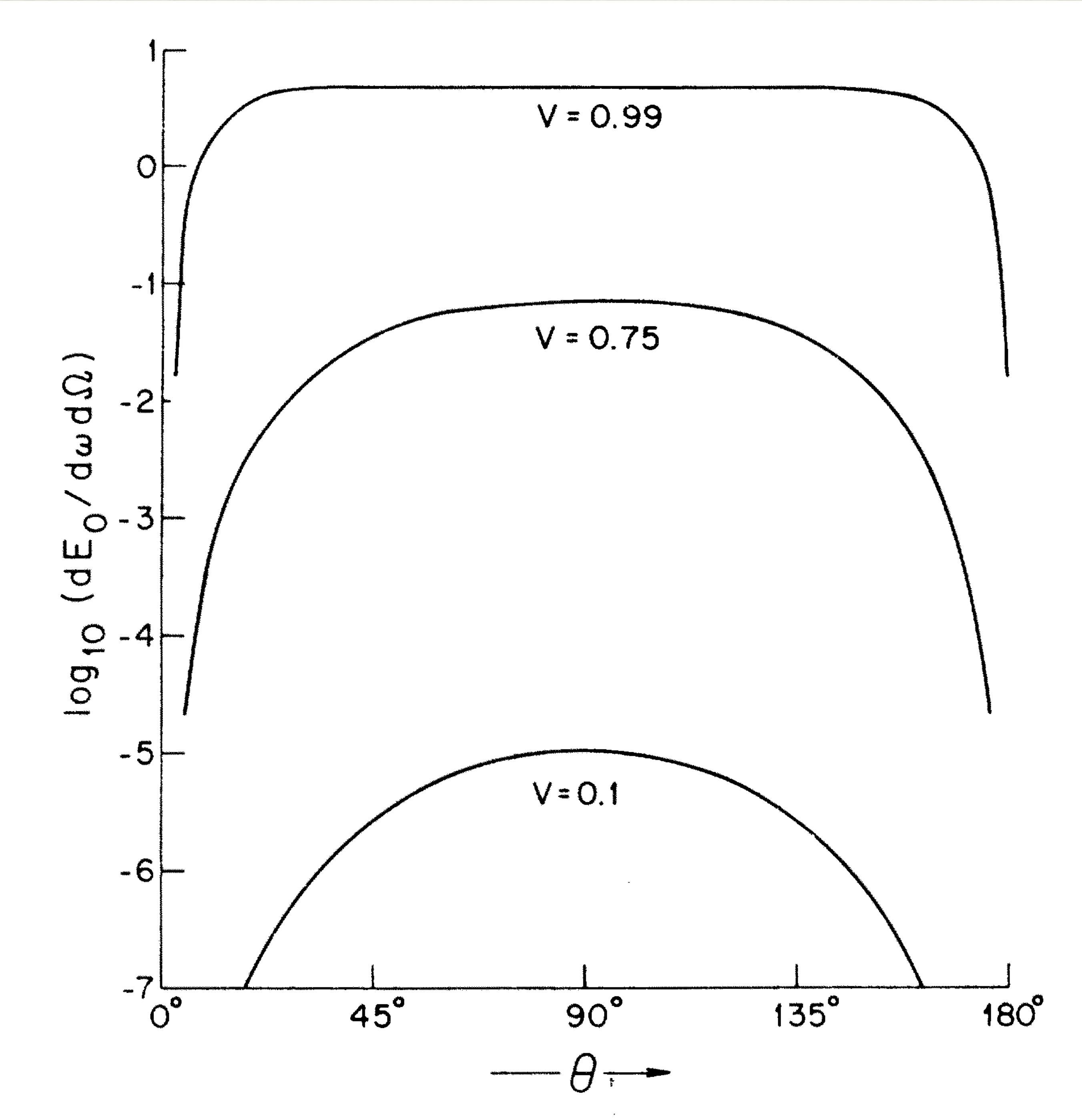
Zero Frequency Limit (ZFL) Smarr '77



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