Gravitational Radiation Reaction and LISA

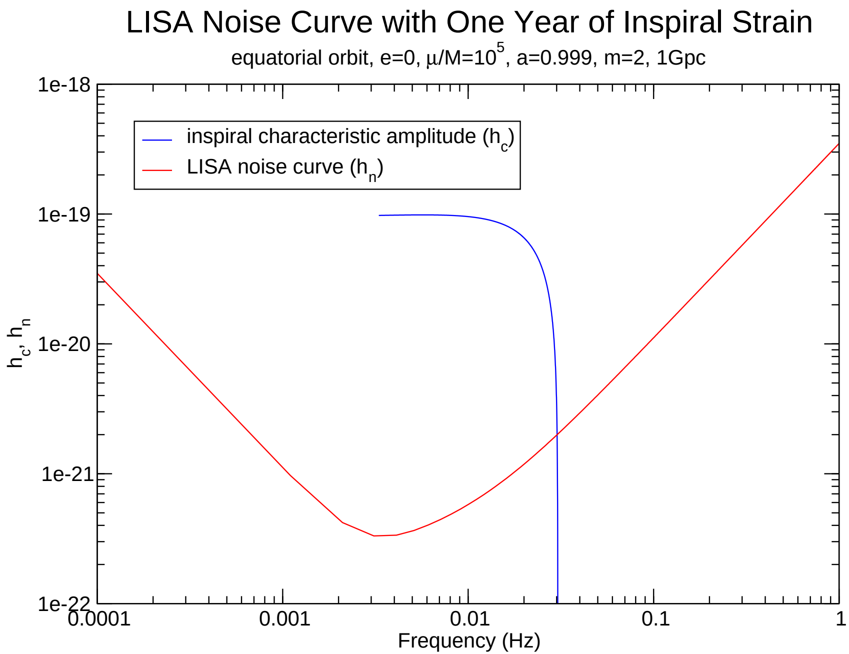
Warren G. Anderson
The University of Texas at Brownsville

Radiation Reaction Workshop,
Yukawa Institute for Theoretical Physics,
Kyoto, Japan, May 30, 2003

Outline

- LISA noise and the need for matched filtering
- The mechanics of matched filtering
- The importance of “Getting it Right”
- Some thoughts on what needs to be done

LISA Noise



Approximate reproduction from Finn and Thorne, PRD 62, 124021 (2000) . Signal to noise ratio for this mode is ~ 23. Close to minimum SNR detectable by matched filtering (Barack and Cutler).

Signal and Noise Vectors

- LISA strain information $h(t)$ is sampled at finite times $h_1 = h(t_1)$, $h_2 = h(t_2), \ldots, h_N = h(t_N)$.

- We can think of the ordered set $\mathbf{h} = (h_1, h_2, \ldots, h_N)$ as a vector in an N-dimensional vector space.

- When no signal is detected by LISA, the strain is entirely from noise $\mathbf{h} = \mathbf{n}$.

- When LISA does detect a signal, the displacement is a sum of the noise displacement and some amount of signal displacement $\mathbf{h} = \mathbf{n} + \mathbf{s}$.

Properties of the Noise

We assume the following properties for LISA noise:

- Each component n_j of the noise vector is drawn from a Gaussian distribution with zero mean: $\langle n_j \rangle = 0$.
- All strain scales are renormalized so the each component n_j has unit variance: $\langle n_j^2 \rangle = 1$.
- Any two components n_j and n_k are uncorrelated: $\langle n_j n_k \rangle = \langle n_j \rangle \langle n_k \rangle$

Properties of the Noise (cont)

As a result, we have the following identities:

- The expected length squared of a noise vector is

$$\langle \mathbf{n}\cdot\mathbf{n}\rangle = \langle \Sigma_{j=1}^{N} n_j^2 \rangle = \Sigma_{j=1}^{N} \langle n_j^2 \rangle = \Sigma_{j=1}^{N} 1 = N$$

- The expected inner product between an arbitrary noise vector $\mathbf{n}$ and any arbitrary uncorrelated vector $\mathbf{t}$ is

$$\langle \mathbf{n}\cdot\mathbf{t}\rangle = \Sigma_{i=1}^{N} \langle n_i \rangle \langle t_i \rangle = 0.$$

- The variance of the inner product between $\mathbf{n}$ and $\mathbf{t}$ is

$$\begin{aligned} \sigma^2(\mathbf{n}\cdot\mathbf{t}) &= \langle (\mathbf{n}\cdot\mathbf{t})^2 \rangle - \langle \mathbf{n}\cdot\mathbf{t}\rangle^2 \\ &= \Sigma_{j=1}^{N} \Sigma_{i\neq j} \langle n_i \rangle \langle n_j \rangle \langle t_i t_j \rangle + \Sigma_{i=1}^{N} \langle n_i^2 \rangle \langle t_i^2 \rangle \\ &= \langle \mathbf{t}\cdot\mathbf{t}\rangle. \end{aligned}$$

