Massive-Field Approach to the Scalar Self Force in Curved Spacetime

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Goals of this talk

- Proposing an alternative regularization method for the scalar self force.

- This method is based on the difference between two retarded scalar fields: a massless scalar field, and a massive scalar field.

- Show that this method, can be used to calculate the mode-sum regularization parameters.

Scalar self force - introduction

Consider a point like object with a scalar charge q , and a given world line $z(\tau)$

and a fixed background spacetime metric $g_{\mu\nu}(x)$

The object induces a scalar field (perturbation) $\phi(x)$ and a scalar force $F_{\mu}(x) \equiv q\phi_{,\mu}(x)$

$\Downarrow$ regularization

$$f_{\mu}^{self}(z_0)$$

Coleman showed that electromagnetic self force
in flat spacetime (ALD term)

$$f_{\mu}^{self}(z_0) = \frac{2}{3} q^2 (\dot{a}_{\mu} - a^2 u_{\mu})$$

can be obtained by replacing the original Green function G

with a new Green function, which depends on a parameter,
such that: $\lim_{\Lambda \to \infty} G_{\Lambda} = G$

This Green function G_{Λ} can be obtained from a fictitious
massive electromagnetic Green function.

The scalar self force in curved spacetime (after regularization):

$$f_{self}^{\mu}(z_0) = (local - term) + \lim_{\tilde{\varepsilon} \to 0} q^2 \int_{-\infty}^{\tau_0 - \tilde{\varepsilon}} \partial^{\mu} G(z_0 \mid z(\tau)) d\tau$$