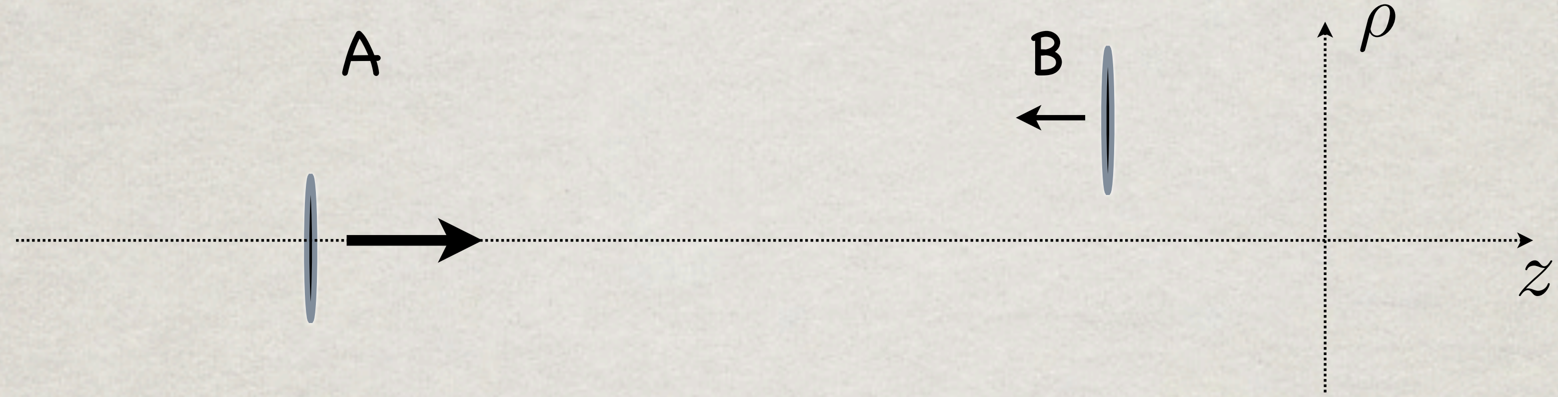

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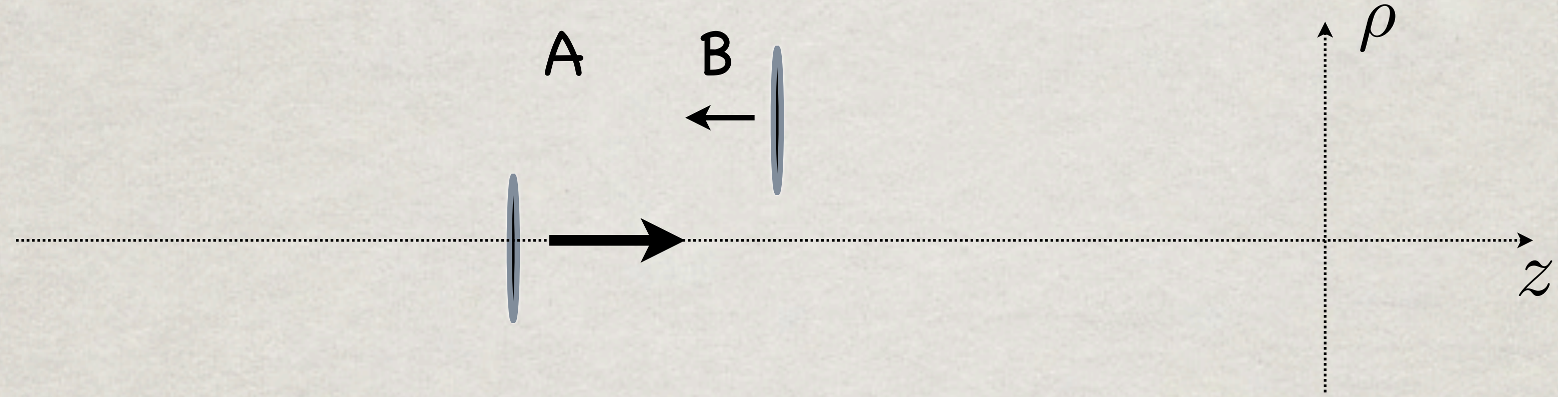

