Mass metrology and the International System of Units

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90 s about the BIPM

- Organized under terms of the Metre Convention (1875).
- Supported by 51 Member States and 28 Associates.
- Carries out a scientific programme overseen by the International Committee for Weights and Measures (CIPM).
- Scientific Sections include Mass, Electricity, Time/Frequency/Gravimetry, Chemistry, Ionizing Radiation.
- Publishes the *SI Brochure*, produced by the Consultative Committee for Units (CCU) of the CIPM.
- Changes to the SI units are recommended by the CCU in cooperation with other relevant technical CCs, for ultimate approval by the General Conference on Weights and Measures (CGPM), which is at the diplomatic level and meets every 4 years [next in 2011].

Topics to be presented

- Reminder of the SI units and quantities.
- Why in particular the kilogram must be redefined.
- The constraints on a new definition.
- The relation between the SI and the fundamental constants of physics (as listed by CODATA); the role of the fine-structure constant.
- When could/should the kilogram be redefined?

In an ideal world, units would be defined by fundamental constants

"Yet, after all, the dimensions of our earth and its time of rotation, though, relative to our present means of comparison, very permanent, are not so by physical necessity. The earth might contract by cooling, or it might be enlarged by a layer of meteorites falling on it, or its rate of revolution might slowly slacken, and yet it would continue to be as much a planet as before.

But a molecule, say of hydrogen, if either its mass or its time of vibration were to be altered in the least, would no longer be a molecule of hydrogen.

If, then, we wish to obtain standards of length, time and mass which shall be absolutely permanent, we must seek them not in the dimensions, or the motion, or the mass of our planet, but in the wavelength, the period of vibration, and the absolute mass of these imperishable and unalterable and perfectly similar molecules."

James Clerk Maxwell, 1870

6/7 of the SI base units at present

- second (s): defined by fixing value of ν_{hfs} , the hyperfine-transition frequency of the caesium atom. [atomic clock]
- metre (m): defined by fixing value of speed of light, c, with additional reference to s. [laser interferometry]
- kilogram (kg): defined by assigning 1 kg to the mass of the international prototype of the kilogram. [artifact definition!]
- ampere (A): defined by fixing the value of μ_0 , the magnetic constant, with additional reference to kg, m, s
- kelvin (K): defined by assigning a value of 273.16 K to the t_{tpw}. [technology is in place to redefine in terms of a fixed value for k_B.]
- mole (mol): defined by the number of ^{12}C atoms in 12 g. This constant, N_A , cannot be fixed without redefining the kilogram.

Definition of the kilogram

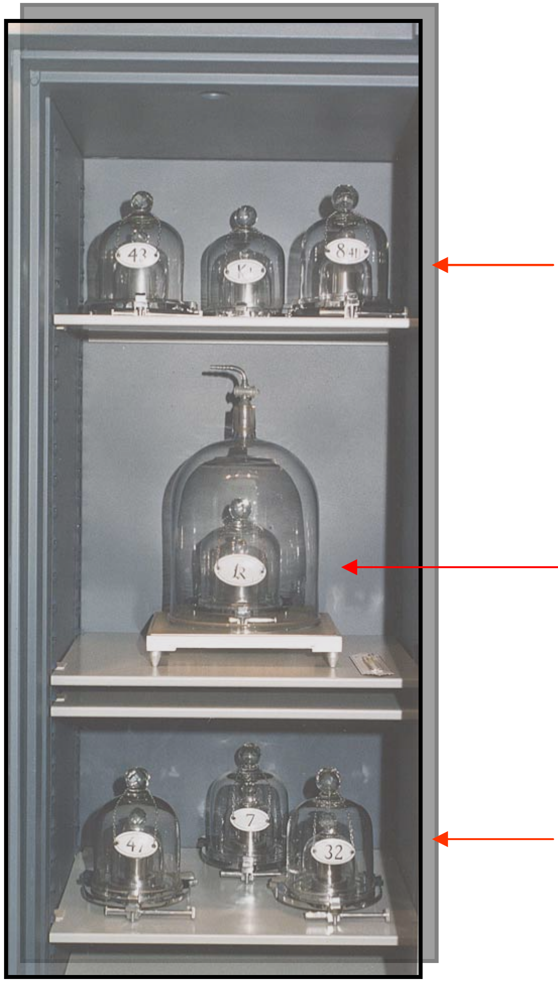
3rd CGPM, 1901 :

"Le kilogramme est l'unité de masse;
il est égal à la masse du prototype international du kilogramme."

"The kilogram is the unit of mass;
it is equal to the mass of the international prototype of the kilogram."

(international prototype manufactured in 1880s, put into service in 1889)

Conservation of the international prototype



official copies (3 of 6)

international prototype

official copies (3 of 6)

1927

from ***La Création du BIPM et son oeuvre***

«Il semble donc que l’unité de masse soit guarantie au cent-millionième près pour plus de 10 000 ans, et cette durée est à peine commencée.

Sans doute, bien avant qu’elle soit écoulée, les travaux exécutés par les métrologistes des siècles futurs auront conduit à des solutions encore plus parfaites.»

Ch.-Ed. Guillaume

BIPM Director
Nobel laureate

What the definition means in practice

The mass in kilograms of any object X is given by :

$$\{m_X\}[\mathrm{kg}] = \left\{\frac{m_X}{m_n}\right\}\cdot\left\{\frac{m_n}{m_{n-1}}\right\}\cdot\ldots\cdot\left\{\frac{m_2}{m_1}\right\}\cdot\left\{\frac{m_1}{m_{\mathcal{K}}}\right\}[\mathrm{kg}]$$

$$\{m_X\}[\mathrm{kg}] = \left\{\frac{m_X}{m_{\mathcal{K}}}\right\}\cdot[\mathrm{kg}]$$

For highest accuracy, $\{\boldsymbol{m_X}/\boldsymbol{m_{\mathcal{K}}}\}$ is a measurement carried out on a precision balance (mass comparator):

$$m_X - m_{\mathcal{K}} = \varepsilon \quad ; \quad \frac{m_X}{m_{\mathcal{K}}} = 1 + \frac{\varepsilon}{m_{\mathcal{K}}}$$

$$\varepsilon << m_{\mathcal{K}}$$

Consequences of 1901 definition and 2008 physics

Even though m_{X} may represent something fundamental--
m_{e}, m_{p}, $m(^{12}\mathrm{C})$, $(\hbar c/G)^{1/2}$ etc., nevertheless

