Hydrodynamic simulations in GR
ー status & perspective ー

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1 Introduction

2 10 yrs ago (June 1993)

3 Achievement in the past decade

4 Current status

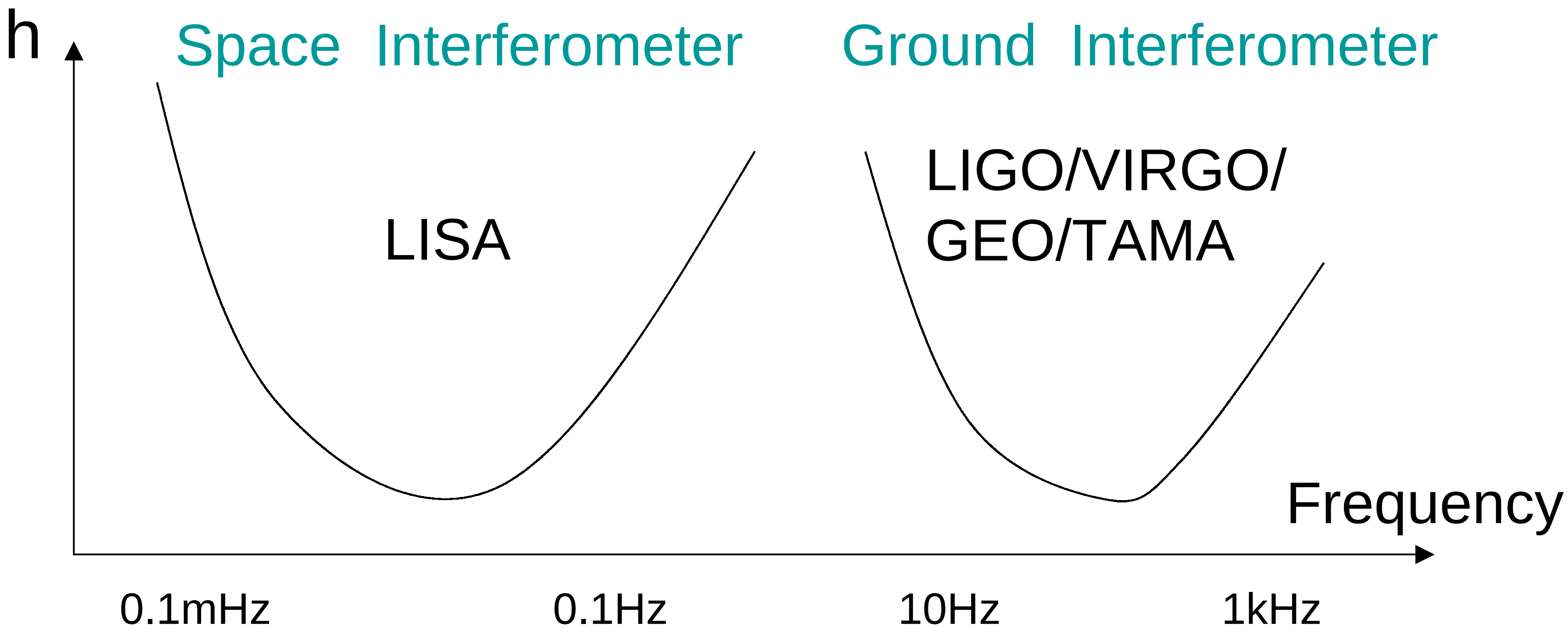
5 Some of latest numerical results:
NS-NS merger & Stellar core collapse

6 Summary & perspective

1: Introduction: roles

A To predict gravitational waveforms:

We know that two types of gravitational wave detectors work now or soon.

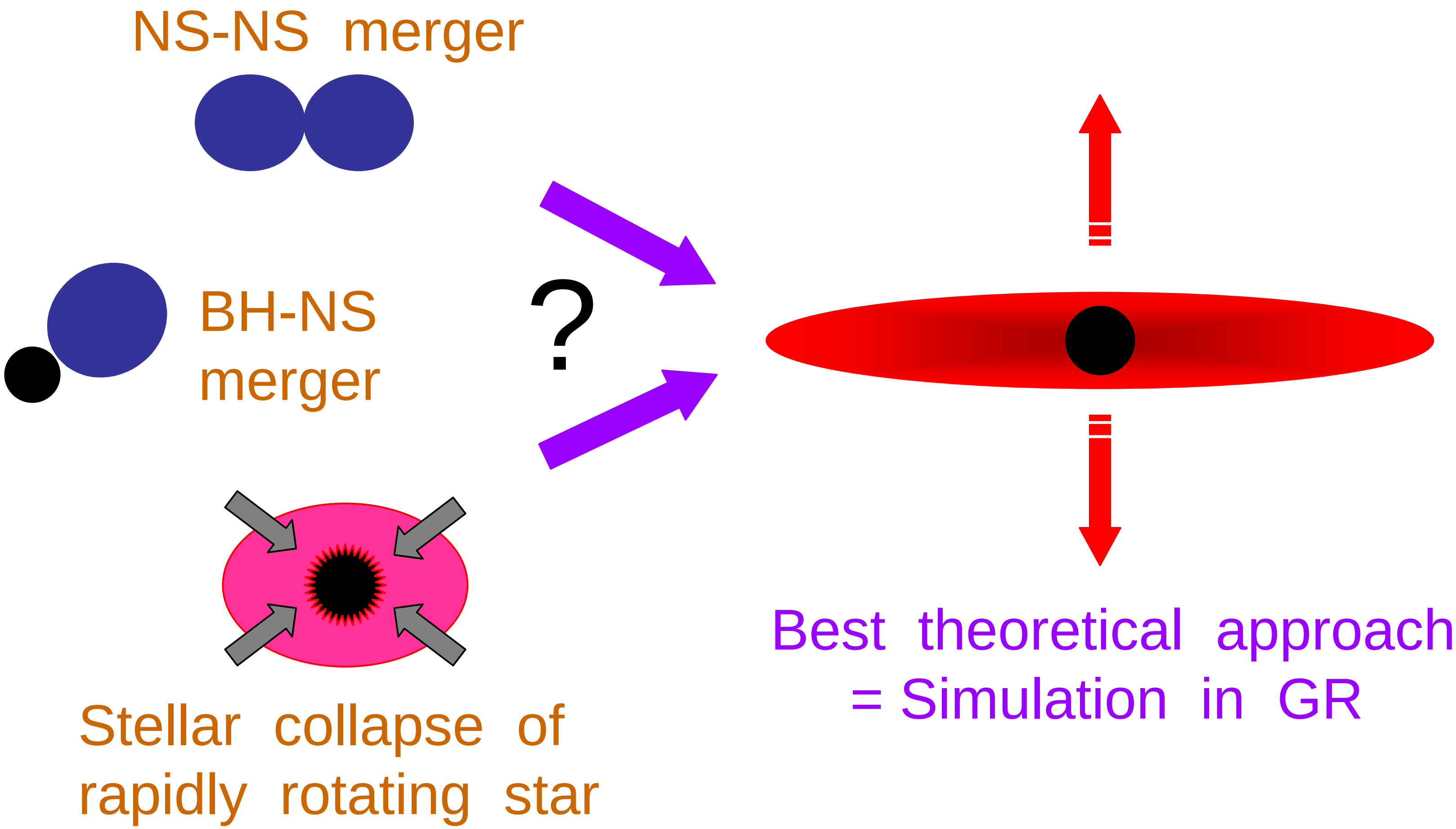


Templates should be made

B Simulate Astrophysical Phenomena

e.g. Central engine of GRBs

= Stellar-mass black hole + disks (Probably)



C Discover new phenomena in GR

e.g.

1: Critical phenomena (Choptuik, …..)

2: Toroidal black hole (Shapiro-Teukolsky)

3: Naked singularity formation (Nakamura, S-T)

etc.

GR hydro phenomena to be simulated

- NS-NS / BH-NS mergers (GW sources/GRB) 3D
- Stellar collapse of massive star to a NS/BH (GW sources/GRB) 2D/3D
- Nonaxisymmetric dynamical instabilities of rotating NSs (GW sources) 3D
- Collapse of supermassive stars to supermassive black holes of mass ~ Million solar-mass (low-frequency GW source) 2D/3D
- Oscillating and rotating NSs (periodic GW sources) 2D/3D
- Accretion induced collapse of a NS to a BH or a BH + disk (or a quark star) (GRB) 2D/3D

In general, 3D simulations are necessary

Necessary elements for hydro simulations in GR

- Einstein's evolution equations solver
- GR Hydrodynamic equations solver
- Appropriate gauge conditions (coordinate conditions)
- Realistic initial conditions
- Gravitational wave extraction techniques
- Apparent horizon (hopefully Event horizon) finder
- Special techniques for handling BHs
- Micro physics (EOS, neutrino processes, B-field …)
- Powerful supercomputers

RED = Indispensable elements

2: 10 yrs ago (June 1993)

- Axially symmetric numerical relativity was actively done mainly for academic issues [head-on collision of two BHs (NCSA), collapse of collisionless matter (Cornell), Critical phenomena (Evans-Abrahams)….], but not for realistic phenomena such as realistic rotating stellar core collapse to NS/BH.

- 3D Numerical relativity had been already started by Nakamura. But, it was in its infancy.

- (I got a position at Osaka June 16, 1993, so I was very happy at that time.)

3D Implementations of 10 yrs ago

- Einstein's evolution equations solver in 3D

→ Ideas for formulation had been already proposed by Nakamura and Bona-Masso (talk later), but only preliminary computations had been done

- GR Hydrodynamic equations solver

→ Old scheme (adding artificial viscosity; not very physical)

- Appropriate gauge conditions (coordinate conditions)

→ Ideas had been already proposed (e.g., Minimal distortion gauge (Smarr & York)), but essentially no computations had been done

- Apparent horizon finder

→ had not been developed (now resolved completely)

- Supercomputers

→ ~ a few Gbytes memory & ~ a few Gflops in speed at best

= Power was comparable to current inexpensive PC ~ $1000 !

3: Achievements in the past decade

Here, focus on progress in main elements:

- Einstein evolution equation solver in 3D
- GR Hydro equation solver
- Appropriate gauge conditions in 3D
- Supercomputers

Progress I

- Formulations for Einstein's evolution equation

(I guess) many people 10 yrs ago believed the standard ADM formalism (e.g., York 1979) works well. BUT:

Standard ADM

Variables in standard ADM formalism:

$$\gamma_{ij}, \quad K_{ij}$$

12 components

Unconstrained free evolution

Numerical simulation becomes unstable even in the evolution of linear GW (Nakamura 87, Shibata 95, Baumgarte-Shapiro (99)

Due to constraint violation instabilities

- New formulations for Einstein's evolution equation :

(i) BSSN formalism

Nakamura (87), Shibata-Nakamura (95), Baumgarte-Shapiro (99)…..

Choose variables:

$$\phi \equiv \frac{1}{12}\ln\left(\det(\gamma)\right)$$

$$\tilde{\gamma}_{ij} \equiv e^{-4\phi}\gamma_{ij}$$

$$K \equiv K_k^k$$

$$\tilde{A}_{ij} \equiv e^{-4\phi}\left(K_{ij} - \frac{1}{3}\gamma_{ij}K\right)$$

$$F_i \equiv \delta^{jk}\partial_j\tilde{\gamma}_{ik}$$

17 components

→

The Important step

Rewrite ADM equations using

$$\left\{\begin{array}{l} constraint\ \ equations \\ \det\left(\tilde{\gamma}_{ij}\right) = 1 \end{array}\right\}$$

↓ Unconstrained free evolution

Stable numerical simulation

(So far no problem in the absence of black holes)

- New formulations for Einstein's evolution equation :

(ii) Hyperbolic formulations

Bona-Masso (92) ……… many references ……..

Kidder-Scheel-Teukolsky (KST) (01)

$$\partial_t g^{ij} + \partial_k Q^{kij} = F^{ij}(g, Q, \ldots)$$

No derivatives

Perhaps robust for BH spacetimes:
But have not succeeded in 2BH merger so far.
(Something is missing.
Need additional ideas (Teukolsky).)

Progress II

- GR Hydro scheme

Trend until the middle of 1990

$\Rightarrow$ Add artificial viscosity to capture shocks

(Wilson 1980, Centrella 1983, Hawley et al. 1984, Stark-Piran 1985, Evans 1986, Nakamura 1993, Shibata 1999 …..)

Schematically,

$$\frac{\partial \rho v_i}{\partial t} + \frac{\partial(\rho v_i v^j + P\gamma_i^{\ j})}{\partial x^j} = \underline{[Viscous\ \ term]_i} + ...$$

Very phenomenological
Not very physical

Drawback : Strong shocks cannot be captured accurately.
& Concern : We do not know if it always gives correct answer for any problems ?

- Hydro scheme: Current trend

High-resolution shock-capturing scheme
= Solve equations using characteristics
(+ Piecewise-Parabolic interpolation
+ Approximate Riemann solver) : very physical !

No artificial viscosity

Developed by Valencia (Ibanez, Marti, Font, …)
& Munich (Mueller …) groups in 1990s.

