Numerical implementation of a local radiation reaction force.

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Motivation

- We want to understand what happens to a CFS-unstable star when the mode amplitude becomes large.
- Almost all of our intuition thus far comes from studying stars in the linear regime.
- To learn more best to mimic Nature: start with a CFS-unstable star with a small mode, and watch it grow.
- To do so need a *local radiation reaction force*.
- Surprisingly, only one attempt at this so far (Lindblom, Tohline & Vallisneri 2002). Strategy is to use Newtonian equations of motion, with the radiation reaction force added to the right hand side.

Problems problems...

There are two main difficulties in implementing a local radiation reaction force.

Problem 1: time derivatives

- Expression for the force may require evaluating a large number of time derivatives, e.g. for Burke-Thorne formalism:

$$\Phi_{RR} = -\frac{G}{5c^5} x_a x_b \frac{d^5 I^{ab}}{dt^5}$$

  where I^{ab} is the mass quadrupole of the source.

- In the linear regime not a problem as an eigenmode analysis is possible, but in a non-linear time evolution, these derivatives must be handled numerically, which will lead can large inaccuracies.
- What to do?

Solution 1: Use results from linear theory

- In the linear regime and for slowly rotating stars, modes have simple descriptions in terms of spherical harmonics. They therefore satisfy certain orthogonality properties with respect to volume integration.

- Lindblom, Tohline & Vallisneri (2002) used these slow-rotation linear results to 'project out' the amplitude and frequency of the mode in the rapid-rotation non-linear regime. Can then use this frequency to approximate the time derivatives:

$$\frac{d^n}{dt^n} \to (i\omega)^n$$

- This approximation assumes that the oscillation timescale is much shorter than the secular mode-growth timescale—probably a good approximation.

- But an error is involved because notions from the linear regime are used; it's not so easy to quantify how serious this error is.

Solution 2: Use the evolution equations to reduce order

- Reduce the number of time derivatives using the equations of motion:

$$I_{ab} = STF \int \rho x_a x_b \, dV$$

$$I^{(1)}_{ab} = STF \int \rho v_a x_b \, dV$$

$$I^{(2)}_{ab} = STF \int \rho (\frac{dv_a}{dt} x_b + v_a v_b) \, dV$$

Eliminate time derivative of velocity using equations of motion:

$$I^{(2)}_{ab} = STF \int 2\rho (v_a v_b - \Phi_{,a} x_b) \, dV$$

- However, further time differentiation leads to time derivatives of Φ appearing in the integrand, which must be eliminated using the Poisson equation:

$$\Delta\Phi = 4\pi G\rho$$

and the continuity equation

$$\frac{\partial\rho}{\partial t} = -\nabla.(\rho\mathbf{v})$$

which combine to give:

$$\Delta\frac{\partial\Phi}{\partial t} = -4\pi G\nabla.(\rho\mathbf{v})$$

- So $I^{(}_{ab}3)$ can be obtained, but at the expense of having to solve a Poisson-like equation.

- Process can be continued, but higher space derivatives of fluid variables need be computed, so there is a limit to how far this trick can take you...

Solution 3: Use a clever post-Newtonian formalism

- Blanchet, Damour & Schaeffer (1990) have found a way of eliminating **all** numerical time derivatives from the computation of the radiation reaction potential at 2.5PN order.
- Their paper has been referenced 42 times, but all of the radiation reaction calculations have looked at either supernovae collapse or binary inspiral - no one has used it to study growth/decay of modes!
- Their procedure involves solving two elliptic equations per timestep (in addition to solving for the Newtonian potential), and also using a velocity-like quantity called the 'momentum per unit rest-mass) w^i rather than the kinematic velocity:

  $$v^i = w^i + \frac{4G}{5c^5} I^3_{ij} w^j$$

  where I^3_{ij} is a functional of w^i.
- The BDS formalism is specially adapted to mass-quadrupole radiation. Rezzola et al. (1999) have found a gauge well suited to mass-current multipole radiation. You can't use the two formalism simultaneously...

Problem 2: Disparity of timescales

- Even for very rapidly rotating (perfect fluid) stars, the radiation reaction growth/decay timescale is much larger that the dynamic mode timescale, e.g. for r-modes:

$$\frac{\tau_{GRR}}{P_{\rm mode}} = 3 \times 10^4 \left(\frac{1.4 M_\odot}{M}\right) \left(\frac{10\,\mathrm{km}}{R}\right)^4 \cancel{\left(\frac{10\,\mathrm{km}}{R}\right)^4} \left(\frac{P_*}{1\,\mathrm{ms}}\right)^5$$

- Given high computational cost of realistic non-linear fluid evolutions, such runs would be prohibitively slow.

Solution: boost strength of radiation reaction

- Can artificially increasing the strength of the radiation reaction forces e.g. by decreasing the code value of the speed of light)
- However, there's no guarantee that this won't swamp other physical effects you might be interested in, e.g. leakage of energy from the unstable mode to stable ones.

The Southampton project

Aim is to test formalism of BDS for linear modes, extend to non-linear regime, and allow for mass-current multipoles as well as mass ones (Faye & Schaeffer, in preparation)

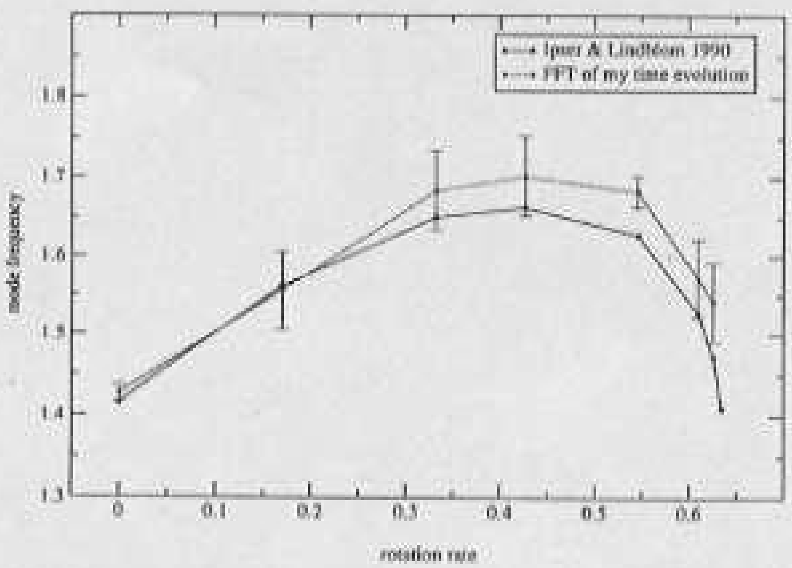

Figure 1: Plot of f-mode frequency verses rotation rate

- Purely Newtonian linear code written and tested. Uses the LORENE code developed by the Paris Meudon group to solve elliptic equations by spectral methods.
- Code works well, see e.g. f-mode plot.
- However, radiation reaction terms still causing trouble...

Summary

- As far as unstable modes are concerned, radiation reaction techniques are still in their infancy.
- Need to find formalisms that allow us to study both mass **and** current quadrupole formalisms simultaneously.
- Problem of high numbers of time derivatives may be under control.
- Problem of large ratio of dynamic timescales more problematic.