Effective field theory of extreme mass ratio inspirals

Chad Galley

Maryland Center for Fundamental Physics,
Department of Physics,
University of Maryland

CRG and B. L. Hu,
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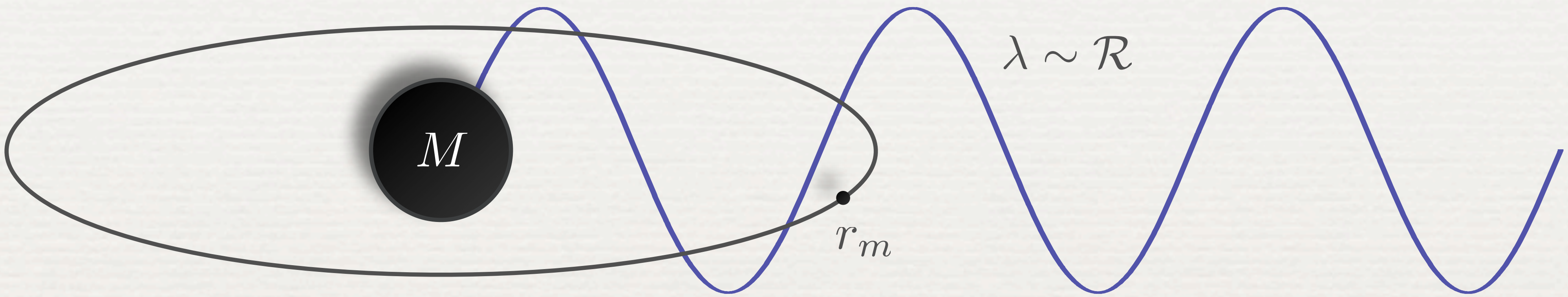
Context

- **Extreme mass ratio inspirals (EMRIs)**



- **Small parameter for perturbative treatment** $\varepsilon = \dfrac{r_m}{\mathcal{R}} \sim \dfrac{m}{M} \ll 1$
  - Test mass in orbit: Geodesic motion of SMBH geometry
  - Small, finite mass: Accelerated motion from MST-QW self-force

$$a^\mu = \frac{m}{2m_{pl}^2} w^{\mu\alpha\beta\nu} \lim_{\epsilon \to 0} \int_{-\infty}^{\tau-\epsilon} d\tau' \, \nabla_\nu D^{ret}_{\alpha\beta\gamma'\delta'}(z^\mu, z^{\mu'}) u^{\gamma'} u^{\delta'} + O(\varepsilon^2)$$

    - Entails interactions with back-scattered waves emitted in the past
    - Intrinsically non-local and history-dependent force
    - Gauge-dependent
- **However, may need corrections to MST-QW self-force...**

Why higher order self-force?

- **Waveforms computed using first-order self-force:**
  - 1 cycle of error in about 100,000 during year-long waveform
  - Sufficient for detection purposes, not measurement -- need "Capra" waveforms
- **Match-filtered search over year-long waveform:**
  - A coherent search is not computationally possible -- need 10^{40} templates
  - Alternatively, search over 3-week intervals and "stitch" together waveforms
- **Could use "kludge" and/or "Teukolsky" waveforms for detection**
  - Use to restrict parameter space of templates
  - Perform refined searches with increasingly longer and more accurate templates
- **To reach LISA's desired precision, may need 2nd order self-force**

Why higher order radiation?