Now used by many groups (including myself)

⇒ Strong shocks & oscillations of stars are computed accurately

⇒ Physical Scheme → No concern on the outputs

⇒ (I believe) This is currently the best choice for simulations of

-- Stellar core collapse

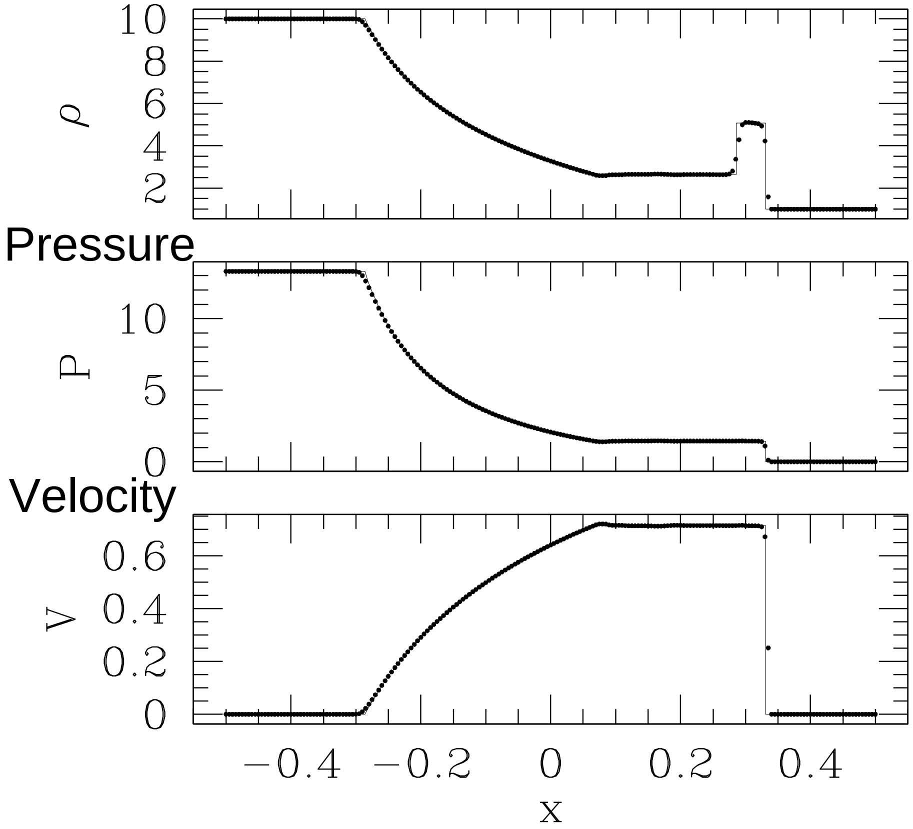
-- NS-NS merger

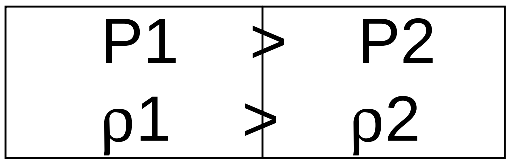
Standard tests for hydro code in special relativity

Riemann Shock Tube

$N = 400$, $\Gamma = 5/3$

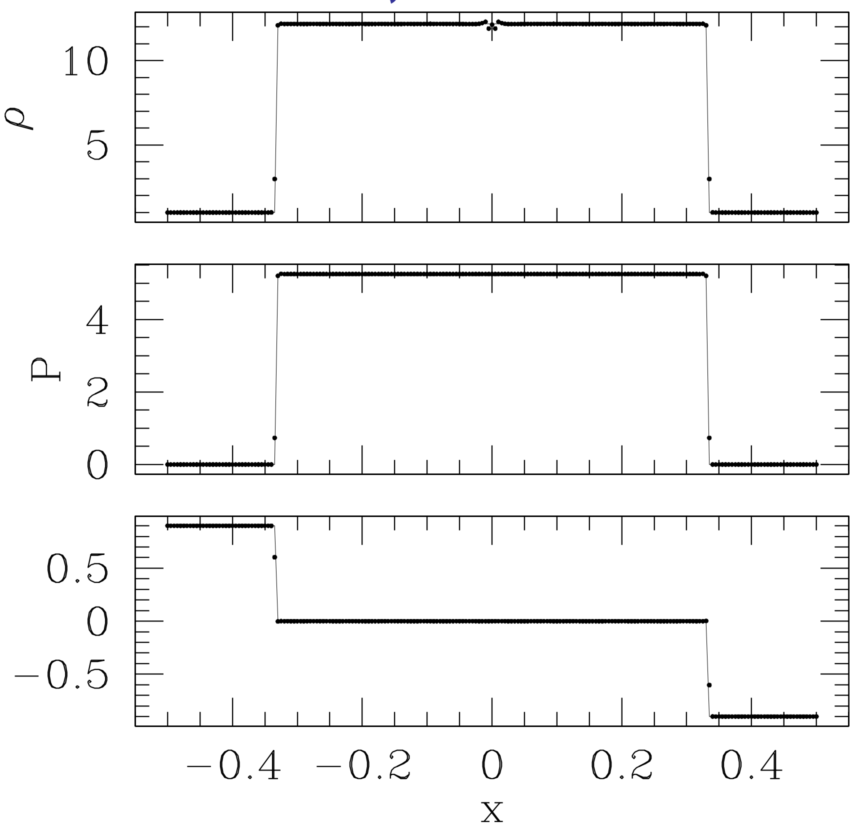
Density

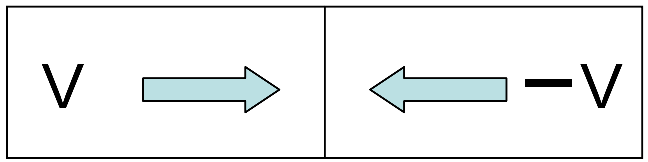




$V = 0.9c.$ **Wall Shock**

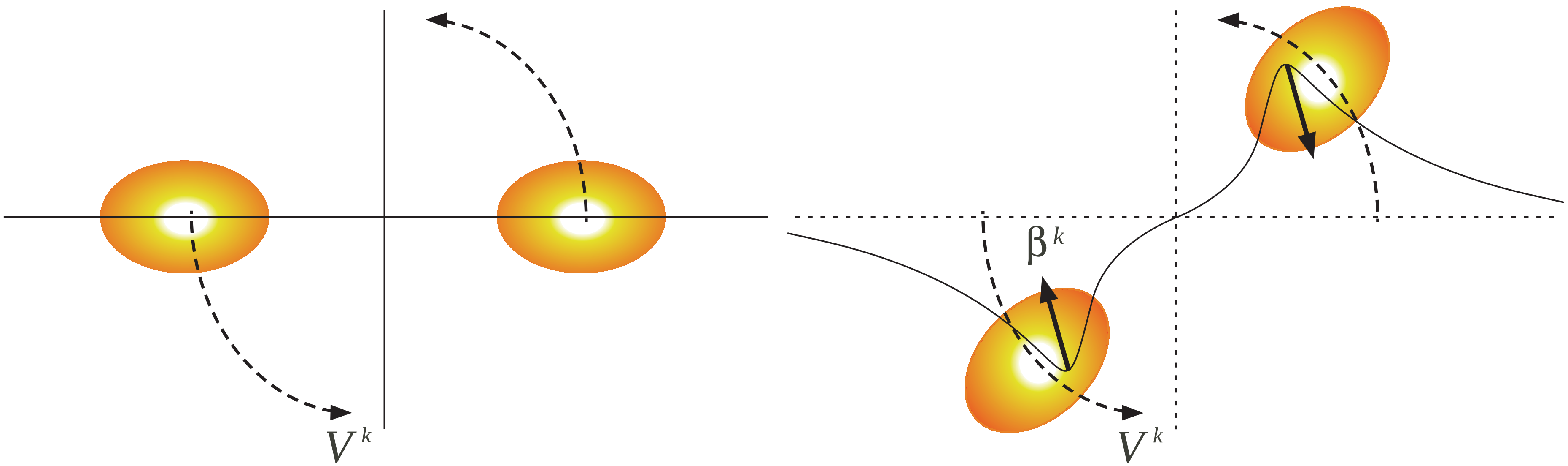
$N = 400$, $\Gamma = 4/3$





Progress III

- Choice of appropriate spatial gauge condition :



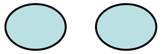
Frame dragging ⟹ Coordinate distortion

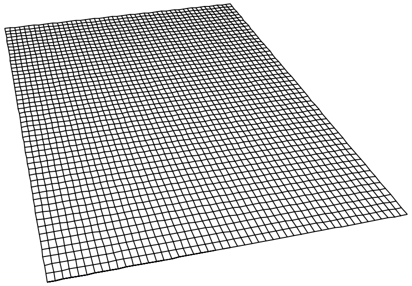
We need to suppress it for long-term evolution.

γ_{xx} on the equatorial plane with zero shift vector

t=0.0

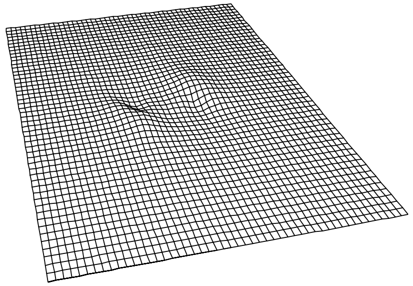
T=0



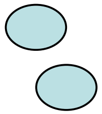


t=4.8

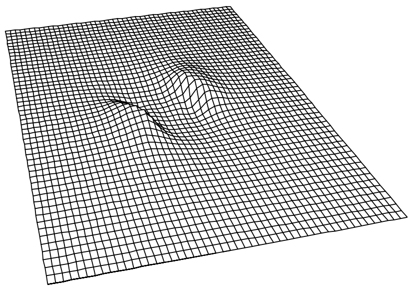
T~P/6



t=8.7



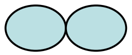
T~P/3

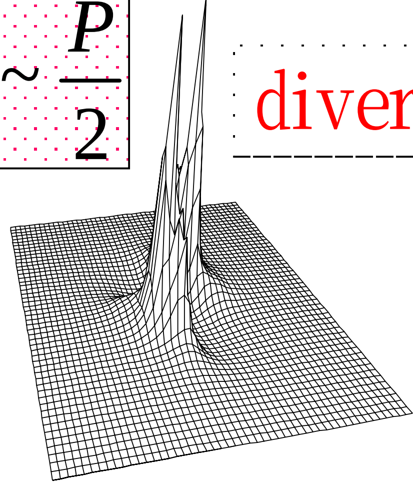


$$\gamma_{xx} - 1 \approx 2 \quad \text{at} \quad t \sim \frac{P}{2}$$

t=12.9

diverge





Distortion monotonically increases to crash

Previous belief: Minimal distortion gauge

(Smarr & York 1978)

Require that an action which denotes the global magnitude of the coordinate distortion is minimized.

$$\text{MD gauge}: \Delta\beta^k + \frac{1}{3} D^k D_j \beta^j = S^k$$

Physically good.
But, computationally time consuming

New Trend: Dynamical gauge (Alcubierre et al 2000, Lindblom & Scheel 2003, Shibata 2003 …..)

Schematic form:

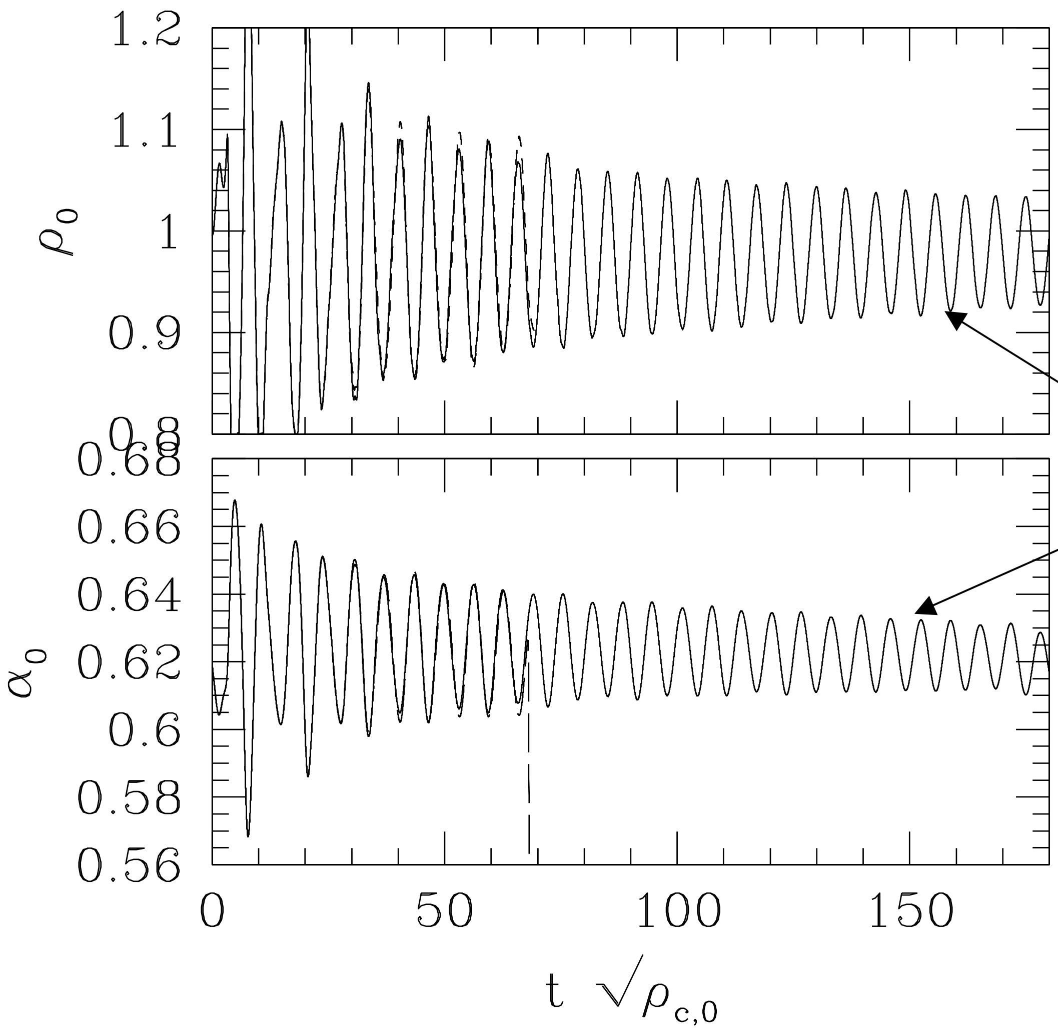
$$\ddot{\beta}^l \approx \Delta\beta^l + \frac{1}{3} D^l D_j \beta^j - S^l$$

Save CPU time significantly !!
Recent numerical experiments show it works well !!

