Improvement of energy-momentum tensor and non-Gaussianities in holographic cosmology

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Overview

- Inflation is good. Maybe too good.
- UV theory? — holography: inflationary spacetime ⟺ 3d QFT
- Holographic description of inflation — immature
- What is the dual 3d QFT?
- Universality of CFT — model independent feature: $T_{\mu\nu}$
- Conformal invariant or scale invariant?
- Our results: Breaking of conf. invariance ⟺ non-Gaussianity

Holographic cosmology

- dS/CFT proposal [Witten] [Strominger]
- Inflation as dS holography with RG flow [Larsen, van der Schaar, Leigh (2002)] [Maldacena (2002)] [many others]
- Power spectrum and bispectrum, assuming particular field content in the 3d QFT [McFadden, Skenderis]
- Power spectrum and bispectrum, including effects of RG flow [Bzowski, McFadden, Skenderis, Garriga, Urakawa, others]

(A)dS/CFT

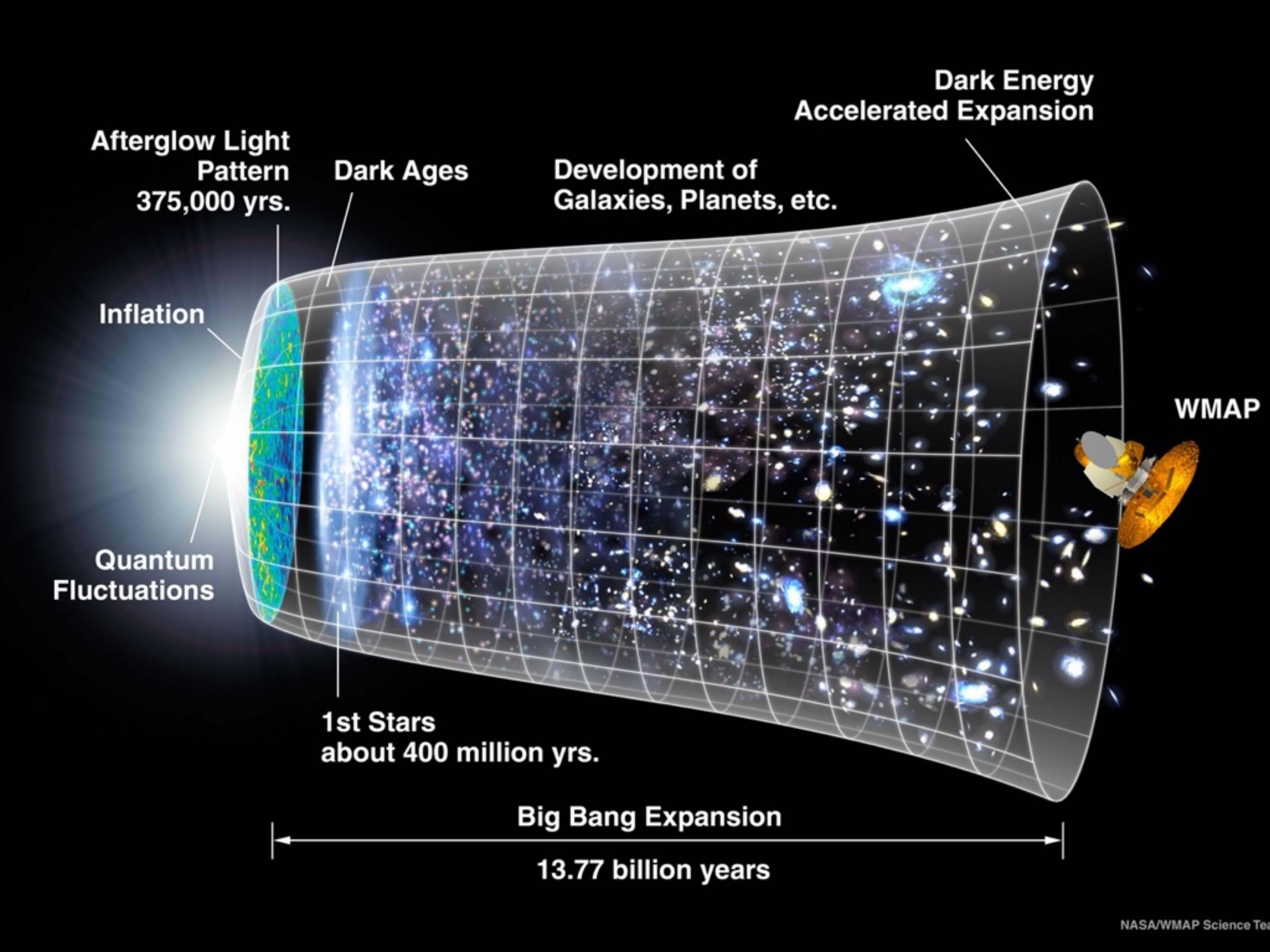
- Strongly coupled/weakly coupled duality
- A tool to compute strongly coupled dynamics using Einstein gravity, or quantum gravitational dynamics using perturbative QFT

