Alternative gravity theories: motion

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Why considering alternative gravity theories?

- **General relativity** passes all tests with flying colors, but useful to contrast its predictions with alternatives.
- Partners to graviton predicted by all unified and extradimensional theories (superstrings).
- ∃ some puzzling experimental issues: **Dark** energy (72%), **dark** matter (24%), Pioneer anomaly…

All alternative theories: wide subject! [*cf.* C.M. Will, Living Rev. Rel. **9**, (2006) 3]
⇒ focus here on scalar-tensor theories & generalizations.

Two hypotheses of

General Relativity

$$S = \underbrace{\frac{c^3}{16\pi G}\int d^4x\sqrt{-g}\,R}_{\substack{\text{Einstein-Hilbert}\\ \text{pure spin 2}}} + \underbrace{S_{\text{matter}}[\text{matter}, g_{\mu\nu}]}_{\substack{\text{Metric coupling}\\ \text{equivalence principle}}}$$

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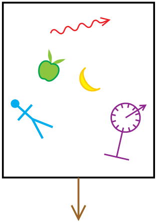
Einstein-Hilbert: pure spin 2
Metric coupling: equivalence principle

freely falling elevator



Modifications of the matter action S_{matter}

- GR: metric coupling $S_{\text{matter}}[\text{matter}, g_{\mu\nu}]$

  Example: $S_{\text{point particle}} = -\int mc\sqrt{-g_{\mu\nu}(x)\, v^\mu v^\nu}\, dt$

- Modified inertia [Milgrom 1994, 1999]:
  Assume $S_{\text{point particle}}(\mathbf{x}, \mathbf{v}, \mathbf{a}, \dot{\mathbf{a}}, \dots)$
  but strong experimental constraints for usual accelerations
  and either nonlocal or unstable [Ostrogradski 1850]

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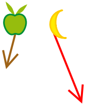

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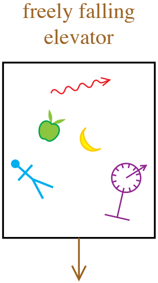

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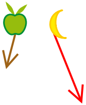

Tests of the metric coupling $S_{\text{matter}}[\text{matter}, g_{\mu\nu}]$

Einstein equivalence principle:

① Constancy of the constants Space & time independence of coupling constants and mass scales of the Standard Model Oklo natural fission reactor $\lvert\dot{\alpha}/\alpha\rvert < 7\times10^{-17}\,\text{yr}^{-1} \ll 10^{-10}\,\text{yr}^{-1}$ (cosmo) [Shlyakhter 76, Damour & Dyson 96]	② Local Lorentz invariance Local non-gravitational experiments are Lorentz invariant Isotropy of space verified at the 10^{-27} level [Prestage et al. 85, Lamoreaux et al. 86, Chupp et al. 89]
③ Universality of free fall Non self-gravitating bodies fall with the same acceleration in an external gravitational field Laboratory: 4×10^{-13} level [Baessler et al. 99]  : 2×10^{-13} level [Williams et al. 04]	④ Universality of gravitational redshift In a static Newtonian potential $g_{00} = -1 + 2\,U(x)/c^2 + O(1/c^4)$ the time measured by two clocks is $\tau_1/\tau_2 = 1 + [U(x_1) - U(x_2)]/c^2 + O(1/c^4)$ Flying hydrogen maser clock: 2×10^{-4} level [Vessot et al. 79–80, Pharao/Aces will give 5×10^{-6}]

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- If metric coupling $S_{\text{matter}}[\text{matter}, g_{\mu\nu}]$ assumed as in GR, then matter must move exactly as in GR for a given metric $g_{\mu\nu}$
$\Rightarrow$ nothing to say about *motion* of massive bodies?

- But S_{gravity} actually tells us how $g_{\mu\nu}$ is generated by matter
$\Rightarrow$ *motion* does depend directly on the dynamics of gravity!

- Clearest illustration: replace $g_{\mu\nu}$ in terms of its material sources
$\Leftrightarrow$ Construct the *Fokker action*

Consider an action

$$S = S_\Phi[\text{fields } \Phi] + S_m[\text{matter sources } \sigma, \Phi]$$

and denote as $\bar{\Phi}[\sigma]$ a solution of $\frac{\delta S}{\delta \Phi} = 0$ for given sources σ

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$$\Rightarrow \frac{\delta S_{\text{Fokker}}[\sigma]}{\delta\sigma} = \left(\frac{\delta S[\sigma,\Phi]}{\delta\sigma}\right)_{\Phi=\bar{\Phi}[\sigma]} + \left(\frac{\delta S[\sigma,\Phi]}{\delta\Phi}\right)_{\Phi=\bar{\Phi}[\sigma]} \frac{\delta\bar{\Phi}[\sigma]}{\delta\sigma}$$

Therefore $S_{\text{Fokker}}[\sigma]$ gives the correct equations of motion for σ

