Comparing the Orbital Decay Rates of Binary Neutron Stars in full General Relativity with those predicted by Post–Newtonian and Quasi–Equilibrium Approximations.

Capra 5, Penn State

31 May – 1 June, 2002

Mark Miller
Washington University
St. Louis

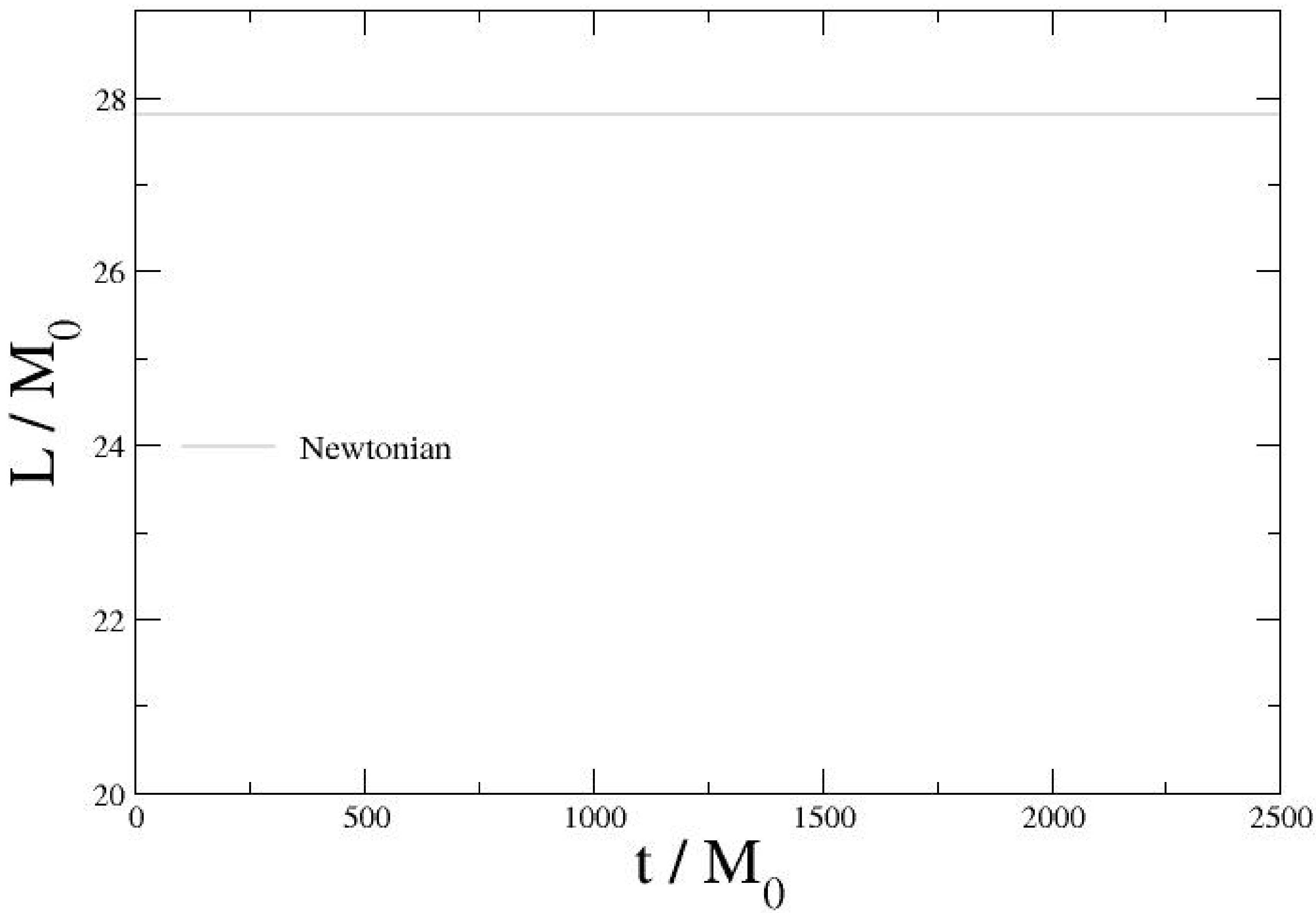
Numerical Relativity is growing up!

- 3+1 Einstein equations coupled to perfect fluid: code able to track orbiting NSs for > 10 orbits.

- Question: ***How do the orbital decay rates for (quasi) circular orbiting NSs in full General Relativity compare to those computed with various GR approximations***?

