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Title:

Instantaneous Forces on Scalar Charges
To First Order in Mass of The Central
Body:
(No Time for Time Averaging)

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$O[M^1]$ G's Fntn for Spherical Spacetime

(1) Scalar wave equation

$$\square_s V = -\left(\frac{r+M}{r-M}\right)V_{,tt} + \frac{1}{(r+M)^2} [(r^2 - M^2)V_{,rr}]_{,r} - \frac{1}{(r+M)^2} L_{op}^2 V ,$$

$$\square_s G(x, x') = -\frac{\delta^4(x-x')}{\sqrt{-g}}$$

(2) Iterative solution for Green's Function

$$G(x, x') = G_o + MG_1 + M^2G_2 + \dots$$

$$\square_s = \square_o + M\square_1 + M^2\square_2 + O[M^3]$$

where

$$\square_o \equiv -\frac{\partial^2}{\partial t^2} + \nabla^2 \quad (\text{flatspace wave operator})$$

$$\square_1 \equiv \frac{2}{r} \left[-\square_o - 2\frac{\partial^2}{\partial t^2} \right]$$

$$\square_2 \equiv \frac{1}{r^2} \left[3\square_o + \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial r^2} \right]$$

$$G_o(x, x') \equiv \frac{1}{4\pi} \frac{\delta[t' - (t - |\mathbf{x} - \mathbf{x}'|)]}{|\mathbf{x} - \mathbf{x}'|}$$

$$\square_o G_o(x, x') \approx \frac{1}{r} \frac{\partial^2}{\partial t^2} G_o(x, x')$$

$$G_1(x, x') = \int G_o(x, x'') \frac{2}{r''} \left[-\square_o - 2\frac{\partial^2}{\partial t''^2} \right] G_o(x'', x') [\sqrt{-g''}]_o d^4x''$$