	Worldline for Compact Object	Gravitational Perturbations
$O(m^0)$	Geodesic	
$O(m^1)$	First order self-force	Leading order radiation
$O(m^2)$	Second order self-force	NLO radiation
$O(m^3)$		NNLO radiation

Include 3rd order gravitational perturbations in "measurement" templates

Effective field theory

- **Systematically include effects from CO's multipole moments**
  - Tidally induced moments from companion SMBH, spin, intrinsic moments, etc.
  - Effacement Principle for EMRIs -- $O(\varepsilon^4)$
- **Use to compute higher order self-force and gravitational radiation**
  - Calculate systematically using Feynman diagrams
  - Generate sufficiently accurate waveforms for LISA's measurement phase
- **Efficiently handles divergences: Dimensional regularization + *Fp***
  - Covariant and gauge invariant
  - Power divergences vanish & log divergences renormalize couplings
  - Extend to curved spacetime using Bunch & Parker's approach

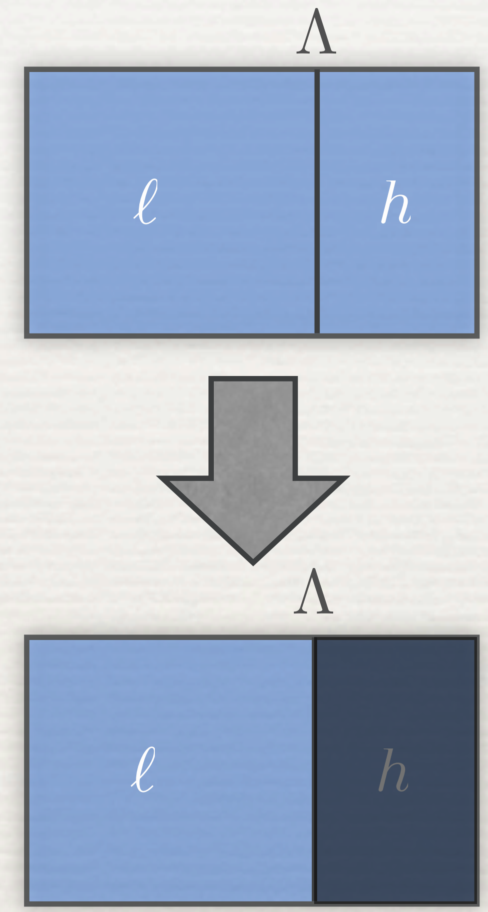
Brief overview of EFT

- **Partition degrees of freedom as either "heavy" or "light" (fast/slow, small/large,...)**
- **Integrate out heavy fields to find influence on light fields at low energies**

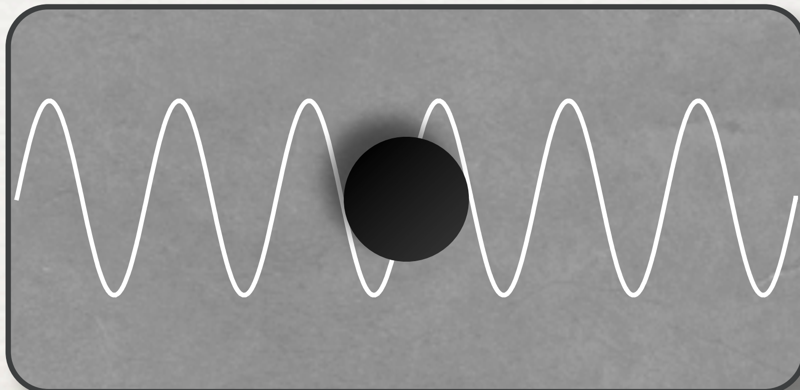
$$\exp\left(iS_{eff}[\ell,\Lambda]\right) = \int \mathcal{D}h_\Lambda \, \exp\left(iS[\ell,h]\right)$$

- **Effective Action:**
  - Real (full theory is unitary) and local (Uncertainty Principle)
  - Integrate out heavy fields if path integral can be performed
  - Include all possible terms that are consistent with symmetries
- **Physical observables at low energies**