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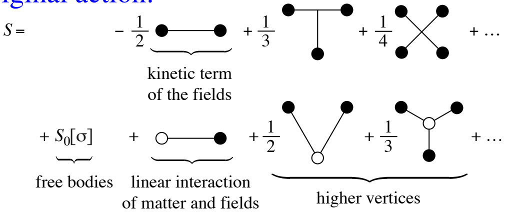

Fokker action in terms of diagrams

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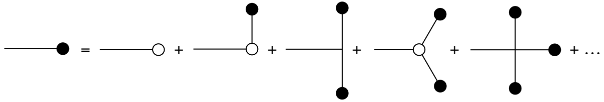

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[Damour & G.E-F, PRD **53** (1996) 5541]

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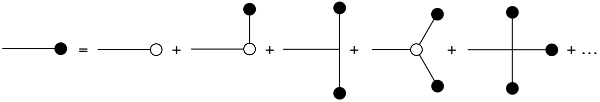

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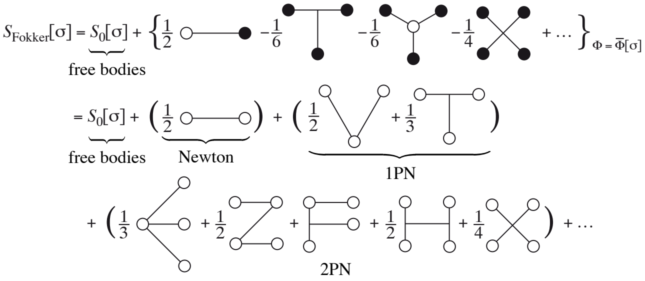

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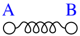
Fokker action for a N-body system

- In general relativity:

$$S_{\text{Fokker}} = -\sum_A \int dt\, m_A c^2 \sqrt{1 - \mathbf{v}_A^2/c^2} \quad \text{(free bodies)}$$



$$+\frac{1}{2}\sum_{A\neq B}\int dt\, \frac{G\, m_A m_B}{r_{AB}}\Big[1 + \frac{1}{2c^2}(\mathbf{v}_A^2 + \mathbf{v}_B^2) - \frac{3}{2c^2}(\mathbf{v}_A \cdot \mathbf{v}_B)$$

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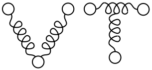

$$-\frac{1}{2}\sum_{B\neq A\neq C}\int dt\, \frac{G^2 m_A m_B m_C}{r_{AB} r_{AC}\, c^2} + \mathcal{O}\left(\frac{1}{c^4}\right)$$

- Most general PPN formalism: 10 parameters including β^{PPN} and γ^{PPN} [Nordtvedt & Will 1972]

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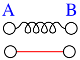

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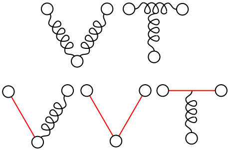

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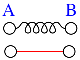

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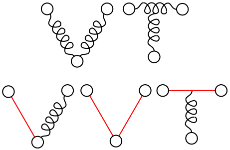

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Matter-scalar interaction $A(\varphi)$:

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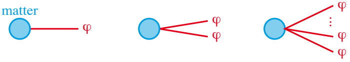

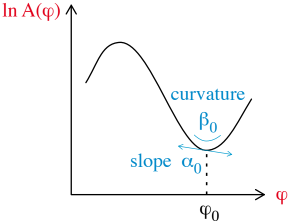

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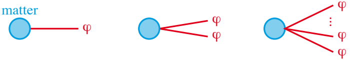

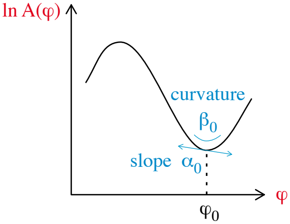

[Damour & G.E-F, CQG **9** (1992) 2093]

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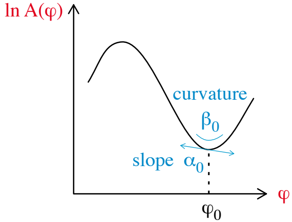

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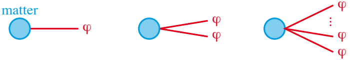
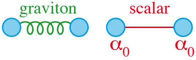
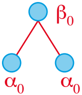


$$G_{\text{eff}} = G_*\,(1 + \alpha_0^2)$$



$$\gamma^{\text{PPN}} - 1 = -\frac{2\,\alpha_0^2}{(1+\alpha_0^2)} \propto -\alpha_0^2 < 0$$

$$\beta^{\text{PPN}} - 1 = \frac{\alpha_0^2\,\beta_0}{2(1+\alpha_0^2)^2} \propto \alpha_0^2\,\beta_0$$