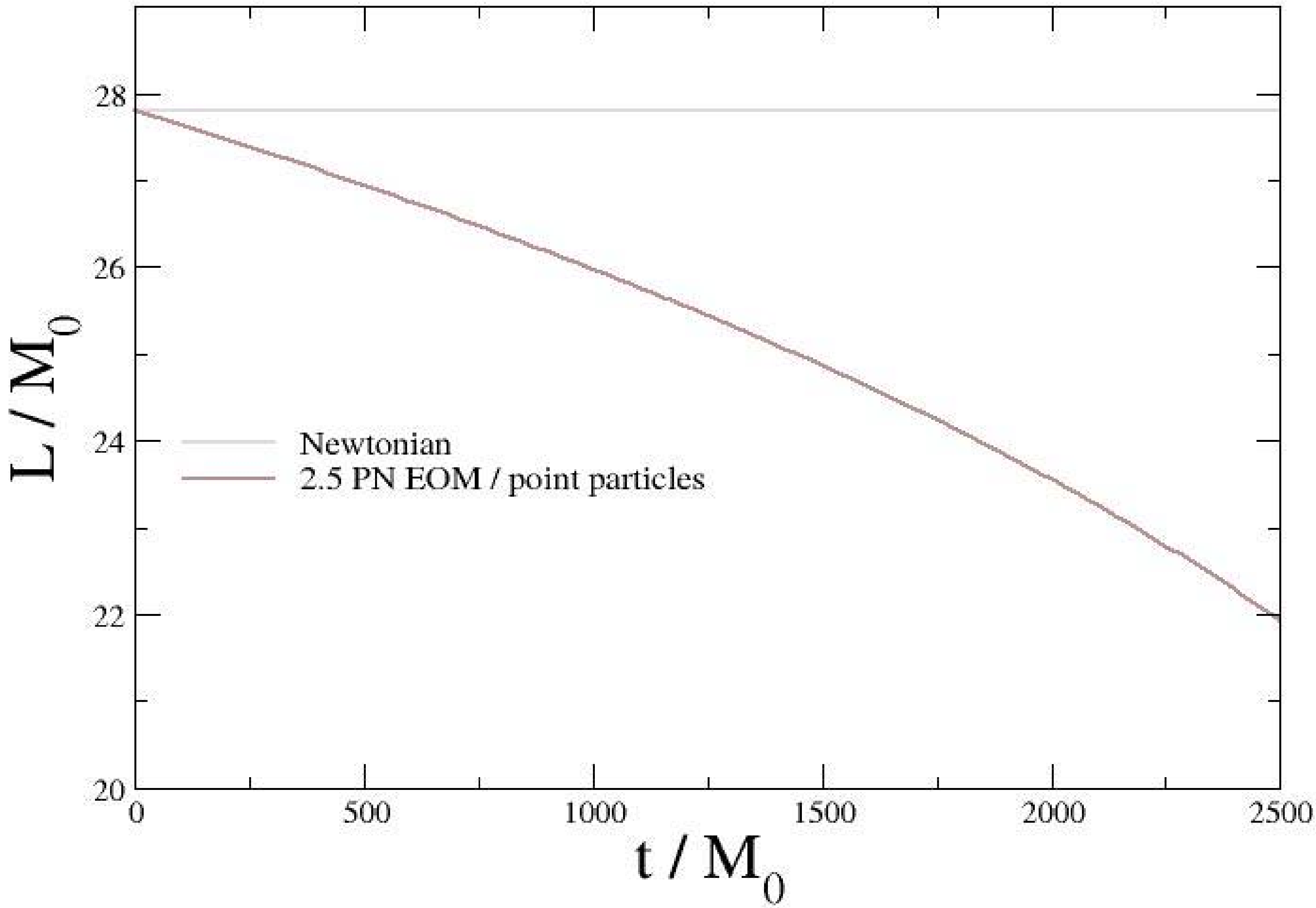
- Review approximations:
  - ➔ Post–Newtonian Approx.
  - ➔ Conformally Flat Quasi–Equilibrium Approx.

- Compare to fully relativistic simulations.



28
26
24
22
20
Newtonian
0
500
1000
1500
2000
2500
L / M0
t / M0

Lincoln & Will: (Phys. Rev. D **42**, 1123 (1990)): 5/2 PN solution to EOM



Conformally Flat QuasiEquilibrium (CFQE) Approximation

Assumptions:

1) conformally flat: $g_{ij} = \phi^4 \delta_{ij}$

2) $Tr(K) = L_t(Tr(K)) = 0$

3) $L_t \tilde{g}_{ij} = 0$

4) approximate Killing vector field:

$$k^a = t^a + \Omega \left(\frac{\partial}{\partial \phi} \right)^a$$

5) co–rotating matter: $u^a \sim k^a$

Equations for solving NS/NS in CFQE approximation:

$$\nabla^2 \phi + \frac{1}{8\phi^7} \tilde{A}_{ij} \tilde{A}^{ij} + 2\pi \phi^5 (\rho h W^2 - P) = 0$$

$$\nabla^2 \beta^i + \frac{1}{3} \partial^i \partial_j \beta^j - \tilde{A}^{ij} \partial_j \left(\frac{2\alpha}{\phi^6}\right) + 16\pi \alpha \phi^4 \rho h W^2 v^i = 0$$

