CONVERGENCE IN NUMERICAL CALCULATIONS OF RADIATION REACTION EFFECTS.

(AND OTHER TOPICS)

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WORK WITH:

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REAL TITLE:

SCRAPS LEFT OVER FROM FOCUS WORKSHOP AND PREVIOUS CAPRA 5 TALKS

REAL TOPICS:

- $\ell=0$ & $\ell=1$ MODES IN KERR.
- CONVERGENCE AND SMOOTHNESS.
- DEBYE POTENTIAL & WEYL CURVATURE
  - TIME vs FREQUENCY DOMAIN
  - SCHWARZSCHILD & KERR
  - ADDING IN SOURCES.

$\ell=0$ & $\ell=1$ MODES IN KERR

REAL ISSUES:

$$\left.\begin{array}{ll} \cdot\ \delta = ? & \text{MODESUM} \\ \cdot\ T^{AV} = ? & \text{MODE SUM} \end{array}\right\} \text{PT. PARTIC}$$

⇒ TENSORS ON SPHERE ?

R.W.: $h_{\theta\theta} \sim \left(\partial_{\theta\theta} + \frac{i}{\sin\theta}\partial_{\theta\phi}\right) Y_{\ell m}$

$h_{rt} \sim Y_{\ell m}$

CHRZ: $h_{nn} \sim \ell^{+}\ell^{+}\,{}_{-2}S_{\ell m} + c.c.$

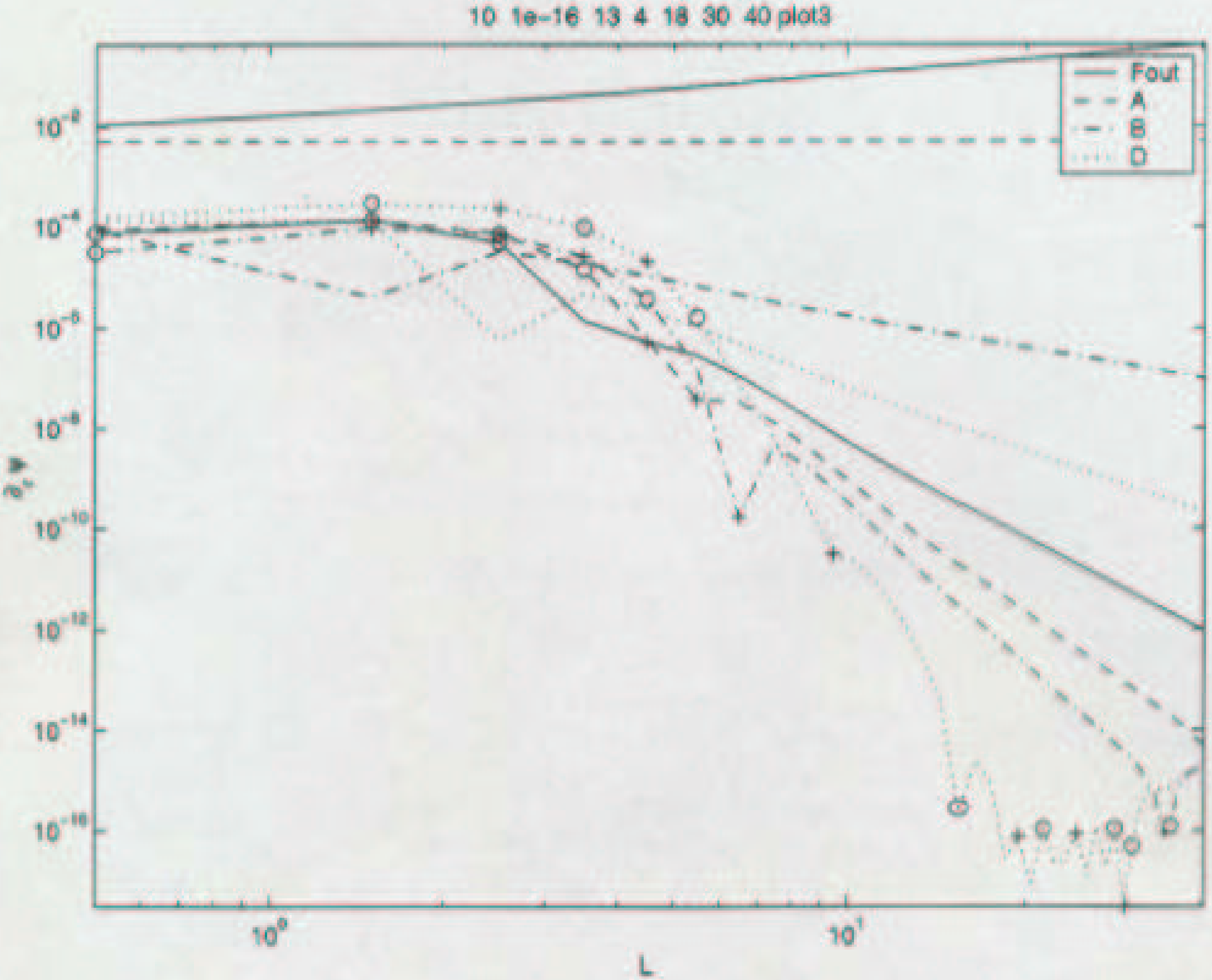
$h_{mm} \sim {}_{-2}S_{\ell m}$

* $h_{n\ell}$ and $h_{m\bar{m}}$ = MISSING IN RAD. G.

KNOW:

δM, δa, TRANSLATIONS, ROTATIONS & GAUGE

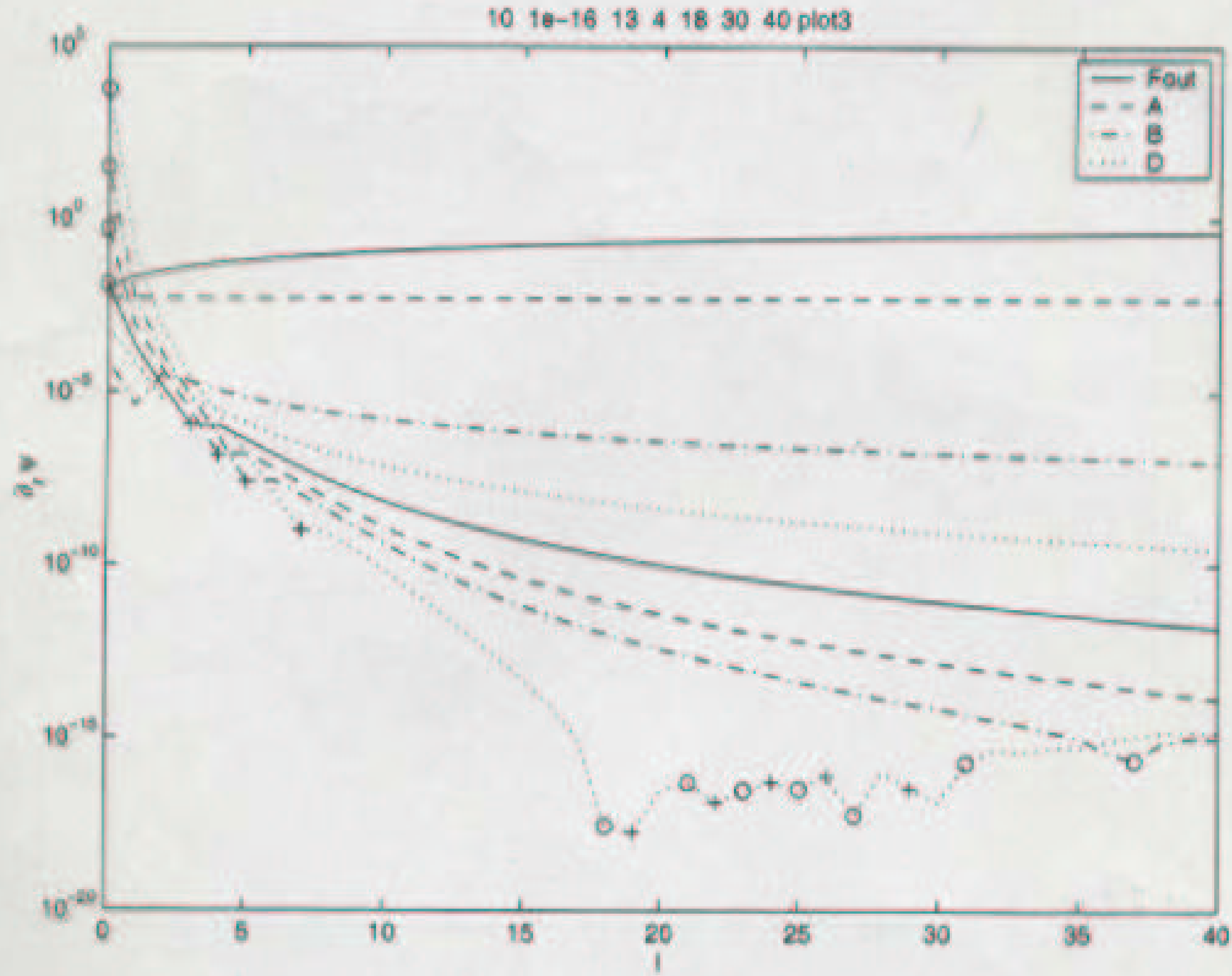
BUT ANG. SEP. REQ. FREQ. DOMAIN


10 1e-16 13 4 18 30 40 plot3
Fout
A
B
D
L

CALCULATING & FITTING.

- A, B, D CALCULATED TO MACHINE PRECISION (LARGE RELATIVE TO F")
- $E_1 - E_4$ CALCULATED TO FITTING ACCURACY. (INCREASING RELATIVE ERROR : $10^{-6} - 10^{-1}$)

	REL. ERR.	ABS. ERR.
A	10^{-16}	10^{-16}
B	10^{-16}	10^{-18}
D	10^{-16}	10^{-22}
E_1 (TAIL)	10^{-6}	10^{-15}
E_2 "	10^{-4}	10^{-15}
E_3 "	10^{-3}	10^{-12}
E_4 "	10^{-1}	10^{-18}