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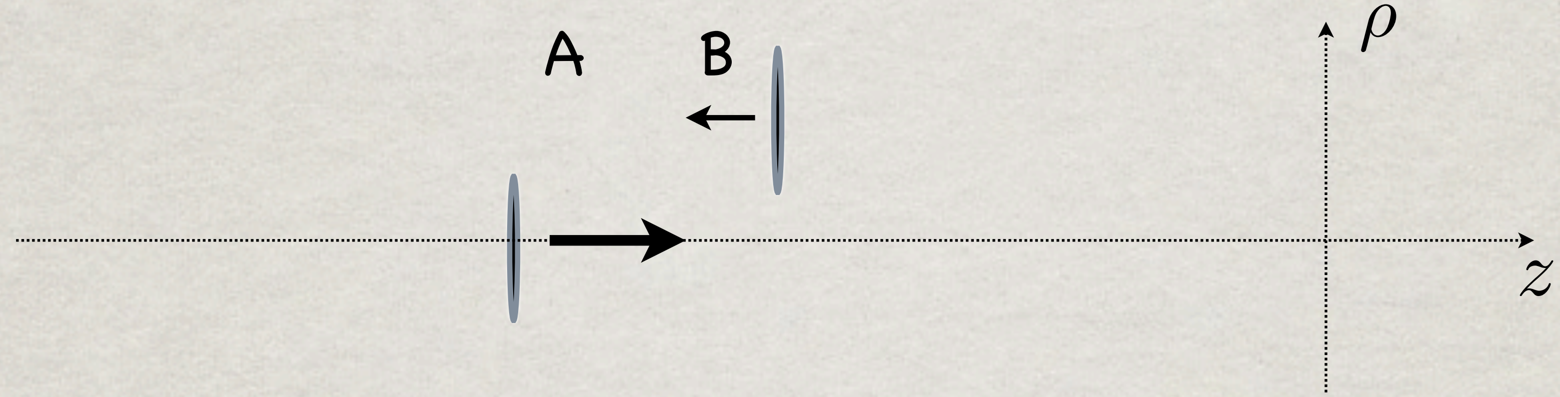

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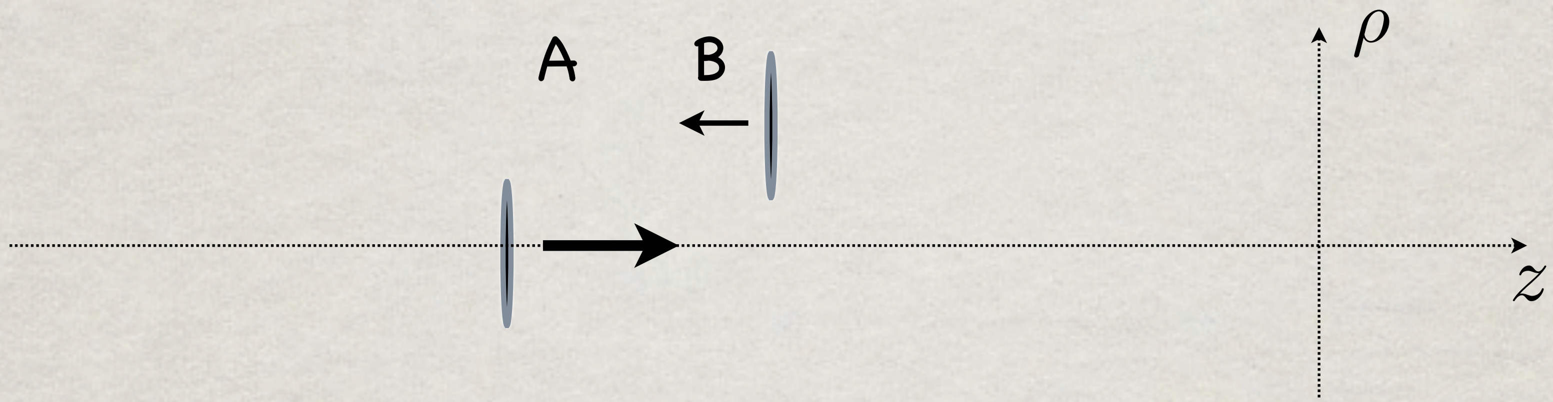