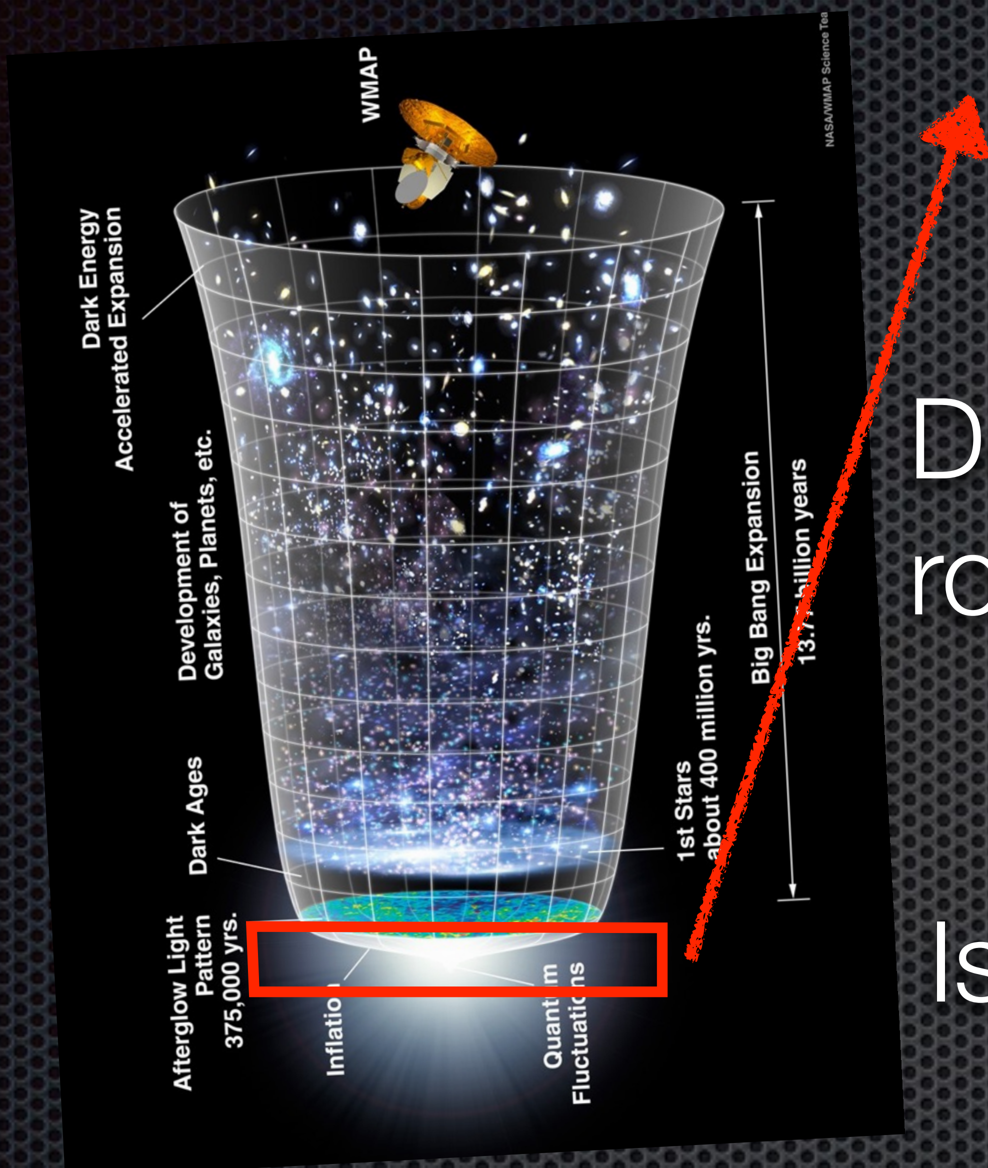
$$\Psi_{\mathrm{dS}}[\underline{g_{ij}(x), g^I(x)}] = Z_{\mathrm{CFT}}[\underline{g_{ij}(x), g^I(x)}]$$