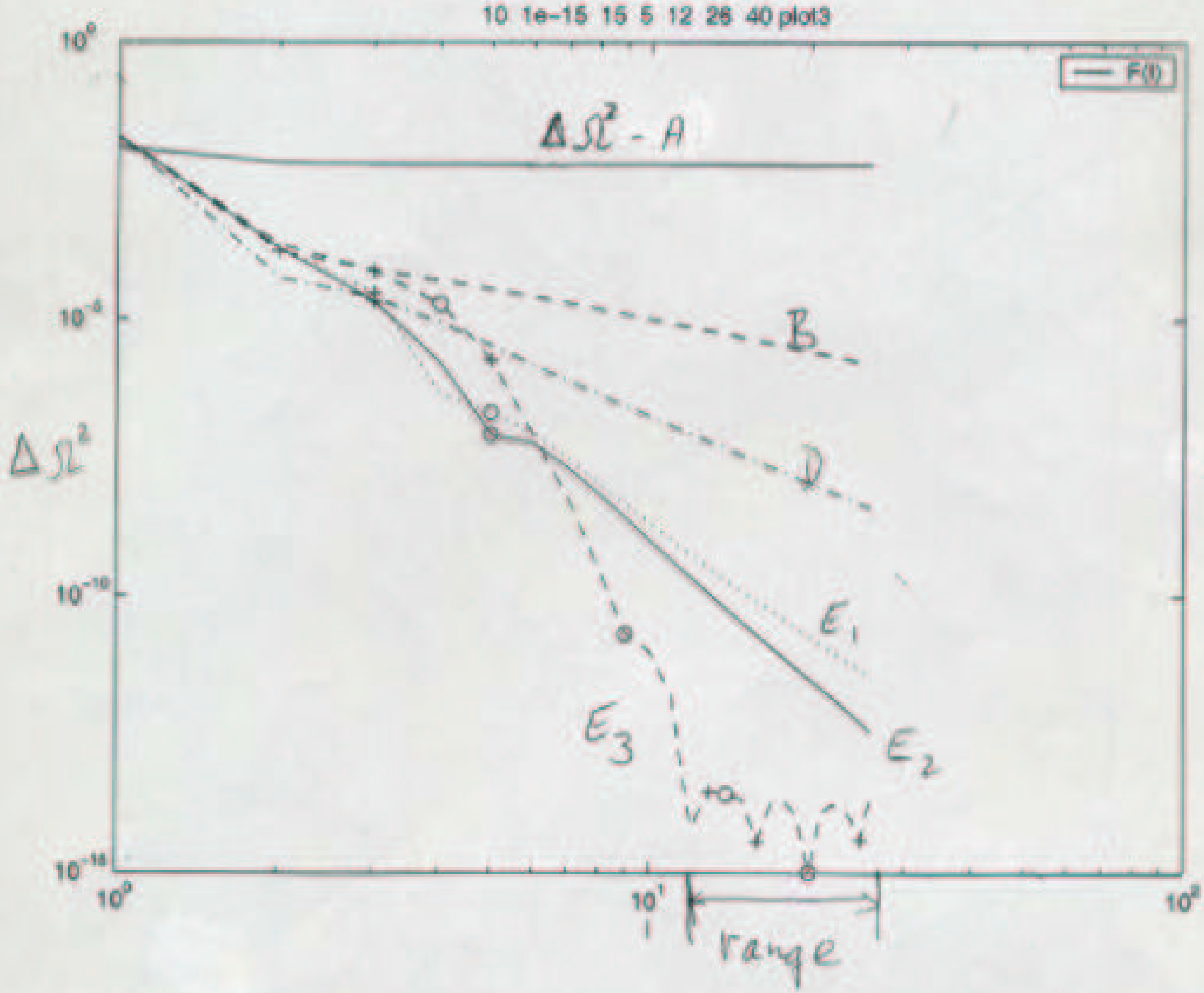


Canlot

f. t

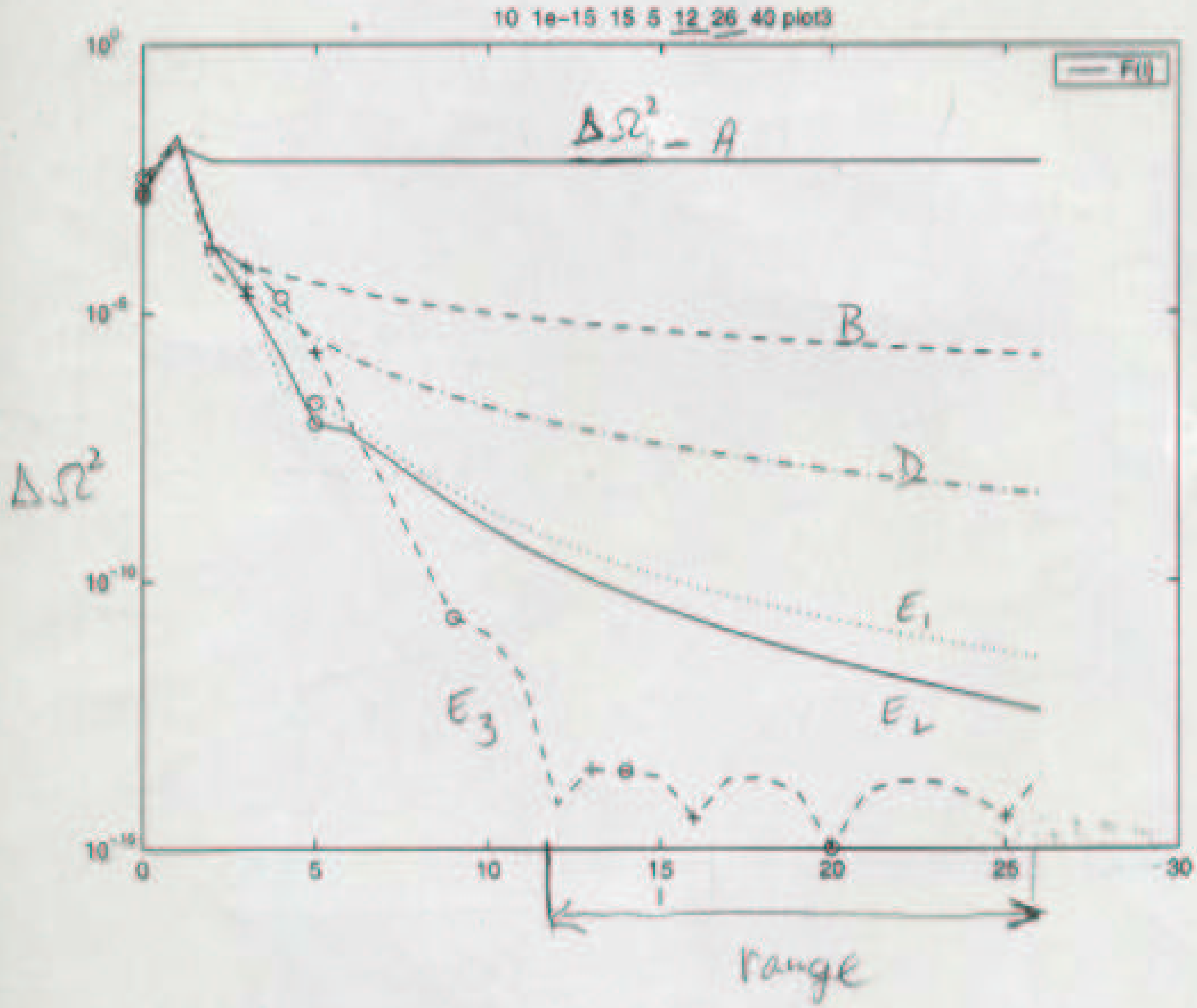
R = 10 m

5 parameters fit for 12 points



R = 10 m

5 parameters Fit 12 points



CONVERGENCE AND SMOOTHNESS.