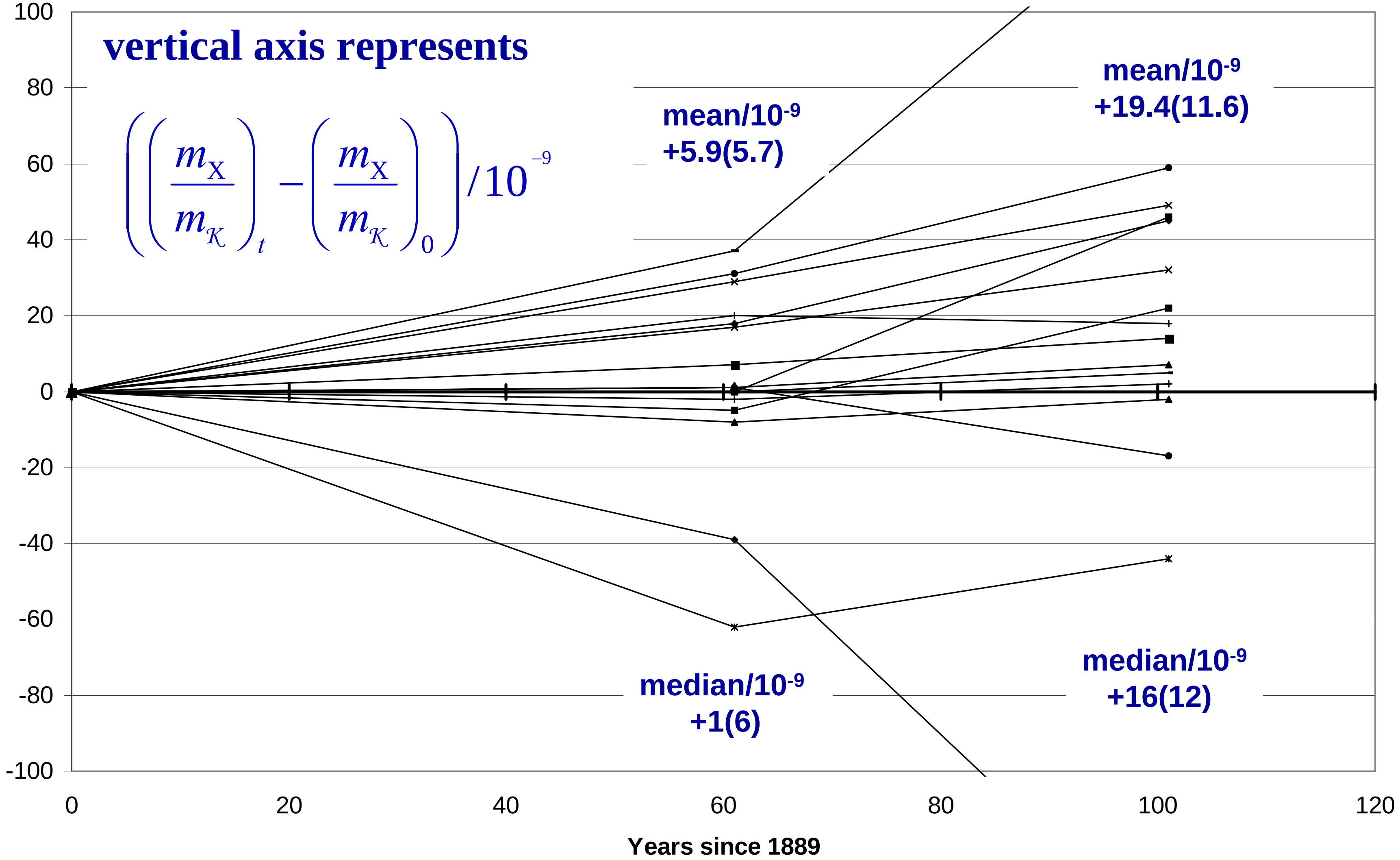
$$\{m_{\mathrm{X}}\}[\mathrm{kg}] = \left\{\frac{m_{\mathrm{X}}}{m_{\mathcal{K}}}\right\}[\mathrm{kg}]$$

This curious situation persists largely because:

Experimental uncertainties of $\{m_{\mathrm{X}}/m_{\mathcal{K}}\}$ are still much larger than the precision of the best commercial balances. The uncertainty is due to the incredible mismatch between 1 kg and, say, m_{e}.

When comparing molecular, atomic, and subatomic masses amongst themselves, it is traditional to use the dalton, Da (also called the atomic mass unit), a non-SI unit which avoids correlations to $m_{\mathcal{K}}$.

Ensemble average of $\mathcal{K}$ and the oldest national prototypes



vertical axis represents

$$\left(\left(\frac{m_{X}}{m_{\mathcal{K}}}\right)_t - \left(\frac{m_{X}}{m_{\mathcal{K}}}\right)_0\right)/10^{-9}$$

mean/10^{-9}
+5.9(5.7)

mean/10^{-9}
+19.4(11.6)

median/10^{-9}
+1(6)

median/10^{-9}
+16(12)

A closer look at the ampere

“The **ampere** is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 metre apart in vacuum, would produce between these conductors a force equal to 2 x 10^{-7} **newton** per metre of length.”

The definition ensures the consistency of SI units. For instance, thanks to this definition, the SI unit “watt” is the same for

$I^2 \cdot R$ (electrical power) and $m \cdot a \cdot v$ (mechanical power)

The ampere definition implicitly fixes the permeability of vacuum, μ_0, (sometimes called the magnetic constant)

$$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{N/A^2} \quad (1\ \mathrm{N/A^2} = 1\ \mathrm{H/m})$$

Why electrical metrologists care about the kilogram definition

Present voltage and resistance metrology rely on quantum-mechanical phenomena and two 'fundamental' constants of physics.

Josephson constant, K_J ,
and
von Klitzing constant, R_K .

According to current knowledge, $K_J = 2e/h$ and $R_K = h/e^2$.

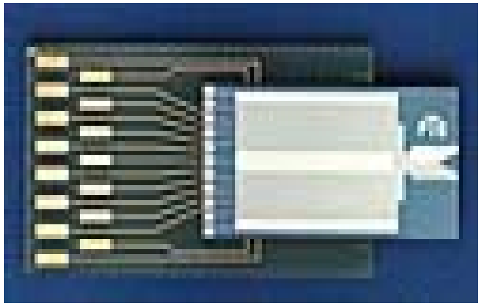
Therefore, fixing the values of *h* and of *e*

- **fixes the value of the Josephson constant,**
- **fixes the value of the von Klitzing constant,**

thereby eliminating the need for conventional (non SI) values, $K_{J\text{-}90}$ and $R_{K\text{-}90}$ that are used today.

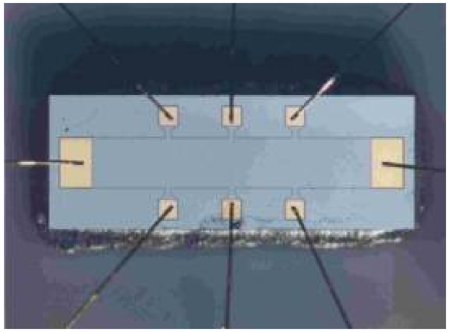
More on the importance of *h* and *e* to electrical metrology

Josephson effect



$$U_J(n) = \frac{n\,f}{K_J}, \qquad K_J = \frac{2e}{h}$$

quantized-Hall effect



$$R_H(i) = \frac{R_K}{i}, \qquad R_K = \frac{h}{e^2}$$

Conventional (non SI values)

$$\frac{h}{e^2} = \frac{\mu_0 c}{2\alpha}$$

$K_{J\text{-}90} \equiv 483\,597.9$ GHz/V

$R_{K\text{-}90} \equiv 25\,812.807\ \Omega$

But…

c **: speed of light in vacuum. This already has a fixed value in the SI.**

μ_0 already has a fixed value in the SI due to the definition of the Ampere

Therefore, it is impossible for *e* , *h* to be fixed as well:

$$\alpha = \frac{\mu_0 c e^2}{2h}$$

A choice must be made.

Many interesting proposals are “on the table”.