T. C. Quinn (2000)

Massive-Field Approach

$$\Box\phi = -4\pi\rho$$

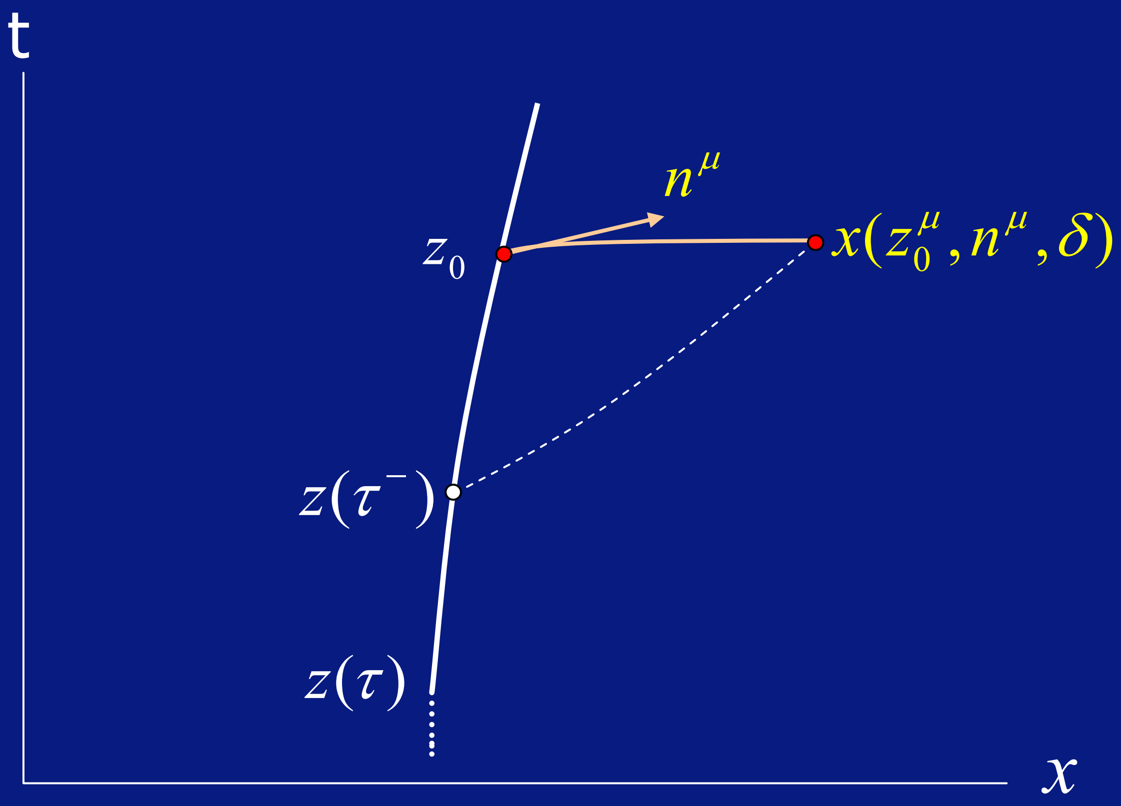
$$(\Box - m^2)\phi_m = -4\pi\rho$$

Auxiliary field

$$\Box\phi \equiv \phi_{;\nu}{}^{\nu}$$

$$\rho(x) = q\int_{-\infty}^{\infty}\frac{1}{\sqrt{-g}}\delta^4(x - z(\tau))d\tau$$

The <u>same</u> charge density for both fields



$$\Delta\phi \equiv \phi - \phi_m \qquad u^{\mu} n_{\mu} = 0 \qquad n^{\mu} n_{\mu} = 1$$

Result

$$\boxed{f_{\mu}^{self}(z_0) = q \lim_{m\to\infty} \left\{ \lim_{\delta\to 0} \Delta\phi_{,\mu}(x) + \frac{1}{2} q \left[m^2 n_{\mu}(z_0) + m a_{\mu}(z_0) \right] \right\}}$$

Calculate $\Delta F_{\mu} \equiv q \Delta \phi_{,\mu}(x)$

Retarded solutions for a charge density of a point particle

$$\phi_m(x) = q \int_{-\infty}^{\infty} G_m(x \,|\, z(\tau)) d\tau \qquad (\Box - m^2) G_m(x \,|\, x') = -\frac{4\pi}{\sqrt{-g}} \delta^4(x - x')$$

$$\phi(x) = q \int_{-\infty}^{\infty} G(x \,|\, z(\tau)) d\tau \qquad \Box\, G(x \,|\, x') = -\frac{4\pi}{\sqrt{-g}} \delta^4(x - x')$$

Assumption: these integrals converge off the world line.

Scalar field

Locally: when x is in a local neighborhood of z

$$G(x|z) = \Theta(\Sigma(x), z)\left[U(x|z)\delta(\sigma) - V(x|z)\theta(-\sigma)\right]$$

$$\phi(x) = q\int_{\tau_1}^{\infty} \overbrace{\underbrace{G(x|z)}}\, d\tau + q\int_{-\infty}^{\tau_1} G(x|z)\, d\tau$$

$$\boxed{\phi(x) = \phi^{dir}(x) + \phi^{tail}(x)}$$

$$\phi^{dir}(x) = qU(x|z(\tau^-))\frac{d\tau}{d\sigma}(\tau^-)$$ diverges as $x \to z(\tau)$

$$\phi^{tail}(x) = -q\int_{\tau_1}^{\tau^-} V(x|z)\, d\tau + q\int_{-\infty}^{\tau_1} G(x|z)\, d\tau$$ regular as $x \to z(\tau)$

Massive scalar field

Locally: when x is in a local neighborhood of z

$$G_m(x|z) = \Theta(\Sigma(x), z)\left[U(x|z)\delta(\sigma) - V_m(x|z)\theta(-\sigma)\right]$$

$$\phi_m(x) = q\int_{\tau_1}^{\infty} G_m \overbrace{\underbrace{(x|z)d\tau}} + q\int_{-\infty}^{\tau_1} G_m(x|z)d\tau$$

$$\phi_m(x) = \phi_m^{dir}(x) + \phi_m^{tail}(x)$$

$$\boxed{\phi_m^{dir}(x) = qU(x|z(\tau^-))\frac{d\tau}{d\sigma}(\tau^-) = \phi^{dir}(x)}$$

$$\phi_m^{tail}(x) = -q\int_{\tau_1}^{\tau^-} V_m(x|z)d\tau + q\int_{-\infty}^{\tau_1} G_m(x|z)d\tau$$