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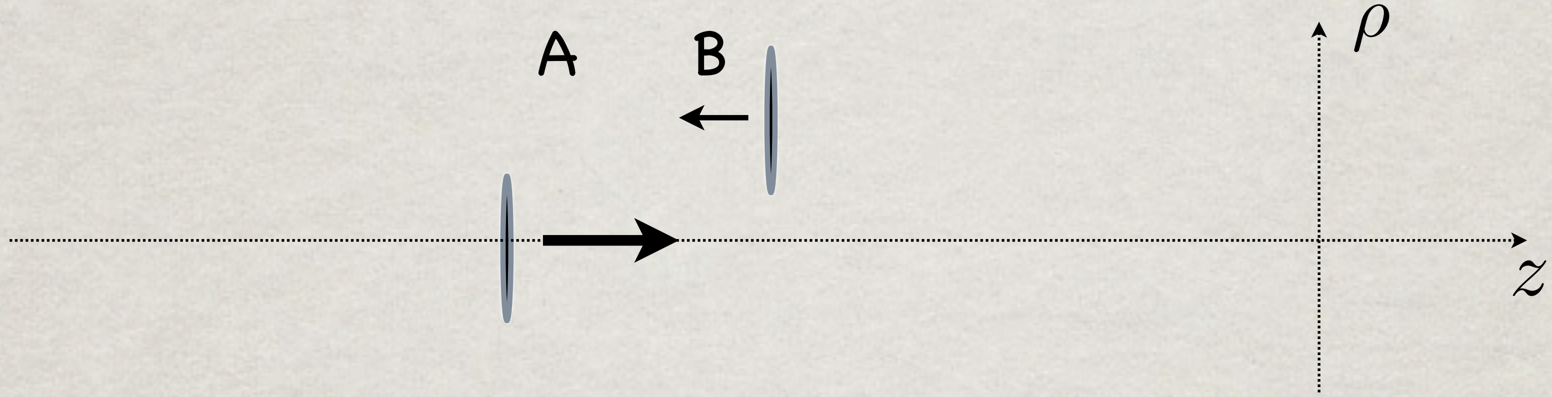

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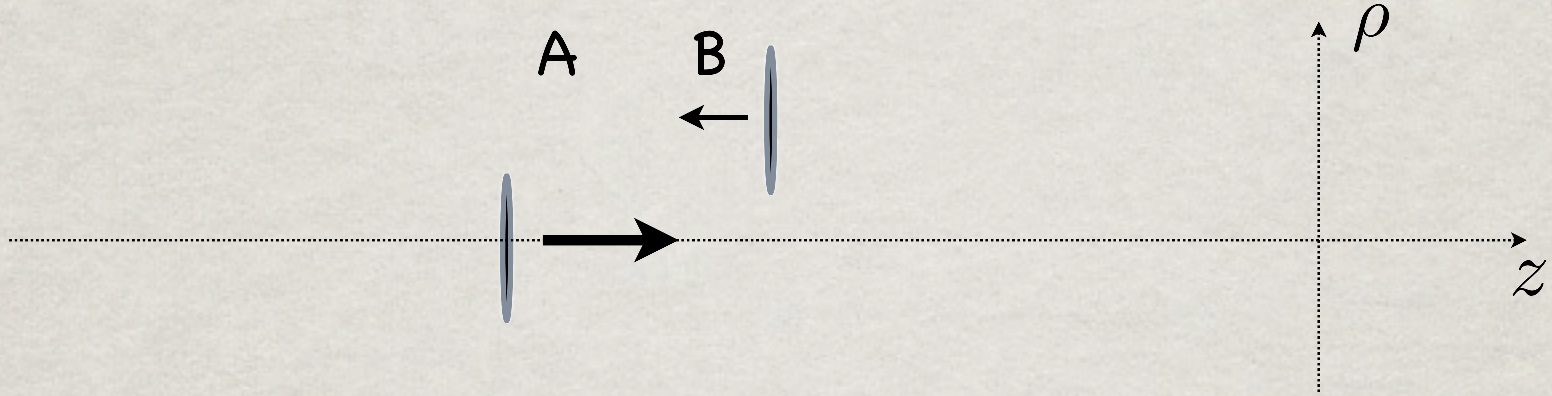