Why chemists care about the kilogram definition

“The **mole** is the amount of substance of a system which contains as many elementary entities as there are atoms in 0.012 **kilogram** of carbon 12; its symbol is ‘mol’.

“When the mole is used, the elementary entities must be specified and may be atoms, molecules, ions, electrons, other particles, or specified groups of such particles.”

This definition is closely linked to the Avogadro Constant

$$N_{\mathrm{A}} = \frac{0.012\left[\mathrm{kg\cdot mol^{-1}}\right]}{m\left({}^{12}\mathrm{C}\right)\left[\mathrm{kg}\right]} = \frac{\textit{molar mass of } x}{\textit{mass of } x}$$

The dalton (example)

$$\{m_{^{12}\mathrm{C}}\}[\mathrm{kg}] = \left\{\frac{m_{^{12}\mathrm{C}}}{m_{\mathcal{K}}}\right\}[\mathrm{kg}] \qquad = 12[\mathrm{u}]$$

$$\{m_\mathrm{e}\}[\mathrm{kg}] = \left\{\frac{m_\mathrm{e}}{m_{^{12}\mathrm{C}}}\right\}\left\{\frac{m_{^{12}\mathrm{C}}}{m_{\mathcal{K}}}\right\}[\mathrm{kg}] \qquad = 12\left\{\frac{m_\mathrm{e}}{m_{^{12}\mathrm{C}}}\right\}[\mathrm{u}]$$

From 2006 CODATA adjustment

quantity	relative uncertainty (ppm)
$m_{^{12}\mathrm{C}}$	0.05
m_e	0.05
$m_\mathrm{e} / m_{^{12}\mathrm{C}}$	0.000 42

(1 u = 1 Da)

$$[\mathrm{u}] = \frac{1}{12}\left\{\frac{m_{^{12}\mathrm{C}}}{m_{\mathcal{K}}}\right\}[\mathrm{kg}]$$

unified atomic mass unit, or dalton

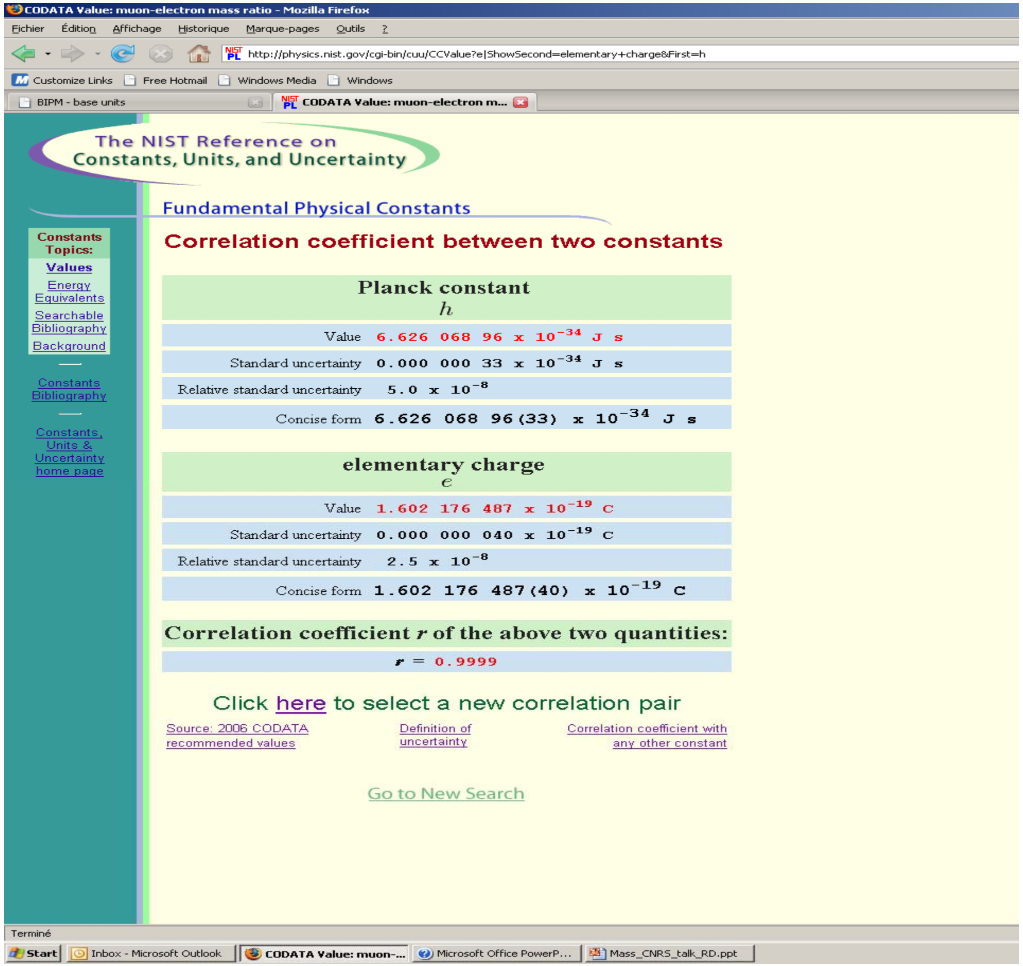
why the present kg definition is inconvenient for physics

- Conceptually peculiar to measure fundamental physical constants in terms of an artefact manufactured in the 19th century.

- Comparison to the international prototype is the dominant uncertainty in the SI values for many <u>fundamental constants</u> of physics; therefore: either large covariances must be taken into account or non-SI units like the dalton must be used.

Corollary: SI values of many fundamental constants change whenever there is a newly-measured link to $\mathcal{K}$., and yet the link to $\mathcal{K}$ is not “fundamental”.

Example: present correlation between *h* and *e*



The NIST Reference on
Constants, Units, and Uncertainty

Constants Topics:
- **Values**
- Energy Equivalents
- Searchable Bibliography
- Background

Constants Bibliography

Constants, Units & Uncertainty home page

Fundamental Physical Constants

Correlation coefficient between two constants

Planck constant
h

Value	$6.626\ 068\ 96 \times 10^{-34}$ J s
Standard uncertainty	$0.000\ 000\ 33 \times 10^{-34}$ J s
Relative standard uncertainty	5.0×10^{-8}
Concise form	$6.626\ 068\ 96(33) \times 10^{-34}$ J s

elementary charge
e

Value	$1.602\ 176\ 487 \times 10^{-19}$ C
Standard uncertainty	$0.000\ 000\ 040 \times 10^{-19}$ C
Relative standard uncertainty	2.5×10^{-8}
Concise form	$1.602\ 176\ 487(40) \times 10^{-19}$ C

Correlation coefficient *r* of the above two quantities:

$r = 0.9999$

Click here to select a new correlation pair

Source: 2006 CODATA recommended values

Definition of uncertainty

Correlation coefficient with any other constant

Go to New Search

If you really need to correct for the correlation, then…

…you need to know where to look for the covariance (e.g., it might not be sufficient to know that the correlation coefficient between *m_e* and *N_A* is approximately -1.0000)