Difference field

$$\boxed{\Delta\phi(x) = \phi - \phi_m = \phi^{tail}(x) - \phi_m^{tail}(x)}$$

$$\Delta V \equiv V - V_m \qquad\qquad \Delta G \equiv G - G_m$$

$$\Delta\phi(x) = -q\int_{\tau_1}^{\tau^-} \Delta V(x\,|\,z)\,d\tau + q\int_{-\infty}^{\tau_1} \Delta G(x\,|\,z)\,d\tau$$

$$\Delta F_\mu(x) \equiv q\Delta\phi_{,\mu}(x)$$

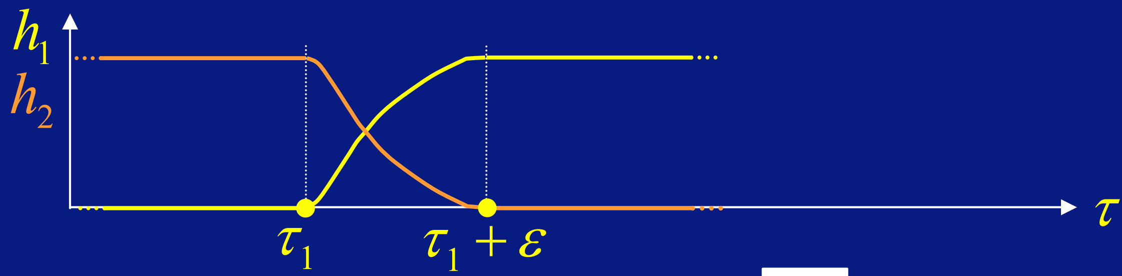
$$\Delta F_\mu(x) = -q^2(\tau^-)_{,\mu}[\Delta V]_{\tau^-} - q^2\int_{\tau_1}^{\tau^-}\Delta V_{,\mu}d\tau + q^2\int_{-\infty}^{\tau_1}\Delta G_{,\mu}d\tau$$

$$\boxed{\begin{aligned}\Delta F_\mu(x) &= F_\mu^{tail}(x) - q^2(\tau^-)_{,\mu}[\Delta V]_{\tau^-} + q^2\int_{\tau_1}^{\tau^-}V_{m,\mu}d\tau - q^2\int_{-\infty}^{\tau_1}G_{m,\mu}d\tau \\ F_\mu^{tail}(x) &\equiv -q^2\int_{\tau_1}^{\tau^-}V_{,\mu}d\tau + q^2\int_{-\infty}^{\tau_1}G_{,\mu}d\tau\end{aligned}}$$

$$\Delta F_{\mu}(x) = F_{\mu}^{tail}(x) - q^2(\tau^-)_{,\mu}[\Delta V]_{\tau^-} + q^2 \int_{\tau_1}^{\tau^-} V_{m,\mu} d\tau - q^2 \int_{-\infty}^{\tau_1} G_{m,\mu} d\tau$$

Smooth splitting $\quad h_1(\tau) + h_2(\tau) = 1$

$h_1(\tau) = 1, \text{ for } \tau \geq \tau_1 + \varepsilon \qquad h_2(\tau) = 1, \text{ for } \tau \leq \tau_1 \qquad h_1(\tau_1) = 0 \qquad h_2(\tau_1 + \varepsilon) = 0$



$$\Delta F_{\mu}(x) = \underset{(1)}{F_{\mu}^{tail}(x)} - \underset{(2)}{q^2(\tau^-)_{,\mu}[\Delta V]_{\tau^-}} + \underset{(3)}{q^2 \int_{\tau_1}^{\tau^-} h_1 V_{m,\mu} d\tau} - \underset{(4)}{q^2 \int_{-\infty}^{\tau_1+\varepsilon} h_2 G_{m,\mu} d\tau}$$