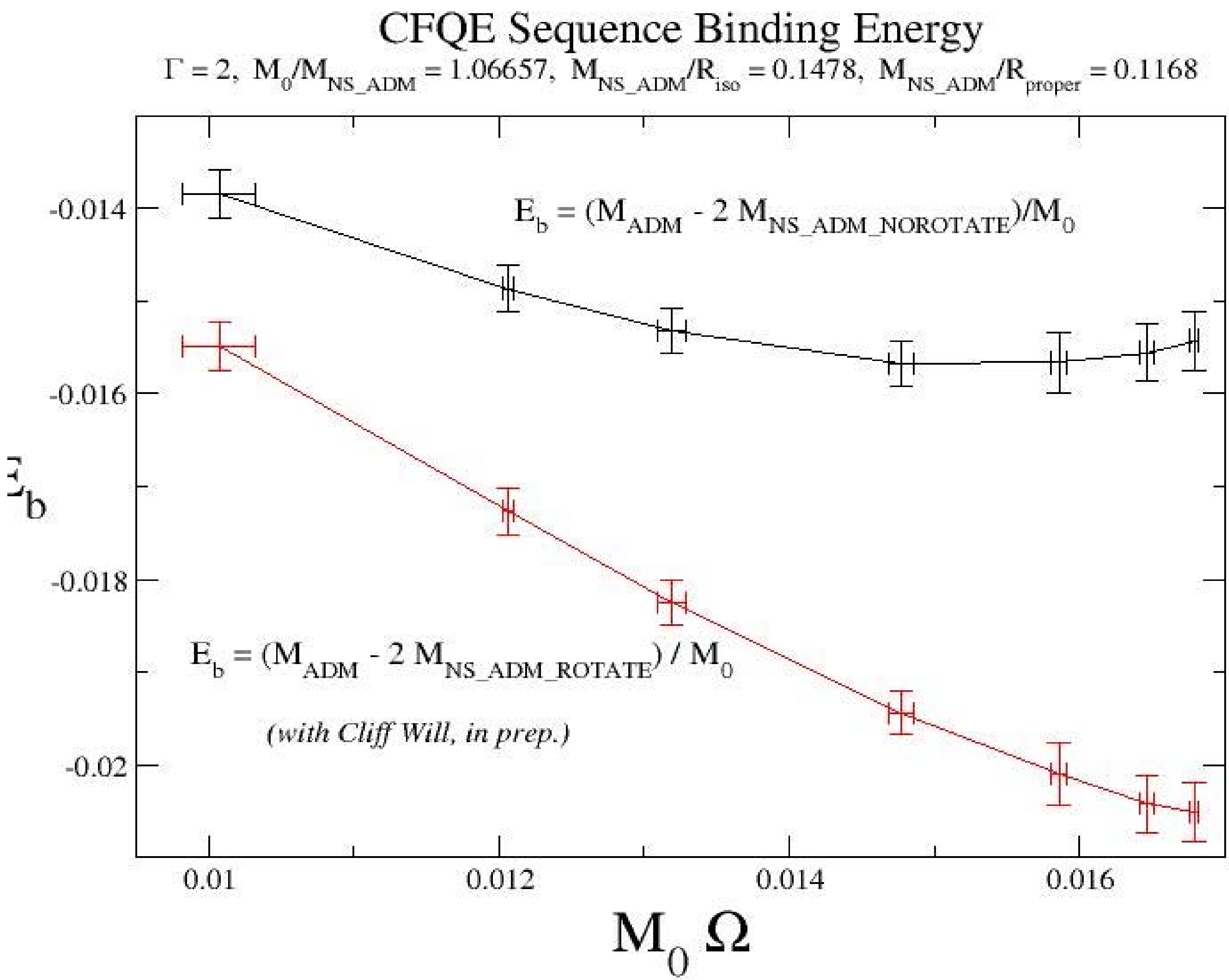
$$\nabla^2 (\alpha\phi) - (\alpha\phi)\left[\frac{7}{8\phi^8} \tilde{A}_{ij} \tilde{A}^{ij} + 2\pi \phi^4 (3\rho h W^2 - 2(\rho + \rho\epsilon) + 3P)\right] = 0$$