Evolution of compact, rapidly rotating & oscillating NS in a dynamical gauge

Central density

Lapse function at r=0



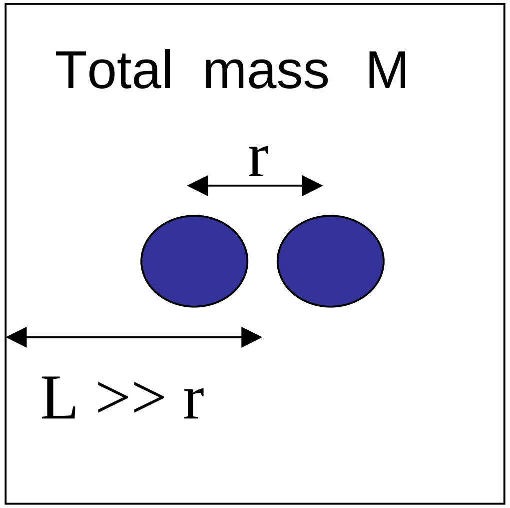
Dynamical gauge

Stable evolution for > 30 oscillation (~ rotation) periods.

Progress IV

Computational resources

Required grid number for accurate extraction of gravitational waveforms



$$\lambda_{GW} \le \lambda_{ISCO} \approx 58\left(\frac{GM}{c^2}\right)\left(\frac{rc^2}{7GM}\right)^{3/2}$$

$$\text{Require} \quad L \ge \lambda_{GW} \quad \& \quad \Delta x \le 0.2\left(\frac{GM}{c^2}\right)$$

$$\Rightarrow \quad \frac{L}{\Delta x} \ge 290\left(\frac{rc^2}{7GM}\right)^{3/2} \quad \& \quad N \ge 580\left(\frac{rc^2}{7GM}\right)^{3/2}$$

Minimum grid number required (in uniform grid):
600 * 600 * 300 (equatorial symmetry is assumed)
$\Rightarrow$ Memory required ~ 200 GBytes (~200 variables)

An example of current supercomputer

FUJITSU FACOM VPP5000 at NAOJ

- Vector-Parallel Machine → max: 60PEs
- Maximum memory → 0.96TBytes (Pragmatically ~ 0.7TBytes)
- Maximum speed → 0.58TFlops
- Our typical run with 32PEs

633 * 633 * 317 grid points = 240 Gbytes memory
(in my code)

About 20000 time steps ~ 100 CPU hours /model

Minimum grid number can be taken

But, we need hypercomputers for well-resolved simul.
(e.g. Earth simulator ~ 10TBytes, ~ 40TFlops)
Or need to develop mesh refinement techniques

4 Current Status

- Einstein evolution equations solver OK
- Gauge conditions (coordinate conditions) OK
- GR Hydrodynamic equations solver OK
- Powerful supercomputer ~OK

but hopefully need hypercomputers

Long-term GR hydro simulations are feasible
(in the absence of BHs)

In the past 5 yrs, computations have been done for

- NS-NS merger (Shibata-Uryu, Miller, …)
- Stellar core collapse (Font, Papadopoulas, Mueller, Shibata)
- Collapse of supermassive star (Shibata-Shapiro)
- Bar-instabilities of NSs (Shibata-Baumgarte-Shapiro)
- Oscillation of NSs (Shibata, Font-Stergioulas, ….)

5a. Latest numerical results by us: NS-NS merger

Current implementation in our group

1. GR : BSSN (or Nakamura-Shibata, but modified year by year; e.g., latest version = Shibata et al. 2003)
   → improve accuracy
2. Gauge : Maximal slicing (K=0) + Dynamical gauge

3. Hydro : High-resolution shock-capturing scheme (Roe-type method with 3rd-order PPM interpolation)

4. Typical grid size : 633 * 633 * 317

EOS & Initial conditions

- Equation of state t = 0 : $P = K\rho^\Gamma$
  t > 0 : $P = (\Gamma - 1)\rho\varepsilon$; Here, $\Gamma = 2$

Compactness $(M/R)_\infty$		Total rest mass M_{*Tot}/M_{*Max} (J=0)	Spin J/M^2	m_2/m_1	Model	Fate
0.14 vs 0.14	●	1.62	0.951	1	M1414	NS
0.16 vs 0.16	●	1.78	0.914	1	M1616	BH
0.13 vs 0.15	●	1.62	0.961	0.901	M1315	NS
0.15 vs 0.17		1.77	0.923	0.925	M1517	BH
0.14 vs 0.18	●	1.76	0.933	0.855	M1418	BH

M_{*Max}: Maximum rest-mass of spherical star in isolation

Unequal mass (new)

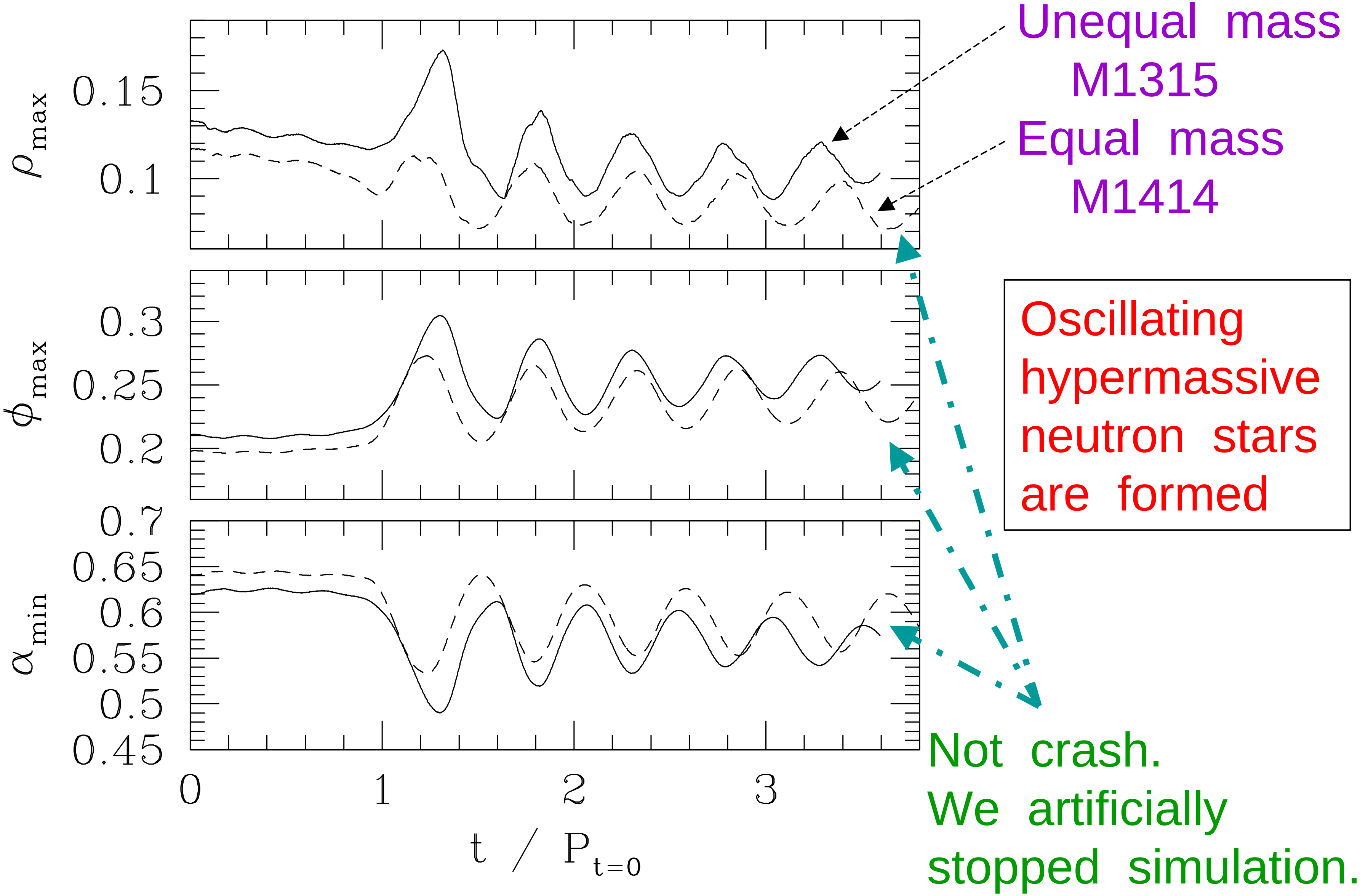
Hypermassive

Note (M/R) = 0.14 & 0.16 mean
R = 15km & 13km if M = 1.4 Solar mass

Animations

- http://esa.c.u-tokyo.ac.jp/~shibata/anim.html

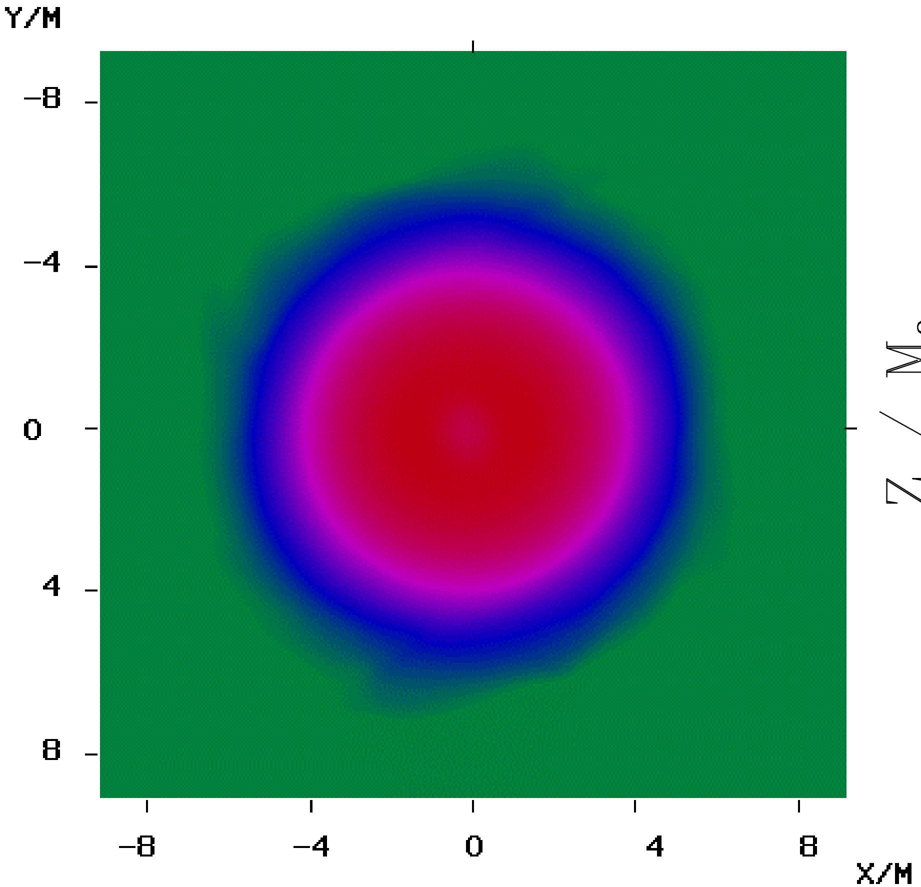
Change of maximum density in NS formation



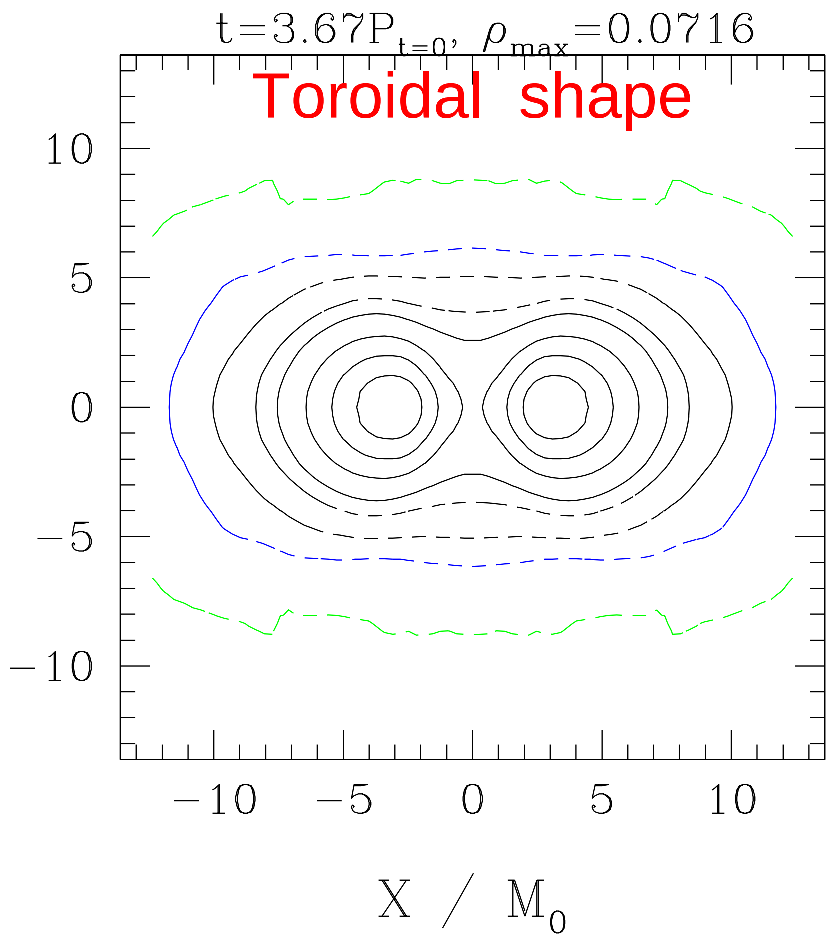
M/R = 0.14 equal mass case : final snapshot

Massive toroidal neutron star is formed

(slightly elliptical)