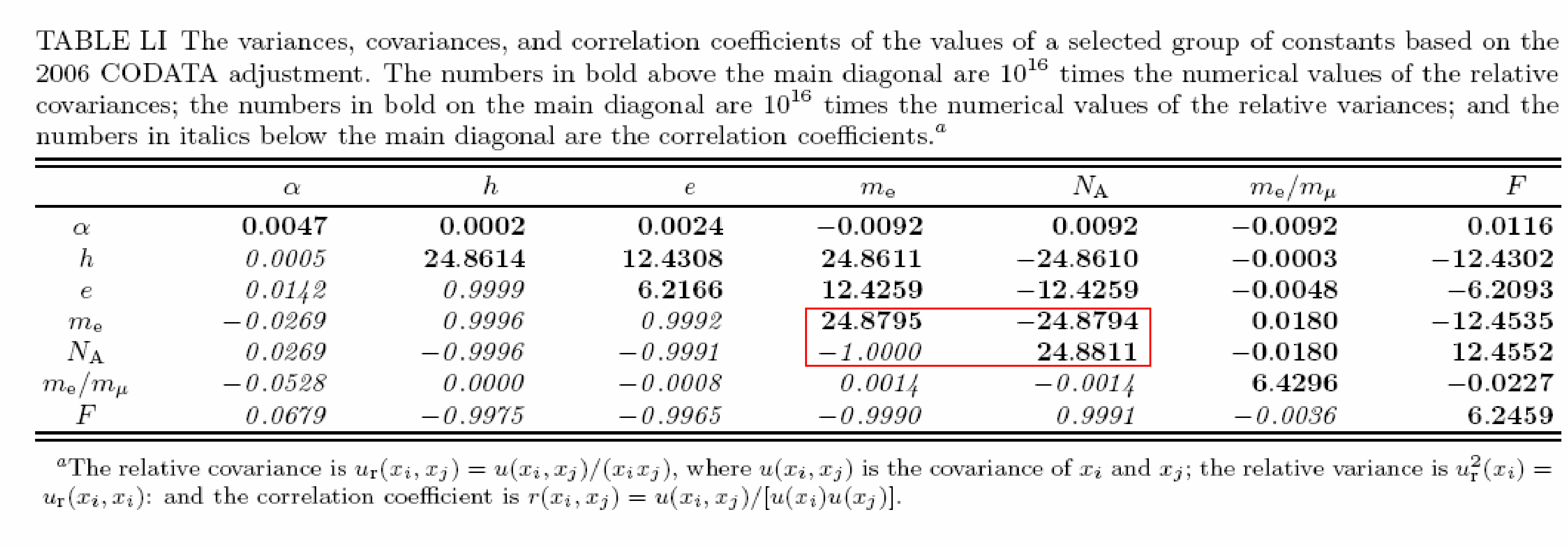


TABLE LI The variances, covariances, and correlation coefficients of the values of a selected group of constants based on the 2006 CODATA adjustment. The numbers in bold above the main diagonal are 10^{16} times the numerical values of the relative covariances; the numbers in bold on the main diagonal are 10^{16} times the numerical values of the relative variances; and the numbers in italics below the main diagonal are the correlation coefficients.[a]

	α	h	e	m_e	N_A	m_e/m_μ	F
α	**0.0047**	**0.0002**	**0.0024**	**−0.0092**	**0.0092**	**−0.0092**	**0.0116**
h	*0.0005*	**24.8614**	**12.4308**	**24.8611**	**−24.8610**	**−0.0003**	**−12.4302**
e	*0.0142*	*0.9999*	**6.2166**	**12.4259**	**−12.4259**	**−0.0048**	**−6.2093**
m_e	*−0.0269*	*0.9996*	*0.9992*	**24.8795**	**−24.8794**	**0.0180**	**−12.4535**
N_A	*0.0269*	*−0.9996*	*−0.9991*	*−1.0000*	**24.8811**	**−0.0180**	**12.4552**
m_e/m_μ	*−0.0528*	*0.0000*	*−0.0008*	*0.0014*	*−0.0014*	**6.4296**	**−0.0227**
F	*0.0679*	*−0.9975*	*−0.9965*	*−0.9990*	*0.9991*	*−0.0036*	**6.2459**

[a] The relative covariance is $u_r(x_i, x_j) = u(x_i, x_j)/(x_i x_j)$, where $u(x_i, x_j)$ is the covariance of x_i and x_j; the relative variance is $u_r^2(x_i) = u_r(x_i, x_i)$: and the correlation coefficient is $r(x_i, x_j) = u(x_i, x_j)/[u(x_i)u(x_j)]$.

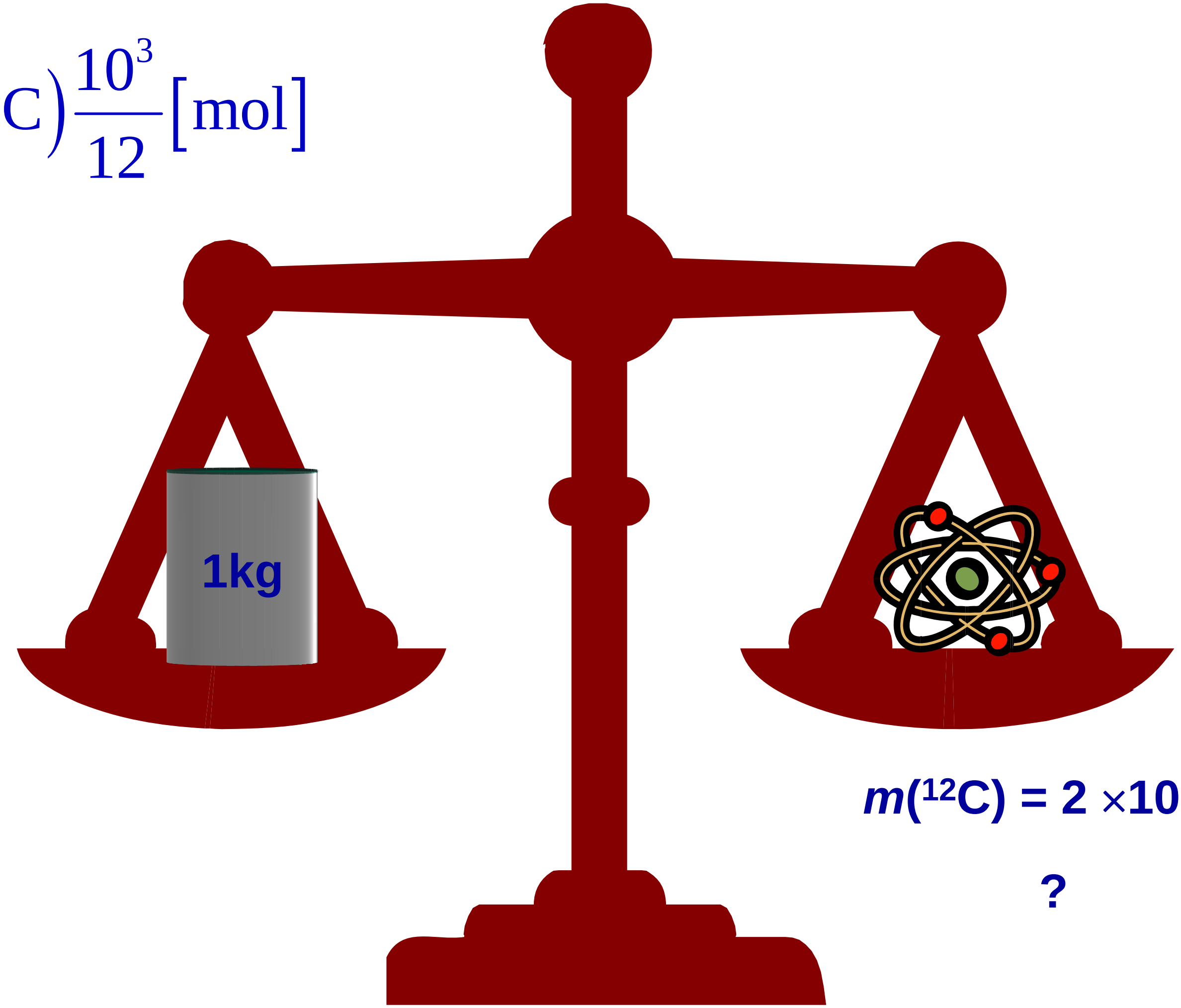
Largest covariances are invariably due to the present uncertainties in linking the artifact kilogram to a fundamental constant.

