Consequences of Analyticity of the Kerr-Schild Geometry from Black Holes to Spinning Particles.

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Kerr-Schild Geometry, and Conformal Field Theory

Black Hole Evaporation ⇔ Conformal Field Theory ⇔ Superstrings:

[1] A. Asthekar, V. Taveras and M. Varadarajan, arXiv:0801.181 [2] A. Strominger, JHEP **02** 009 (1998). [3] S. Carlip, Phys. Rev. Lett. **82**, 2828 (1999). [4] O. Coussaert and M. Henneaux, Phys. Rev. Lett. **72**, 183 (1994). [5] M. Banados, Phys. Rev. Lett. **82**, 2030 (1999). [6] J.A. Cardy, Nucl. Phys **B270**, 186 (1986).

Superstring Theory ⇔ Conformal Field Theory:

[1] V.G. Knizhnik and A.B. Zamolodchikov, Nucl. Phys. **B247**, 83 (1984). [2] A.A. Belavin, A.M. Polyakov and A.B. Zamolodchikov, Nucl. Phys. **B241**, 333 (1984).

Kerr-Schild Geometry ⇔ Conformal Field Theory ⇔ Superstrings:

[1]G.C. Debney, R.P. Kerr and A.Schild, J. Math. Phys. **10**, 1842 (1969). [2] A.B., *Twistorial Analyticity and Three Stringy Structures of the Kerr Spinning Particle.* Phys. Rev. D **70**, 086006 (2004). [3] A.B., Grav. & Cosmol. **11**, 301 (2005); [4] A.B., Int. J. Geom. Methods Mod. Phys. **4**, 437 (2007); [5] A.B., Phys. Rev. D **70**, 086006 (2004); [6] A.B., *String-like Structures in Complex Kerr Geometry.* (gr-qc/9303003,) In: "Relativity Today", Ed. by R.P.Kerr and Z.Perjés,1994, p.149.

Kerr-Schild Formalism. Debney, Kerr and Schild, J. Math. Phys. **10** (1969) 1842

Complex Projective Angular Coordinate:

$$Y = e^{i\phi} \tan\frac{\theta}{2} \in CP^1$$

The Kerr-Schild ansatz for metric,

$$g_{\mu\nu} = \eta_{\mu\nu} + 2he^3_\mu e^3_\nu, \quad \sqrt{-g} = 1.$$

$\eta_{\mu\nu}$ - auxiliary Minkowski space-time.

Principal Null Direction

$$e^3 = du + \bar{Y} d\zeta + Y d\bar{\zeta} - Y\bar{Y} dv$$

in the null Cartesian coordinates $\sqrt{2}\zeta = x + iy$, $\sqrt{2}\bar{\zeta} = x - iy$, $\sqrt{2}u = z - t$, $\sqrt{2}v = z + t$.

Null tetrad $e^a, a = 1, 2, 3, 4$. Real directions e^3 and $e^4 = dv + he^3$, and two complex conjugate directions

$$e^1 = d\zeta - Y dv, \quad e^2 = d\bar{\zeta} - \bar{Y} dv.$$

The Kerr Theorem. The geodesic and shear-free (GSF) null congruences ($Y_{,2} = Y_{,4} = 0.$) are determined by function $Y(x)$ which is a solution of the equation $F(Y, l_1, l_2) = 0$, where F is an arbitrary analytic function of the projective twistor coordinates

$$Y, \quad l_1 = \zeta - Yv, \quad l_2 = u + Y\bar{\zeta}$$

Integration of the Einstein-Maxwell field equations for the geodesic and shear-free congruences was fulfilled in DKS, leading to the following form of the function

$$h = \frac{1}{2} M(Z + \bar{Z}) - \frac{1}{2} A\bar{A}Z\bar{Z} \ , \qquad (1)$$

of the Kerr-Schild ansatz:

$$g_{\mu\nu} = \eta_{\mu\nu} + 2h e^3_\mu e^3_\nu. \qquad (2)$$

Necessary functions $Z = P/\tilde{r}$, $Y(x)$ and parameters are determined by the generating function F.

$$PZ^{-1} = \tilde{r} = - \quad dF/dY \qquad (3)$$

is complex radial distance, factor P is connected with Killing vector or the boost of the source.

There was obtained a system of differential equations for functions A, and M.