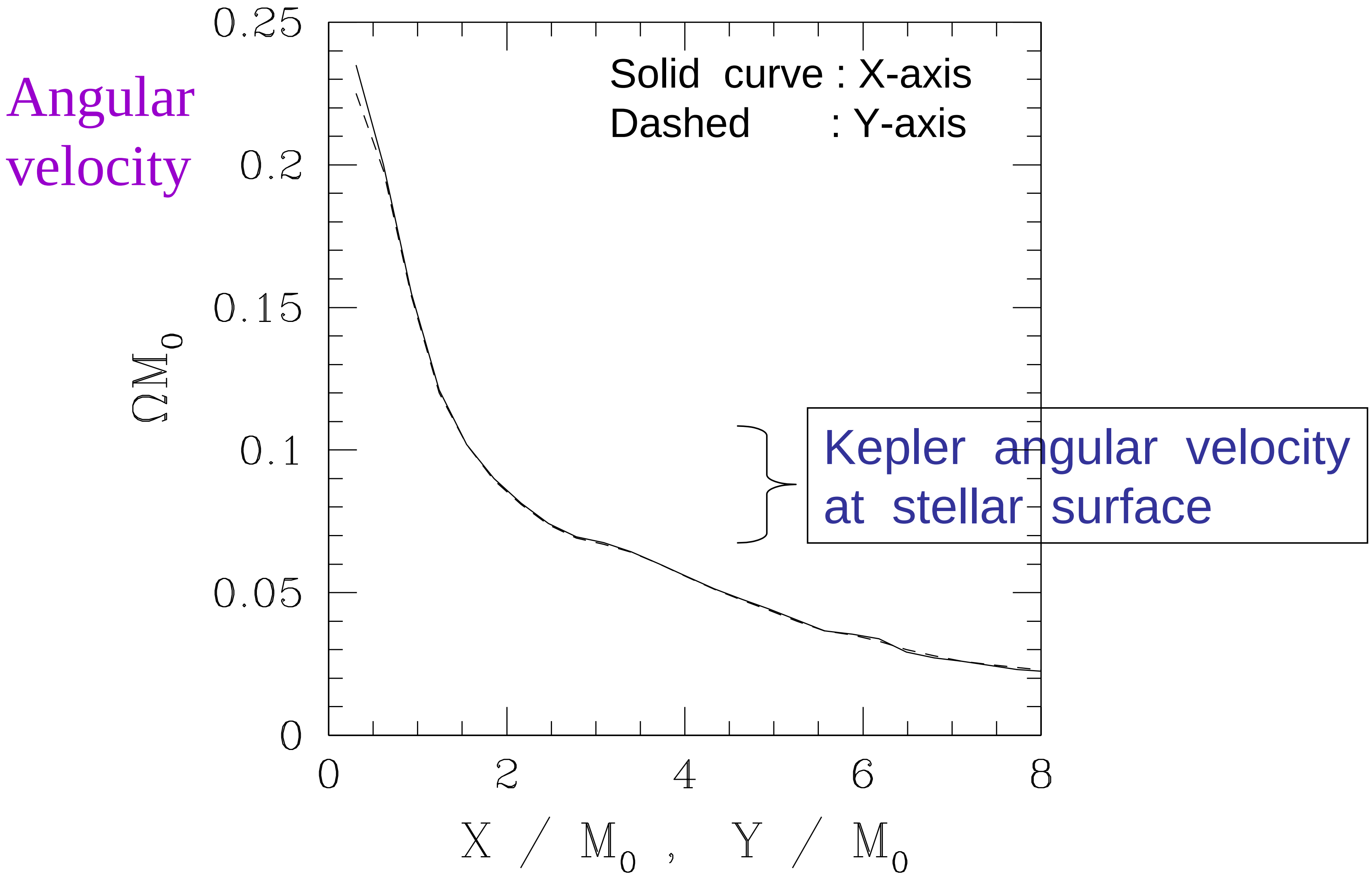


X – Y contour plot



X – Z contour plot

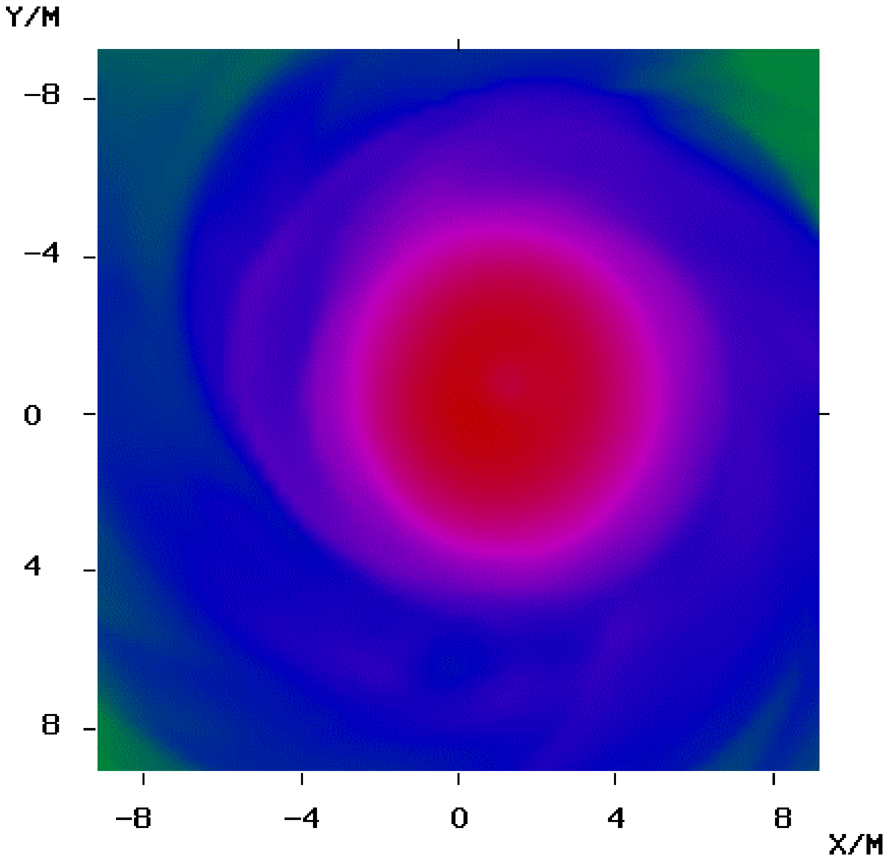
Formed Massive toroidal NS is differentially and rapidly rotating



Comparison between equal and unequal mass mergers

M/R = 0.13 vs 0.15:

Massive NS + disk

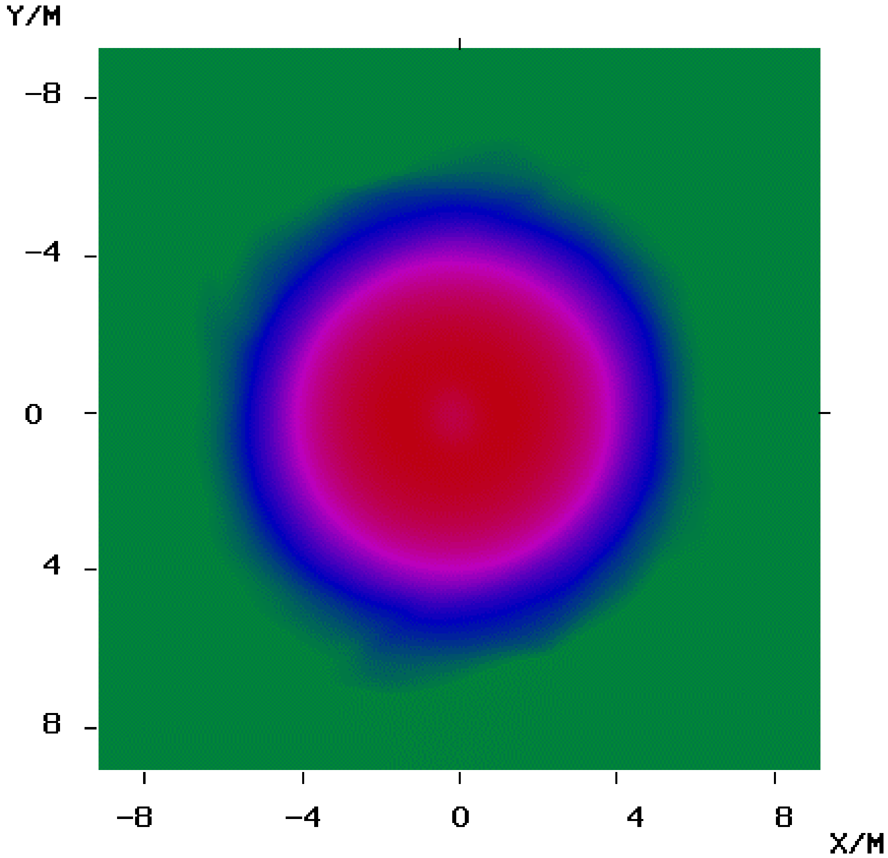


Unequal-mass case

Mass ratio ~ 0.901

M/R = 0.14 vs 0.14:

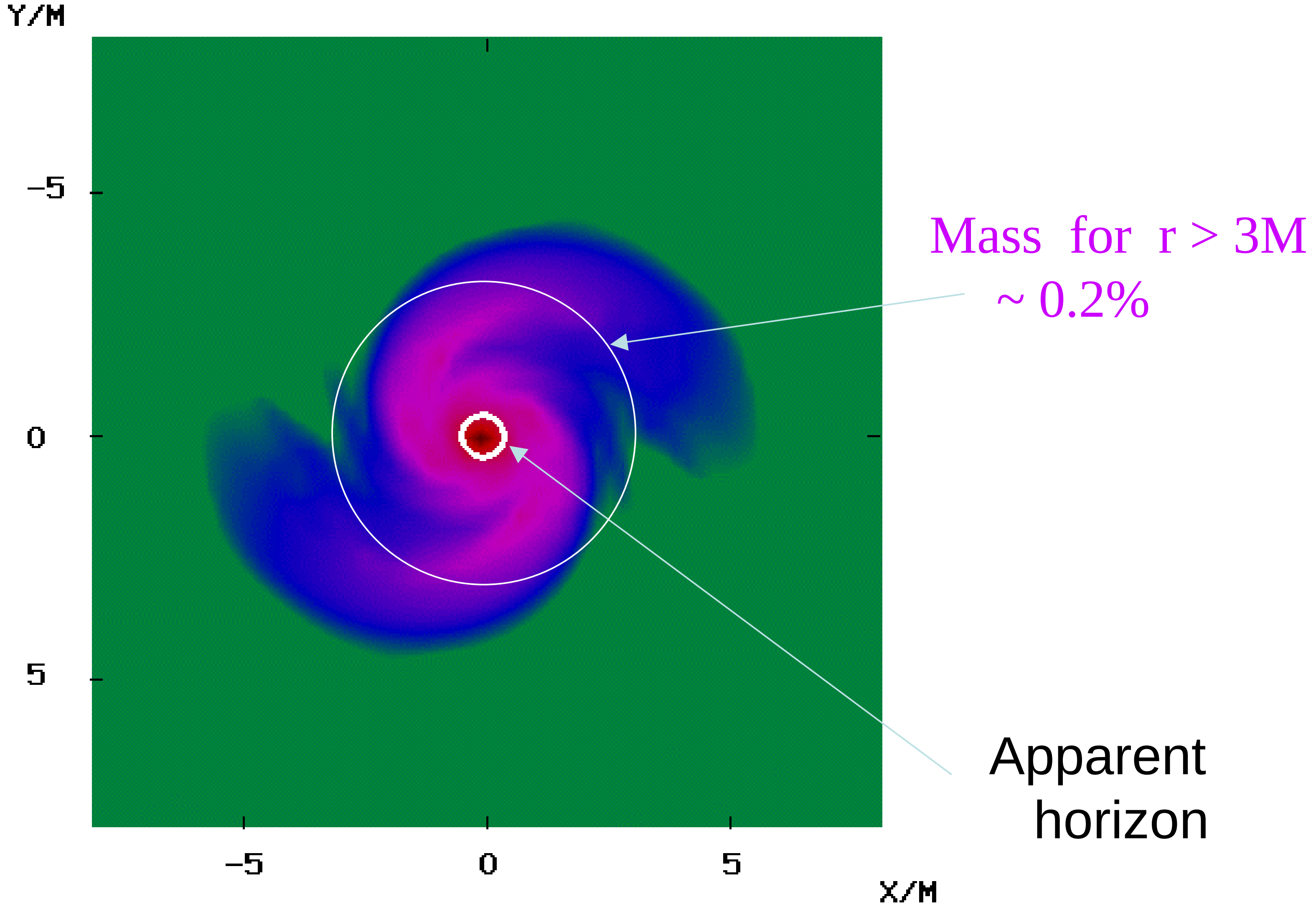
Massive NS



Equal-mass case

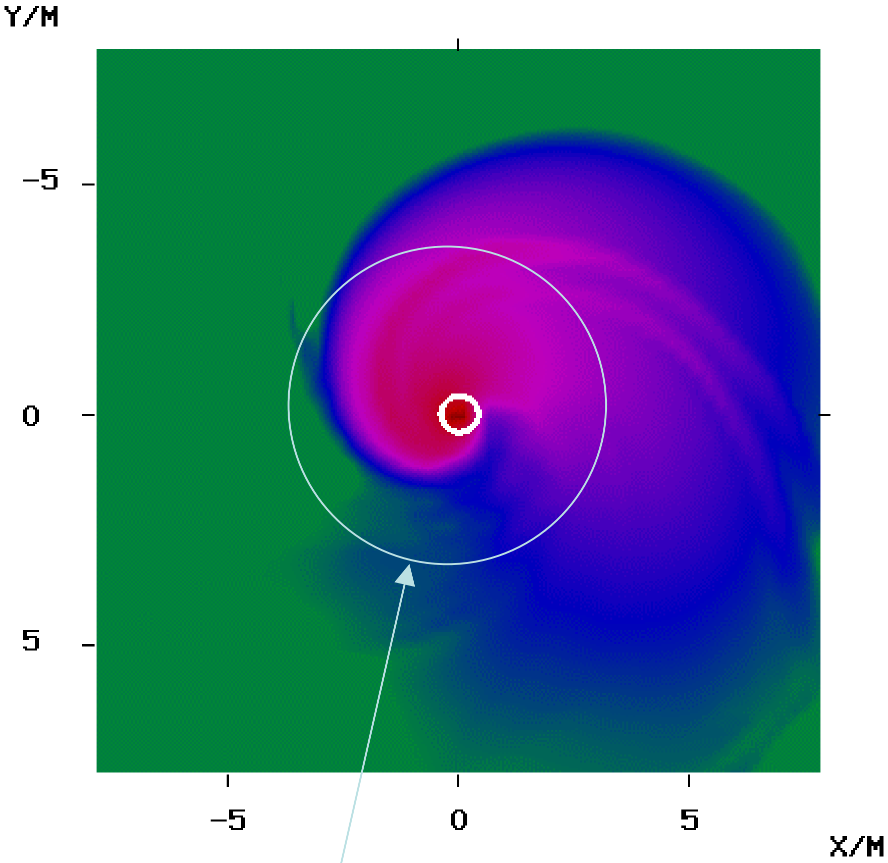
Black hole formation case: M/R = 0.16

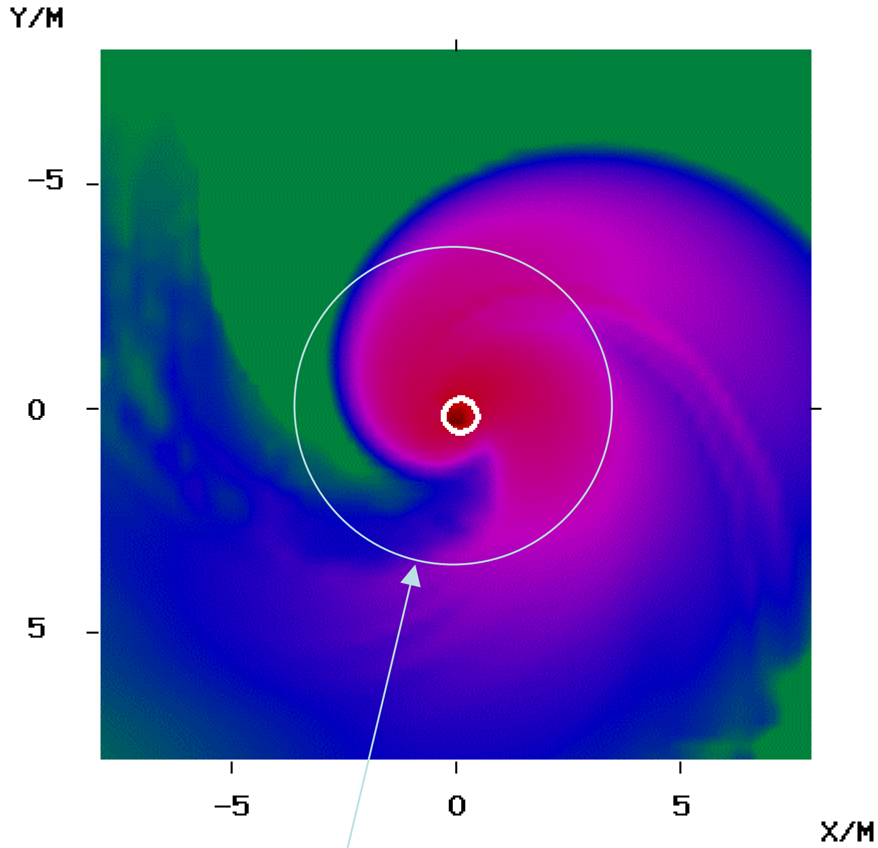
Equal-mass case



Disk mass for unequal-mass merger

M1517: Mass ratio 0.925 → M1418: Mass ratio 0.855





Mass for r > 3M
~ 2%

→

Mass for r > 3M
~ 5%

Products of mergers for $\Gamma = 2$

Equal – mass cases

- Low mass cases

  Hypermassive neutron stars

  of nonaxisymmetric & quasiradial oscillations.

- High mass cases

  Direct formation of Black holes

  with very small disk mass

Unequal – mass cases (mass ratio ~ 0.9)

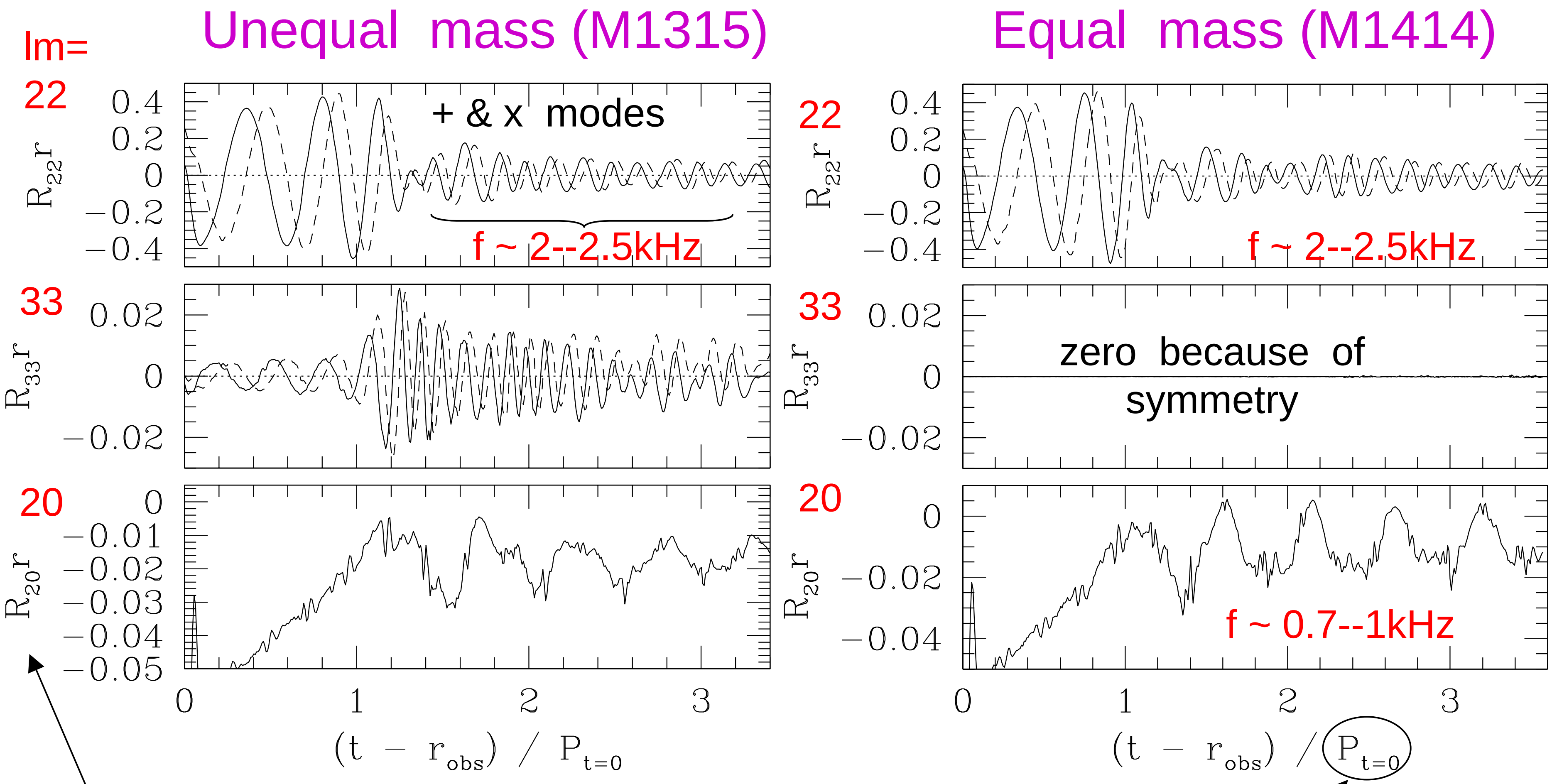
- Likely to form disks of mass

  ~ several percents of total mass

  → BH(NS) + Disk

  → Maybe a candidate for short GRB

Gravitational waves in NS formation

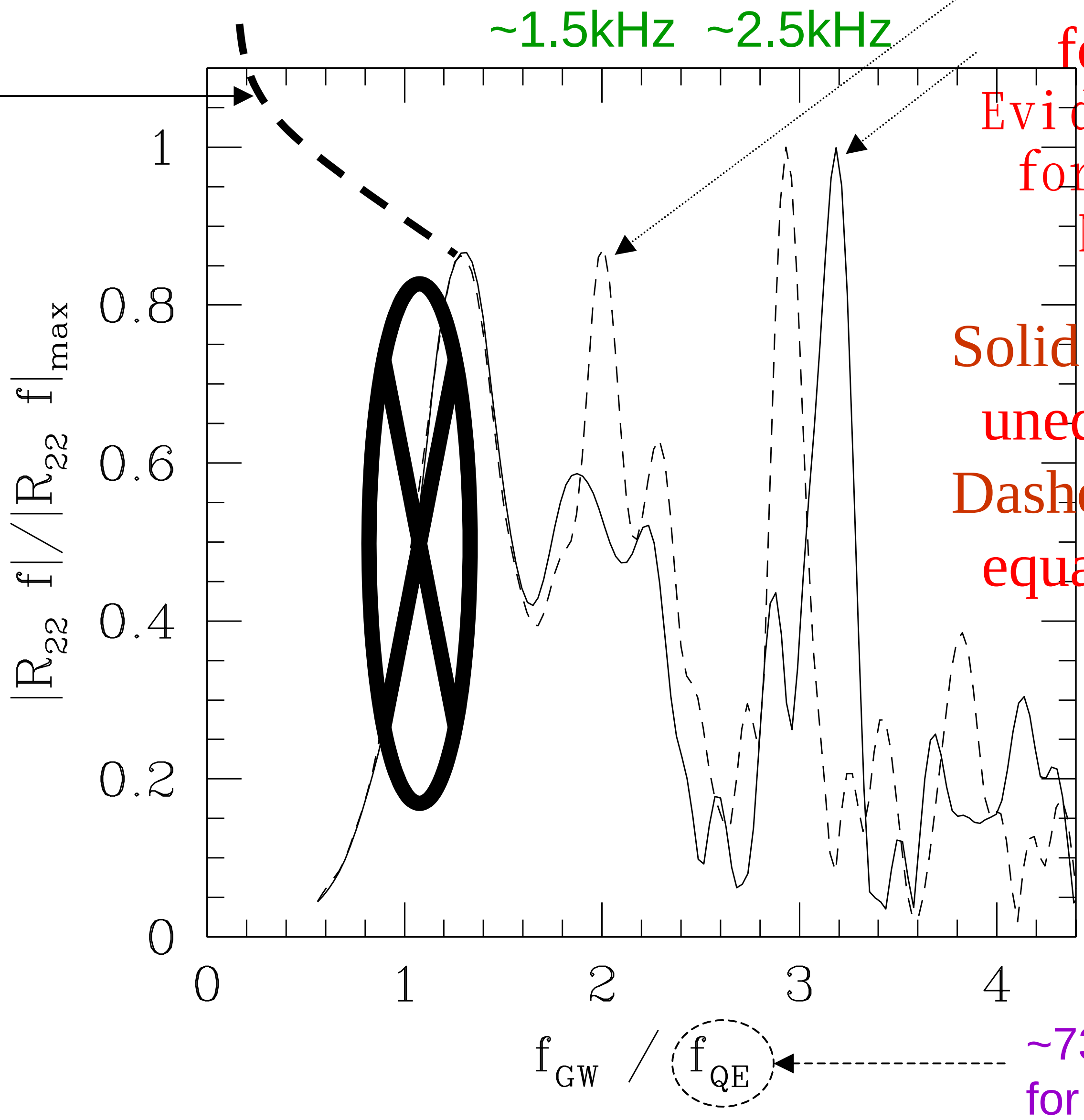


Gauge inv. variables
with (l, m) = (2, 2), (3, 3) & (2, 0)

P ~ 2.7msec (M/2.8solar)

