Investigating consequences of the self-force

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- A "tableau" description of matched asymptotic expansions
- A "shorthand" notation to describe higher-order perturbation theory in GR

- μ is the mass of a small black hole.
- $\mathscr{R}$ is a length scale of the curvature of the background geometry.

$\mu \ll \mathscr{R}$ and also to higher order in $r/\mathscr{R}$.

g	$\sim$			$(r^2/\mathscr{R}^2)$		$(r^3/\mathscr{R}^3)$		$(r^4/\mathscr{R}^4)$		
η	&	0	&	${}_2H$	&	${}_3H$	&	${}_4H$	&	$= g^0 + h_\mathsf{R}$
μ/r	&	$\mu/\mathscr{R}$	&	$\mu r/\mathscr{R}^2$	&	$\mu r^2/\mathscr{R}^3$	&	$\mu r^3/\mathscr{R}^4$	&	$= h_\mathsf{S}^{\mu}$
μ^2/r^2	&	$\mu^2/r\mathscr{R}$	&	$\mu^2/\mathscr{R}^2$	&	$\mu^2 r/\mathscr{R}^3$	&	$\mu^2 r^2/\mathscr{R}^4$	&	$= h_\mathsf{S}^{\mu^2}$
μ^3/r^3	&	$\mu^3/r^2\mathscr{R}$	&	$\mu^3/r\mathscr{R}^2$	&	$\mu^3/\mathscr{R}^3$	&	$\mu^3 r/\mathscr{R}^4$	&	$= h_\mathsf{S}^{\mu^3}$
$\vdots$		$\vdots$		$\vdots$		$\vdots$		$\vdots$		
g^{Schw}		0		${}_2h$		${}_3h$		${}_4h$		

- In the matching region, each term in the tableau is much larger than every term below or to the right.

Second order perturbation theory with a point mass

Define the parts of the Einstein tensor of various orders in h by

$$G(g+h) = G(g) + G^{(1)}(g,h) + G^{(2)}(g,h) + G^{(3)}(g,h) + \dots$$

- $G^{(1)}(g,h)$ looks like a wave operator on h
- $G^{(2)}(g,h)$ looks like "$\nabla h \nabla h$" or "$h \nabla\nabla h$".

At second order solve $G(g) + G^{(1)}(g,h) + G^{(2)}(g,h) = 8\pi T$ or

$$G(g+h^{\mathsf{R}}) + G^{(1)}(g+h^{\mathsf{R}}, h^{\mathsf{S}}) + G^{(2)}(g+h^{\mathsf{R}}, h^{\mathsf{S}}) = 8\pi T$$

$$G(g) + G^{(1)}(g,h^{\mathsf{R}}) + G^{(2)}(g,h^{\mathsf{R}}) + G^{(1)}(g+h^{\mathsf{R}}, h^{\mathsf{S}}) + G^{(2)}(g+h^{\mathsf{R}}, h^{\mathsf{S}}) \\ = 8\pi T$$

This results in

$$G^{(1)}(g,h^{\mathsf{R}}) = -G^{(2)}(g,h^{\mathsf{R}}) - G^{(1)}(g+h^{\mathsf{R}}, h^{\mathsf{S}}) - G^{(2)}(g+h^{\mathsf{R}}, h^{\mathsf{S}}) + 8\pi T$$

Schematically this is

$$ {}_g\Box h^{\mathsf{R}} = \nabla h^{\mathsf{R}}\,\nabla h^{\mathsf{R}} + [{}_{(g+h^{\mathsf{R}})}\nabla^2 h^{\mathsf{S}} - {}_g\nabla h^{\mathsf{S}}{}_g\nabla h^{\mathsf{S}} - 16\pi T] + O(h^3) $$

The righthand side is C^0 at $O(\mu)$ and finite but discontinuous at $O(\mu^2)$.

Numerical relativists could solve this nonlinear field equation, while simultaneously having μ move along a geodesic of $g+h_R$.

Note that there is no mention of any gauge choice.