Experimental

How do we link the kg to a fundamental constant?

A large mismatch between 1 kg and the mass of a carbon-12 atom

$$\frac{1}{N_{\mathrm{A}}} = m\left({}^{12}\mathrm{C}\right)\frac{10^{3}}{12}[\mathrm{mol}]$$



$m(^{12}\mathrm{C}) = 2 \times 10^{-26}$ kg

?

N_A by the X-ray Crystal Density (XRCD) method

***n* = number of ^{28}Si atoms in the sphere:**

$$n = \left\{\frac{m_{\mathrm{sph}}}{m_{^{28}\mathrm{Si}}}\right\} = \left\{\frac{m_{\mathrm{sph}}}{m_{\mathcal{K}}}\right\}\left\{\frac{m_{\mathcal{K}}}{m_{^{12}\mathrm{C}}}\right\}\left\{\frac{m_{^{12}\mathrm{C}}}{m_{^{28}\mathrm{Si}}}\right\}$$

$$n = 8\frac{V}{a^3}$$

V = volume of sphere

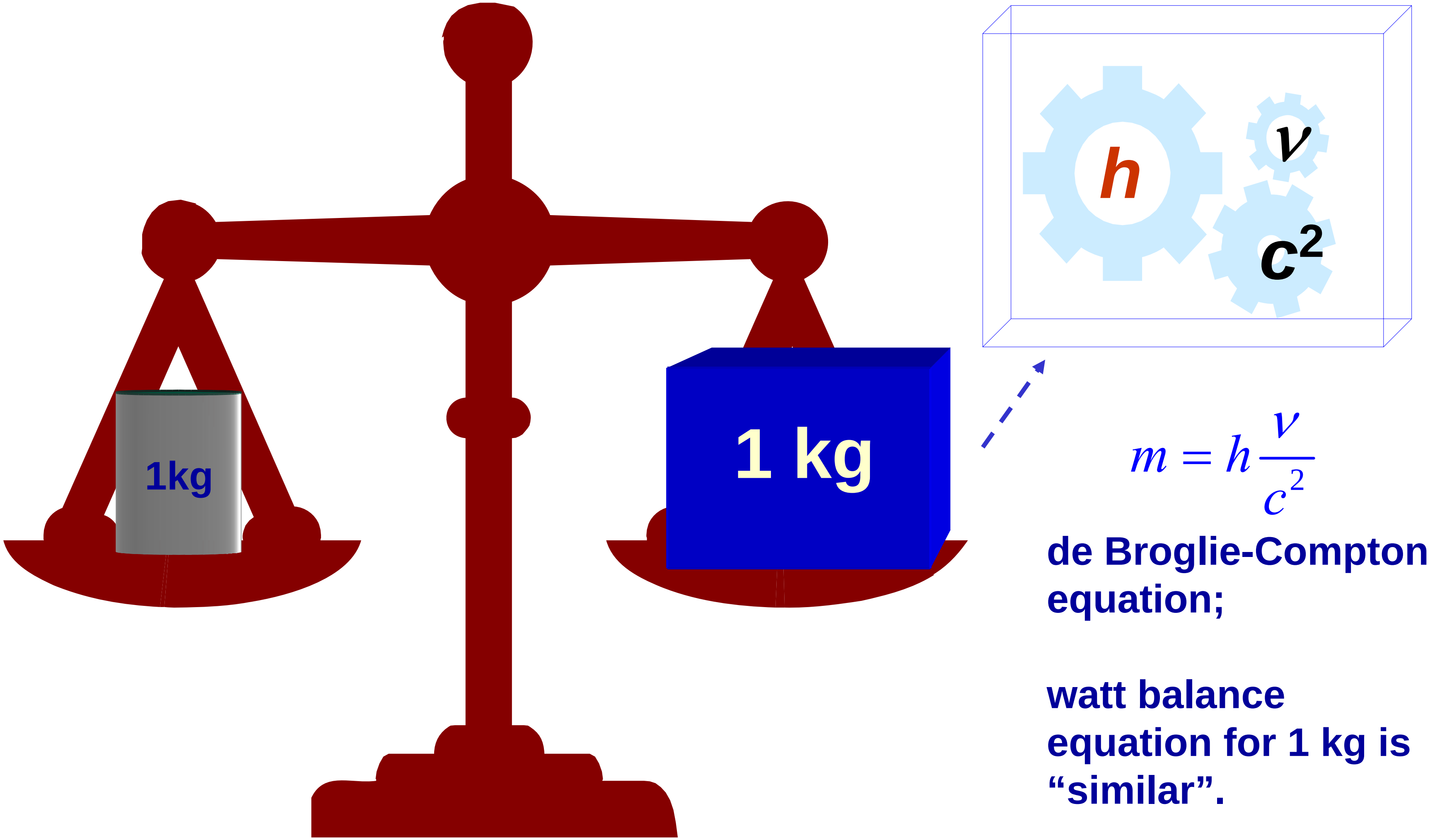
a^3 = volume of unit cell

$$\left\{\frac{m_{^{12}\mathrm{C}}}{m_{\mathcal{K}}}\right\} = \left\{\frac{m_{\mathrm{sph}}}{m_{\mathcal{K}}}\right\}\left\{\frac{m_{^{12}\mathrm{C}}}{m_{^{28}\mathrm{Si}}}\right\}\frac{a^3}{8V}$$

mass comparator ; mass spectrometer
X-ray interferometer; optical interferometer
purity: chemical, atomic, crystallographic

$$\frac{1}{N_{\mathrm{A}}} = \left\{\frac{m_{^{12}\mathrm{C}}}{m_{\mathcal{K}}}\right\}\frac{10^3}{12}[\mathrm{mol}] = \left\{\frac{m_{^{12}\mathrm{C}}}{m_{\mathcal{K}}}\right\}[\mathrm{kg}]\cdot\frac{1}{12}\left[\frac{\mathrm{mol}}{\mathrm{g}}\right]\cdot 10^3\left[\frac{g}{\mathrm{kg}}\right]$$

How the Planck constant can be linked to the kilogram



$$m = h\frac{\nu}{c^2}$$

de Broglie-Compton equation;

watt balance equation for 1 kg is "similar".