Fourier spectrum for NS formation

Inspiral waveform f^{-1/6} is absent



Emitted by formed NS

Evidence for formation of NS

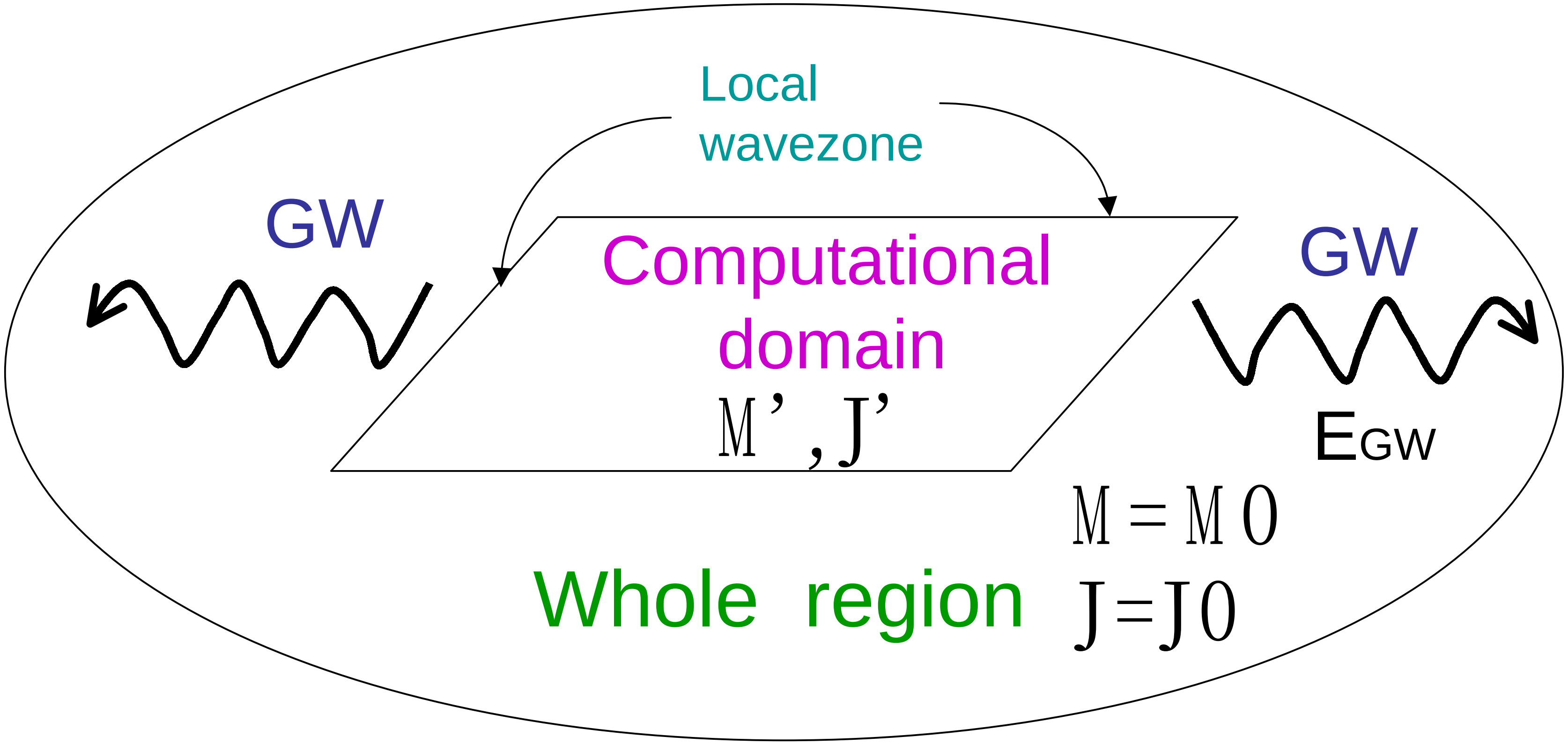
Solid curve: unequal mass

Dashed curve: equal mass

~730Hz for M~2.8solar

Computation of mass and angular momentum

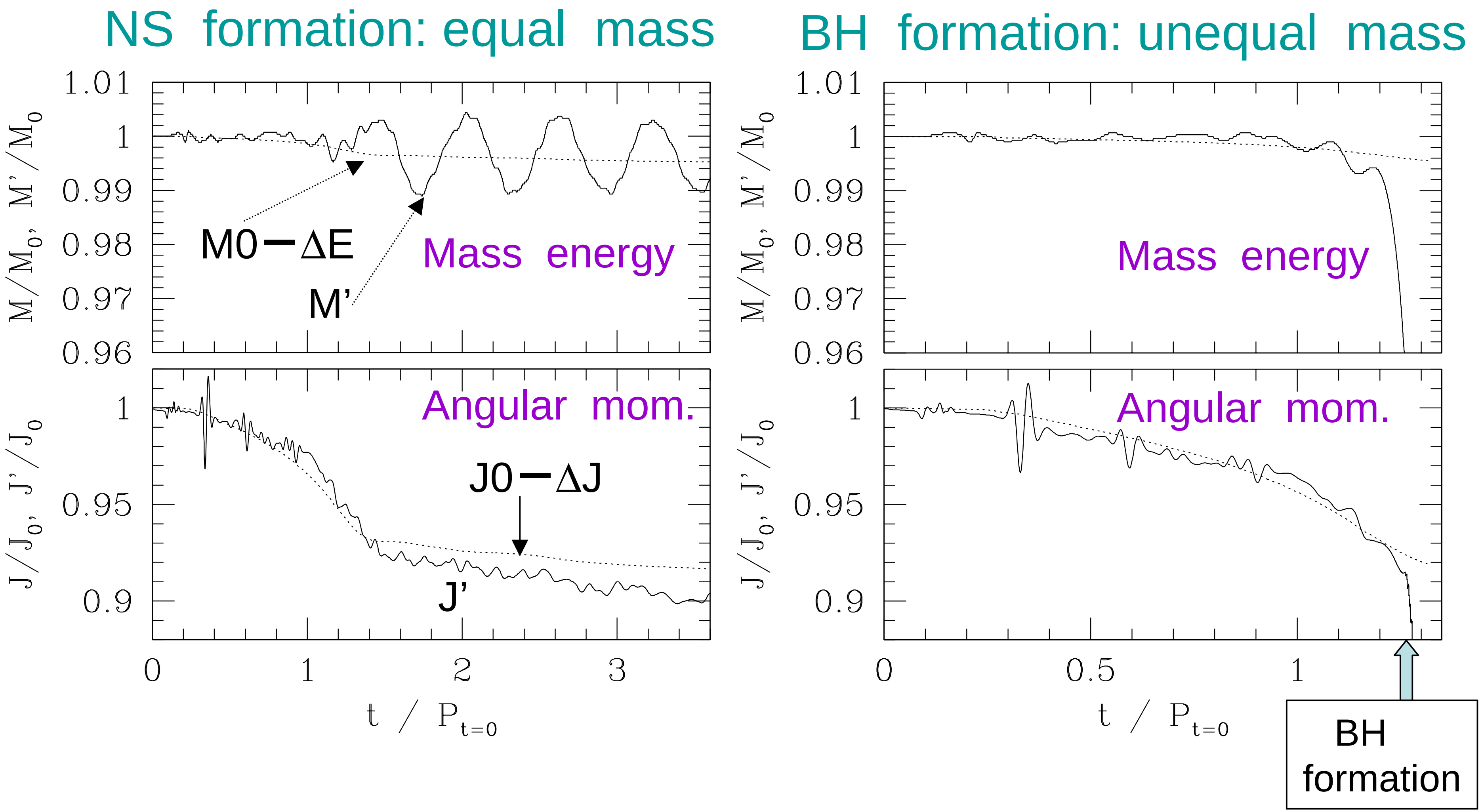
-- Check of the conservation --



$M0 - E_{GW} = M'$ & $J0 - J_{GW} = J'$

should be satisfied

Radiation reaction : OK within ~ 1%



Solid curves : computed from data sets in finite domain.
Dotted curves: computed from fluxes of gravitational waves

5b. Axisymmetric (2D) simulations for stellar core collapses

WHY WE SHOULD REVISIT 2D SIMULATIONS NOW

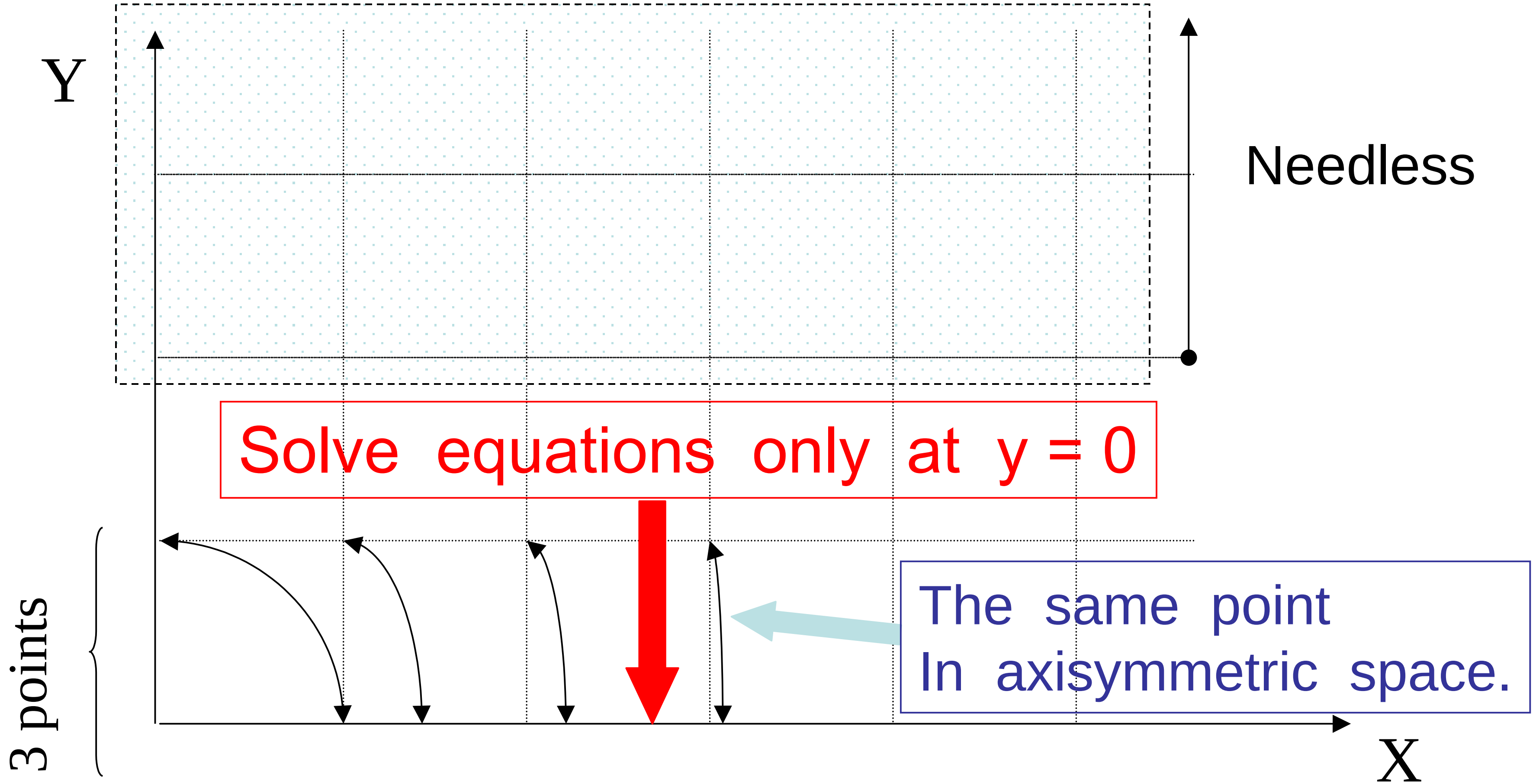
→ Many unsolved important problems such as realistic stellar core collapse & formation of BH/NS ← NOT YET DONE

Coordinate singularities prevent accurate and longterm stable simulations in axisymmetric cases, but

→ Cartoon method (no coordinate singularity) proposed by Potsdam Group enables a long-term, stable, and accurate simulation (talk later) : Technology has been developed

Computational resources are large enough to perform 2D simulations with (3~5000)^2 grid points : well-resolved simulations have become feasible very recently

Review of the cartoon method



- Use Cartesian coordinates : No coordinate singularity
- Prepare a 3D code for the Einstein eqs., and impose axisymmetric boundary condition at y=+,-Δy
- Total grid number = N∗3∗N for (x, y, z)

Stellar core collapse

- Parametric (fairly realistic) EOS

  (Following Mueller, Dimmelmeier, Font 2002)

$$P = P_{\text{Polytrope}} + P_{\text{Thermal}}$$

$$P_{\text{Thermal}} = \left(\Gamma_{\text{Thermal}} - 1\right) \rho \varepsilon_{\text{Thermal}}$$

$$P_{\text{Polytrope}} = \begin{cases} K_1 \rho^{\Gamma_1} & \rho \le \rho_{\text{Nuc}} \\ K_2 \rho^{\Gamma_2} & \rho \ge \rho_{\text{Nuc}} \end{cases}$$

$$\varepsilon_{\text{Thermal}} = \varepsilon - \varepsilon_{\text{Polytrope}}$$

$$\rho_{\text{Nuc}} \approx 2 \times 10^{14} g \cdot \text{cm}^{-3}$$

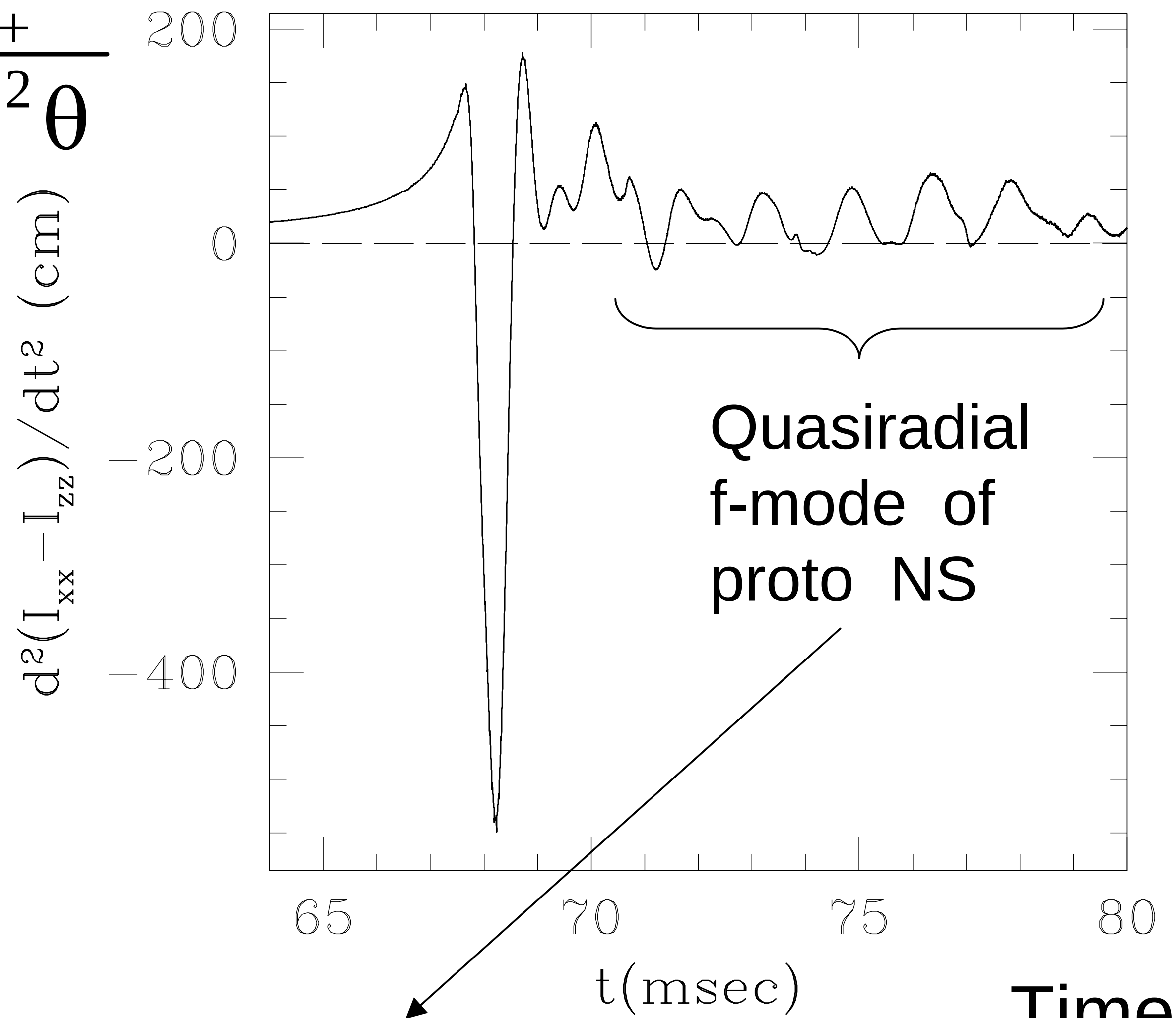
$$\Gamma_1 \sim \frac{4}{3} \qquad \Gamma_2 \ge 2 \qquad \Gamma_{\text{Thermal}} = 1.5$$

Give a rotating star of $\Gamma = 4/3$ & $\rho \sim 1.e10$ g/cc at t=0

Grid size = (2500, 2500) for (x, z)

Gravitational waveform
(by Quadrupole formula)

$$\frac{h_+}{\sin^2\theta}$$



h ~ 10^-23
at 10Mpc

Time (msec)

Characteristic frequency = 600 -- 800Hz

6 Summary

1. Rapid progress in particular in the past 5 yrs
2. Scientific (quantitative) runs are feasible now.
3. Accurate and longterm simulations are feasible for many phenomena in the absence of BHs : NS-NS merger, Stellar collapse, Bar-instabilities of NSs ….
4. (I think) numerical implementations for fundamental parts have been almost established (for the BH-absent spacetimes)

Issues for the near future

1 Several (technical) Issues still remain :

- Grid numbers are still not large enough in 3D

→ We would need Mesh-Refinement (AMR/FMR) or hypercomputer (~10TBytes, ~10TFlops)

- Computation crashed due to grid stretching around BH horizon

→ We need to develop excision techniques.