$$\int \mathcal{D}\ell \, \exp\left(iS_{eff}[\ell,\Lambda]\right)$$

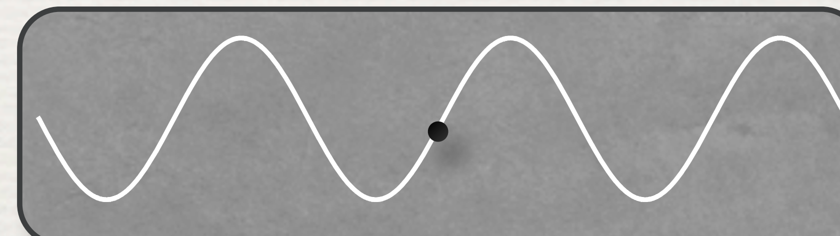


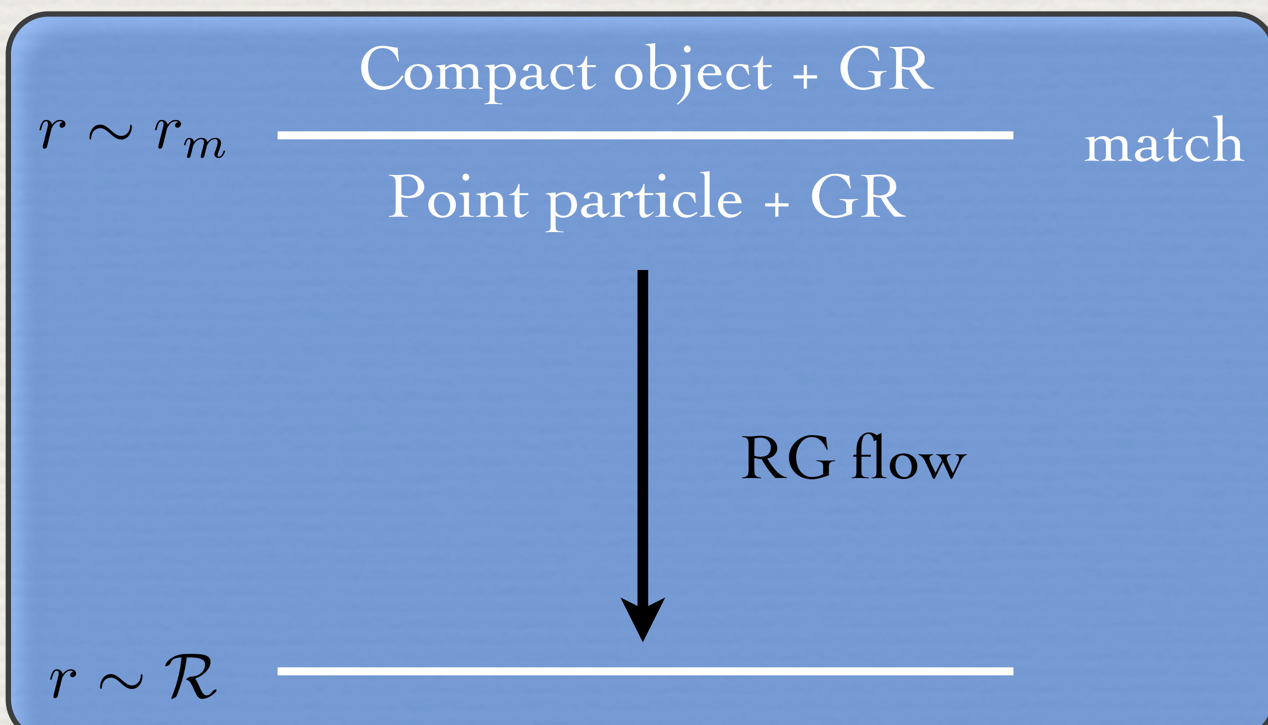
EFT of a compact object



Integrate out short distance gravitational perturbations

Resulting theory describes a particle interacting with long wavelength perturbations





RG flow of effective particle couplings (induced moments)

$$\epsilon \equiv \frac{r_m}{\mathcal{R}} \qquad \mathcal{R}^{-4} = R_{\mu\alpha\nu\beta}R^{\mu\alpha\nu\beta}$$

Effective point particle

Integrate out short wavelength gravitational perturbations → **Include all terms consistent with symmetries:**