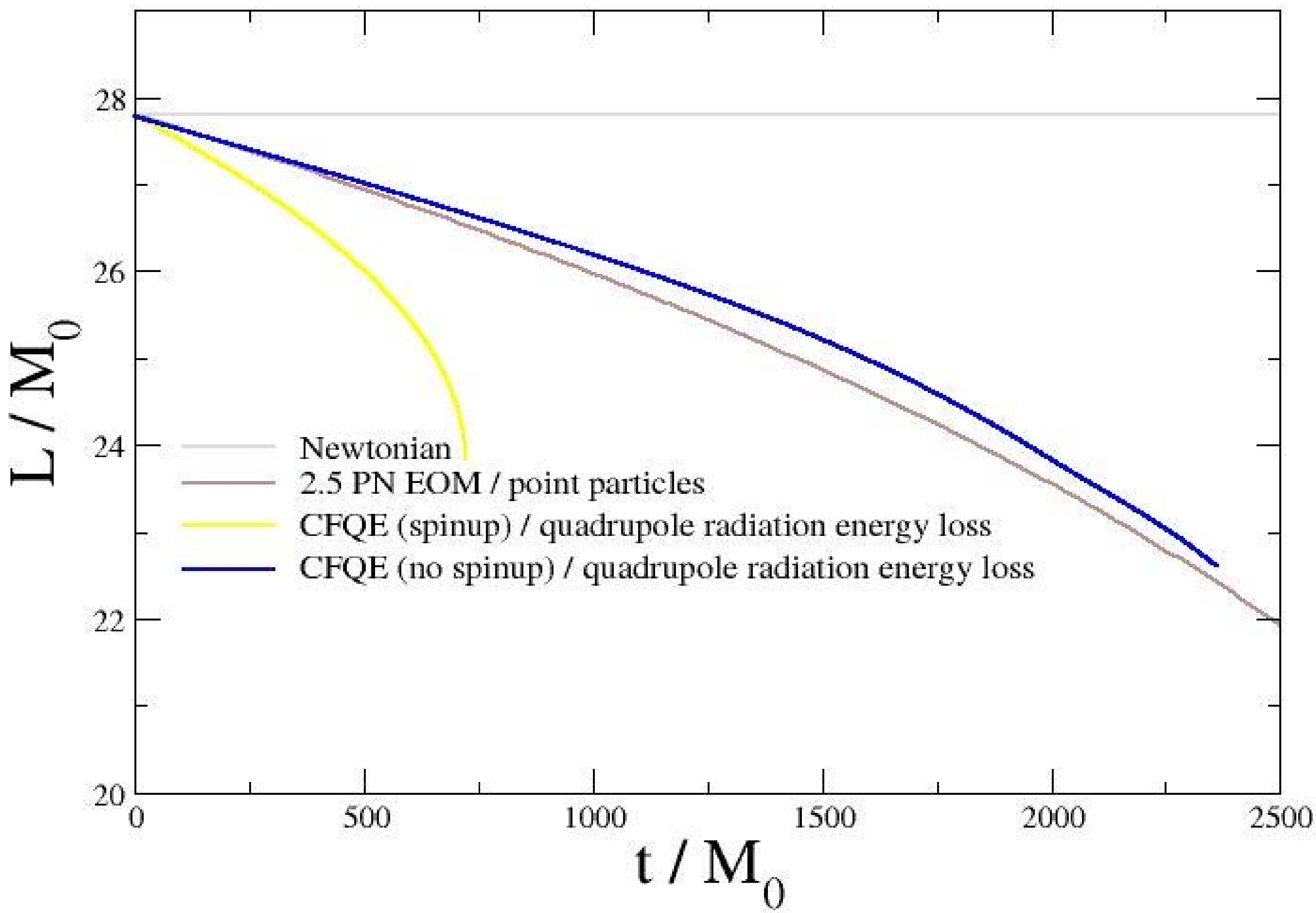
$$\frac{\rho u^t}{\rho + \rho\epsilon + P} = constant$$

Numerical Algorithm: specify ρ_c and relative separation

Constant M_0 CFQE Sequences:

$$E_b = \frac{E_{ADM} - 2M_{NS}}{M_0}$$


CFQE Sequence Binding Energy
$\Gamma = 2$, $M_0/M_{NS\_ADM} = 1.06657$, $M_{NS\_ADM}/R_{iso} = 0.1478$, $M_{NS\_ADM}/R_{proper} = 0.1168$
$E_b = (M_{ADM} - 2\,M_{NS\_ADM\_NOROTATE})/M_0$
$E_b = (M_{ADM} - 2\,M_{NS\_ADM\_ROTATE}) / M_0$
(with Cliff Will, in prep.)
E_b
-0.014
-0.016
-0.018
-0.02
0.01
0.012
0.014
0.016
$M_0\,\Omega$



Newtonian
2.5 PN EOM / point particles
CFQE (spinup) / quadrupole radiation energy loss
CFQE (no spinup) / quadrupole radiation energy loss
L / M0
t / M0

General Relativistic Hydrodynamics:

$$T^{ab} = (\rho + \rho\epsilon + P) u^a u^b + P g^{ab} \qquad \nabla_a T^{ab} = 0$$

$$J^a = \rho u^a \qquad \nabla_a J^a = 0$$

High Resolution Shock Capturing Methods
2nd order coupling to spacetime evolution
Rigorous consistency code tests: (Font, Miller, Suen Tobias: PRD 61, 044011 (2000))