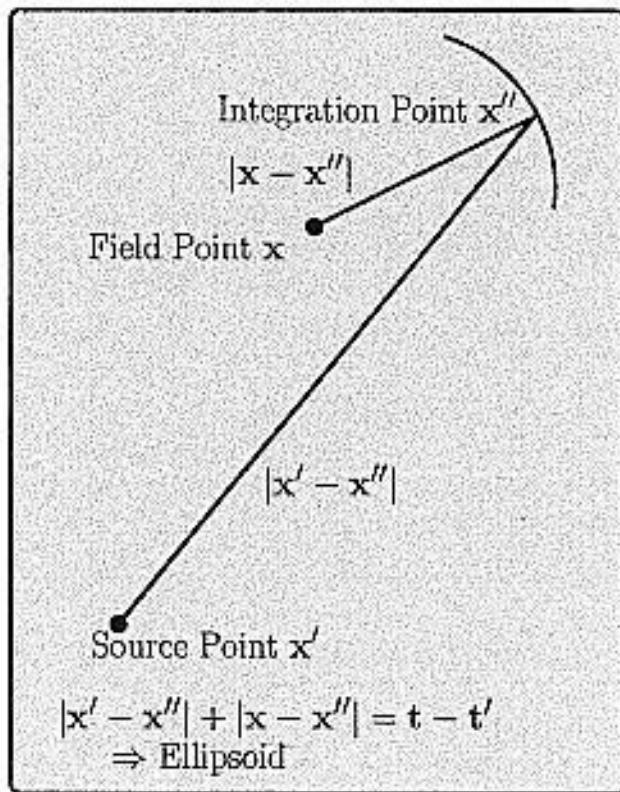
Green's function Integral

Some Details

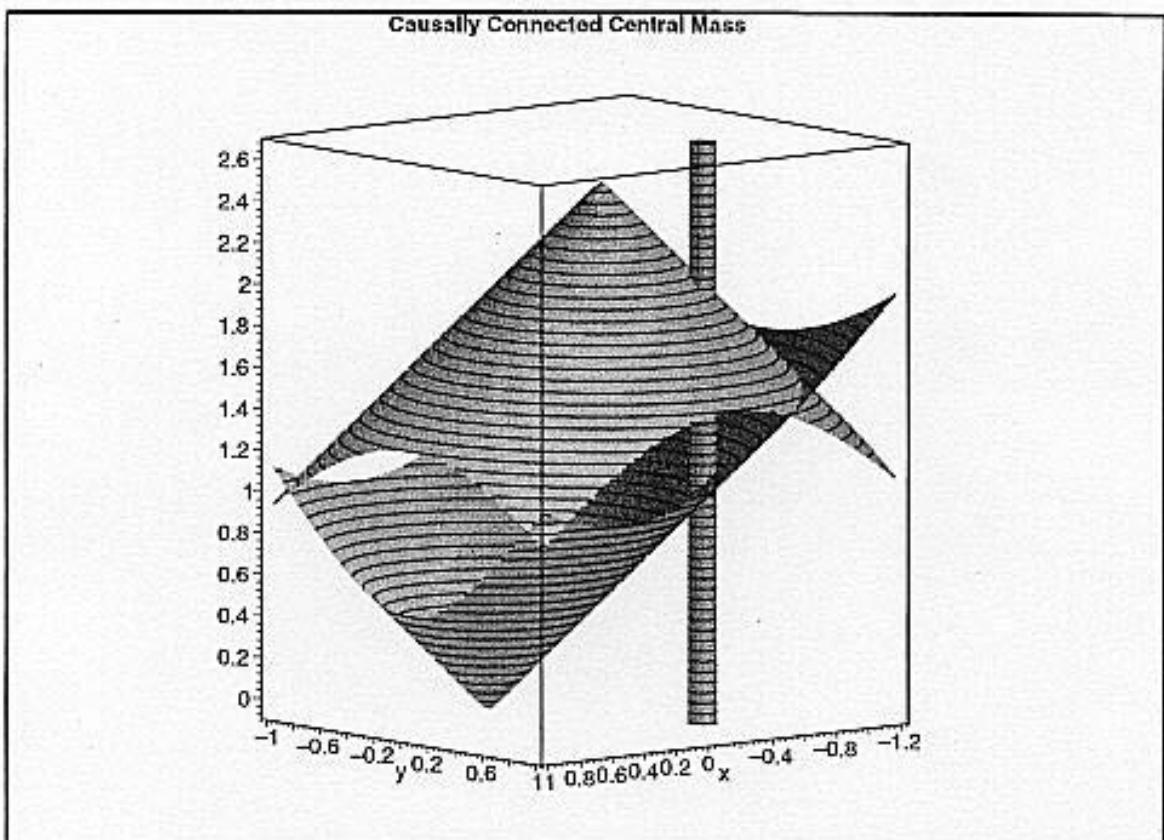
$$G_1(x, x')$$

$$= \frac{1}{4\pi} \frac{\partial^2}{\partial t'^2} \int \frac{1}{r''} \frac{\delta[t'' - (t - |\mathbf{x} - \mathbf{x}''|)]}{|\mathbf{x} - \mathbf{x}''|} \frac{1}{4\pi} \frac{\delta[t' - (t - |\mathbf{x} - \mathbf{x}'|)]}{|\mathbf{x} - \mathbf{x}'|} \sqrt{-g} d^3x'$$

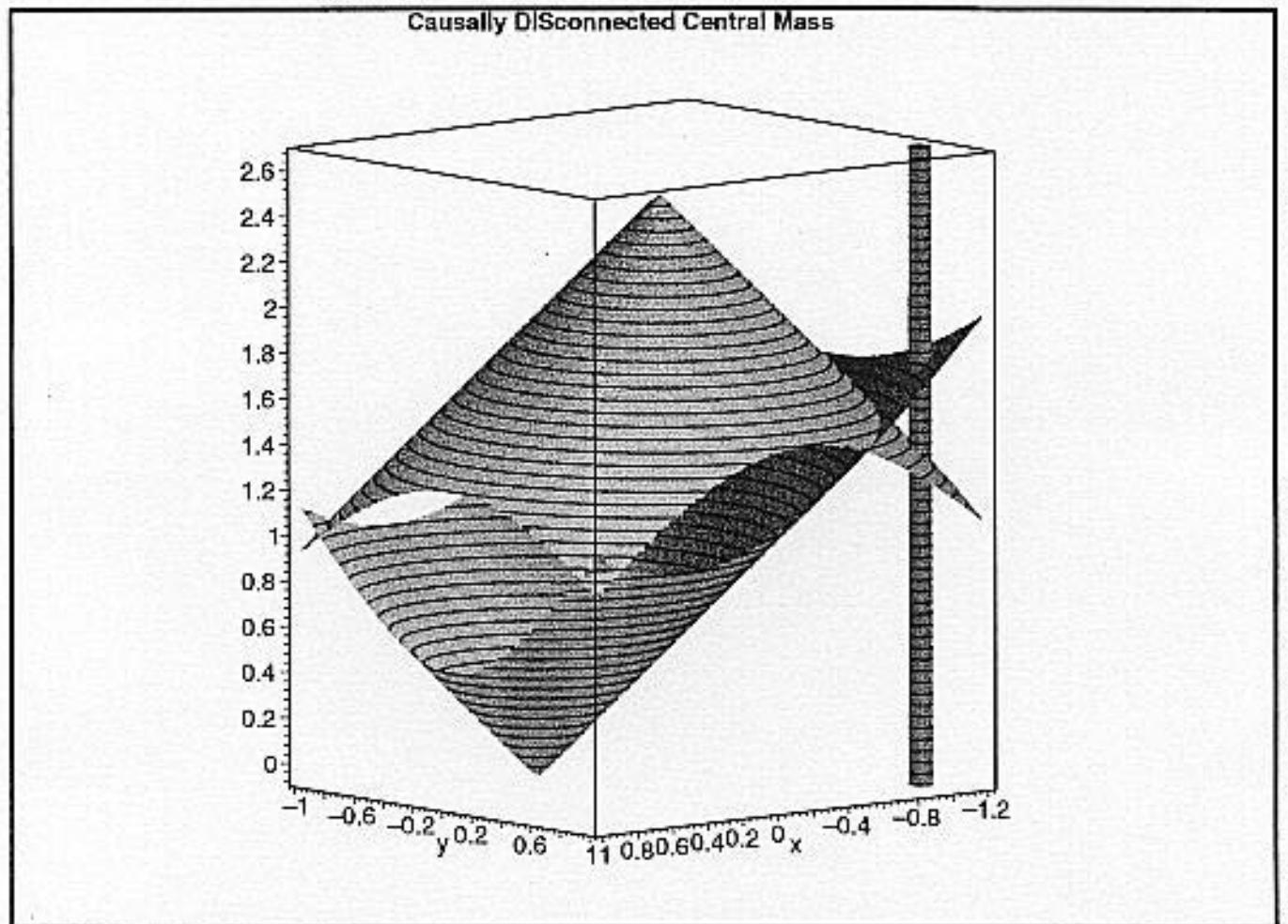
$$\frac{\partial^2}{\partial t'^2} \frac{1}{(4\pi)^2} \int \frac{1}{r''} \frac{\delta[(t - t') - (|\mathbf{x} - \mathbf{x}''| + |\mathbf{x}' - \mathbf{x}''|)]}{|\mathbf{x} - \mathbf{x}''| |\mathbf{x}' - \mathbf{x}''|} \sqrt{-g} d^3x'$$