Watt Balance

Part 1

$$m_{\mathcal{K}} g = I \cdot f\left(\vec{B},\vec{r}\right)$$

g is local grav. accel.; I is a current

Part 2

$$U = v \cdot f\left(\vec{B},\vec{r}\right)$$

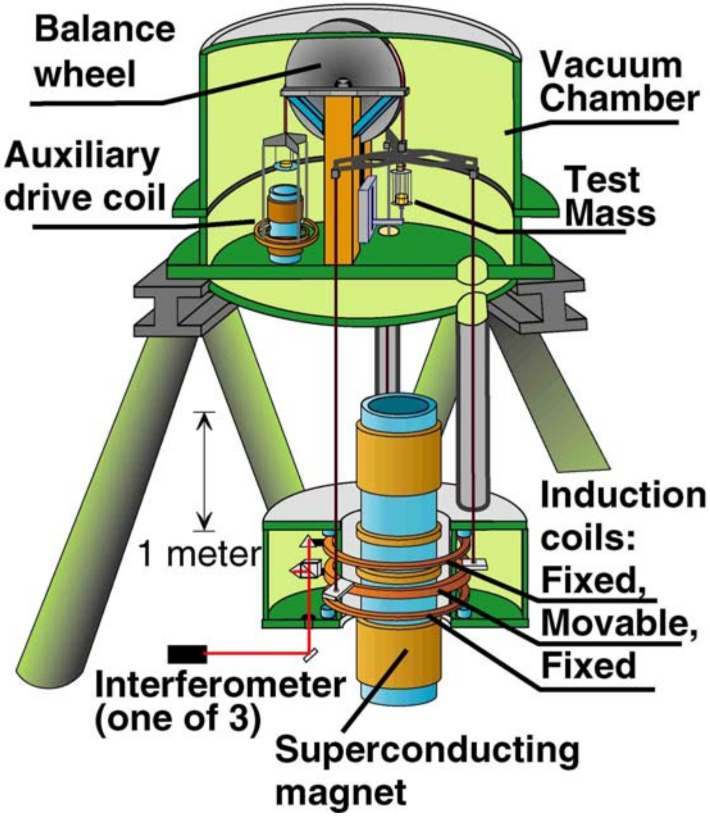
U is a voltage; v is a velocity

$$m_{\mathcal{K}} g v = IU$$

[Watt] = [Watt] $I = \frac{U'}{R}$

Using quantum electrical devices,

$$m_{\mathcal{K}} = \frac{h}{4}\left(\frac{\nu_1 \nu_2}{g v} \cdot \prod_{\text{i}} (\text{integers})\right)$$



Schematic of NIST apparatus

(courtesy of NIST)

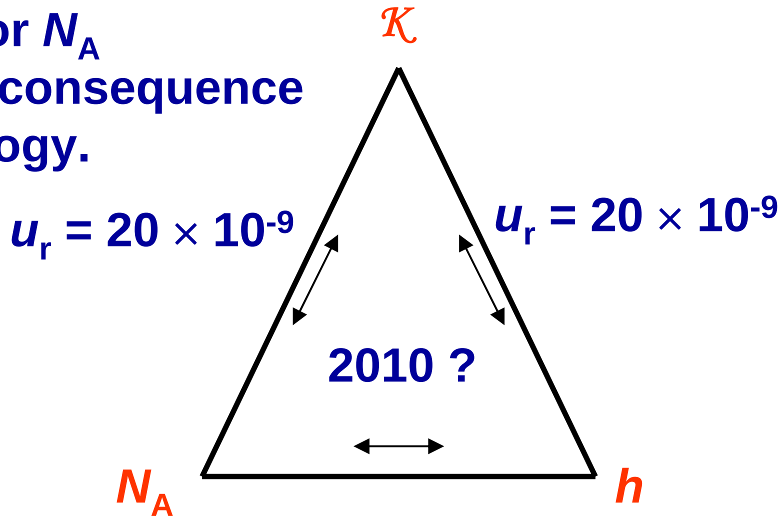
The international prototype, N_A, and h form a metrological triangle

$$N_{\mathrm{A}}\, h = 0.012 \frac{m_{\mathrm{e}}}{m\left({}^{12}\mathrm{C}\right)} \frac{c\alpha^{2}}{2R_{\infty}}$$

$$u_{\mathrm{r}}\left(N_{\mathrm{A}}\, h\right) = 1.4 \times 10^{-9}$$

From CODATA 2006

Whether the “new” kg is defined by h or N_A should have no consequence for mass metrology.

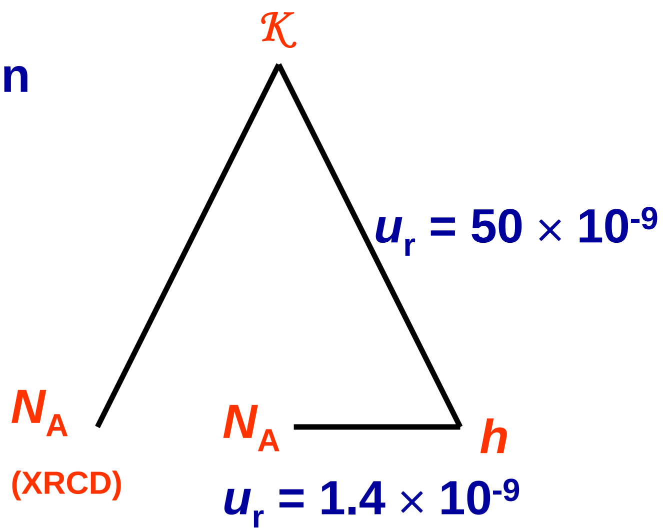


$u_r = 1.4 \times 10^{-9}$ (already achieved)

what we are working toward

At present, the triangle does not close

CODATA 2006
recommendation



$\mathcal{K}$

$u_r = 50 \times 10^{-9}$

N_A

(XRCD)