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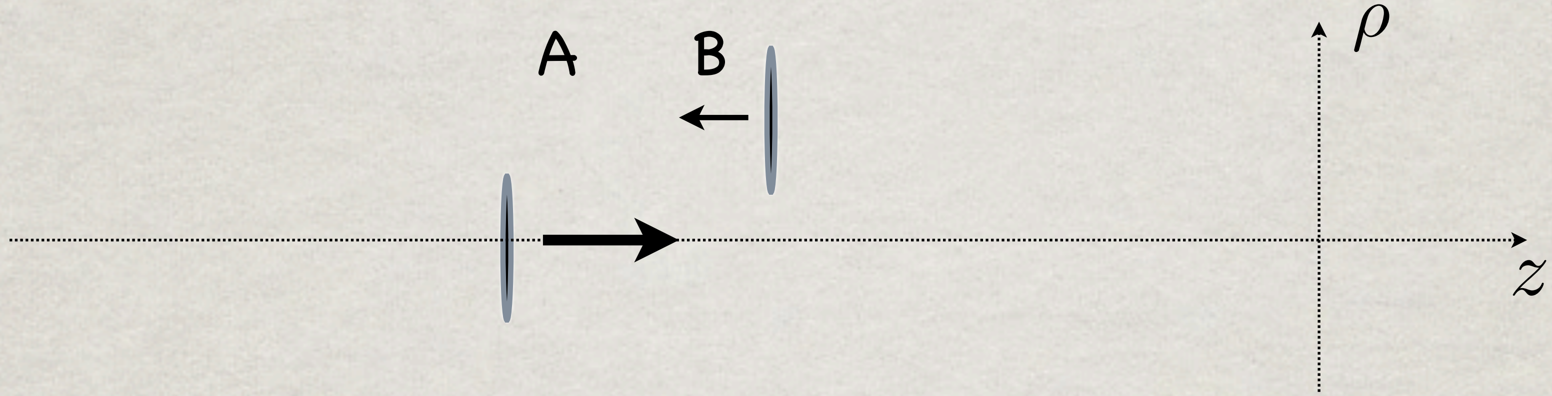

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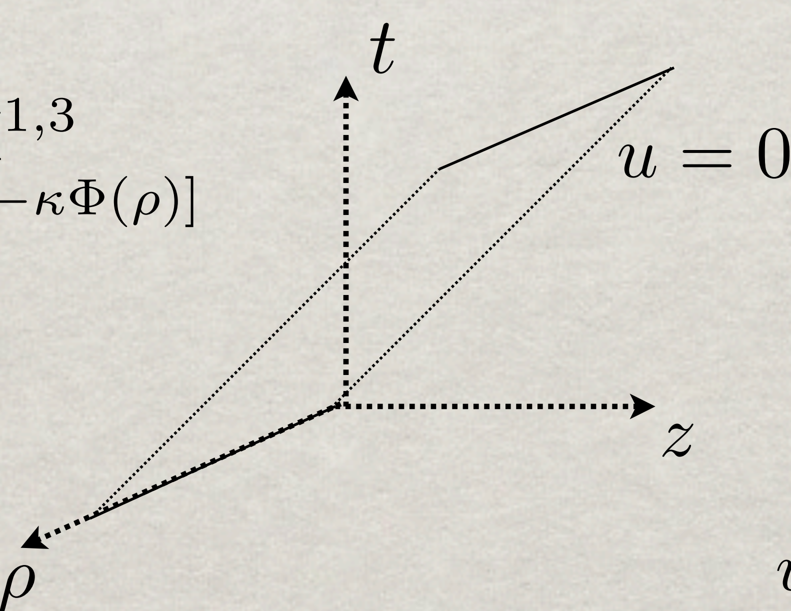