Einstein's Equations in 3+1 form:

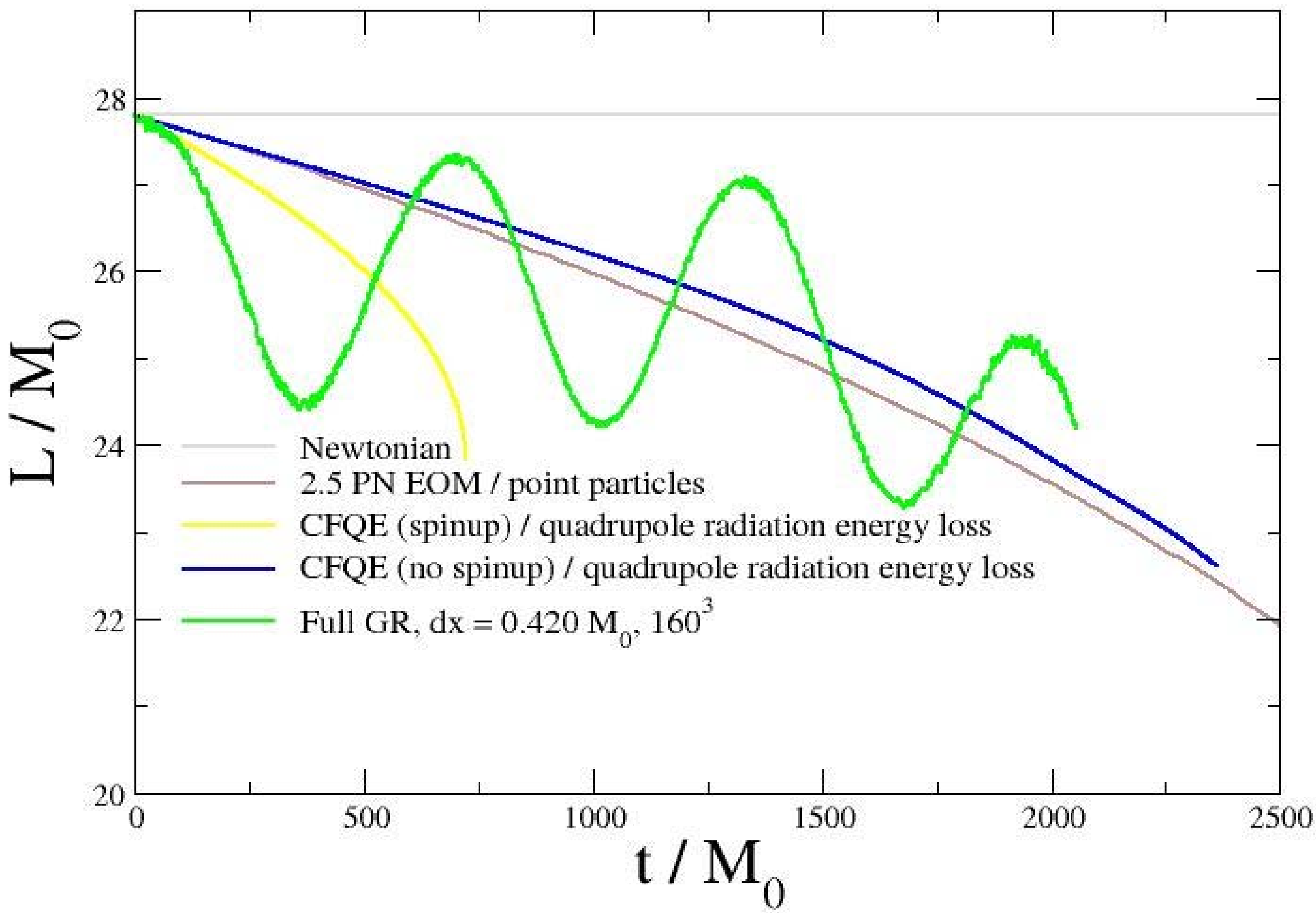
Shibata and Nakamura, PRD 52 (1995).

Baumgarte and Shapiro, PRD 59 (1999).