Electromagnetic sector:

$$A,_2 - 2Z^{-1}\bar{Z}Y,_3 A = 0, \quad A,_4 = 0, \tag{4}$$

$$\mathcal{D}A + \bar{Z}^{-1}\gamma,_2 - Z^{-1}Y,_3 \gamma = 0, \quad \gamma,_4 = 0, \tag{5}$$

where $\mathcal{D} = \partial_3 - Z^{-1}Y,_3 \partial_1 - \bar{Z}^{-1}\bar{Y},_3 \partial_2$.

The strength tensor of self-dual electromagnetic field is given by the tetrad components

$$\mathcal{F}_{12} = AZ^2, \quad \mathcal{F}_{31} = \gamma Z - (AZ),_1 \ . \tag{6}$$

Gravitational sector:

Real function M.

$$M,_2 - 3Z^{-1}\bar{Z}Y,_3 M = A\bar{\gamma}\bar{Z}, \tag{7}$$

$$\mathcal{D}M = \frac{1}{2}\gamma\bar{\gamma}, \quad M,_4 = 0. \tag{8}$$

Integration: For any holomorphic $F(Y) \Rightarrow$ GSF congruence $\Rightarrow$ a family of algebraically special solutions.

The Kerr-Newman solution is a very important particular case:

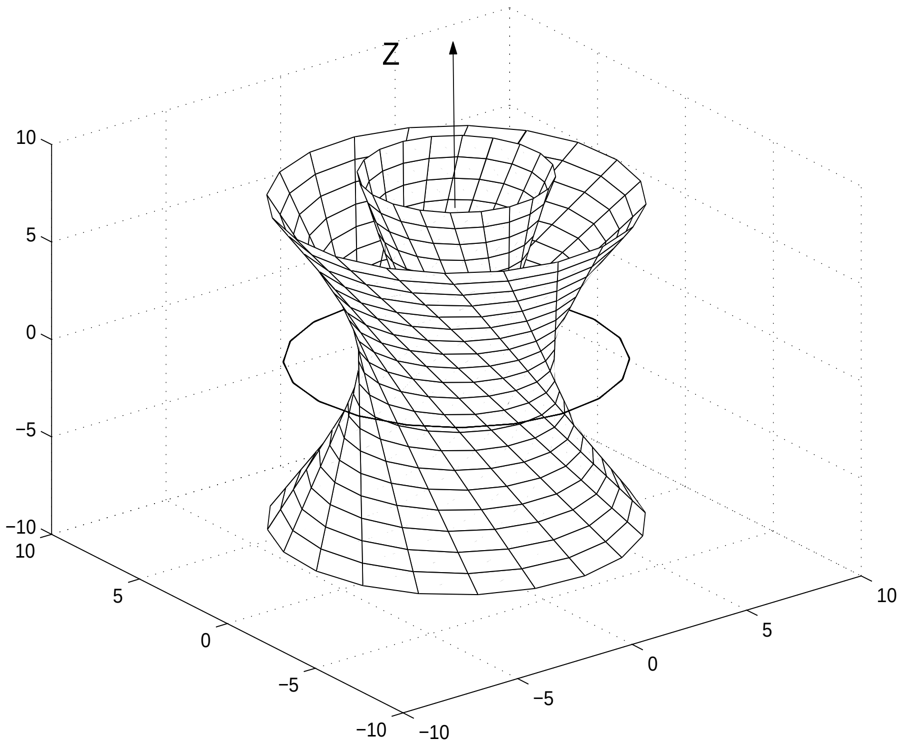
Metric $g_{\mu\nu} = \eta_{\mu\nu} + 2hk_\mu k_\nu$, where $h = \frac{mr - e^2/2}{r^2 + a^2 \cos^2\theta}$.

Electromagnetic field $A_\mu = \frac{er}{r^2 + a^2 \cos^2\theta} k_\mu$.

Kerr congruence $k_\mu(x) = e^3_\mu(Y)\sqrt{2}/(1 + Y\bar{Y})$ which is determined by function $Y(x)$ (solution of the eq. $F(Y, x) = 0$.

Singular ring at $r + ia\cos\theta \equiv \partial_Y F = 0$.