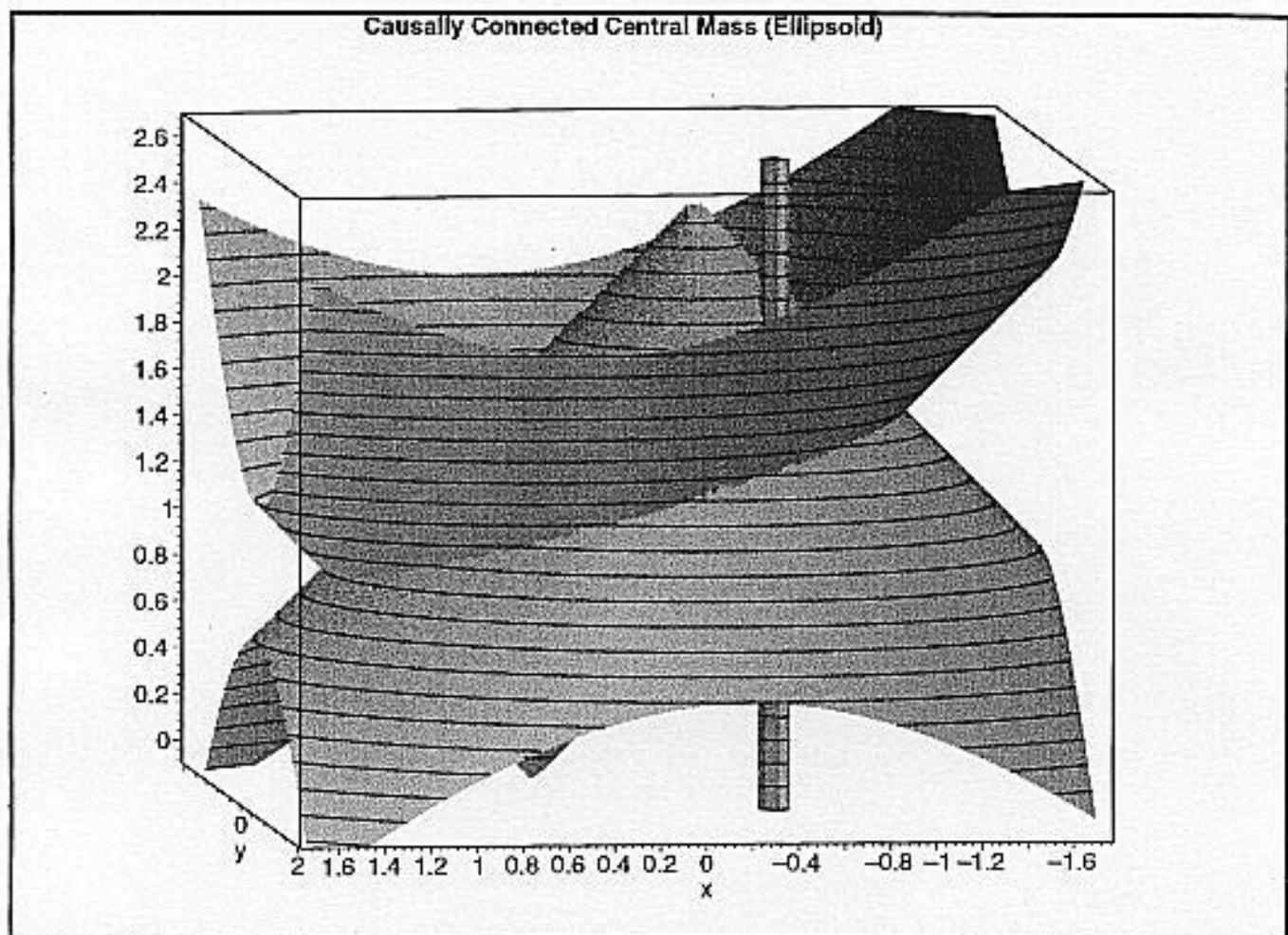
Ellipsoid of Integration (Mass inside)