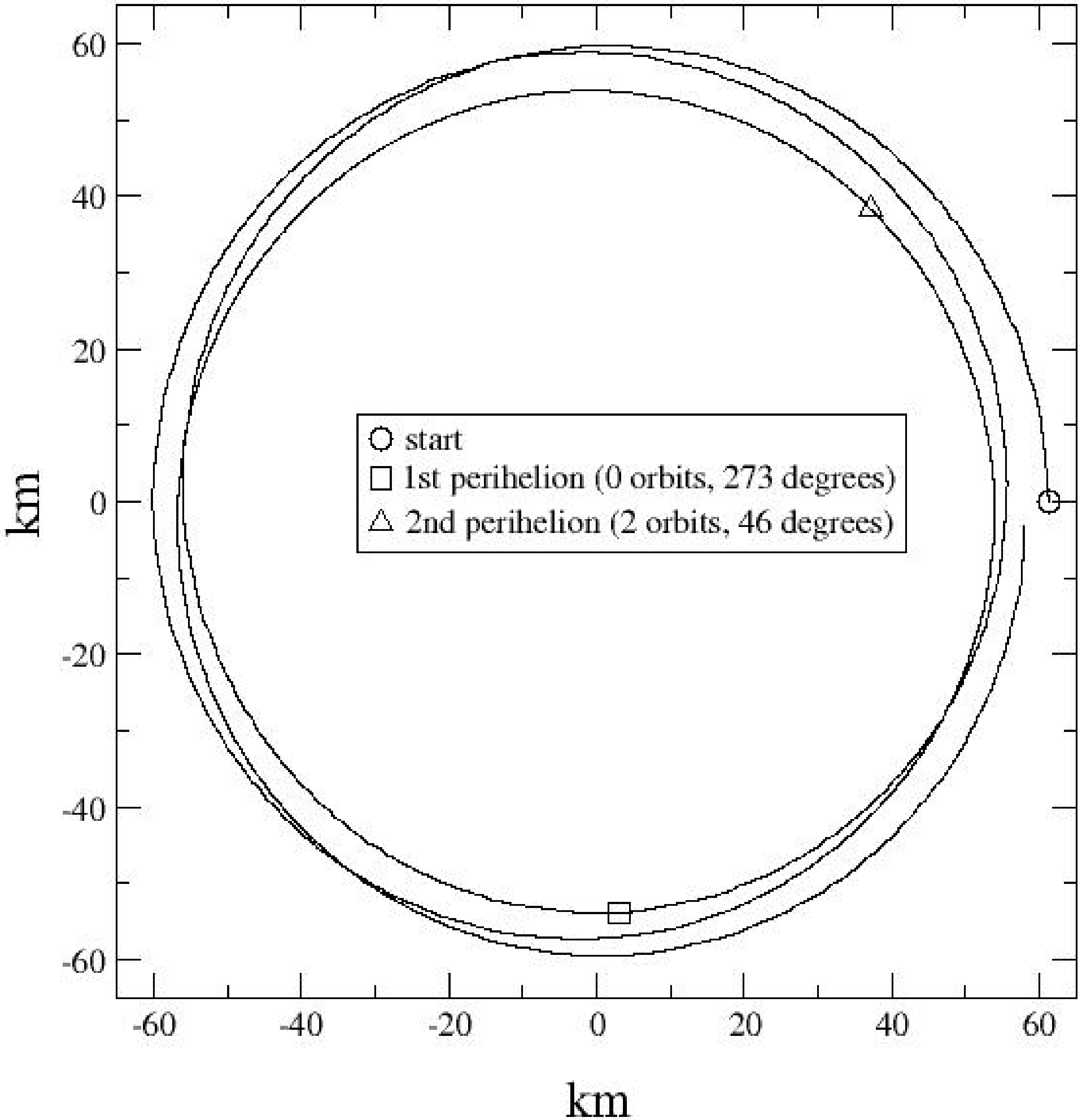
Alcubierre, Brugmann, Miller, Suen, PRD 60 (1999)

$$g_{ij} = e^{4\phi} \tilde{g}_{ij}, \quad det(\tilde{g}_{ij}) = 1$$

$$K_{ij} = \frac{1}{3} g_{ij} K + e^{4\phi} \tilde{A}_{ij}, \quad K = g^{ij} K_{ij}, \quad \Gamma^i = \partial_j \tilde{g}^{ij}$$



L / M0
t / M0
28
26
24
22
20
0
500
1000
1500
2000
2500
Newtonian
2.5 PN EOM / point particles
CFQE (spinup) / quadrupole radiation energy loss
CFQE (no spinup) / quadrupole radiation energy loss
Full GR, dx = 0.420 M0, 160^3


start
1st perihelion (0 orbits, 273 degrees)
2nd perihelion (2 orbits, 46 degrees)
km
km
-60
-40
-20
0
20
40
60