The Kerr congruence is vortex of null lines (twistors)



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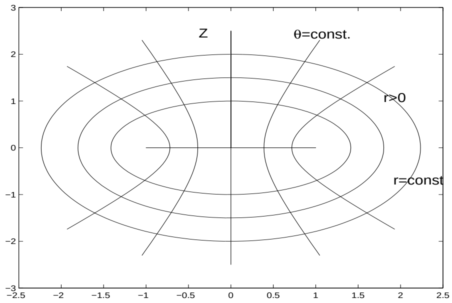
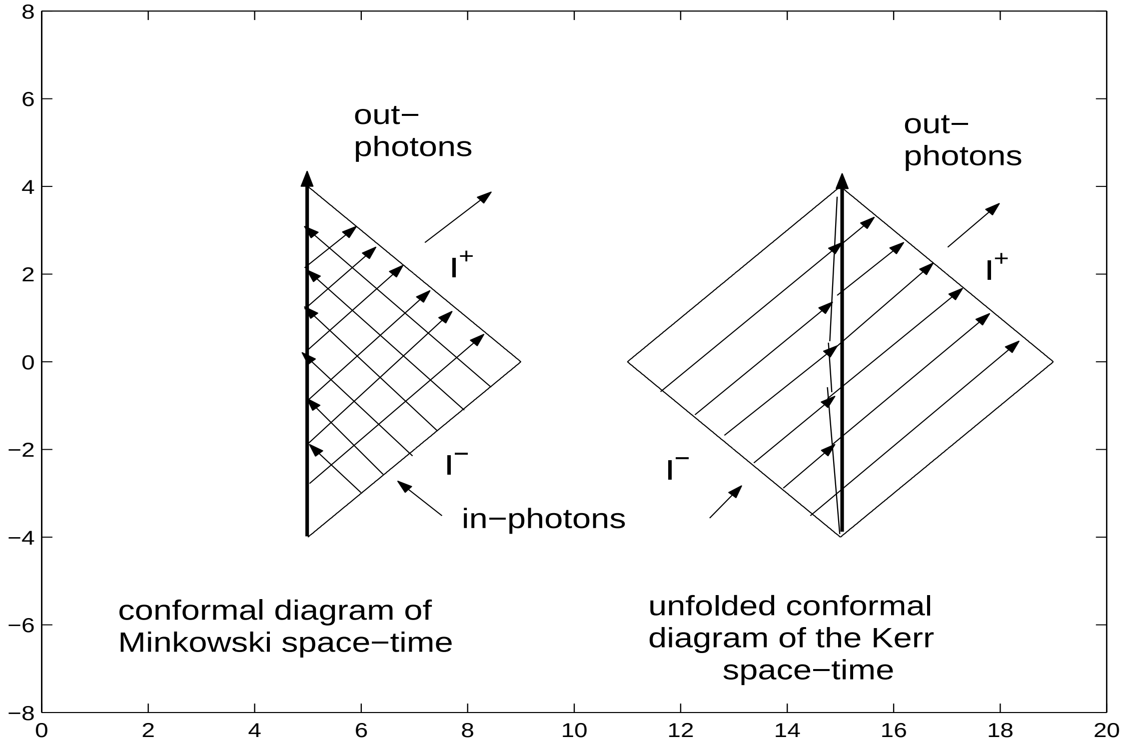
The Kerr singular ring and the Kerr congruence.

The Kerr singular ring is a branch line of space on two sheets: "negative" and "positive" where the fields change their signs and directions. Congruence covers the space-time twice. **Problem of the Kerr source** ⇒ twosheetedness !!!



Twosheeted oblate coordinate system.



Twosheetedness of the Kerr space-time. The 'in' and 'out' electromagnetic fields are positioned on different sheets, they are **aligned to Kerr congruence** and don't interact with each other.

Electromagnetic field is to be aligned with the Kerr congruence:

$$F^{\mu\nu} k_\mu = 0.$$

Kerr -Schild form of metric $g^{\mu\nu} = \eta^{\mu\nu} - 2hk^\mu k^\nu$, where

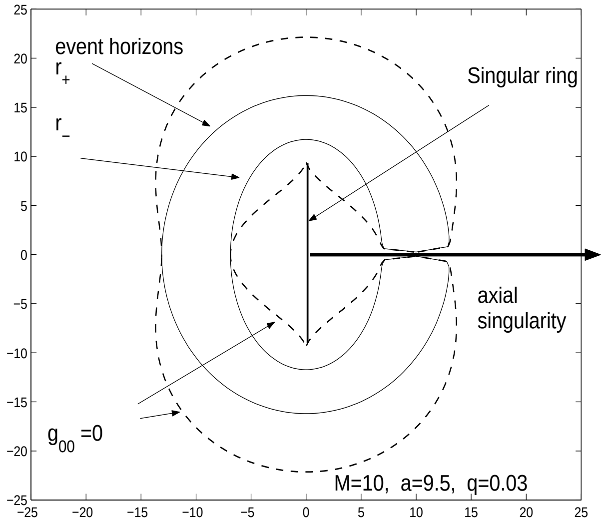
$$h = 2\frac{mr - |\psi|^2}{r^2 + a^2 \cos^2\theta}$$

General stationary solutions, $\psi(Y)$ is arbitrary holomorphic function of Y.