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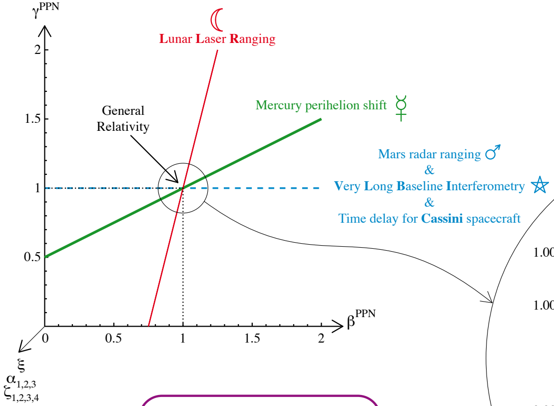
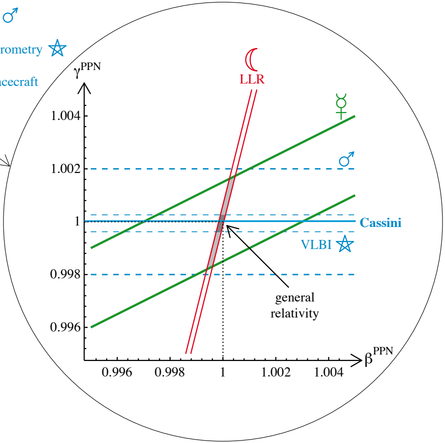
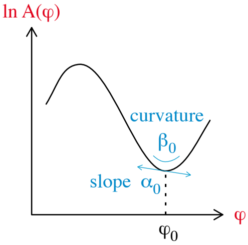
Solar-system constraints on PPN parameters

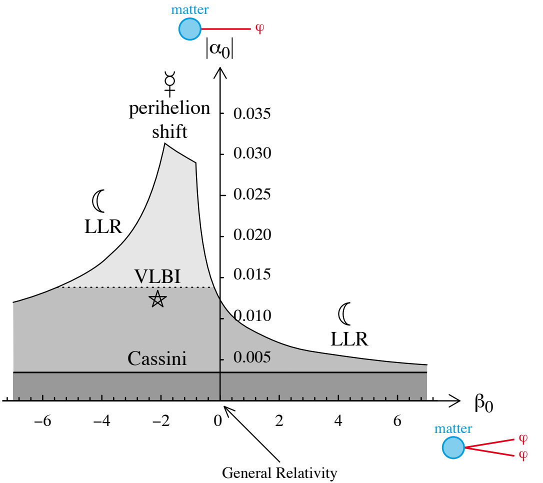




GENERAL RELATIVITY
is essentially the only
theory consistent with
weak-field experiments

Solar-system constraints on scalar-tensor gravity



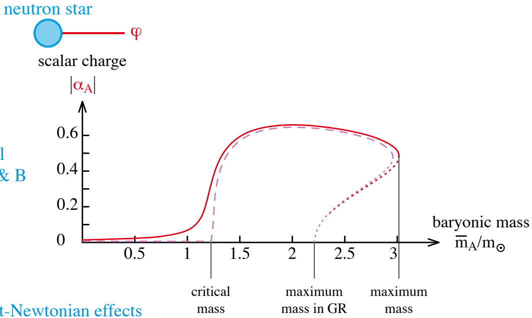


Vertical axis ($\beta_0 = 0$) : Jordan–Fierz–Brans–Dicke theory $\alpha_0^2 = \dfrac{1}{2\,\omega_{BD} + 3}$

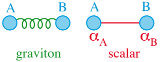
Horizontal axis ($\alpha_0 = 0$) : perturbatively equivalent to G.R.

Strong-field predictions

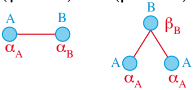
Nonperturbative strong-field effects:



- $G^{\text{eff}}_{AB} = G\,(1 + \alpha_A \alpha_B)$ — depends on internal structure of bodies A & B



- Similarly for $(\gamma^{\text{PPN}} - 1)$ and $(\beta^{\text{PPN}} - 1)$ $\Rightarrow$ all post-Newtonian effects

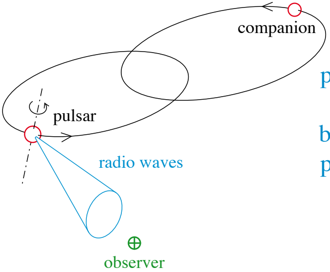


- Energy flux $= \dfrac{\text{Quadrupole}}{c^5} + O\left(\dfrac{1}{c^7}\right)$ spin 2

$$+ \frac{\text{Monopole}}{c}\left(0 + \frac{1}{c^2}\right)^2 + \frac{\text{Dipole}}{c^3} + \frac{\text{Quadrupole}}{c^5} + O\left(\frac{1}{c^7}\right) \quad \text{spin 0}$$