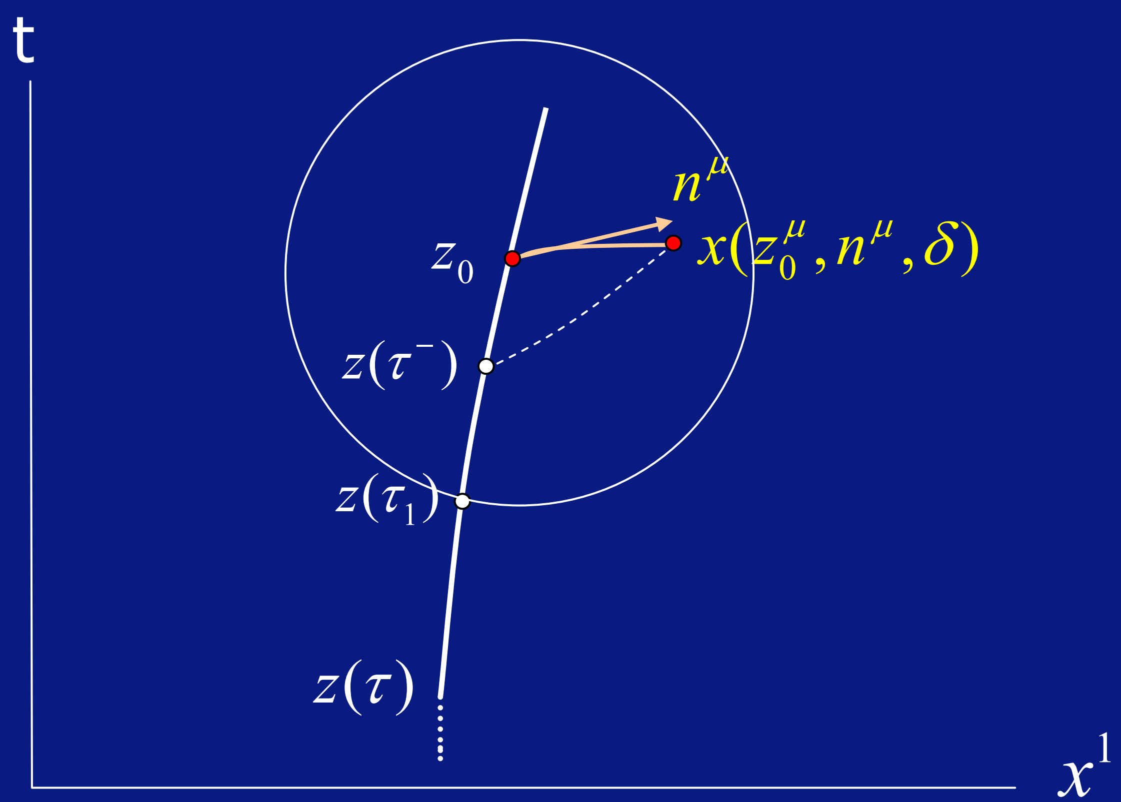
I. Calculating the limit $\delta \to 0$

II. Then calculating the asymptotic form as $m \to \infty$

(1)

$$F_\mu^{tail}(x) \equiv -q^2 \int_{\tau_1}^{\tau^-} V_{,\mu} d\tau + q^2 \int_{-\infty}^{\tau_1} G_{,\mu} d\tau$$

$$\lim_{\delta \to 0} F_\mu^{tail} = \lim_{\tilde{\varepsilon} \to 0} q^2 \int_{-\infty}^{\tau_0 - \tilde{\varepsilon}} \partial_\mu G(z_0 \mid z(\tau)) d\tau$$



$$\Delta F_{\mu}(x) = F_{\mu}^{tail}(x) \boxed{- q^2 (\tau^-)_{,\mu} [\Delta V]_{\tau^-} + q^2 \int_{\tau_1}^{\tau^-} h_1 V_{m,\mu} \, d\tau} - q^2 \int_{-\infty}^{\tau_1+\varepsilon} h_2 G_{m,\mu} \, d\tau$$

(2)+(3)

Hadamard expansion

$$V(x|z) = \sum_{n=0}^{\infty} \frac{\sigma^n}{n!} v_n(x|z) \qquad V_m(x|z) = \sum_{n=0}^{\infty} \frac{\sigma^n}{n!} \tilde{v}_n(x|z)$$

Recurrence differential equations for $v_n, \tilde{v}_n$ ➡ $\tilde{v}_n = \sum_{k=-1}^{n} (m^2/2)^{n-k} \frac{v_k}{(n-k)!}$

$$\boxed{V_m = mU \frac{J_1(ms)}{s} + \sum_{n=0}^{\infty} v_n J_n(ms) \left(\frac{-s}{m} \right)^n} \qquad s(x|z) \equiv \sqrt{-2\sigma(x|z)}$$