General coordinate transformations
Reparameterizations
SO(3) rotations (e.g., spherical CO)

$$S_{pp}[z] = -m \int d\tau + c_R \int d\tau \, R + c_V \int d\tau \, R_{\alpha\beta} \dot{z}^\alpha \dot{z}^\beta + c_E \int d\tau \, \mathcal{E}_{\alpha\beta} \mathcal{E}^{\alpha\beta} + c_B \int d\tau \, \mathcal{B}_{\alpha\beta} \mathcal{B}^{\alpha\beta} + \cdots$$

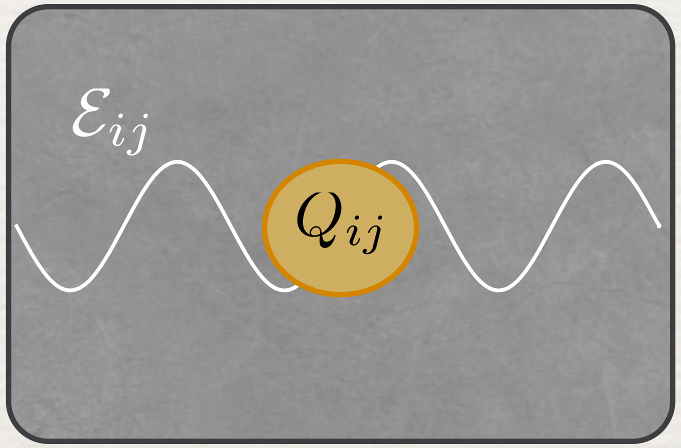
Ricci tensor terms can be removed by:

Using leading order classical equations of motion
Field redefinition of metric

$$S_{pp}[z] = -m \int d\tau + c_E \int d\tau \, \mathcal{E}_{\alpha\beta} \mathcal{E}^{\alpha\beta} + c_B \int d\tau \, \mathcal{B}_{\alpha\beta} \mathcal{B}^{\alpha\beta} + \cdots$$

Non-minimal couplings

- **To make predictions:**
  - Treat non-minimal couplings as free parameters to be fit by data
  - Match observables in effective theory to those of the long wavelength limit of the full theory (explicit example later)
- **Example: Neutron star**
  - External, adiabatic quadrupole tidal field induces a quadrupole moment on NS
  - Let NS be in weak-field region of a SMBH



$$Q_{ij}^{(n)} = -\lambda_n \mathcal{E}_{ij}$$

Thorne, PRD **58**, 124031 (1998)

$$S_{pp}[z] \approx \int dt \left(\frac{1}{2} m \mathbf{v}^2 + \frac{mM}{r} \right) - \frac{1}{2} \sum_n \int dt\, Q_{ij}^{(n)} \mathcal{E}_{ij} + \sum_n \int dt \frac{\dot{Q}_{ij}^{(n)} \dot{Q}_{ij}^{(n)} - \omega_n^2 Q_{ij}^{(n)} Q_{ij}^{(n)}}{4 \lambda_n^2 \omega_n^2}$$

Flanagan & Hinderer, PRD **77**, 021502(R) (2008)

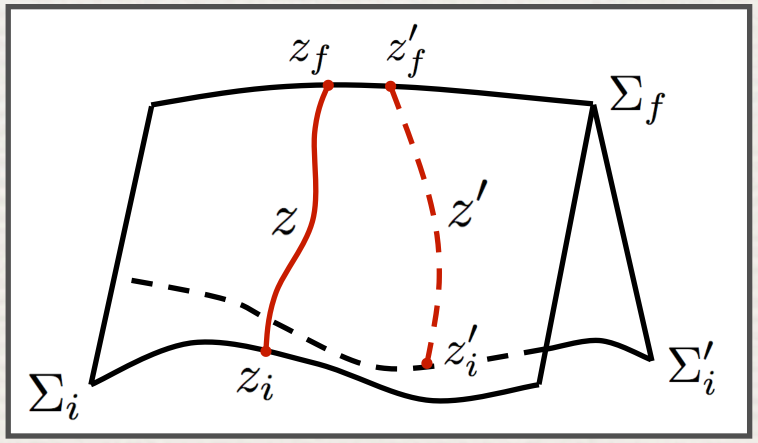
$$\Rightarrow \quad c_E \approx -\frac{\lambda}{4} = -\frac{1}{6} \frac{k_2 r_m^5}{G}$$