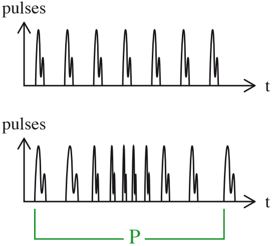
Dipole $\propto (\alpha_A - \alpha_B)^2$

Pulsar timing



pulsar = (very stable) clock

binary pulsar = moving clock



- Time of flight across orbit $\propto \dfrac{\text{size of orbit}}{c}$ ("Roemer time delay")

– orbital period P
– eccentricity e
– periastron angular position ω
– …

} "Keplerian" parameters

Pulsar timing (continued)

- Redshift $\propto \frac{G\, m_B}{r_{AB}\, c^2}$ + second order Doppler effect $\propto \frac{\vec{v}_A^{\,2}}{2\, c^2}$ ("Einstein time delay")

– parameter γ_{Timing}

- Time evolution of Keplerian parameters

– periastron advance $\dot{\omega}$ (order $\frac{1}{c^2}$)

– gravitational radiation damping $\dot{P}$ (order $\frac{1}{c^5}$)

} "post-Keplerian" observables
[PSR B1913+16 • Hulse & Taylor]

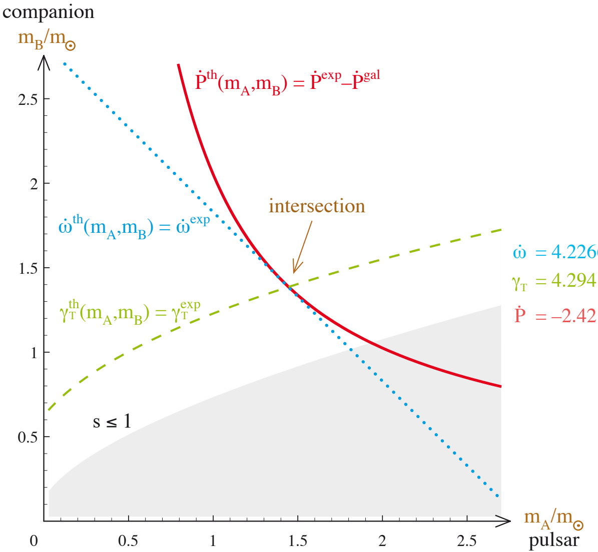
3 observables	–	2 unknown masses m_A, m_B	=	1 test

Plot the three curves [strips]

$$\gamma_{\text{Timing}}^{\text{theory}}(m_A, m_B) = \gamma_{\text{Timing}}^{\text{observed}}$$
$$\dot{\omega}^{\text{theory}}(m_A, m_B) = \dot{\omega}^{\text{observed}}$$
$$\dot{P}^{\text{theory}}(m_A, m_B) = \dot{P}^{\text{observed}}$$

} " γ - $\dot{\omega}$ - $\dot{P}$ test "

The Hulse-Taylor binary pulsar (PSR B1913+16) in GR



$\dot{\omega} = 4.22661°/\text{yr}$

$\gamma_T = 4.294$ ms

$\dot{P} = -2.421 \times 10^{-12}$

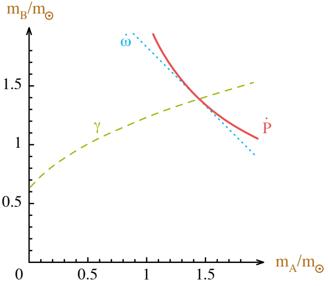
$\xrightarrow{\text{GR}}$

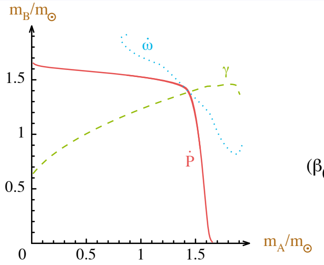
$m_A = 1.4408\ m_\odot$

$m_B = 1.3873\ m_\odot$

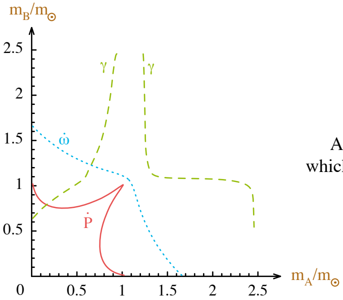
The Hulse-Taylor binary pulsar in scalar-tensor theories

General relativity
passes the test



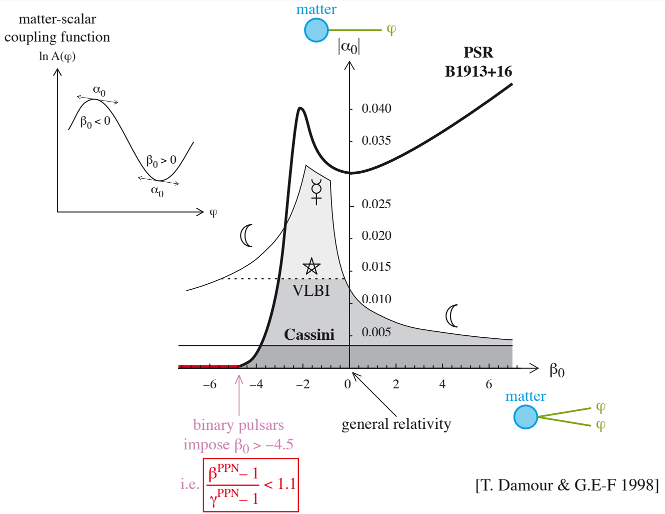


A scalar–tensor theory
which **passes the test**
($\beta_0 = -4.5$, α_0 small enough)