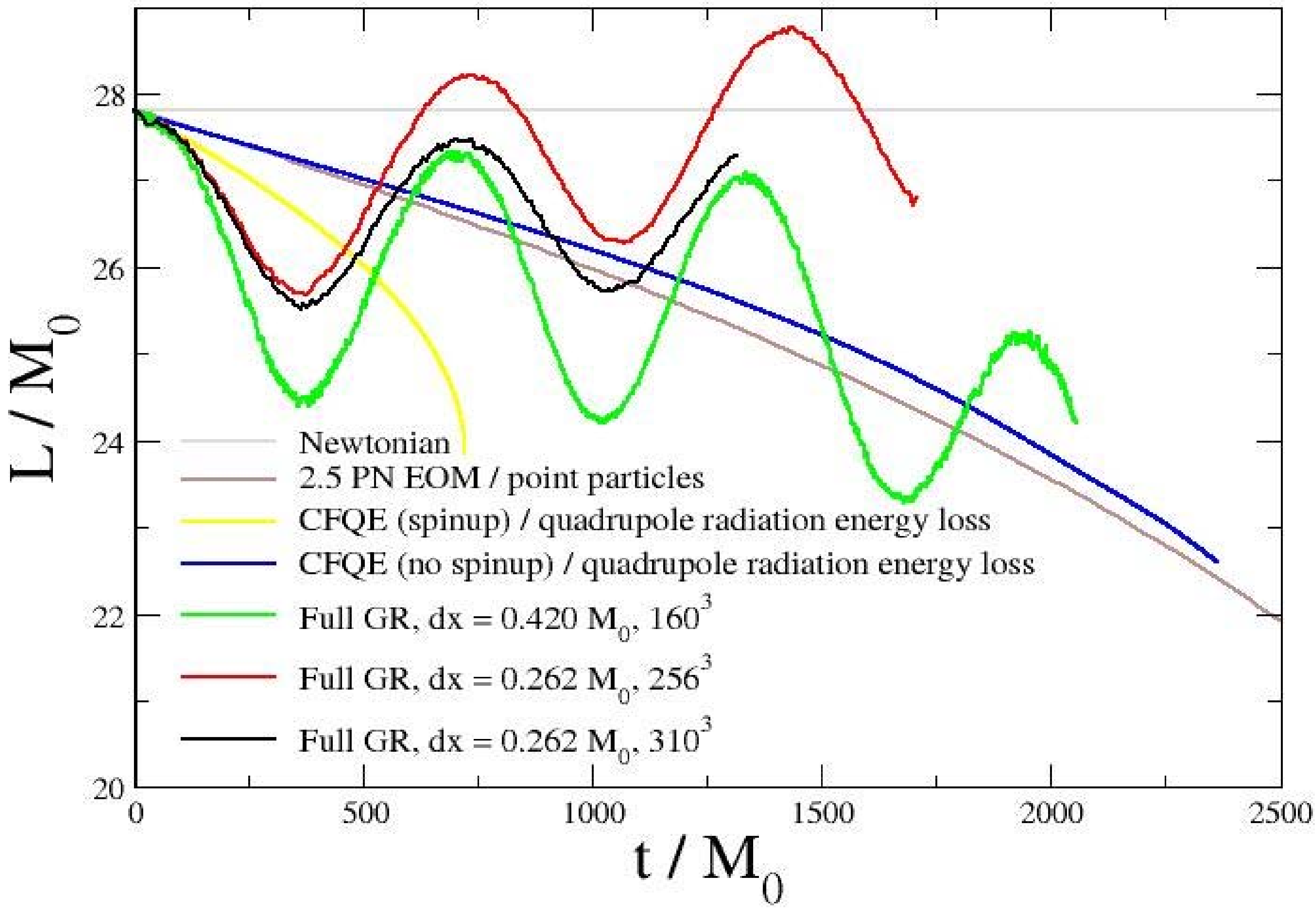
Convergence: used to extract physics from the code!