Ellipsoid of Integration (Mass Outside)



Cone Intersection: Ellipsoid of Integration



Green's function for Spherical Spacetime

Green's Function

$$G(x, x') = \frac{1}{4\pi} \left\{ \frac{\delta[t' - (t - |\mathbf{x} - \mathbf{x}'|)]}{|\mathbf{x} - \mathbf{x}'|} + M \left[+2 \frac{\delta'[t' - (t - |\mathbf{x} - \mathbf{x}'|)]}{|\mathbf{x} - \mathbf{x}'|} \ln \left(\frac{r + r' + |\mathbf{x} - \mathbf{x}'|}{r + r' - |\mathbf{x} - \mathbf{x}'|} \right) \right. \right.$$
$$\left. \left. + 4 \frac{\delta[t - t' - (r + r')]}{(t - t')^2 - |\mathbf{x} - \mathbf{x}'|^2} - 8 \frac{\Theta[t - t' - (r + r')](t - t')}{[(t - t')^2 - |\mathbf{x} - \mathbf{x}'|^2]^2} \right] \right\} + O[M^2]$$

Flatspace Green's Function

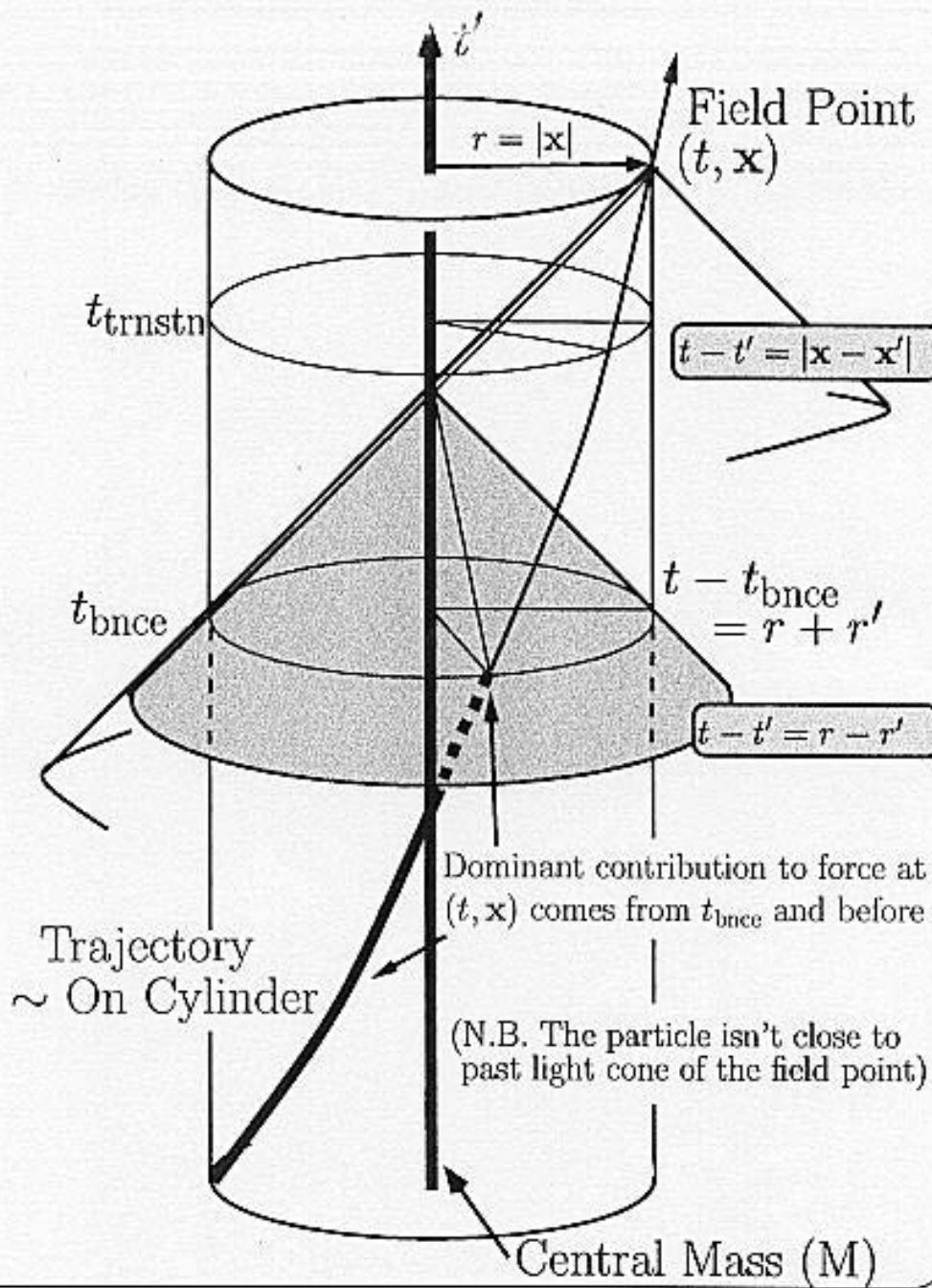
Corrections to the null cones

Shapiro Time-Delay

Tail (support on the interior of cones)

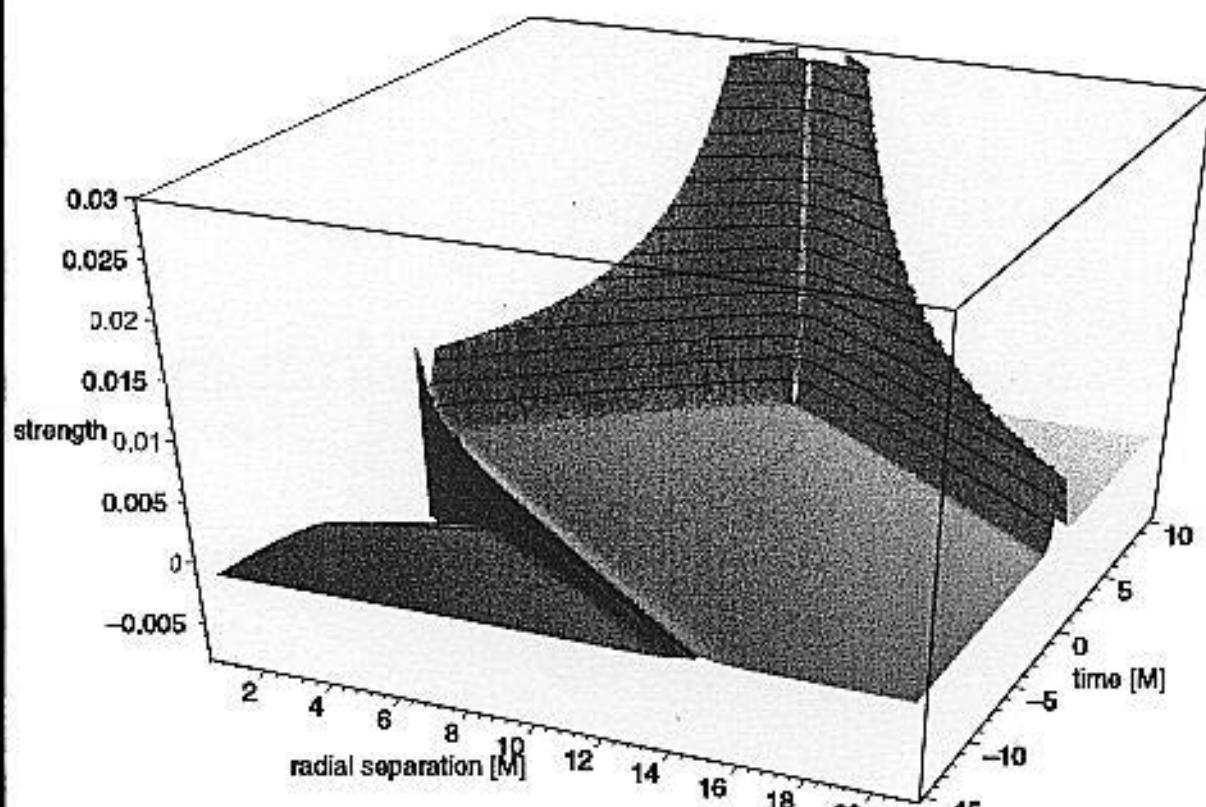
This has been shown ($r \rightarrow \infty$) to give the same radiation as the standard perturbation theory [Leonard&Poisson]