A scalar–tensor theory
which **does not pass the test**
($\beta_0 = -6$, any α_0)

PSR B1913+16 constraints on scalar-tensor gravity

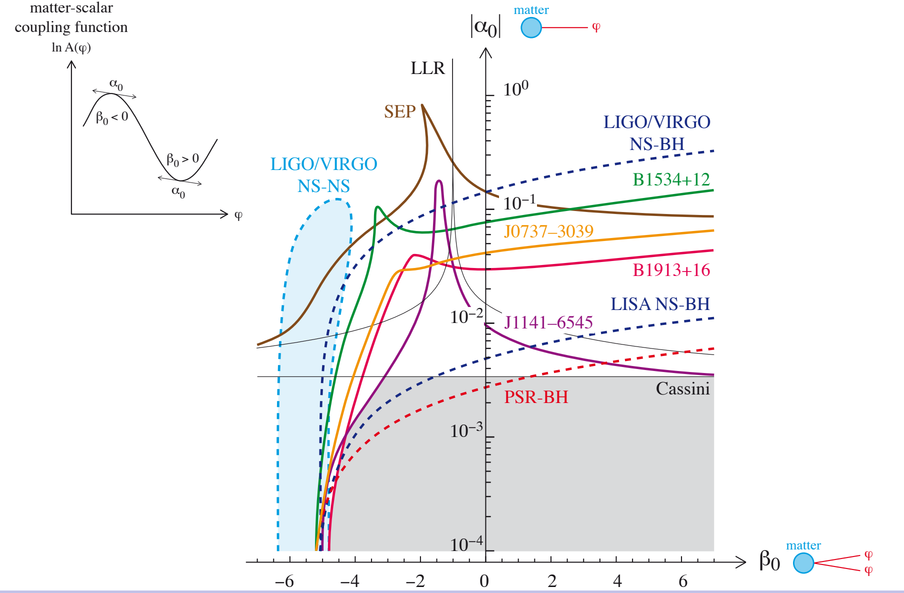


[T. Damour & G.E-F 1998]

Vertical axis ($\beta_0 = 0$) : Jordan–Fierz–Brans–Dicke theory $\alpha_0^2 = \frac{1}{2\,\omega_{BD} + 3}$

Horizontal axis ($\alpha_0 = 0$) : perturbatively equivalent to G.R.

All pulsar constraints on scalar-tensor gravity



Black holes in scalar-tensor theories

- Nonperturbative strong-field effects for compact bodies
  $\Rightarrow$ even larger deviations from G.R. for black holes?

- No! because no-hair theorem $\Rightarrow$ scalar charge $\alpha_{\text{black hole}} = 0$
- Collapsing stars radiate away their scalar charge when forming a black hole

- Black holes at equilibrium do not feel the scalar field
  and move thus exactly as in general relativity
  $\Rightarrow$ no scalar-field effect observable in a binary black-hole system

- On the other hand, large emission of dipolar scalar waves
  $\propto \frac{(\alpha_{\text{BH}} - \alpha_{\text{NS}})^2}{c^3} = \mathcal{O}\left(\frac{1}{c^3}\right)$ in a black hole–neutron star binary

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Extended bodies in general relativity

- Point-particle in general relativity:
  $$S_{\text{point particle}} = -\int mc\, ds = -\int mc \sqrt{-g_{\mu\nu}(x)\, v^\mu v^\nu}\, dt$$

- Effective action for an extended body:
  $$S_{\text{extended body}} = S_{\text{point particle}} + \int \left(k_1 R + k_2 R_{\mu\nu} u^\mu u^\nu + \cdots\right) c\, ds$$
  [Goldberger & Rothstein, PRD **73** (2006) 104029]

- But an action contribution $\propto$ lowest-order field equations
  $\Leftrightarrow$ *local* field redefinition: $S[\psi + \varepsilon] = S[\psi] + \varepsilon \dfrac{\delta S}{\delta \psi} + \mathcal{O}(\varepsilon^2)$

  $\Rightarrow$ the R and $R_{\mu\nu}$ terms above have no observable consequences
  [Damour & G.E-F, PRD **58** (1998) 042001]
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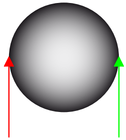

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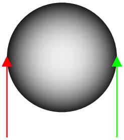