$$(L^2-1)^{-1} \quad \text{vs} \quad L^{-2}$$

cf $|\sin\theta/2|$ vs $|\theta/2|$

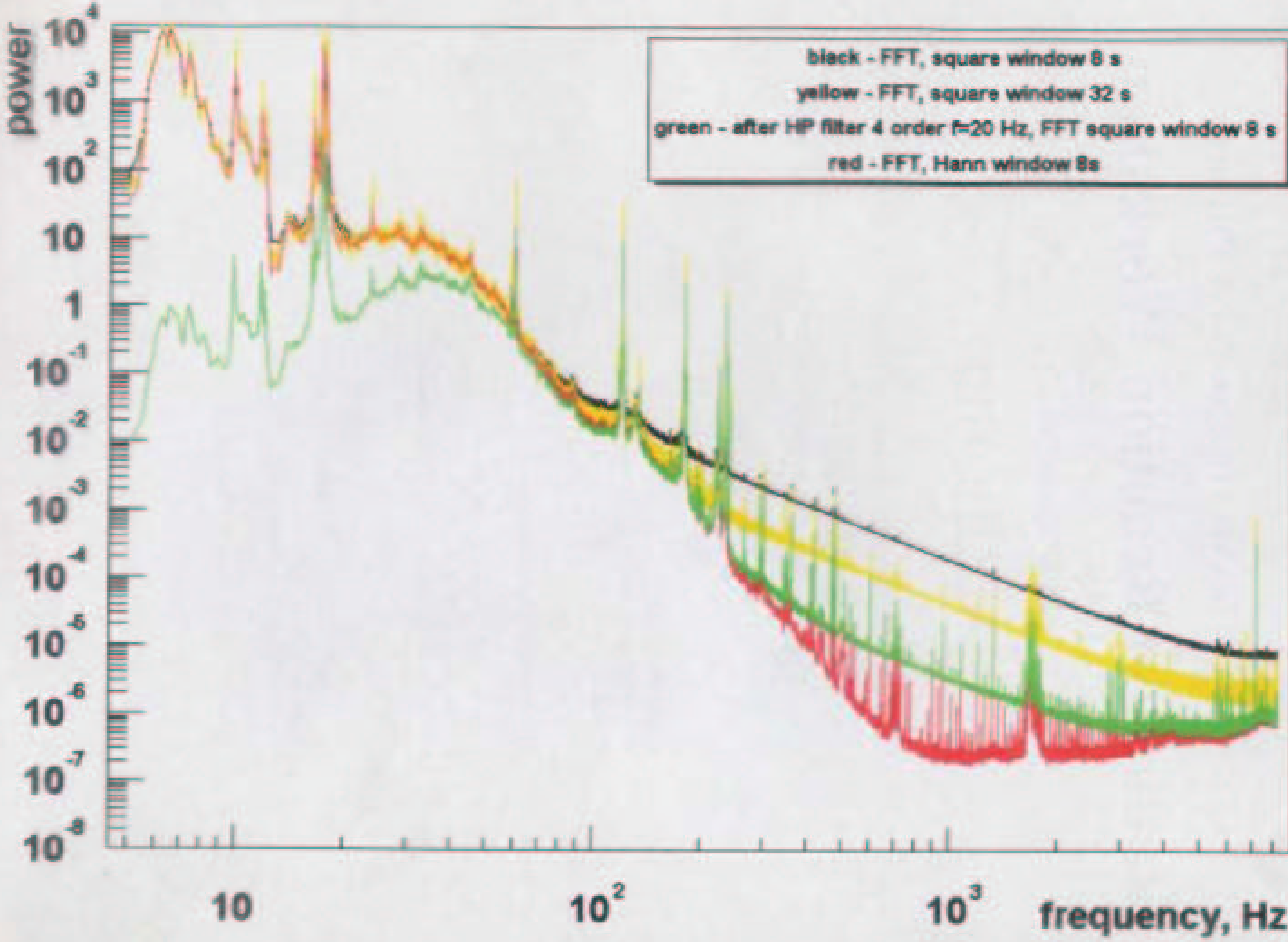
$\downarrow$ C^∞ on sphere $\downarrow$ C^0 at antipode.