There is an infinite set of the *exact solutions !*, in which function $\psi(Y)$ is holomorphic, but singular at the set of points $\{Y_i, i = 1, 2, ...\}$, $\psi(Y) = \sum_i \frac{q_i}{Y(x) - Y_i}$. These solutions have a set of lightlike beams in the corresponding angular directions ϕ_i, θ_i, (singular pp-strings) along corresponding rays of the Kerr congruence.

How act such beams on the BH horizon? Singular beams lead to formation of the holes in the black hole horizon, which opens up the interior of the "black hole" to external space.

Black holes with holes in the horizon A.B., E.Elizalde, S.R.Hildebrandt and G.Magli, Phys. Rev. **D74** (2006) 021502(R)



Singular beam forms a small hole in the horizon.

The boundaries of **ergosphere** (punctured surfaces) are determined by $g_{00} = 0$. The **event horizons**,r_+ and r_-, are two null surfaces. These surfaces turn out to be joined by a tunnel, forming a simply connected surfaces.

Wave excitations propagating in the direction Y_i are described by means of the function

$$\psi_i(Y,\tau) = q_i(\tau)\,\mathrm{exp}\{i\omega\tau\}\frac{1}{Y - Y_i}, \tag{9}$$

where the extra dependence is on the retarded time τ. The right hand sides of the gravitational KS equations are proportional to γ and $\gamma\bar{\gamma}$ and are quite small for low-frequency aligned wave excitations, since the function γ will be of the order: $\gamma \sim \dot{\psi} \sim i\omega\psi$.

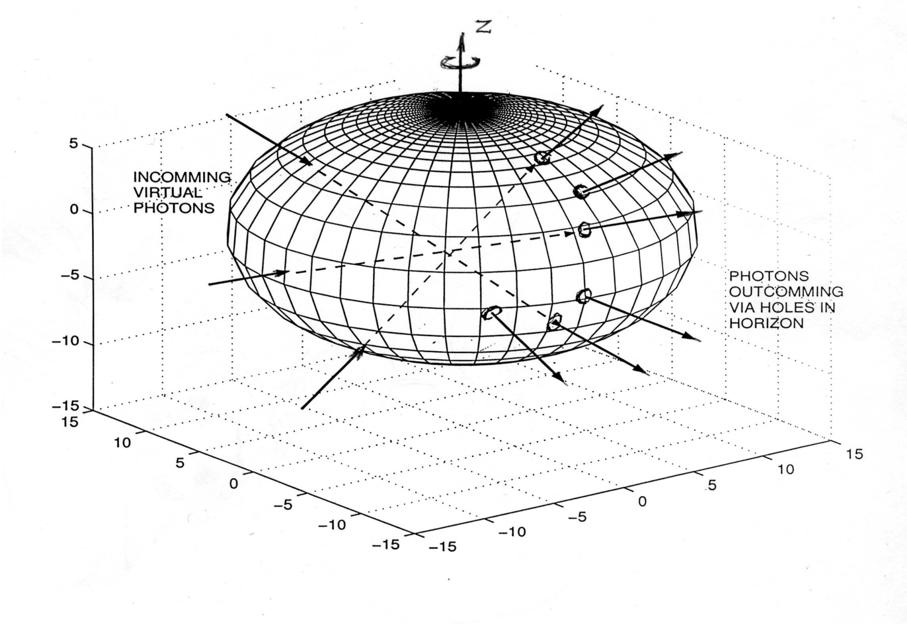
Electromagnetic field deforms horizon! Beams perforate horizon!

$$H = \{mr - \psi(Y)^2/2\}/(r^2 + a^2\cos^2\theta).$$

Solutions tend in the low-frequency limit to exact stationary Kerr-Schild solutions with singular beams.

Beam-like basis functions.