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Extended bodies in general relativity (continued)

- First observable effects for nonspinning extended bodies in GR:

$$S_{\text{extended body}} = S_{\text{point particle}} + \int \Big(k_3 C^2_{\mu\nu\rho\sigma} + k_4 C_{\mu\nu\rho\alpha} C^{\mu\nu\rho}{}_{\beta} u^\alpha u^\beta + k_5 C_{\mu\alpha\nu\beta} C^{\mu}{}_{\gamma}{}^{\nu}{}_{\delta} u^\alpha u^\beta u^\gamma u^\delta + \cdots \Big) c\, ds$$

[Goldberger & Rothstein 2006]

- By dimensionality, $kC^2 \sim m \Rightarrow k \sim m\, R_{\text{adius}}{}^4 \sim m \left(\frac{Gm}{c^2}\right)^4$ for a compact body (such that $Gm/Rc^2 \sim 1$)
  $\Rightarrow$ Corrections to motion $\propto \frac{kc^2}{c^4} \sim \mathcal{O}\left(\frac{1}{c^{10}}\right)$

- Consistent with Newtonian reasoning:
  Extra force felt by a body A because it deforms a companion B
  $\sim \frac{Gm_A}{r^2_{AB}} \left(\frac{R_B}{r_{AB}}\right)^5 = \mathcal{O}\left(\frac{1}{c^{10}}\right)$, if $\frac{Gm_B}{R_B c^2} \sim 1$

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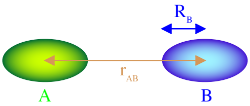

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Extended bodies in scalar-tensor gravity

- Point-particle in scalar-tensor theories:
  $S_{\text{point particle}} = -\int m(\varphi) c\, ds = -\int m(\varphi) c \sqrt{-g_{\mu\nu}(x)\, v^\mu v^\nu}\, dt$
  where $d \ln m(\varphi)/d\varphi = \alpha(\varphi)$ is the scalar charge

- Effective action for an extended body:

  The function $m(\varphi)$ gets replaced by the functional

  $m[\varphi, g_{\mu\nu}] = m(\varphi) + I(\varphi) R + J(\varphi) R_{\mu\nu}\, u^\mu u^\nu + K(\varphi) \Box\varphi +$
  $L(\varphi) \nabla_\mu \partial_\nu \varphi\, u^\mu u^\nu + M(\varphi) \partial_\mu\varphi \partial_\nu\varphi\, u^\mu u^\nu + N(\varphi) g^{\mu\nu} \partial_\mu\varphi\partial_\nu\varphi + \cdots$

  [Damour & G.E-F 1998]

- Use of lowest-order field equations (local field redefinitions)
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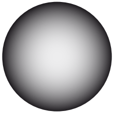

- Dimensional analysis: $N \sim m\,R_{\text{adius}}{}^2$
- Weakly self-gravitating body: $N = \frac{1}{6}\beta_0 \times I_{\text{nertia moment}}$
  [Nordtvedt, PRD **49** (1994) 5165]
- Compact bodies ($Gm/Rc^2 \sim 1$):
  A priori effects of order $\mathcal{O}\left(\frac{1}{c^2}\right) \times R_{\text{adius}}{}^2 = \mathcal{O}\left(\frac{1}{c^6}\right)$
  $\gg \mathcal{O}(1/c^{10})$ finite-size effects in general relativity
- But nonperturbative strong-field effects can occur
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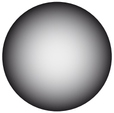
Extended bodies in scalar-tensor gravity (continued)



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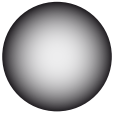

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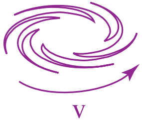
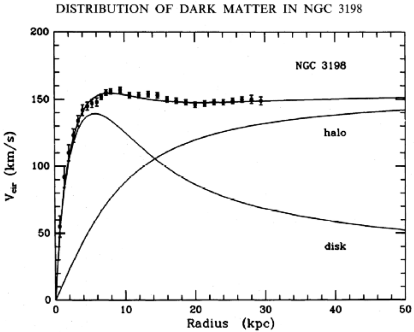
Large-distance modifications of gravity?