$$Q_{numerical} = Q_{exact} + O((\Delta x)^n)$$

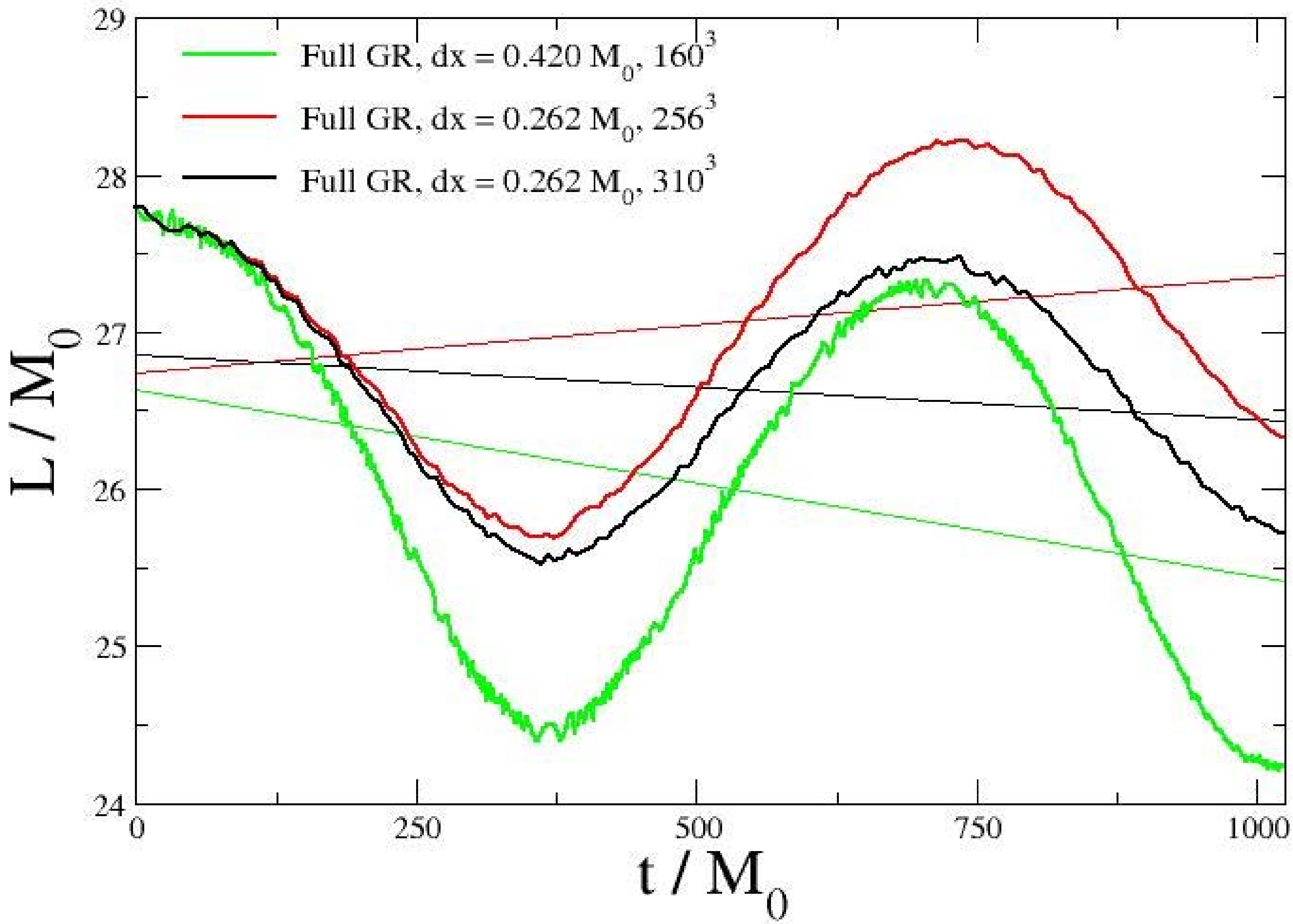
Typical case: 2 sources of error (truncation, boundary)

$$Q_n = Q_e + c_1 (\Delta x)^2 + \frac{c_2}{(r_{bound})^2} + \ldots$$

$$Error = max\left\{ |Q_n - Q_e| \ , \ \frac{c_2}{(r_{bestbound})^2} \ , \ c_1 (\Delta x_{best})^2 \right\}$$



L / M_0
t / M_0
28
26
24
22
20
0
500
1000
1500
2000
2500
Newtonian
2.5 PN EOM / point particles
CFQE (spinup) / quadrupole radiation energy loss
CFQE (no spinup) / quadrupole radiation energy loss
Full GR, dx = 0.420 M_0, 160^3
Full GR, dx = 0.262 M_0, 256^3
Full GR, dx = 0.262 M_0, 310^3



Full GR, dx = 0.420 M_0, 160^3
Full GR, dx = 0.262 M_0, 256^3
Full GR, dx = 0.262 M_0, 310^3
L / M_0
t / M_0
24
25
26
27
28
29
0
250
500
750
1000

Convergence + Richardson extraplation = Physics

$$\left(\frac{dL}{dt}\right)_n = \left(\frac{dL}{dt}\right)_e + c_1(\Delta x)^2 + \frac{c_2}{(r_{bound})^2} + \ldots$$

$$\left(\frac{dL}{dt}\right)_e = -0.0014 \pm \left\{ \begin{array}{c} 0.0011 = c_1(\Delta x_{best})^2 \\ 0.0021 = c_2/(r_{bestbound})^2 \end{array} \right\}$$

$$\left(\frac{dL}{dt}\right)_{Newt} = 0 \qquad \left(\frac{dL}{dt}\right)_{2.5PN} = -0.0019$$

$$\left(\frac{dL}{dt}\right)_{CFQE,\, no\, spinup} = -0.0016$$

Conclusions:

◆ Numerical relativity has matured to the point of being able to track, in a stable fashion, many orbital time periods for relativistic NSs.

◆ Available computational resources exist in order to plot results on the timescales of multiple orbits ***and*** their error bars on the same graph!

Future Plans:

◆ long and short timescale analysis of corotational CFQE approx.

◆ irrotational and/or 20 ms NS rotation CFQE initial data + realistic EOS –> <u>*Realistic Waveforms*</u>

◆ more of the same for BH/NS binary system