There is a different derivation, see F. G. Friedlander

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(2)+(3) ➡

I. Calculating the limit $\delta \to 0$

II. Then calculating the asymptotic form as $m \to \infty$

⬇

$$q^2 \left[-\frac{1}{2}(m^2 n_\mu + m a_\mu) + \frac{1}{3}(\dot{a}_\mu - a^2 u_\mu) + (\frac{1}{6} R_\mu{}^\nu u_\nu + \frac{1}{6} R^{\eta\nu} u_\eta u_\nu u_\mu - \frac{1}{12} R u_\mu) \right]$$

(4) $-q^2 \int_{-\infty}^{\tau_1+\varepsilon} h_2 G_{m,\mu}(x \mid z)\, d\tau$

Simplified problem: The world line is within a convex domain (no - caustics), spacetime is globally hyperbolic.

Use asymptotic expansion (Friedlander)

$$G_m(x \mid z) = \Theta(\Sigma(x), z)\left\{U\delta(\sigma) - [V^{asym}{}_m + O(m^{-1/2})]\theta(-\sigma)\right\}$$

$$V_m^{asym} = U\sqrt{\frac{2m}{s^3\pi}}\cos(ms - \frac{3\pi}{4})$$

$$\lim_{m\to\infty}\lim_{T\to-\infty}(-q^2)\int_T^{\tau_1+\varepsilon} h_3 h_2 G_{m,\mu}(z_0 \mid z)\, d\tau$$

$$0$$

(4) $-q^2 \int_{-\infty}^{\tau_1+\varepsilon} h_2 G_{m,\mu}(x \mid z)\, d\tau$ A more general case : allow caustics

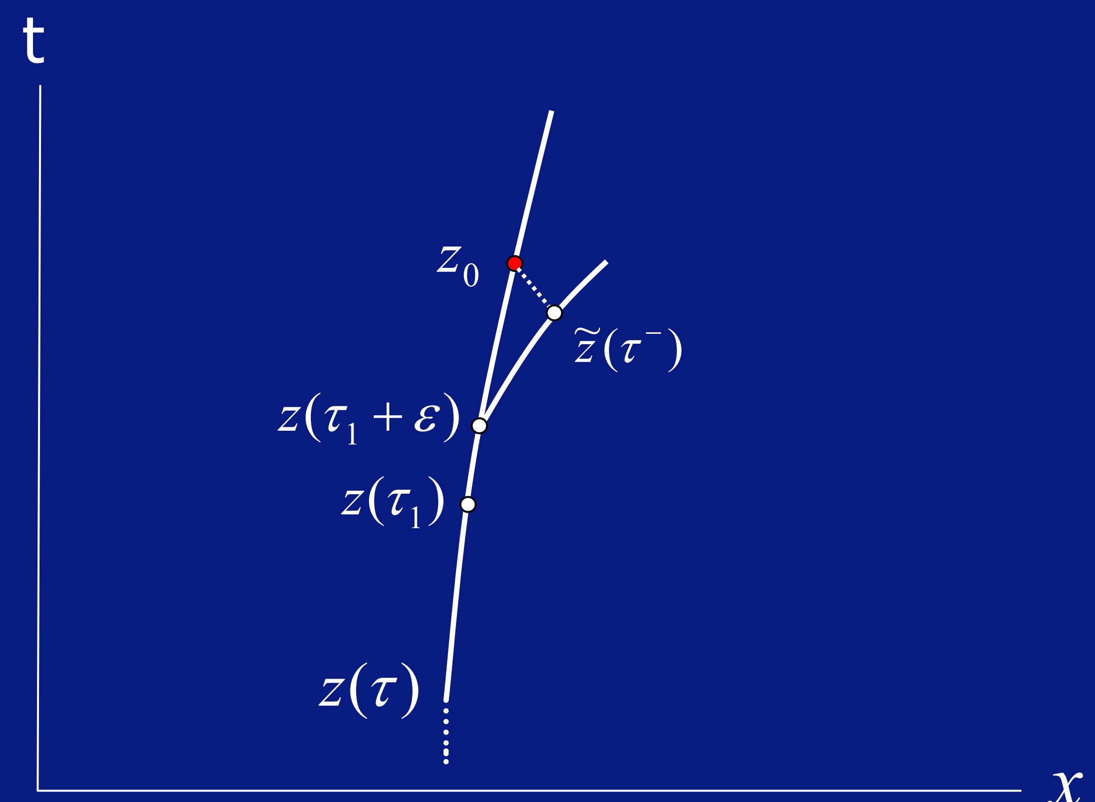
$$F_{(m)\mu}(z_0) \equiv q\phi_{m,\mu}(z_0) = (\text{depends on } [\tau_1+\varepsilon, \tau^-]) + q^2 \int_{-\infty}^{\tau_1+\varepsilon} h_2 G_{m,\mu}\, d\tau$$

