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TALK #1 A SIMPLE CONTEXT FOR RADIATION

REACTION

a scalar charge in flat-duct

$$ds^2 = \alpha(\gamma)(-dy^2 + dx^2 + dy^2 + dz^2)$$

$$\alpha(\gamma) \propto \gamma$$

Green's function is simple (Waylen, Caldwell),
 a tail term. Radiation reaction could be solved
 exactly. Gravitational case not much more complicated.

Various ideas could be tested :

- regularization of mode sum
- normal neighbourhood expansion
- Wiseman matching

Real questions could be answered :

Is a particle in a nonremoving geodesic
 driven toward comotion?

71

TALK #2: RADIATION REACTION BEYOND PERTURBATION

(with W. Israel and

$$g_{\alpha\beta} = \underbrace{g_{\alpha\beta}^{\text{back}} + g_{\alpha\beta}^{\text{tail}}}_{\text{smooth}} + \underbrace{g_{\alpha\beta}^{\text{direct}}}_{\text{singular: } \mu/r}$$

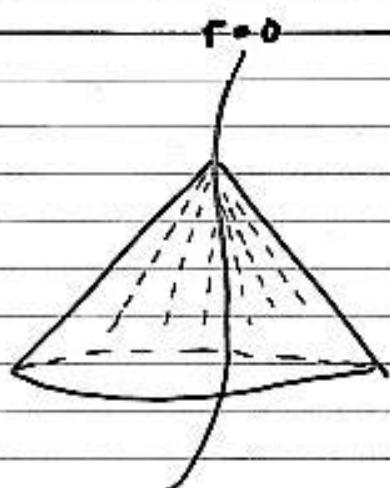
$$F_{\alpha\beta} = S(\text{world line})$$

$$\text{world line} = \text{geodesic of } g_{\alpha\beta}^{\text{back}} + g_{\alpha\beta}^{\text{tail}}$$

Do similar statements exist in the exact theory?

Consider an arbitrary world line in a spacetime $g_{\alpha\beta}$

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$$\text{world line: } r=0$$

$$l^\alpha = \frac{dx^\alpha}{dr} = \text{tangent to null cone generators}$$

l^α is determined by world line

Consider the following proposal for the full metric:

$$\tilde{g}_{\alpha\beta} = \underbrace{g_{\alpha\beta}}_{\text{"gap + h_{\alpha\beta}"}} + H \underbrace{ds^2_{\text{dp}}}_{\text{"direct"}}$$

(generalized Kerr-Schild)

The exact field equations for $\tilde{g}_{\alpha\beta}$ are linear in H .

If we make $\tilde{R}_{\alpha\beta} = \text{finite for } r \neq 0$? What does

$$\tilde{R}_{\alpha\beta} = O(1/r^2) + O(1/r) + O(1/r) + \text{finite}$$

$$\text{If } H = 2\mu/r, \text{ then } O(1/r^3) = 0$$

$$\text{If motion is geodesic, then } O(1/r^2) = 0$$

Putting the $O(1/r)$ terms to zero requires restrictive conditions on background ...

Could something like this be made to work?

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2- LEAVER'S METHOD

The radiation field ψ satisfies

$$\left(-\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial r^{(0)}^2} - V \right) \psi = 0$$

$$V = \left(1 - \frac{r_0}{r} \right) \left(\frac{Q(t)}{r^2} - \frac{(S-1)r_0}{r^3} \right)$$

Initial data is provided by $\psi(0, r)$, $\dot{\psi}(0, r)$. Future evolution is determined by retarded Green's function g which satisfies wave equation with $\delta(t)\delta(r^*-r')$ on right hand side.