From Whence the Field and the Force

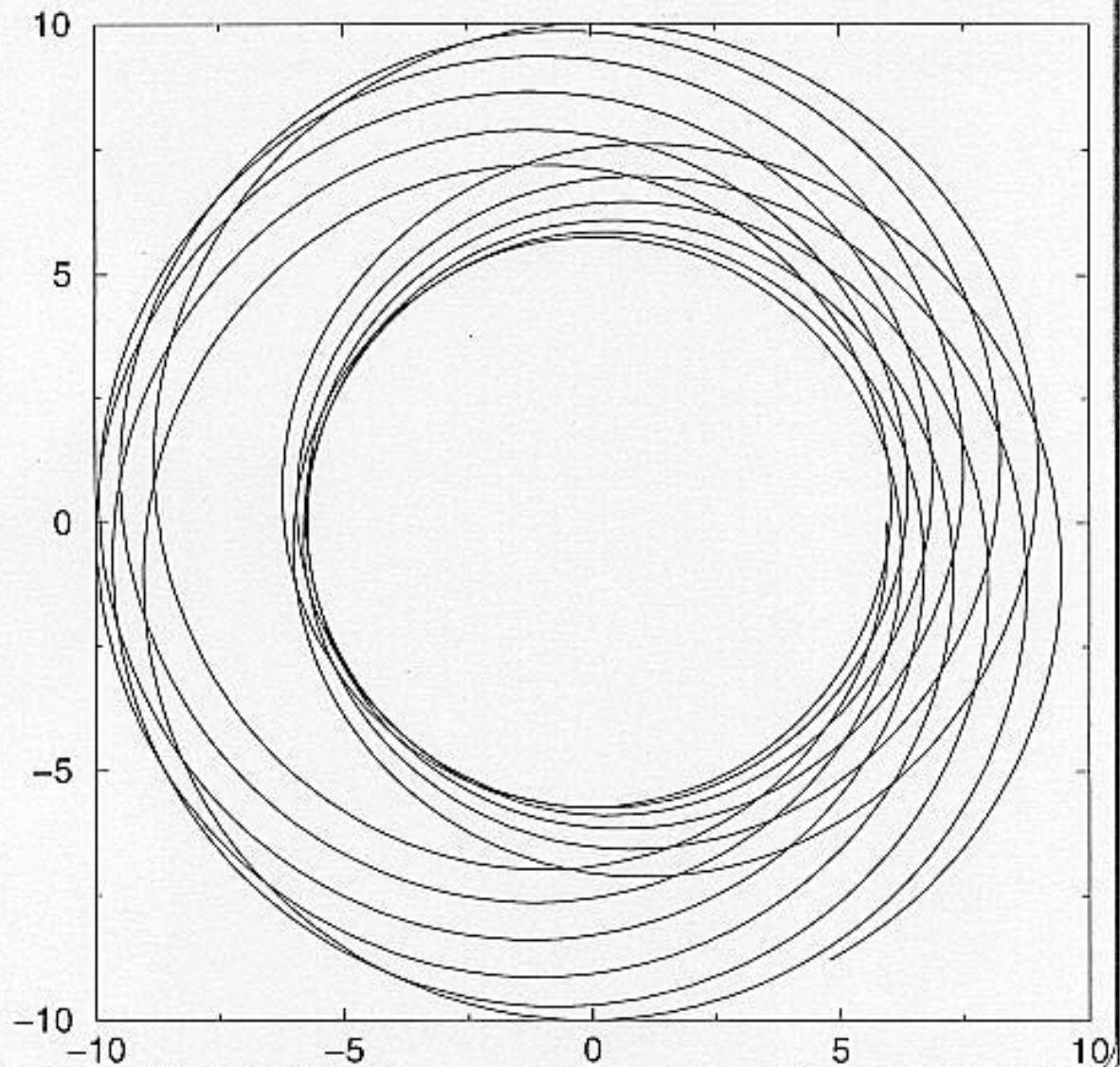


A Look At the Green's Function

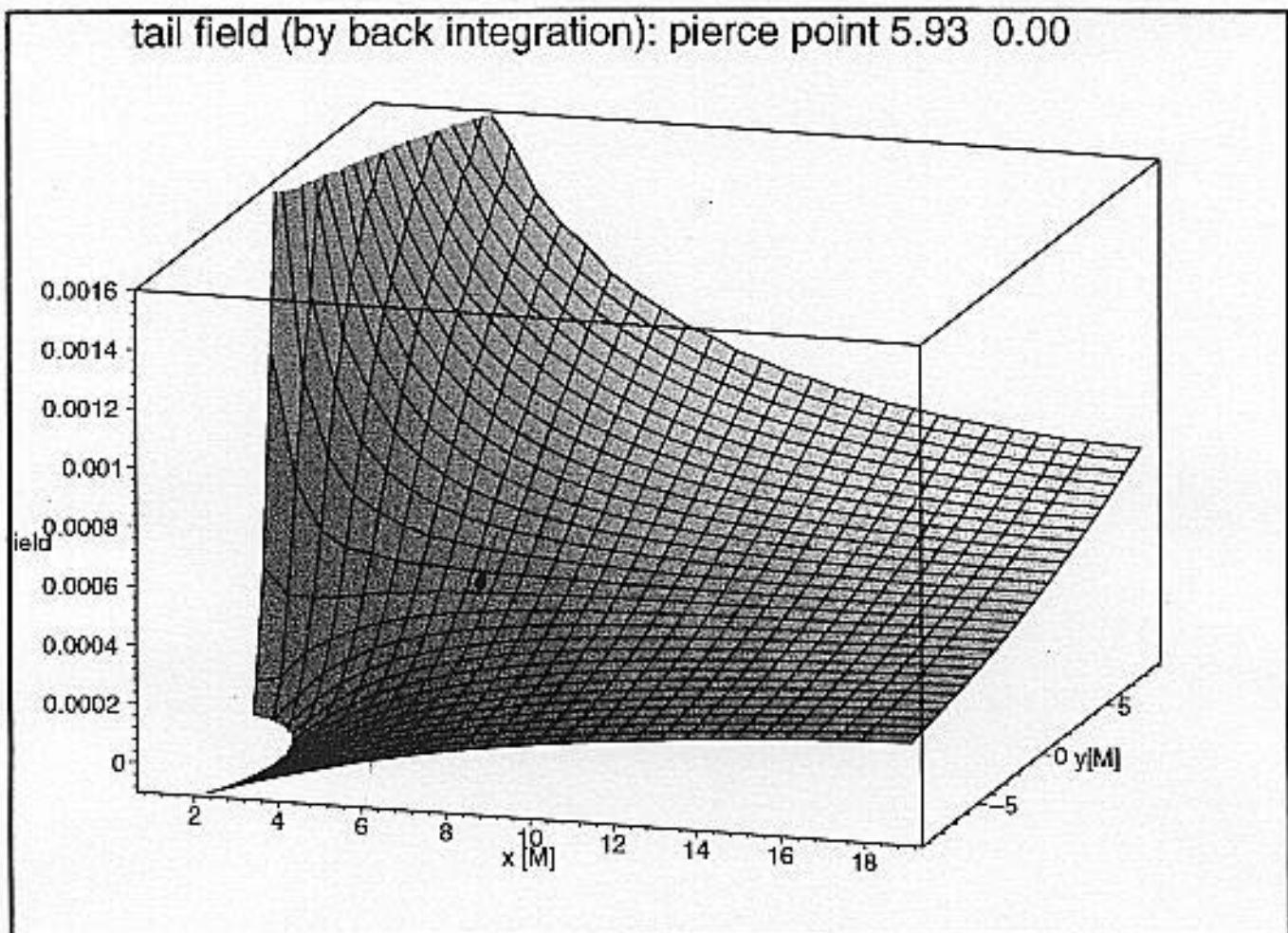
causal struture of greensfunction (Heaviside enhanced 200 times)



The “Geodesic History”