Matched Filtering

- In linear algebra, one uses the dot product of two vectors $\mathbf{a}$ and $\mathbf{b}$, $\mathbf{a}\cdot\mathbf{b} := a_1 b_1 + a_2 b_2 + \ldots + a_N b_N$ to determine how much of vector $\mathbf{a}$ is contained in vector $\mathbf{b}$.
- We therefore create a *template vector* , $\mathbf{t}$, of the signal we seek, normalized so that $\mathbf{t}\cdot\mathbf{t} = 1$.
- For a signal to be detectable in the data, we need $\mathbf{t}\cdot\mathbf{h} \gg \sigma(\mathbf{t}\cdot\mathbf{n})$.
- We define the matched filter signal-to-noise ratio to be

$$SNR := (\mathbf{t}\cdot\mathbf{h})/\sigma(\mathbf{t}\cdot\mathbf{n}) = \mathbf{t}\cdot(\mathbf{n}+\mathbf{s})/\sqrt{\mathbf{t}\cdot\mathbf{t}}.$$

- The expected value of the SNR is therefore

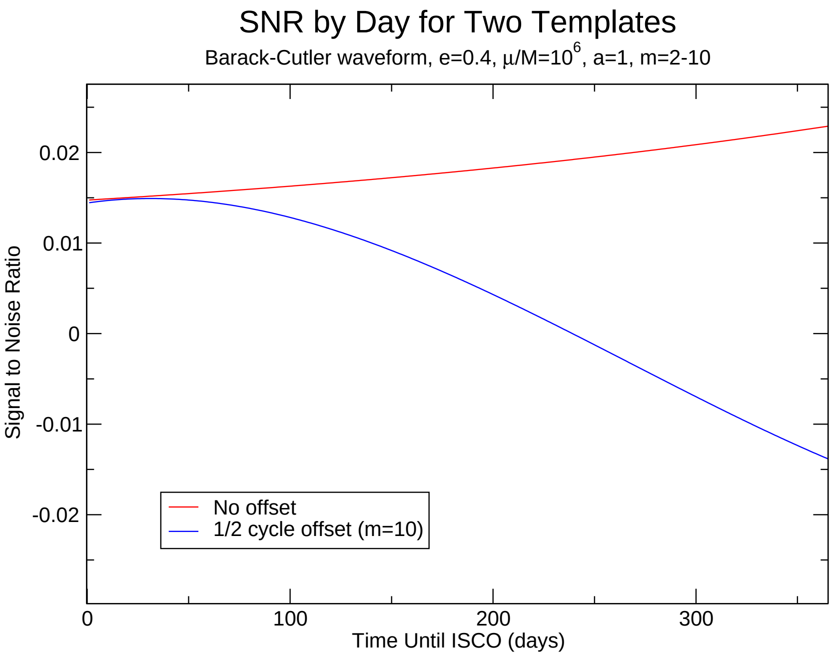
$$\langle \text{SNR} \rangle = (\langle \mathbf{t}\cdot\mathbf{n} \rangle + \mathbf{t}\cdot\mathbf{s})\,/\sqrt{\mathbf{t}\cdot\mathbf{t}} = (\mathbf{t}\cdot\mathbf{s})/\sqrt{\mathbf{t}\cdot\mathbf{t}}.$$

- If we get the template perfect, then $\mathbf{s} = \alpha\mathbf{t}$, and

$$\langle \text{SNR} \rangle >= \alpha(\mathbf{t}\cdot\mathbf{t})/\sqrt{\mathbf{t}\cdot\mathbf{t}} = \alpha$$

or in other words, we need an $\text{SNR} \sim 10$.

SNR Examples



Note that the SNR for the mismatch becomes negative at late times, thus cancelling some of the signal-to-noise we had developed in the earlier section.

We Need You!

Ideally, we need high precision waveforms including all effects. However, if these are not available, we might be able to get by with approximate waveforms. Some investigations that might be needed:

- Can we calculate highly accurate radiation reaction waveforms (Mino) ?
- At what time (frequency) might conservative terms start to contribute significantly? Can we compare approximate calculations with conservative terms to radiation reaction waveforms?
- If conservative terms will be important, is there some way to obtain them separately?
- I have been addressing detection. Parameter estimation generally requires even more accurate waveforms.
- Communication between gravitational theorists and data analysts will be important.

Cautionary Addendum

In order to make this treatment more accessible for non-experts, I have “swept some things under the rug”. Here are some warnings for those who might want to take this treatment too seriously:

- In the treatment of matched filtering, I have implicitly assumed that the noise is white, that is, that the noise has the same power at all frequencies. LISA noise is not white, as we see in the first graph. However, I can use the formulae for white noise providing I weight the different frequency contributions to the templates appropriately. Thus, the template would not be a pure waveform (however, if you give me a waveform, I can easily construct a template).

- For the second graph, the results are a bit misleading. The current strategy is not to do a single one year matched filter, but to break up the signal into different portions. This will allow one to match different portions with templates from slightly different systems, thereby saving some of the signal-to-noise ratio.

- Even if one were to do a single template, what I have done is not exactly what would be done, because I have not maximized over phase (slid the template along in time to find the maximum SNR). This would likely put the maximum SNR somewhere further from the beginning. However, ultimately the results would be qualitatively the same.