$$-\int \psi(0, r') g(t, r', r) dr'$$

The Fourier transform of the Green's function

$$\tilde{g}(w, r, r') = \frac{\psi^{in}(r') \psi^{up}(r)}{2i\omega A^{in}}$$

$$\psi^{in} \sim e^{-i\omega r^*} \quad r \rightarrow r_0$$

$$\psi^{up} \sim e^{+i\omega r^*} \quad r \rightarrow \infty$$

$$- \quad \rightarrow \infty$$

$$\psi^{in} = A^{up} \psi^{up} + A^{dn} \psi^{dn}$$

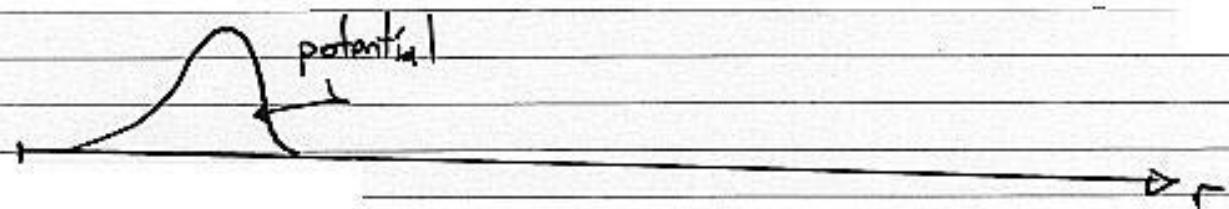
→ Need to calculate ψ_{in} , ψ_{up} , ψ_{dn} , and invert Fourier transform.

- Calculation of wave functions

Solve $(\frac{\delta^2}{\delta r^{*2}} + \omega^2 - V) \psi = 0$ in two

different zones : near zone ($\omega r \ll 1$) and weak-field zone ($r \gg \lambda$). These zones overlap when $\omega r \ll \omega \tau \ll 1$.

⇒ low frequency expansion $\omega r \ll 1$



WEAK-FIELD ZONE

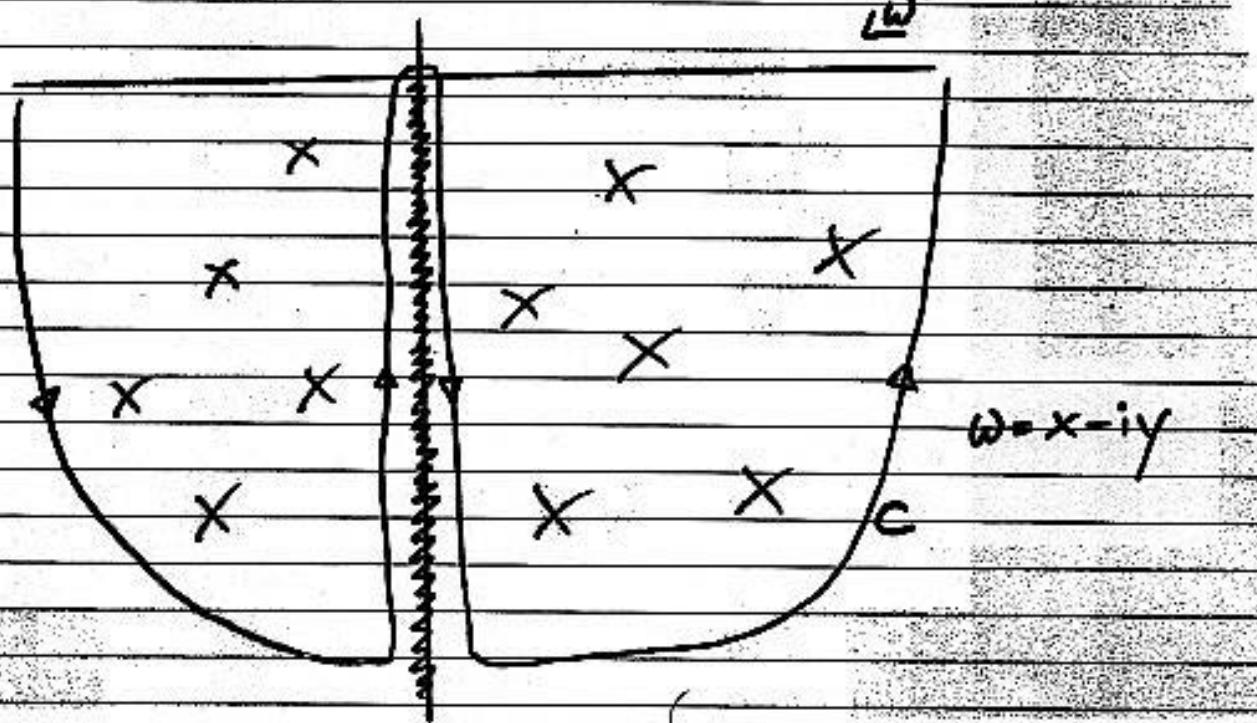
Near zone : $\psi \sim$ hypergeometric function

Weak-field zone : $\psi \sim (wr)^{1/2} e^{i wr} +$ mess of Bessel functions

The solutions are matched in the overlap.