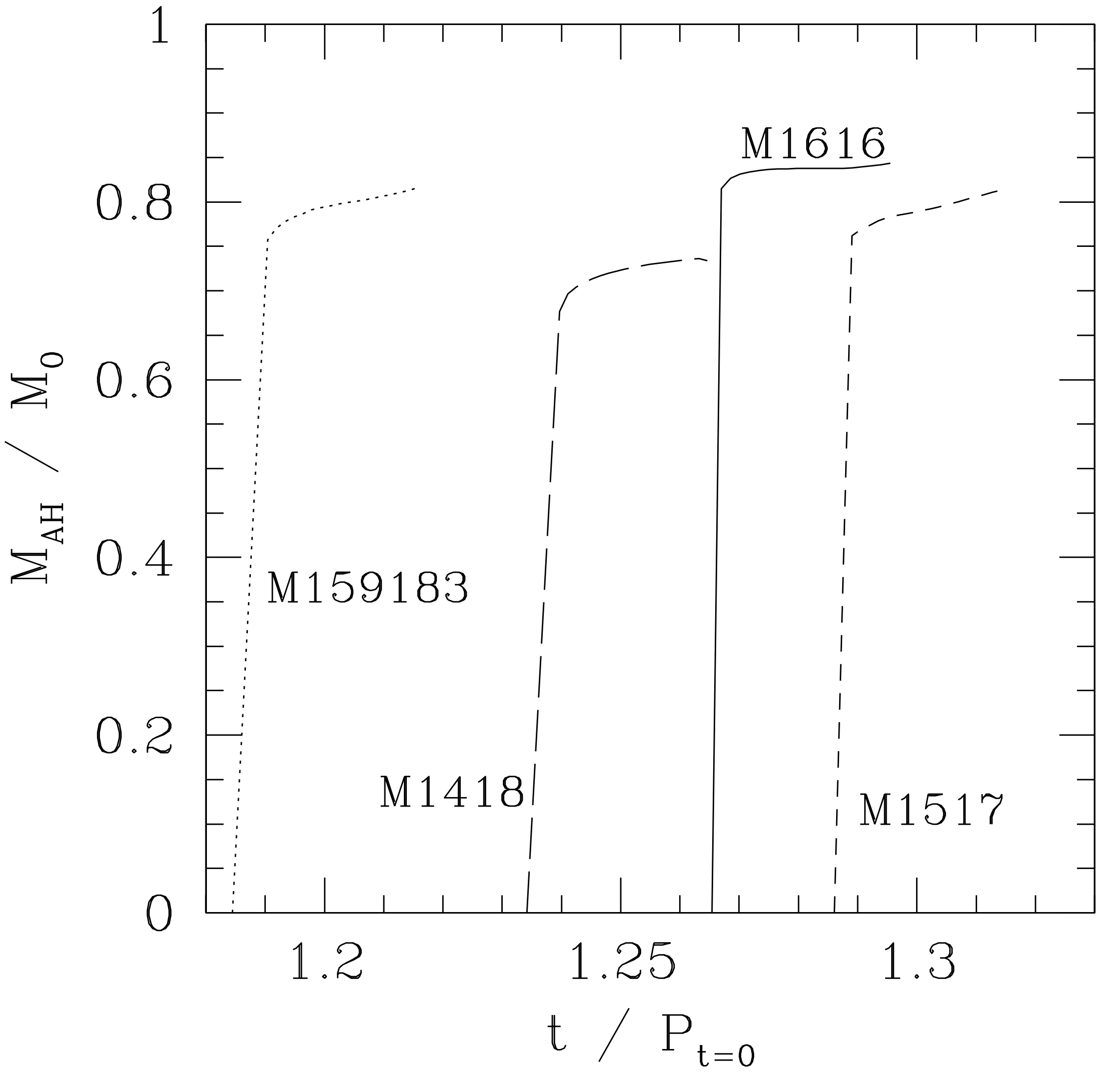
If we succeed,

→ Enable to simulate BH-NS & longterm simulation for stellar collapses to BH

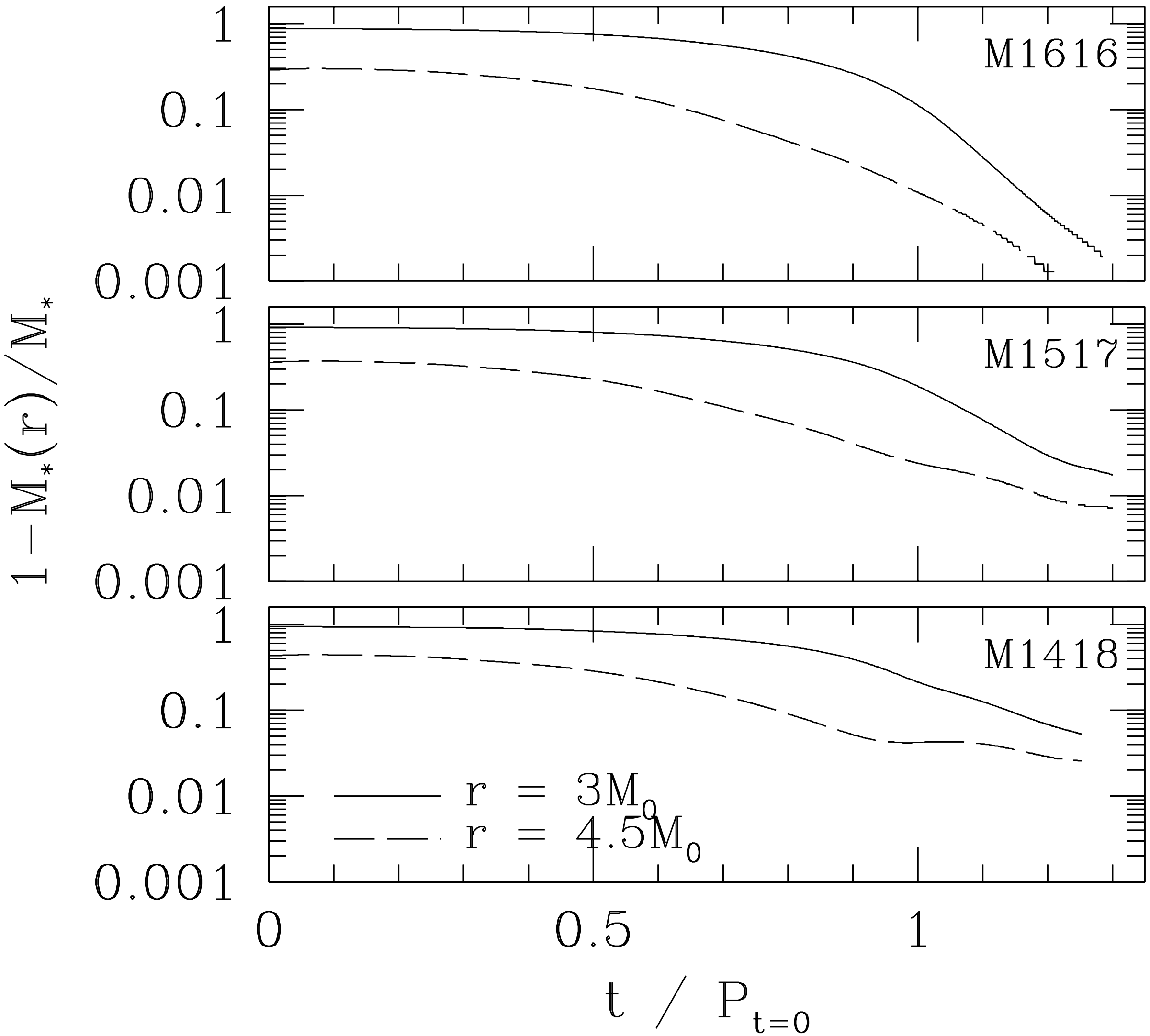
2 Incorporate more realistic physics

- More realistic EOS (e.g., Lattimer-Swesty …)
- Neutrino cooling (Ruffert-Janka in Newton)
- Magnetic fields (Illinois group starts a project)

Evolution of apparent horizon mass



Mass fraction outside a sphere for BH formation case



Equal mass

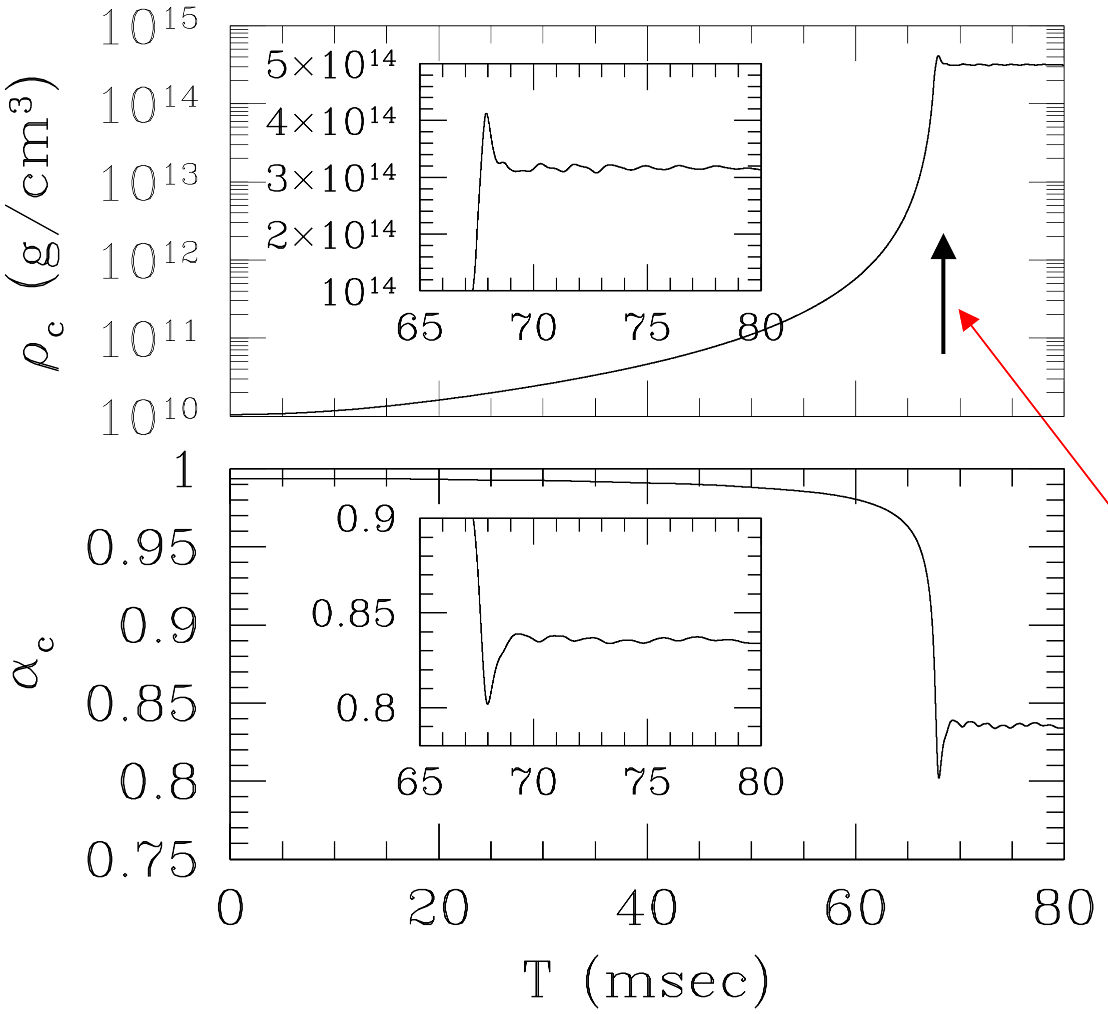
Mass-ratio =0.925

Mass-ratio =0.855

Collapse from a rigidly rotating initial condition with central density ~ 1e10 g/cc