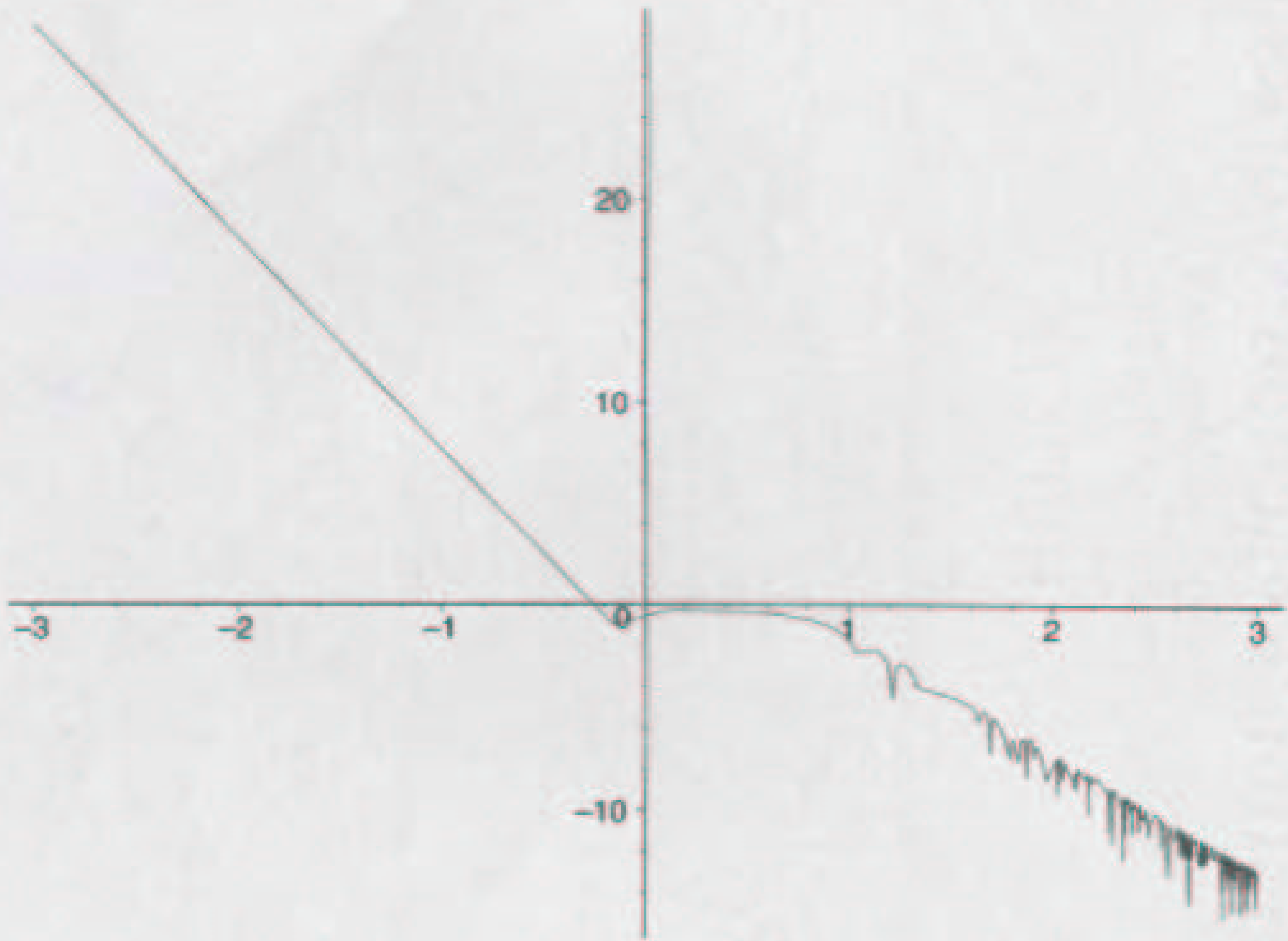
IN BOTH CASES <u>DIFFERENTIABILITY</u> IMPROVES WITH <u>INCREASED FALL-OFF</u>

$$|\sin\theta/2| - |\theta/2| \sim |\theta^3|$$

(1-1 RELATION !!!)

<u>**ASIDE**</u>: EXACTLY SAME ISSUE ARISES IN DATA ANALYSIS.


power
10^4
10^3
10^2
10
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10
10^2
10^3
frequency, Hz
black - FFT, square window 8 s
yellow - FFT, square window 32 s
green - after HP filter 4 order f=20 Hz, FFT square window 8 s
red - FFT, Hann window 8s



20
10
0
−10
−3
−2
−1
1
2
3

DEBYE POTENTIALS

$$h_{\mu\nu} = D_{\mu\nu}(\psi_D)$$

$$\Rightarrow \psi_{0,4} = \mathcal{O}^4(\psi_D).$$

SCHWARZSCHILD VACUUM

$\psi_D(\psi_4)$ CAPRA 4

$\psi_D(\psi_0)$ FOCUS WORKSHOP.
& CAPRA 5 (A.O.)