Closed-Time-Path formalism

- **Start with quantum theory for worldline and metric perturbations**
- **CTP (or in-in) generating functional** Schwinger (1961), Keldysh (1964)

$$Z\left[j^1_\mu, j^2_\mu, J^1_{\mu\nu}, J^2_{\mu\nu}\right] = \left\langle 0, \mathrm{in}\middle|\hat{U}^\dagger(j^2, J^2)\hat{U}(j^1, J^1)\middle|0, \mathrm{in}\right\rangle$$

  - Doubling of degrees of freedom $\hat{z}^\mu \to \hat{z}^\mu_1, \hat{z}^\mu_2 \quad \hat{h}^{\mu\nu} \to \hat{h}^{\mu\nu}_1, \hat{h}^{\mu\nu}_2$
  - Initial-value formulation of QFT, valid even in curved spacetime
  - Generates expectation values and correlations of quantum operators
  - Guarantees real and causal equations of motion for expectation values of quantum operators



- **Compare with in-out construction**

$$Z\left[j_\mu, J_{\mu\nu}\right] = \left\langle 0, \mathrm{out}\middle|\hat{U}(j, J)\middle|0, \mathrm{in}\right\rangle$$

  - Useful for scattering processes between asymptotic in and out states
  - In curved spacetime, in-out fails to give real and causal dynamics

CTP generating functional

- **Effective point particle & full spacetime geometry**

$$S_{tot}[z,\bar{g}] = S[\bar{g}] + S_{pp}[z,\bar{g}]$$

- **Long wavelength gravitational perturbations on a vacuum background spacetime**

$$\bar{g}_{\mu\nu} = g_{\mu\nu} + h_{\mu\nu}/m_{pl}$$

$$\Downarrow$$

$$S_{tot}[z,h] = S^{(2)} + S^{(3)} + \cdots + S^{(0)}_{pp} + S^{(1)}_{pp} + S^{(2)}_{pp} + \cdots$$

- **"Integrate out" the gravitational perturbations at scale of background curvature**

$$Z[j^\mu_a, J^{\mu\nu}_a] = \int_{CTP} \mathcal{D}z^\mu_a \, \exp\left\{ iS^{(0)}_{pp}[z^\mu_a] + i\int d\lambda \, j^a_\mu z^\mu_a + iS_{int}\left[z_a, -i\frac{\delta}{\delta J^{\mu\nu}_a} \right] \right\} Z_0[J^{\mu\nu}_a]$$

$$Z_0[J^a_{\mu\nu}] = \exp\left\{ -\frac{1}{2} J^a_{\alpha\beta} \cdot D^{\alpha\beta\gamma'\delta'}_{ab} \cdot J^b_{\gamma'\delta'} \right\} \qquad D^{\alpha\beta\gamma'\delta'}_{ab} = \begin{pmatrix} D^{\alpha\beta\gamma'\delta'}_F & D^{\alpha\beta\gamma'\delta'}_- \\ D^{\alpha\beta\gamma'\delta'}_+ & D^{\alpha\beta\gamma'\delta'}_D \end{pmatrix}$$

Gravitational waves

- **Generating functional for connected correlation functions:**

$$W[j^a_\mu, J^a_{\mu\nu}] = -i \ln Z[j^a_\mu, J^a_{\mu\nu}]$$

- **Derivatives yield (true) expectation values of quantum operators**
  - Example: One-point functions of metric perturbations and worldline coordinates

$$\left\langle \hat{h}^{\mu\nu}_a(x) \right\rangle = \frac{\delta W}{\delta J^a_{\mu\nu}} \qquad \left\langle \hat{z}^\mu_a(\lambda) \right\rangle = \frac{\delta W}{\delta j^a_\mu}$$