$\exists$ evidences for dark matter:

- $\Omega_\Lambda \approx 0.7$ (SNIa) and $\Omega_\Lambda + \Omega_m \approx 1$ (CMB) $\Rightarrow$ $\Omega_m \approx 0.3$, at least $10\times$ greater than estimates of baryonic matter.
- Rotation curves of galaxies and clusters: almost rigid bodies





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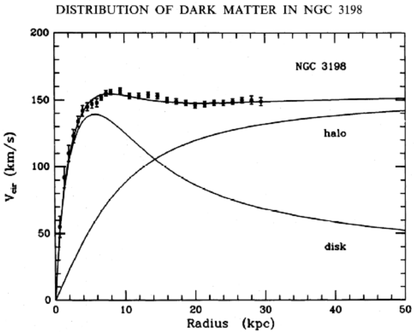

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Milgrom's MOND proposal [1983]

MOdified Newtonian Dynamics
for small accelerations (i.e., at large distances)

$$a = a_N = \frac{GM}{r^2} \quad \text{if } a > a_0 \approx 1.2 \times 10^{-10}\,\text{m.s}^{-2}$$

$$a = \sqrt{a_0 a_N} = \frac{\sqrt{GMa_0}}{r} \quad \text{if } a < a_0$$

- Automatically recovers the Tully-Fisher law [1977]

$$v_\infty^4 \propto M_{\text{baryonic}}$$

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[Sanders & McGaugh, Ann. Rev. Astron. Astrophys. **40** (2002) 263]

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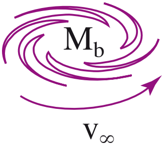

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Modified inertia or gravity?

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Most promising framework: generalized scalar-tensor theories

Relativistic AQUAdratic Lagrangians
[Bekenstein (TeVeS), Milgrom, Sanders]

$$S = \frac{c^3}{16\pi G}\int d^4x\sqrt{-g}\left\{R - 2f(\partial_\mu\varphi\partial^\mu\varphi)\right\}$$
$$+S_{\text{matter}}\left[\text{matter} \; ; \; \tilde{g}_{\mu\nu} \equiv A^2(\varphi)g_{\mu\nu} + B(\varphi)U_\mu U_\nu\right]$$

- A "k-essence" kinetic term can yield the $\frac{\sqrt{GMa_0}}{r}$ MOND force
- Matter coupled to the scalar field
- "Disformal" term (almost) necessary to predict enough lensing

Consistency conditions on $f(\partial_\mu\varphi\partial^\mu\varphi)$

Hyperbolicity of the field equations + Hamiltonian bounded by below

- $\forall x, \quad f'(x) > 0$
- $\forall x, \quad 2\,x f''(x) + f'(x) > 0$

N.B.: If $f''(x) > 0$, the scalar field propagates faster than gravitons, but still causally
$\Rightarrow$ no need to impose $f''(x) \leq 0$

These conditions become much more complicated *within matter*

[J.-P. Bruneton & G. Esposito-Farèse, Phys. Rev. D **76** (2007) 124012]

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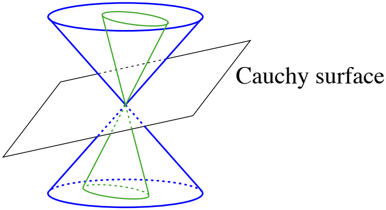

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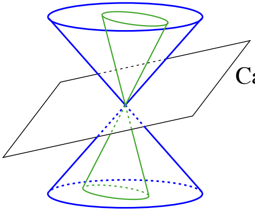

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Difficulties

- Complicated Lagrangians (unnatural)
- Fine tuning (≈ fit rather than predictive models):
  Possible to predict different lensing and rotation curves
- Discontinuities: can be cured

- In TeVeS [Bekenstein], gravitons & scalar are slower than photons
  ⇒ gravi-Cerenkov radiation suppresses high-energy cosmic rays [Moore *et al.*]
  Solution: Accept slower photons than gravitons

- ∃ preferred frame (ether) where vector $U_\mu = (1, 0, 0, 0)$
  Maybe not too problematic if U_μ is dynamical
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Post-Newtonian constraints

- Solar-system tests ⇒ matter *a priori* weakly coupled to φ
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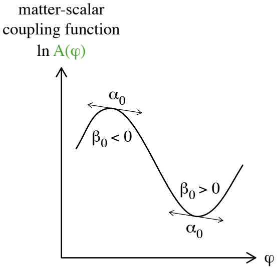

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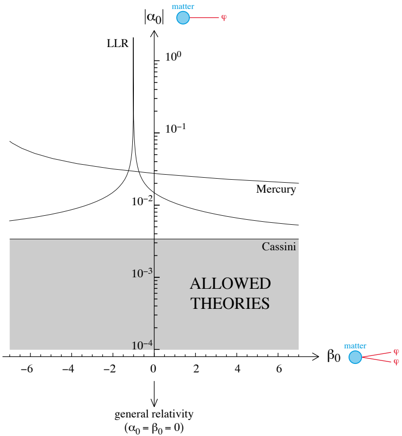

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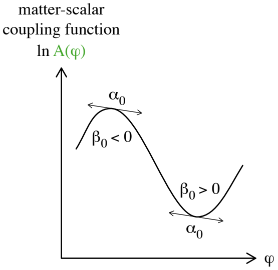