boundary conditions sources of $T^{ij}(x), \mathcal{O}_I(x)$

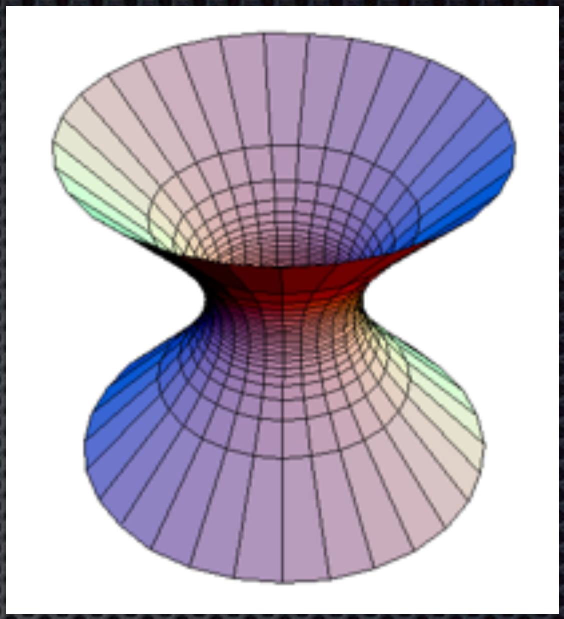
- Dictionary: boundary value of metric = source of EM tensor in the boundary theory
- Metric fluctuations $\Longleftrightarrow$ correlators of the boundary EM tensor

Is the CMB conformal invariant,
or just scale invariant?


Dark Energy
Accelerated Expansion
Afterglow Light
Pattern
375,000 yrs.
Dark Ages
Development of
Galaxies, Planets, etc.
Inflation
WMAP
Quantum
Fluctuations
1st Stars
about 400 million yrs.
Big Bang Expansion
13.77 billion years
NASA/WMAP Science Tea



4d (approximate)
de Sitter spacetime



De Sitter group is *SO*(4,1):
rotations in the ambient spacetime.

Isomorphic to *3d conformal group*

Conformal transformation on inflationary spacetime

- FRW metric: $ds^2 = \dfrac{-d\tau^2 + (dx^i)^2}{H^2\tau^2}, \quad -\infty < \tau < 0$
  - P_i: translation in 3d space (homogeneity)
  - M_{ij}: rotation in 3d space (isotropy)
  - D: simultaneous scaling $\tau \to \lambda\tau, \quad x^i \to \lambda x^i$

    (➔ scale invariance)
  - K_i: nonlinear transformation $\tau \to \tau + 2(\boldsymbol{b}\cdot\boldsymbol{x})\tau,$

$$x^i \to x^i + (\boldsymbol{\tau}^2 - \boldsymbol{x}^2)b^i + 2(\boldsymbol{b}\cdot\boldsymbol{x})x^i$$

(➔ ?)

Observables

- Scale invariance: 7 parameters (P_i, M_{ij}, D)
- Conformal invariance: 10 parameters (P_i, M_{ij}, D, K_i)
  - Conformal invariance impose strong constraints on correlation functions (power spectrum, bispectrum, trispectrum, etc.) of primordial fluctuations
  - Gravitons and curvatons: conformal
    [Maldacena Pimentel 2011] [Creminelli 2011]
  - Inflaton fluctuations: only scale invariant

Correlation functions

for quasi-primary field of dimension Δ

$$\phi(x) \to \phi'(x') = \left|\frac{\partial x'}{\partial x}\right|^{-\Delta/d} \phi(x)$$

Poincaré + scaling | Conformal

2pt correlators

Poincaré + scaling:

$$\langle \phi_1(x_1)\phi_2(x_2)\rangle = \frac{C_{12}}{|x_1 - x_2|^{\Delta_1+\Delta_2}}$$

Conformal:

$$\langle \phi_1(x_1)\phi_2(x_2)\rangle = \frac{C_{12}}{|x_1 - x_2|^{2\Delta_1}} \quad (\Delta_1 = \Delta_2)$$

$$= 0 \quad (\Delta_1 \neq \Delta_2)$$

3pt correlators

Poincaré + scaling:

$$\langle \phi_1(x_1)\phi_2(x_2)\phi_3(x_3)\rangle = \frac{C_{123}}{x_{12}^a x_{23}^b x_{31}^c}$$

$$x_{ij} = |x_i - x_j|, \quad a + b + c = \Delta_1 + \Delta_2 + \Delta_3$$

Conformal:

$$\langle \phi_1(x_1)\phi_2(x_2)\phi_3(x_3)\rangle$$

$$= \frac{C_{123}}{x_{12}^{\Delta_1+\Delta_2-\Delta_3} x_{23}^{\Delta_2+\Delta_3-\Delta_1} x_{31}^{\Delta_3+\Delta_1-\Delta_2}}$$

Energy-momentum tensor

- EM tensor: conserved current of translation
- In presence of rotation (or Lorentz) symmetry, EM tensor can be made symmetric (Belinfante tensor)
- If in addition scaling current conserved and Virial $V^j = \partial_i L^{ij}$ exists, EM tensor can be made traceless
- Traceless EM tensor ➔ classical conformal symmetry (invariance of the action) $x^i \to x^i + \epsilon^i$ $\quad \delta g_{ij} = \partial_i \epsilon_j + \partial_j \epsilon_i = \frac{2}{d} g_{ij} \partial_k \epsilon^k$

$$\delta S = \int d^d x T^{ij} \partial_i \epsilon_j = \frac{1}{2} \int d^d x T^{ij} \underline{(\partial_i \epsilon_j + \partial_j \epsilon_i)} = \frac{1}{d} \int d^d x T^i{}_i \partial_j \epsilon^j$$