$m \to \infty$
assumption

0

$m \to \infty$

0



(1)+(2)+(3)+(4)

Result

$$f_{\mu}^{self}(z_0)=q\lim_{m\to\infty}\left\{\lim_{\delta\to 0}\Delta\phi_{,\mu}(x)+\frac{1}{2}q\left[m^2 n_{\mu}(z_0)+m a_{\mu}(z_0)\right]\right\}=$$

$$F^{tail}{}_{\mu}(z_0)+q^2\left[\frac{1}{3}(\dot{a}_{\mu}-a^2u_{\mu})+(\frac{1}{6}R_{\mu}{}^{\nu}u_{\nu}+\frac{1}{6}R^{\eta\nu}u_{\eta}u_{\nu}u_{\mu}-\frac{1}{12}Ru_{\mu})\right]$$

Application: calculation of the mode-sum regularization parameters.

Notation change $m \to \Lambda$

Multipole expansion

$$\phi = \sum_{l=0}^{\infty} \phi^l \qquad \phi_\Lambda = \sum_{l=0}^{\infty} \phi_\Lambda{}^l$$

$$n_\mu(z_0) \Rightarrow n_\mu(x) = \sum_{l=0}^{\infty} n^l{}_\mu(x) \qquad a_\mu(z_0) \Rightarrow a_\mu(x) = \sum_{l=0}^{\infty} a^l{}_\mu(x)$$

Substitute in f_μ^{self}

$$f_\mu^{self}(z_0) = q \lim_{m\to\infty} \left\{ \lim_{\delta\to 0} \Delta\phi_{,\mu}(x) + \frac{1}{2} q \left[\Lambda^2 n_\mu(z_0) + \Lambda a_\mu(z_0) \right] \right\}$$

$$h_\mu^l \equiv A_\mu L - B_\mu - C_\mu / L$$

$$f_\mu^{self}(z_0) = \sum_{l=0}^{\infty} \lim_{\delta\to 0} (q\phi_{,\mu}^l - h_\mu^l) - D_\mu$$

$$D_\mu = \lim_{\Lambda\to\infty} \sum_{l=0}^{\infty} \left\{ \lim_{\delta\to 0} [q\phi_{\Lambda,\mu}^l - \frac{1}{2} q^2 (\Lambda^2 n_\mu^l + \Lambda a_\mu^l)] - h_\mu^l \right\}$$

Static particle in Schwarzschild spacetime

$$\phi^{lm}_{\Lambda,r^* r^*} - (V^l + \Lambda^2 f)\phi^{lm}_{\Lambda} = -4\pi\rho^{lm} f \qquad f \equiv 1 - \frac{2M}{r}$$

For large value of l - use WKB approximation to solve for ϕ^{lm}_{Λ}

$$\Downarrow$$

$$A_{r^+} = -\frac{q^2}{r_0^2}\frac{1}{\sqrt{f(r_0)}} \qquad B_{r^+} = q^2\frac{\frac{M}{r_0} - 1}{2r_0^2 f(r_0)} \qquad C_{r^+} = 0$$

For large value of Λ (any value of l) - use WKB approximation and take $\Lambda \to \infty$

$$\Downarrow$$

$$D_{r^+} = 0$$

Conforms with results obtained by Barack and Ori (2000)

References

S. Coleman, Electromagnetism: Paths to Research, ed. D. Teplitz, (Plenum, New-York, 1982).