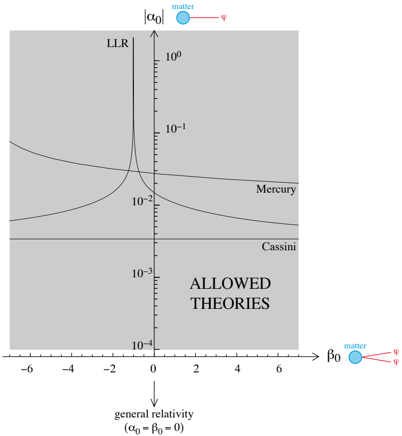

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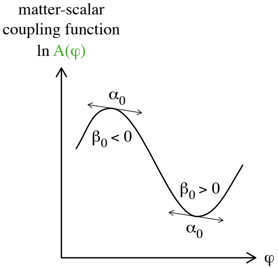

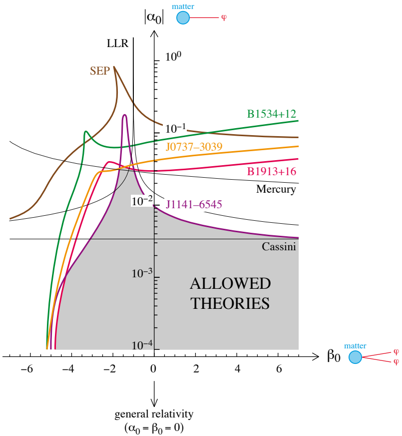

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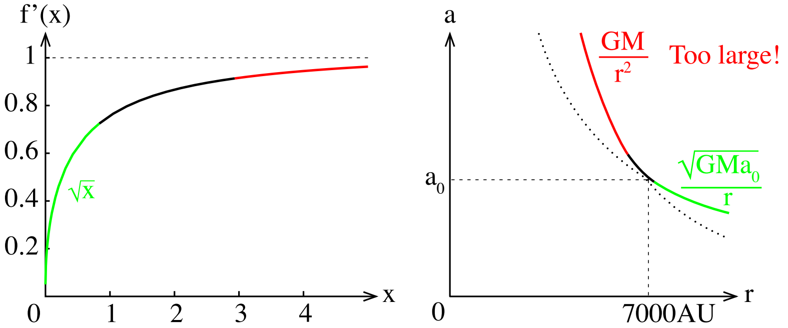

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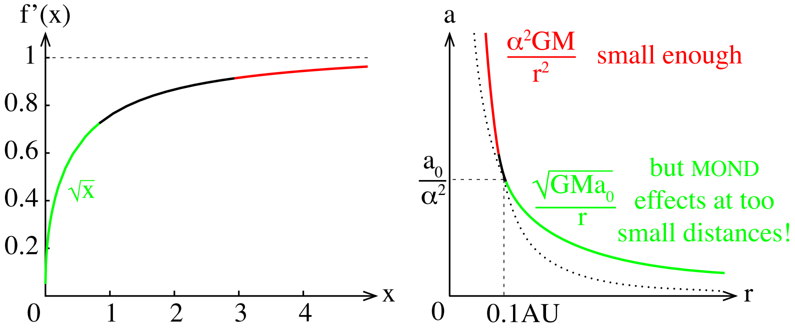

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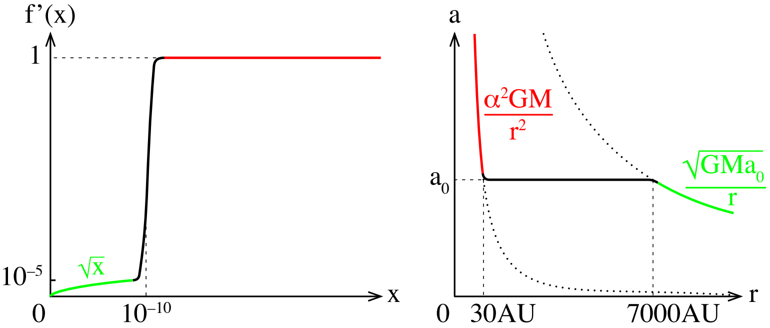

Quite unnatural! (and not far from being experimentally ruled out)

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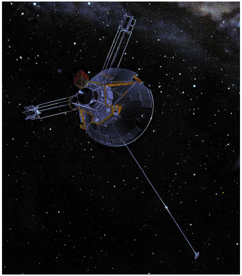
- Extra acceleration $\sim 8.5 \times 10^{-10}\,\mathrm{m.s}^{-2}$ towards the Sun between 30 and 70 AU
- Simpler problem than galaxy rotation curves ($M_{\text{dark}} \propto \sqrt{M_{\text{baryon}}}$), because we do not know how this acceleration is related to $M_{\odot}$
- $\Rightarrow$ *several* stable & well-posed solutions



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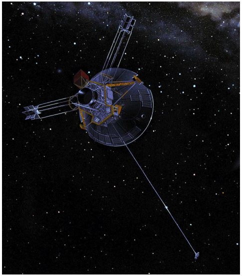

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