EM tensor and symmetries

- Poincaré = translation + rotation

conserved current

improvement term

$$T^{ij} = T_c^{ij} + \partial_k B^{kij} + \frac{1}{2}\partial_k \partial_\ell X^{k\ell ij}$$

Symmetric and traceless EM tensor

- Scaling symmetry + virial $V^j = \partial_i L^{ij}$

Trace identity (local Callan-Symanzik equation):

$$T^i{}_i = \beta^I \mathcal{O}_I + \partial_i J^i + \kappa^\alpha \Box \mathcal{O}_\alpha$$

Our work

- Holographic cosmology with EM tensor improvement
- Recall the trace identity: $T^i{}_i = \beta^I \mathcal{O}_I + \partial_i J^i + \kappa^\alpha \Box \mathcal{O}_\alpha$

  =0 in exact dS

  improvement term
- Action: $S = \frac{1}{2}\int d^3x \sqrt{g}\Big(g^{ij}\partial_i\phi^I\partial_j\phi^I + \xi R(\phi^I)^2\Big)$

  $I = 1, 2, \cdots, N_\xi$

  ξ=0: minimal coupling; ξ=⅛: conformal coupling
- The improvement term affects the observables
- Computed power spectrum and bispectrum including the improvement term in exact dS

Power spectra

- Holographic computation
  - Scalar power spectrum $$\Delta_{\mathrm{S}}^2(k) = \frac{k^3}{2\pi^2}\langle\langle\zeta(k)\zeta(-k)\rangle\rangle = \frac{16}{\pi^2 N_\xi (1-8\xi)^2}$$
  - Tensor power spectrum $$\Delta_{\mathrm{T}}^2(k) = \frac{k^3}{2\pi^2}\langle\langle\gamma_{ij}^*(k)\gamma^{ij}(-k)\rangle\rangle = \frac{512}{\pi^2 N_\xi}$$
  - Tensor/scalar ratio $$r \equiv \frac{\Delta_{\mathrm{T}}^2(k)}{\Delta_{\mathrm{S}}^2(k)} = 32(1-8\xi)^2$$
- Observation
  - [Planck (2013)] $\Delta_{\mathrm{S}}^2(k_0) = 2.215 \times 10^{-9}$ $\quad k_0 = 0.05\ \mathrm{Mpc}^{-1}$
  - [BICEP2 (2014)] $r = 0.20^{+0.07}_{-0.05}$ $\quad N_\xi \gg 1, \quad \left|\xi - \frac{1}{8}\right| \approx 10^{-2}$

Central charge of the holographic universe [Larsen Strominger]: $C_T = \frac{3}{32}\frac{N_\xi}{\pi^2} \approx 10^9.$

Non-Gaussianities

$$\langle\langle \zeta_{k_1}\zeta_{k_2}\zeta_{k_3}\rangle\rangle = f_{\rm NL}^{\rm local} B_\zeta^{\rm local} + f_{\rm NL}^{\rm equil} B_\zeta^{\rm equil} + f_{\rm NL}^{\rm ortho} B_\zeta^{\rm ortho}$$

- Holographic computation with improvement term

$$f_{\rm NL}^{\rm local} = 0, \quad f_{\rm NL}^{\rm equil} = \frac{5}{36}(1+24\xi), \quad f_{\rm NL}^{\rm ortho} = -\frac{10}{9}\xi$$

- Observation [Planck 2013]

$$f_{\rm NL}^{\rm local} = 2.7 \pm 5.8, \quad f_{\rm NL}^{\rm equil} = -42 \pm 75, \quad f_{\rm NL}^{\rm ortho} = -25 \pm 39$$

  - Holographic computation is consistent with observational constraint (but hopeless to detect in near future)
  - Similar to in-in formalism sub-horizon computation of NG

[Bzowski, McFadden, Skenderis (2009-2013)]: $f_{\rm NL}^{\rm local} = f_{\rm NL}^{\rm ortho} = 0, \quad f_{\rm NL}^{\rm equil} = \frac{5}{36}$

bosons, fermions, gauge fields (ξ = 0, ⅛)

Summary

- Holography may help us understand the primordial fluctuations better.
- Improvement of EM tensor — scale invariant but not necessarily conformal invariant density fluctuations
- Equilateral and orthogonal type non-Gaussianities of O(1) predicted (but no local type)

Thank you for your attention.