Plane waves don't exist on the Kerr's twosheeted background. Twistor-string conjecture of scattering by Nair and Witten (2003) leads to beam-like basis wave functions for ingoing vacuum photons which are supported on twistor lines. Basis $|in>$ - state is a wave function on twistor space $Z^A \in Tw = CP^3$. Amplitude of scattering and stress-energy tensor $< T^{\mu\nu} >_{vac} = \sum_i T^{\mu\nu}(\phi_i)$ are determined by the system of Kerr-Schild equations via the vacuum in-fields γ_{in} related with the basis primary holomorphic field $\phi(Y) = \sum_i \phi_i(Y) = \sum_i \frac{q_i}{Y - Y_i}$. The both fields contain beams performing horizon.



Algebraically special solutions $\Rightarrow$ beam-like basis functions $\Rightarrow$ Horizon is not irresistible with respect to outgoing beams!

Twistorial structure of interaction.

The beams propagate along the null *twistor* lines of the Kerr principal null congruence.

The case of quadratic Kerr's generating functions F(Y) is well studied. A.B. and G. Magli, Phys.Rev.**D 61**044017 (2000).

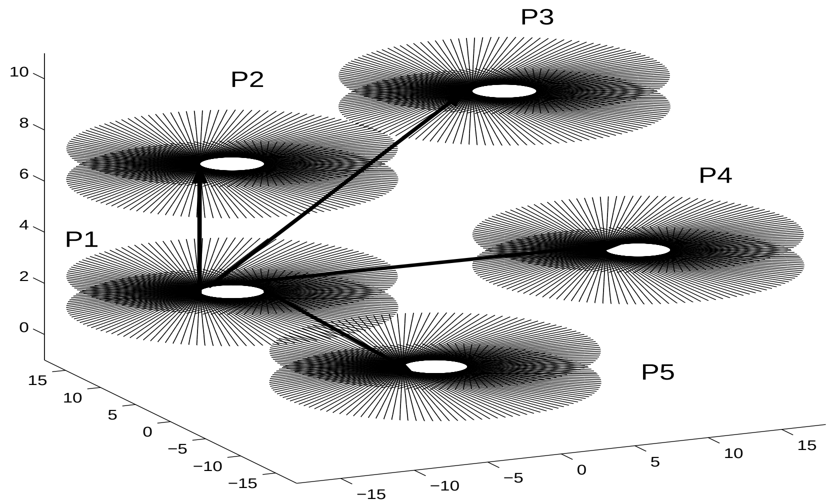
$$F = AY^2 + BY + C, \Rightarrow$$

two solutions of the equation $F = 0$ for the function $Y(x) \Rightarrow$ twosheetedness of the Kerr geometry.

The functions $F(Y)$ of higher degrees are formed as a product of quadratic blocks corresponding to n different particles

$$F(Y) = \prod_i^n F_i(Y).$$

The particles i and j are positioned on different Riemannian sheets of the function $F(Y)$ and interact with each other only via a *common* twistor line of the i-th and j-th congruences, by forming a singular null string connecting these particles.



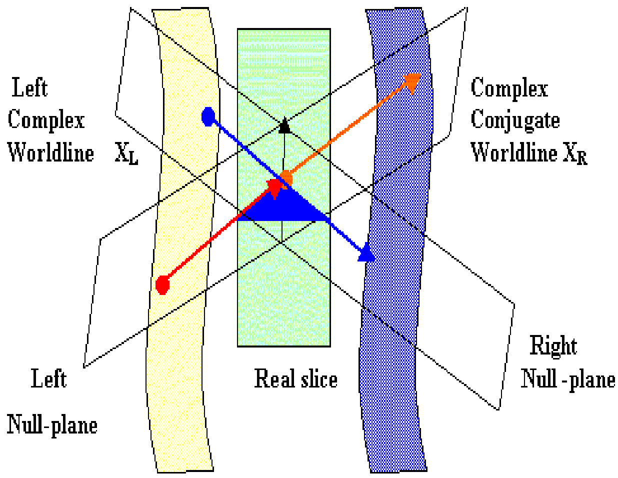
Interaction of multiparticle solutions.

Rimannian multi-sheeted structure of space-time. What can be observable? Only correlation functions!

Conformal field theory as a starting point for 2D Quantum Gravity. A.B.Zamolodchikov and Al.B.Zamolodchikov (1995), J.Teshner (2003). Conformal Bootstrap. Multipoint correlation functions as observable for 2D Quantum Gravity.

4D Kerr-Schild Gravity $\Rightarrow$ $S^2 \oplus (1+1)$.

2D Conformal field theory on $S^2 = G(Y) \otimes \bar{G}(Y)$ $\Rightarrow$ Twistorial Complex Null Planes $\Rightarrow$ a way to 4D Quantum Gravity!



Left and Right complex null planes.