Density
at r = 0

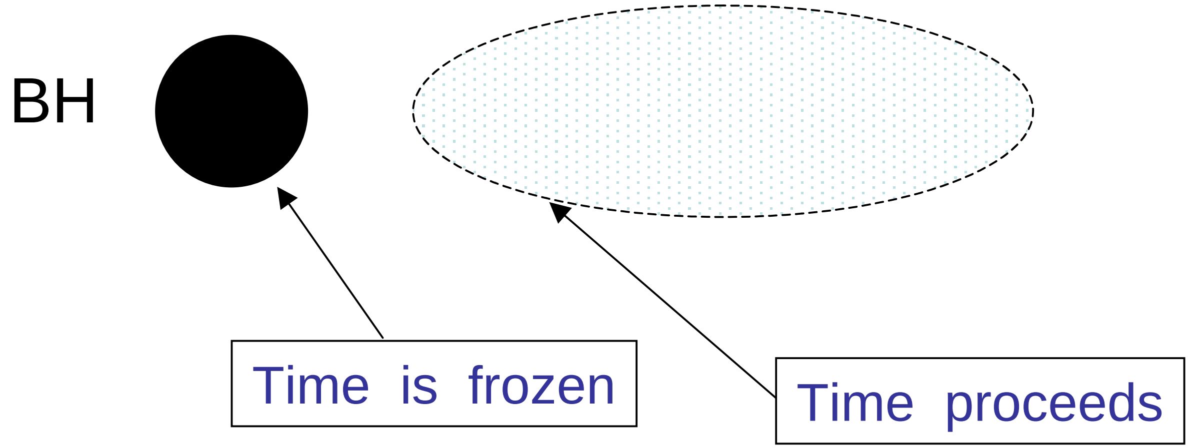
Lapse
at r = 0

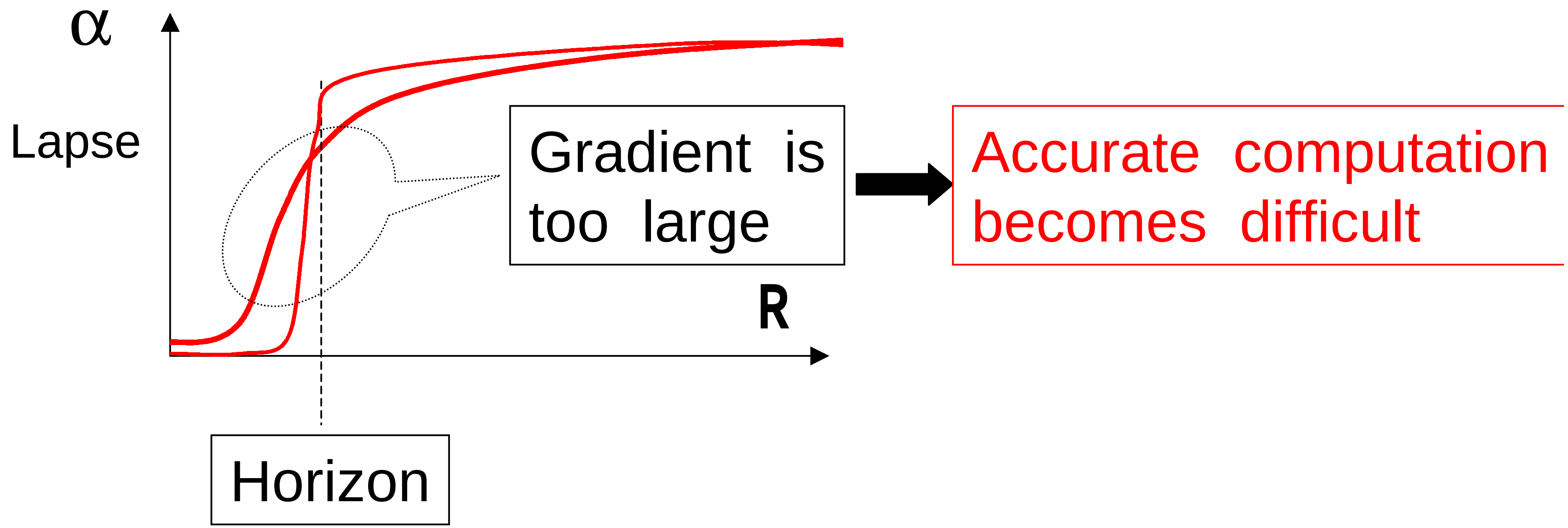


At t = 0,
T/W = 0.009
ρ (r=0) = 1.e10
M = 1.49 Solar
J/M^2 = 1.14

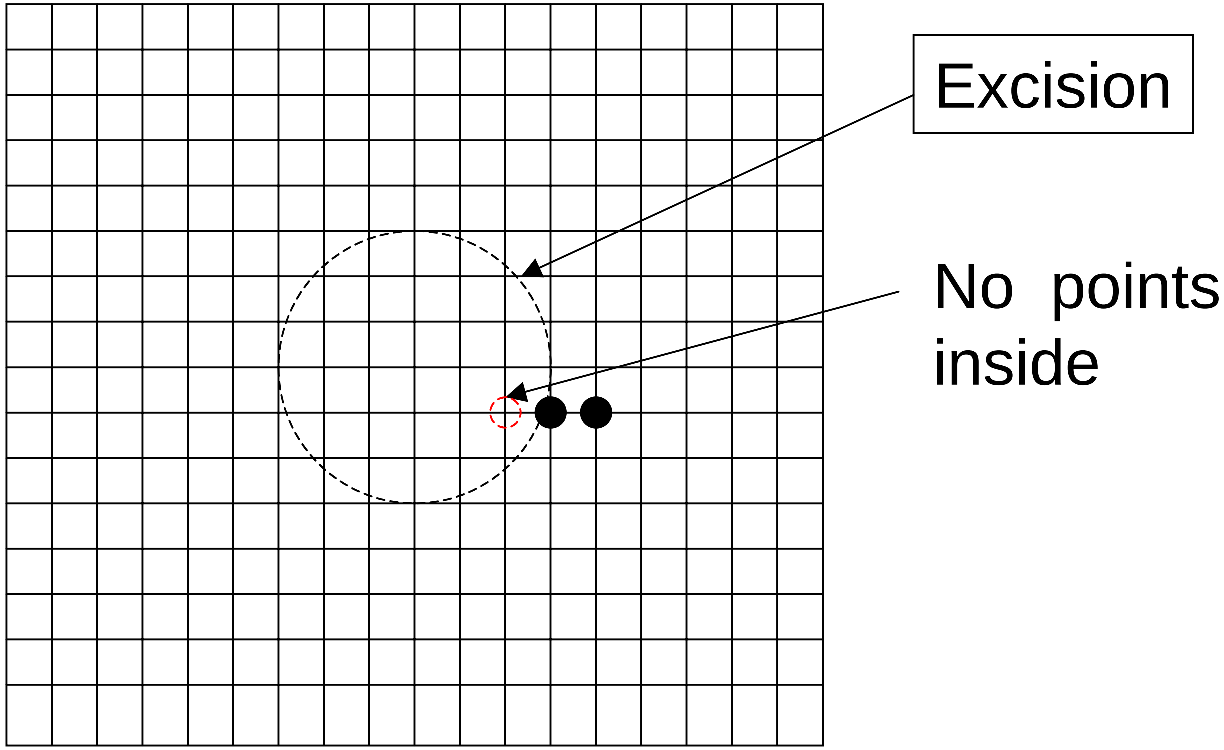
Animation
is started here.

Unsolved Issue : Handling BHs





A solution = Excision (Unruh)

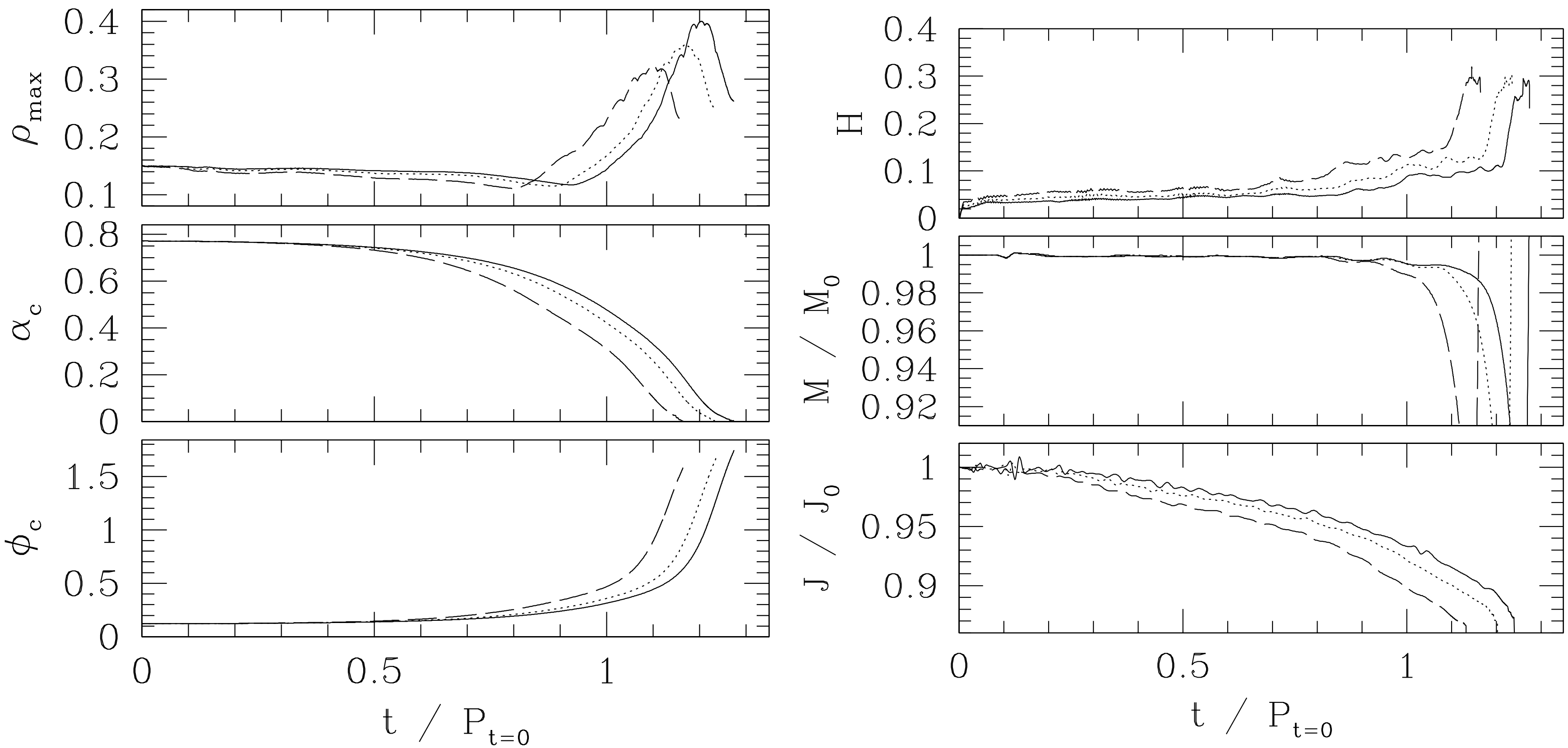


Appropriate formulation, gauge, boundary conditions?

-- 1BH → OK (Cornell, Potsdam, Illinois…)

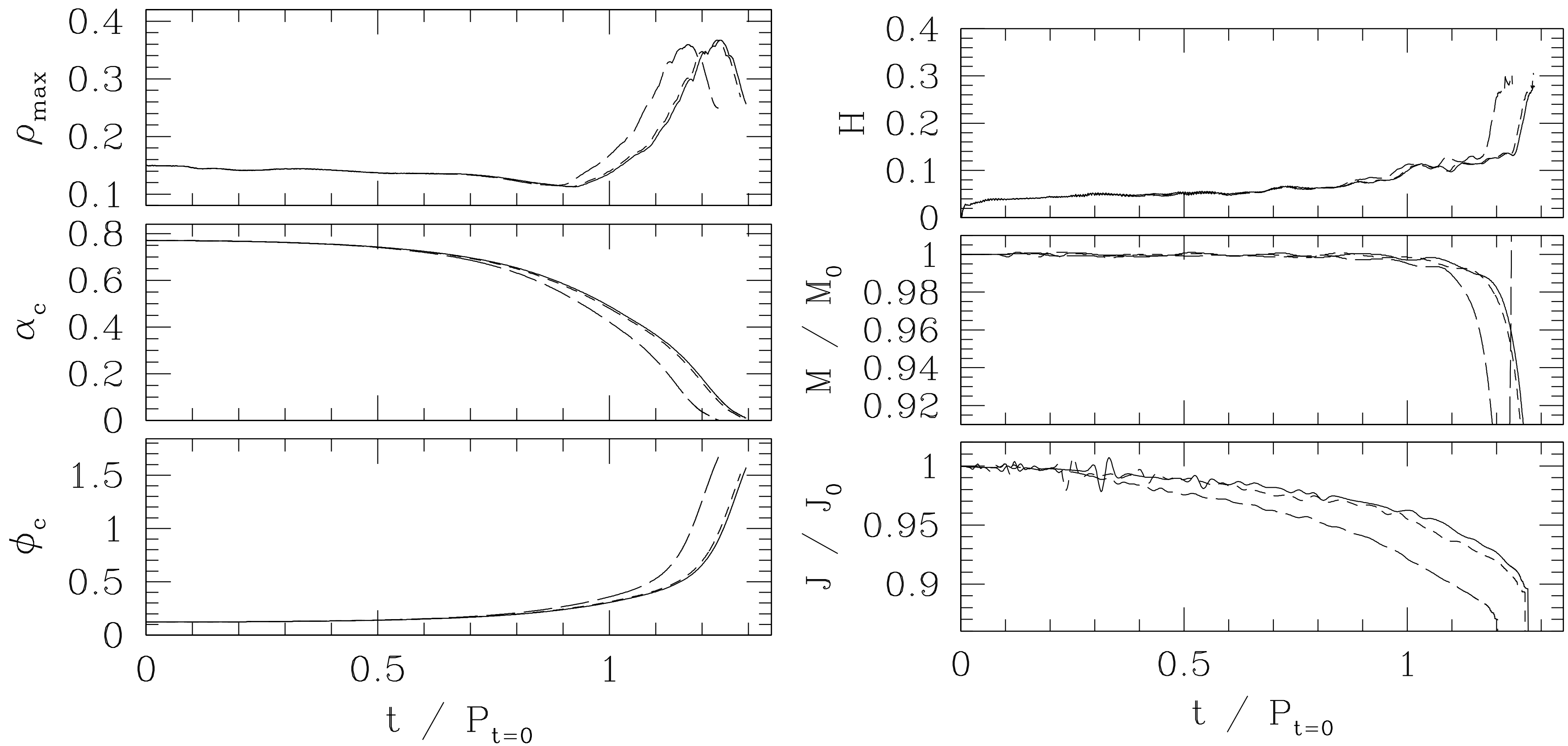
-- 2BH → No success for a longterm simulation

Convergence wrt grid spacing



BH formation case

Convergence wrt outer bondaries



BH formation case