Inversion of Fourier transform

late times, it is advantageous to deform the contour into the lower half plane:



$$g(t) = \int_C \tilde{g}(w) e^{-ixt} e^{-yt} dw + \sum \text{residues}$$

The branch cut arises because $\tilde{g}(w)$ is multivalued in the complex plane. The poles represent the quasi-normal modes. Their contribution is exponentially damped. At late times,

$$g(t) \sim \frac{1}{i} \int_0^\infty [\tilde{g}(w) - \tilde{g}(we^{2\pi i})] e^{-yt} dy$$

$$\omega = -iy$$

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- Putting it all together

$$\alpha(t) \sim \frac{1}{2} (-1)^{\ell} \operatorname{Re} \int_0^{\infty} h(r) h(r') e^{-rt} dy + O(r^{\ell})$$

$$\text{In weak-field zone} \quad h(r) \approx 2(-1)^{\ell+1} / (wr) (e/wr)$$

$$\text{Near event horizon} \quad h(r) \approx 2(\ell+1)! (\ell+1)! (-\omega r_e)^{\ell+1} e$$

All that's left to do is to integrate wrt y ...

- For r, r' both in weak field region

$$\underline{\alpha}(t, r, r') = \omega(-1)^{\ell} \operatorname{Re} \frac{(2\ell+2)!}{(2\ell+1)!} \frac{(rr')^{\ell+1}}{x^{2\ell+3}} G(x, y)$$

$$G(x, y) = \sum_{m=0}^{\infty} \frac{(2m+1)!}{m! (2m+1)!} x^{2m} F(-m, -\ell; -m, \ell + \frac{3}{2}; y^2/x^2)$$

$$\begin{matrix} & & & & & \\ J & , & & & & \end{matrix}$$

If $r' \ll r$ (initial data has compact support), then

$$m = \frac{1}{m! (\ell+1)!}$$

$$= \frac{1}{(1-x^2)^{\ell+2}}$$

$$\Rightarrow g(t, r, r') = 2(-1)^t \operatorname{Re} \frac{(2\ell+2)!!}{(2\ell+1)!!} \frac{(rr')^{\ell+1}}{\sqrt{r^{2\ell+3}} (1-r'^2/t^2)^{\ell+2}}$$

$$= (-1)^t \operatorname{Re} \frac{(t+1)!}{(2\ell+1)!!} r^{\ell+1} \frac{(1-u/v)^{\ell+1} (1+u/v)}{v^{\ell+2}}$$

- For $r=r_e$, r' in weak-field region

$$g(v, r') = 2(-1)^t \operatorname{Re} \frac{(t-s)!(t+s)!(2\ell+2)!!}{(2\ell)!(2\ell+1)!!} \frac{(rer')^{\ell+1}}{\sqrt{r^{2\ell+3}} (1-r'^2/v^2)^{\ell+2}}$$

FIELD DECAY

Near i^+ : $\psi \sim 2(-1)^{\ell+1} \operatorname{Re} I_{A\ell} \frac{(2\ell+2)!!}{(2\ell+1)!!} \frac{r^{\ell+1}}{r^{2\ell+3}}$

At S^+ : $\psi \sim (-1)^{\ell+1} \operatorname{Re} I_{A\ell} \frac{(2\ell+1)!!}{(2\ell+1)!!} \frac{1}{v^{\ell+2}}$

At H^+ : $\psi \sim 2(-1)^{\ell+1} \operatorname{Re} \frac{e^{i\theta}}{(2\ell)!(2\ell+1)!!} \frac{(2\ell)!(2\ell+1)!(2\ell+2)!!}{(2\ell+1)!(2\ell+1)!!} I_{A\ell} \frac{1}{v^{2\ell+3}}$

$I_{A\ell} = \int \psi(r) r^{\ell+1} dr$