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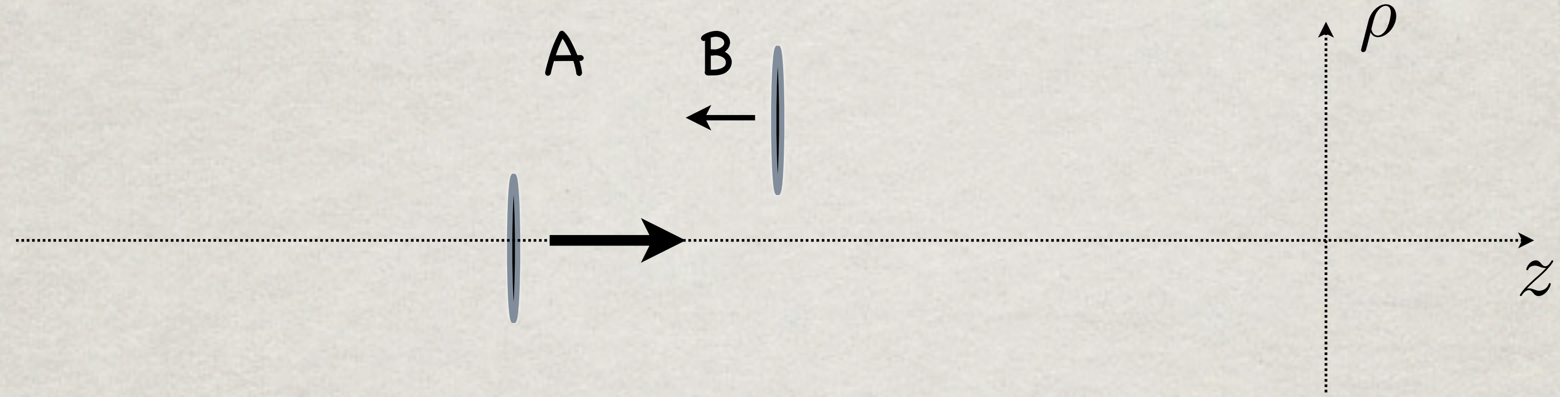

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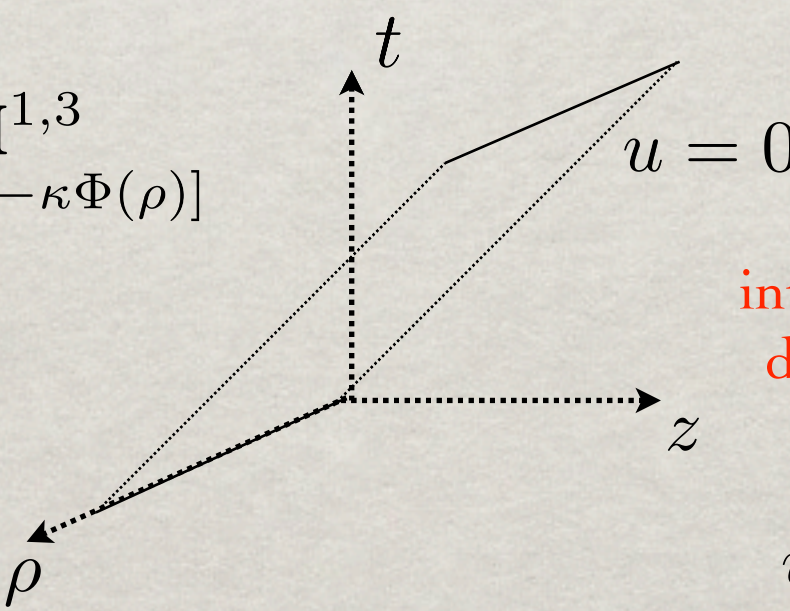