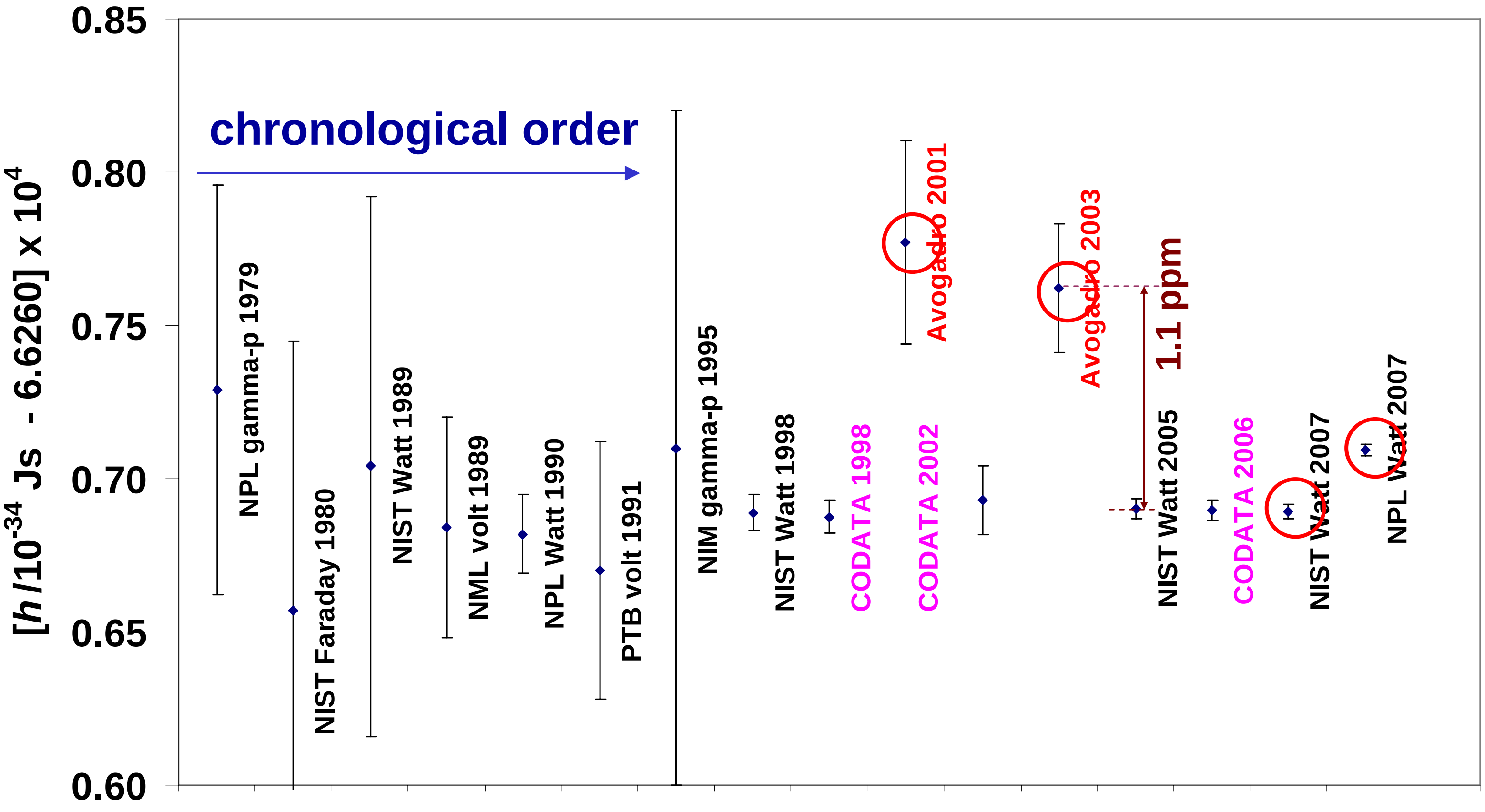
N_A

h

$u_r = 1.4 \times 10^{-9}$

CIPM, CCU, CCM, CCEM agree
that this situation must be clarified
before a redefinition of the kilogram

History of measurements of the Planck constant

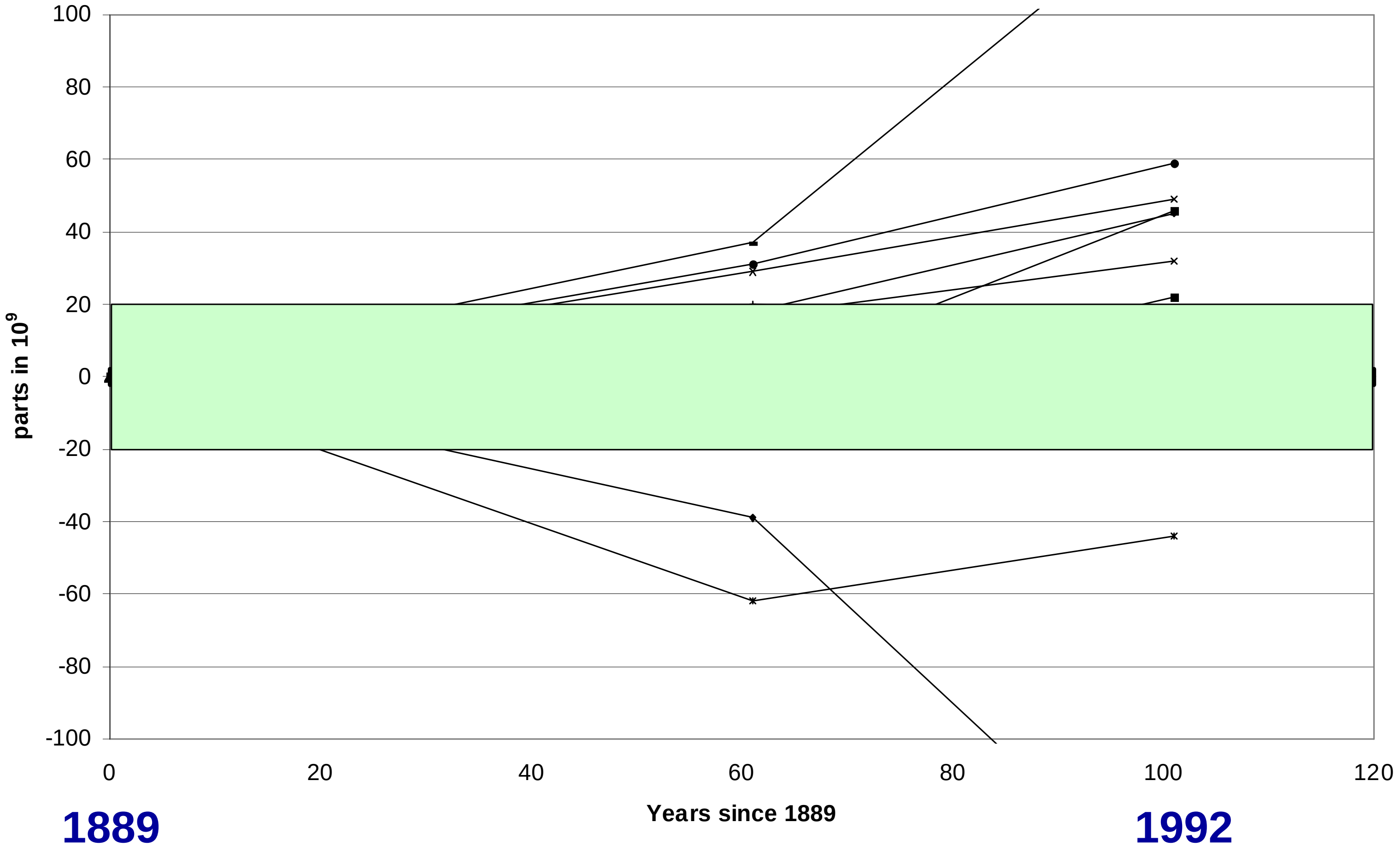


***N_A* and *h* are related by**

$$N_{\mathrm{A}}\ h = 0.012 \frac{m_{\mathrm{e}}}{m\left(^{12}\mathrm{C}\right)} \frac{c\alpha^2}{2R_\infty} \qquad u_{\mathrm{r}}\left(N_{\mathrm{A}}\ h\right) = 1.4\times10^{-9}$$