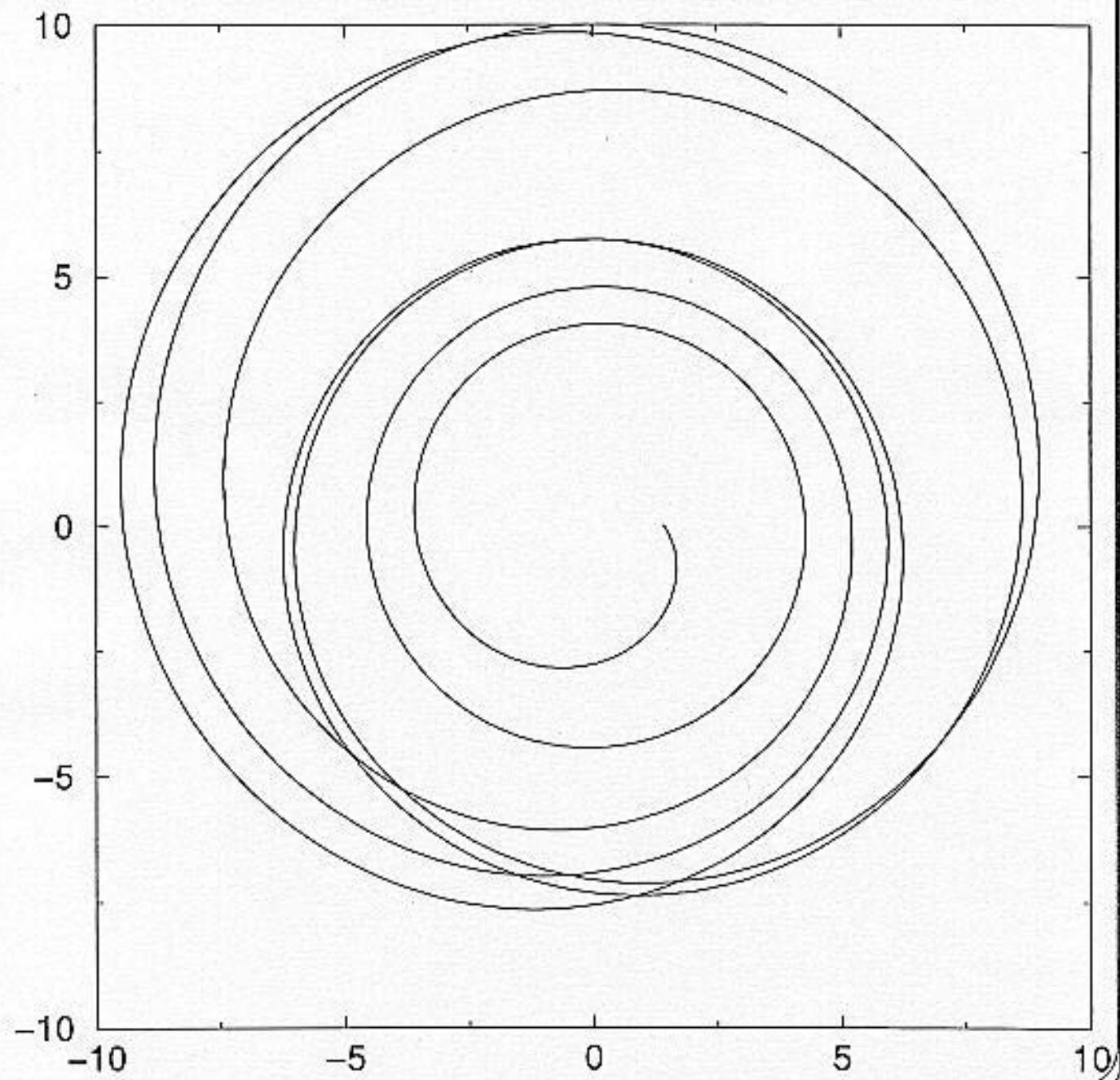


The Tail Field Delivers The Force



Note the Tail Field is Smooth!

The Decaying Orbit



To The Future:

- (1) This Technique of Computing the $O[M^1]$ works for the leading order “Kerr” part of the metric.
- (2) Do the same calculation for a Mass Charge.
 - Deal with the “Gauge” Issue.
 - Recover the 1-PN Equations of Motion
- (3) Continue with Multipole Calculation of Force.
- (4) Use the actual mode functions.

Mode Decomposition of $O[M]$ G's Fnctn

$$G(x, x') = \frac{1}{4\pi} \left\{ \frac{\delta[t' - (t - |\mathbf{x} - \mathbf{x}'|)]}{|\mathbf{x} - \mathbf{x}'|} \right. \\ + M \left[+2 \frac{\delta'[t' - (t - |\mathbf{x} - \mathbf{x}'|)]}{|\mathbf{x} - \mathbf{x}'|} \ln \left(\frac{r + r' + |\mathbf{x} - \mathbf{x}'|}{r + r' - |\mathbf{x} - \mathbf{x}'|} \right) \right. \\ \left. + 4 \frac{\delta[t - t' - (r + r')]}{(t - t')^2 - |\mathbf{x} - \mathbf{x}'|^2} \right. \\ \left. - 8 \frac{\Theta[t - t' - (r + r')](t - t')}{[(t - t')^2 - |\mathbf{x} - \mathbf{x}'|^2]^2} \right\} + O[M^2]$$

$$= \int e^{-i\omega(t-t')} \sum_{lm} \left\{ \frac{i\omega}{2\pi} j_l(\omega r_<) h_l^{(1)}(\omega r_>) \right. \\ - \frac{2M\omega^2}{\pi} [h_l^{(1)}(\omega r_<) h_l^{(1)}(\omega r_>) \int_0^{wr_<} x(j_l(x))^2 dx \\ + j_l(\omega r_<) h_l^{(1)}(\omega r_>) \int_{\omega r_<}^{wr_>} x h_l^{(1)}(x)(j_l(x)) dx \\ + j_l(\omega r_<) j_l(\omega r_>) \int_{\omega r_>}^{\infty} x(h_l^{(1)}(x))^2 dx] \\ \times Y_{lm}^*(\theta', \phi') Y_{lm}(\theta, \phi) \} d\omega + O[M^2]$$

How Many Modes Do You Have to Sum?

Relative Error: $V_{\text{tail}}/V_l(\text{flatspace})$

$$t - t_{\text{trnstn}} = \sqrt{2} r$$