USES TIME DOMAIN (e.g. 2^{ND} ORDER

USES ANGULAR SEPARATION

WEYL APPEARS AS SOURCE FOR ψ_D

FIND HOMOGENEOUS SOLUTIONS

ARE ALGEBRAICALLY SPECIAL.

SOURCES

- SEE WALD
- START WITH SCHWARZSCHILD
  (HAVE RESULT FOR METRIC ALREADY, BUT 'GAUGE' ISSUES REMAIN)
- IN PROGRESS, " " " " .

KERR

- CHARACTERIZE A.S. WITHOUT SEP.
- USE ψ_0 AND ψ_4 EQUATIONS.
- PERHAPS, START WITH POTENTIAL.
-
-
-
- HOPE SPRINGS ETERNAL (ALMOST!)

is a solution of the Teukolsky equation for ψ_0 where Eqs. (6.13) and (B11) of Ref. 6 have been solution for ψ_4 associated with this solution for ψ_0 is

$$
\begin{aligned}
\psi_4 = {} & (\bar\delta - \bar\tau + 3\alpha + \bar\beta)(\bar\delta - \bar\tau + 2\alpha + 2\bar\beta)(\bar\delta + \alpha + 3\bar\beta - \bar\tau)(\bar\delta + 4\bar\beta + 3\bar\tau)\bar\psi_G \\
& + \{(\bar\delta - \bar\tau + 3\alpha + \bar\beta)(\bar\delta - \bar\tau + 2\alpha + 2\bar\beta)(\delta + \bar\alpha + 3\beta - \tau)(\delta + 4\beta + 3\tau) \\
& + (\Delta + \bar\mu + 3\gamma - \bar\gamma)(\Delta + \bar\mu + 2\gamma - 2\bar\gamma)(D - \rho + 3\epsilon - \bar\epsilon)(D + 3\rho + 4\epsilon) \\
& - \tfrac{1}{2}[(\Delta + \bar\mu + 3\gamma - \bar\gamma)(\bar\delta - 2\bar\tau + 2\alpha) + (\bar\delta - \bar\tau + 3\alpha + \bar\beta)(\Delta + 2\bar\mu + 2\gamma)] \\
& \times [(D + \bar\rho - \rho + \bar\epsilon + 3\epsilon)(\delta + 4\beta + 3\tau) + (\delta + 3\beta - \bar\alpha - \bar\pi - \tau)(D + 3\rho + 4\epsilon)]\}\psi_G .
\end{aligned}
$$

Equations (15)–(18) are the electromagnetic and gravitational Starobinsky-Teukolsky relations[3,7] for perturbations of an arbitrary vacuum type-D space-time. For the Kerr metric they may be simplified (by manipulations along the lines of Appendix C of Ref. 6) to yield relations between the radial and angular solutions for φ_0 and φ_2 and for ψ_0 and ψ_4 although I have not completed the reduction of Eq. (18), which evidently requires considerable algebra.

Finally, it is worth noting that Chrzanowski derived his results by postulating a factorized form of the Green's function for electromagnetic and gravitational perturbations of Kerr. Now that his final results are rigorously established, one may reverse the steps of Chrzanowski's argument (taking into account the known factorized form of the Green's function for the Teukolsky equation)

in these problems through numerous di
during the past three years.

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[1]S. A. Teukolsky, Astrophys. J. 185, 635
Teukolsky derived his equations for φ_0 and
D space-times, but the same derivation wo
algebraically special space-times. The equ
ψ_0 contains an extra source term if $\lambda \neq 0$ in
ground space-time, as given in Eq. (6.10) o

[2]E. T. Newman and R. Penrose, J. Math.
(N.Y.) 3, 566 (1962), and 4, 998(E) (1963).

[3]S. Chandrasekhar, Proc. Roy. Soc. Lond
A 358, 421, 441 (1978).