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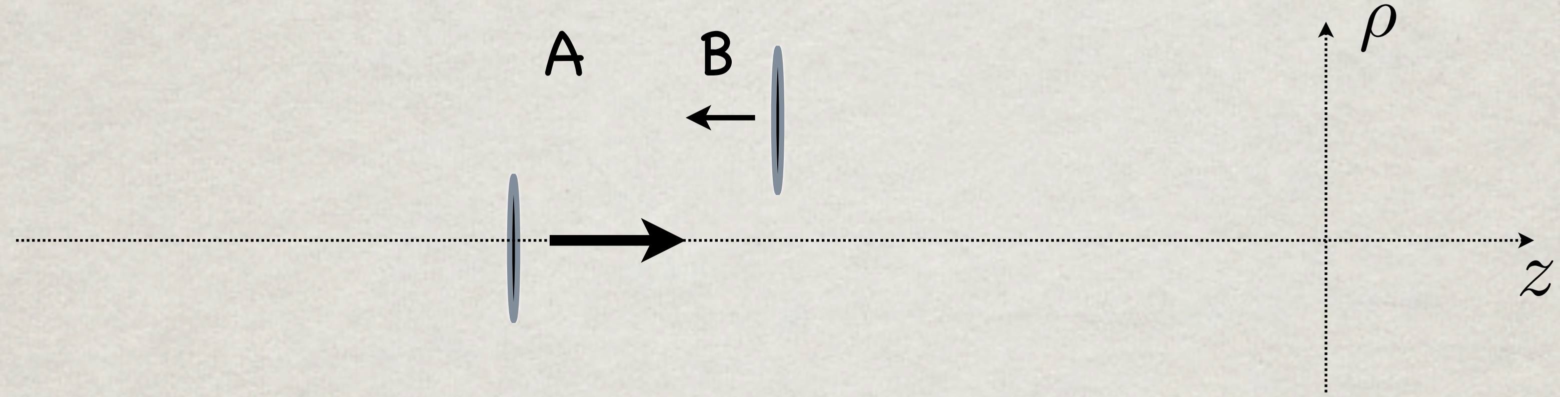

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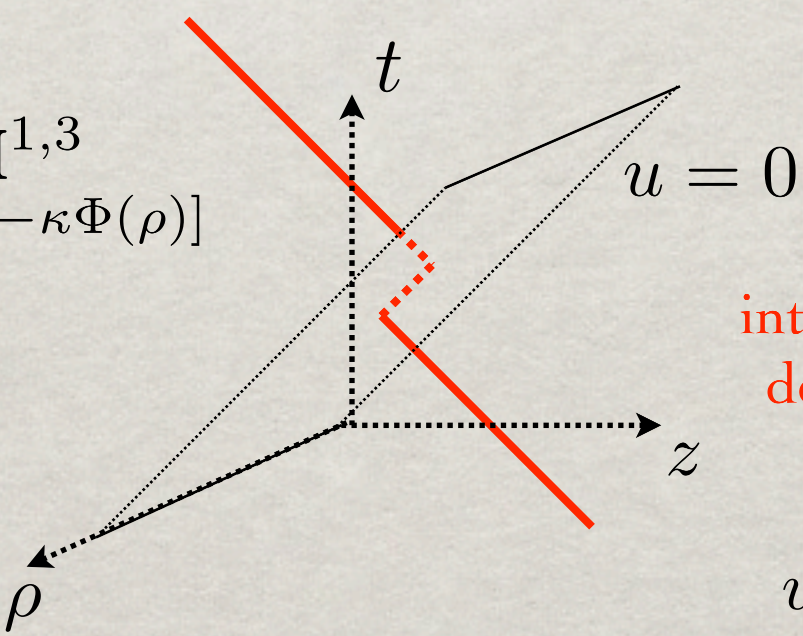

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