Changes in mass of national prototypes manufactured in 1889

CODATA/NPL_WB: 280 x 10^{-9} ; CODATA/XRCD: 1000 x 10^{-9}



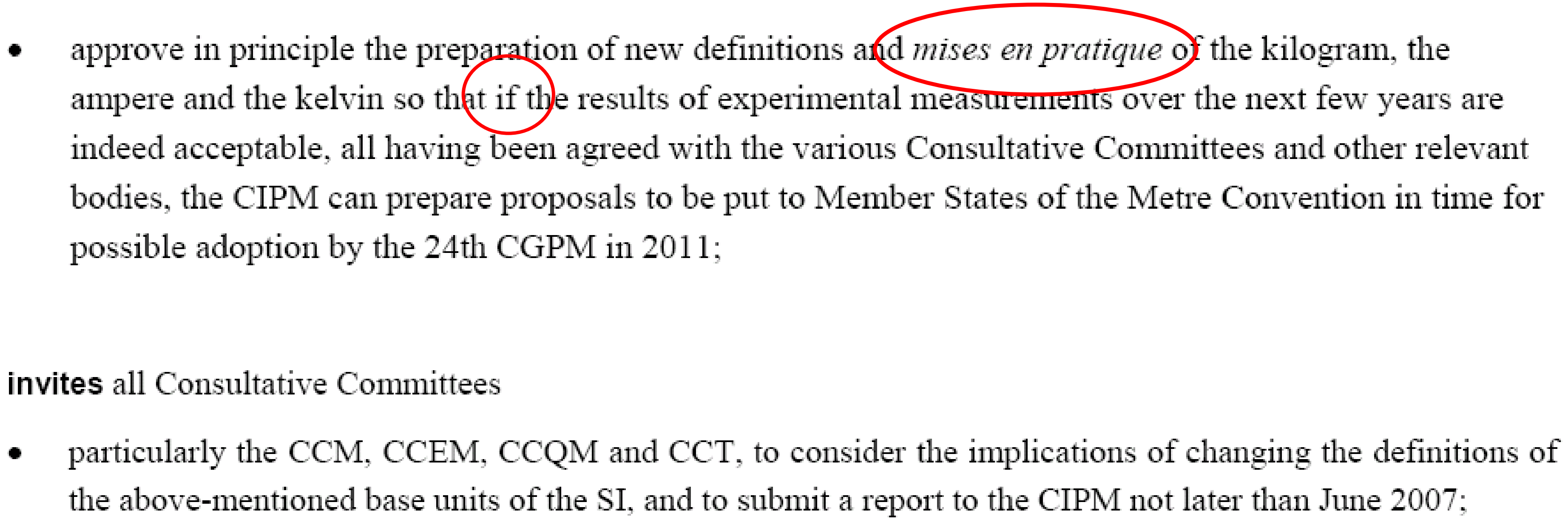
1889 **1992**

Steps to redefine the kilogram

CCU, CCM, etc. send proposals/counter proposals to the CIPM.

The CIPM takes action, or it does not.

In 2005, the CIPM approved a Recommendation whose major points for mass metrologists are:



- approve in principle the preparation of new definitions and *mises en pratique* of the kilogram, the ampere and the kelvin so that if the results of experimental measurements over the next few years are indeed acceptable, all having been agreed with the various Consultative Committees and other relevant bodies, the CIPM can prepare proposals to be put to Member States of the Metre Convention in time for possible adoption by the 24th CGPM in 2011;

invites all Consultative Committees

- particularly the CCM, CCEM, CCQM and CCT, to consider the implications of changing the definitions of the above-mentioned base units of the SI, and to submit a report to the CIPM not later than June 2007;

Some consequences for mass metrologists

With m_{X} defined, the present uncertainty of $\{\boldsymbol{m_{\mathrm{X}}/m_{\mathcal{K}}}\}$ will be attributed to the mass of the international prototype and will propagate to all other, macroscopic, masses derived from the international prototype.

Here X might be the mass, now fixed, of a fundamental constant such as m_{e}; or the product of a fundamental constant, such as h, and an appropriate factor; or…

Since the international prototype becomes a derived mass standard, the possibility is open to replace it by a better artefact or by the average of a group of artefacts.

Some consequences for everybody

- There will be no discontinuity in the kilogram, therefore no immediate consequences to measurements.
- The relative uncertainties of all mass standards, including the international prototype, will have an additional (but identical) component.
- Because the relative uncertainty component is exactly the same for all mass standards and all masses derived from mass standards, this component does not increase the uncertainty of comparisons between mass standards.
- Therefore, the consequence for end users will be negligible.
- Nevertheless, we should forge the strongest possible experimental links to a new definition to ensure that values of macroscopic masses remain traceable to the SI to sufficient accuracy.
- It seems unlikely that the public will understand the new definition of the kilogram. (This is a challenging problem in communications.)

Change the underlying structure of the SI ?

Instead of a unit system derived from base units and their definitions, it is perhaps more reasonable to define a new, quantum-based SI through fixed values of a set of basis constants. For example:

$\nu_{\mathrm{hfs}} = 9\ 192\ 631\ 770\ \mathrm{s}^{-1}$

$c = 299\ 792\ 458\ \mathrm{m}^{1}\ \mathrm{s}^{-1}$

$h = \{X\}\ \mathrm{kg}\ \mathrm{m}^{2}\ \mathrm{s}^{-1}$

$e = \{Y\}\ \mathrm{A}^{1}\ \mathrm{s}^{1}$

$k_{\mathrm{B}} = \{Z\}\ \mathrm{K}^{-1}\ \mathrm{kg}^{1}\ \mathrm{m}^{2}\ \mathrm{s}^{-2}$

Choose the basis set such that the result of experiments expressed in SI will have the lowest possible uncertainty; therefore future changes are to be expected.

Look here for an elementary, yet rigorous treatment.

(P.J. Mohr, *Metrologia* 2008 **45** 129-133)

To summarize…

1. Fix ν_{hfs}, c, h, μ_0, k_{B} ?

Consequences: the only exact electric constant is, as now, the vacuum impedence.

$$\mu_0 c = 376.7\ldots\ \Omega\text{, exact}$$

At the kg level, the practical link to the SI is through the watt balance or XRCD of ^{28}Si; $u_{\mathrm{r}}(e) = (1/2)u_{\mathrm{r}}(\alpha)$; $u_{\mathrm{r}}(m_{\mathrm{e}}) \approx 2u_{\mathrm{r}}(\alpha)$

[Note: if G replaces ν_{hfs}, we have Planck units.]

2. Fix ν_{hfs}, c, h, e, k_{B} ?

Consequences: Conversion factors for quantum standards used in electrical metrology have no uncertainty; $u_{\mathrm{r}}(\mu_0) = u_{\mathrm{r}}(\alpha)$

At the kg level the practical link to the SI is through the watt balance or XRDS ^{28}Si; $u_{\mathrm{r}}(m_{\mathrm{e}}) \approx 2u_{\mathrm{r}}(\alpha)$.

Conclusions

- Artifacts are inherently unstable with respect to fundamental physical constants.
- There is no experimental evidence (so far) that $m_{\mathcal{K}}$ is changing with respect to the fundamental physical constants.
- Present results between Watt Balance and XRCD values of h (or N_A) disagree at the 10^{-6} level ($>3u_\mathrm{c,r}$).
- Two most recent Watt Balance results disagree by 3×10^{-7} ($>6u_\mathrm{c,r}$).
- An immediate redefinition of the kilogram would benefit electrical metrology and the entire CODATA community.
- Many groups are active in this research and progress is being made.
- The transition to a quantum-based SI is now inevitable, probably by 2011.

Reflections

The SI is intended to be used in science, technology, commerce, and daily life.

This universality is an ideal that can never be attained.

"…the reality is that scientists must be conversant in many languages"

– J.D. Jackson *Classical Electrodynamics*, 3rd Ed.

However, we can and we shall move to a quantum-based SI.