- **Waveform of gravitational waves given by**

$$h^{rad}_{\alpha\beta}(x) = \frac{1}{2} \sum_{a=1}^{2} \left\langle \hat{h}^a_{\alpha\beta}(x) \right\rangle$$

  - Possesses a perturbative expansion expressable as Feynman diagrams
  - Needed for computing templates for matched filter searches in LISA

CTP effective action & self-force

- **Why effective action?**
  - Real & causal equations of motion $\Rightarrow$ $$0 = \frac{\delta \Gamma}{\delta \langle \hat{z}_a^\mu(\lambda) \rangle} \bigg|_{z_1 = z_2}$$
  - Self-consistent description of particle and field dynamics
- **Effective action is Legendre transform of generating functional**

$$i\Gamma[\langle \hat{z}_a^\mu \rangle] = -im \int d\tau + \left(\begin{array}{c} \text{sum of all 1PI} \\ \text{connected diagrams} \end{array} \right)$$

  - "1PI" -- one-particle irreducible diagrams
  - "connected" diagrams are those that are contiguous
- **For small quantum corrections only need tree-level diagrams**

$$i\Gamma\big[\langle \hat{z}_a^\mu \rangle\big] = -im \sum_{a=1}^{2} \int d\tau_a + \left(\begin{array}{c} \text{sum of all tree-level} \\ \text{connected diagrams} \end{array} \right) + \cancel{\left(\begin{array}{c} \text{1PI graviton} \\ \text{loop corrections} \end{array} \right)}$$

Review

- **Generating functional**

$$Z$$

- **Gravitational waves from log of generating functional**

$$W = -i \ln Z$$

- **Self-force from effective action**

$$\Gamma = \text{Legendre transform of } W$$

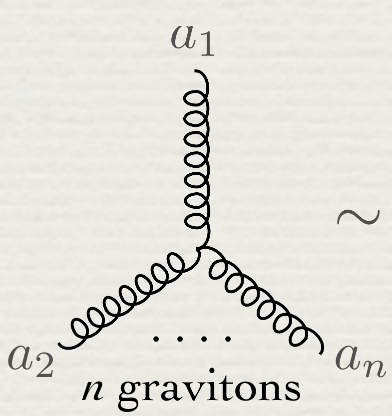
Power counting rules

- **Interaction terms can be interpreted diagrammatically**
  - Streamlines perturbative calculations
  - How does one know what diagrams appear at a given order?
- **Power counting rules:** $x^\mu \sim \mathcal{R}$ $\quad \frac{m}{m_{pl}} \sim \sqrt{\epsilon L}$

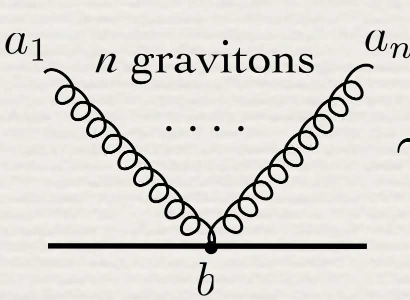
$$h_{\mu\nu} \sim \frac{1}{\mathcal{R}} \qquad L \sim m\mathcal{R}$$

- **Power counting the interactions:** $S_{int}[z,h] = S^{(3)} + \cdots + S^{(1)}_{pp} + S^{(2)}_{pp} + \cdots$

$$S^{(n)} = \int d^4x\, g^{1/2} \frac{V^{(n)}}{n!} h^n \qquad\qquad S^{(n)}_{pp} = \int d\tau \frac{V^{(n)}_{pp}}{n!}\, h^n$$



$$\sim \left(\frac{L}{\varepsilon}\right)^{1-\frac{n}{2}}$$



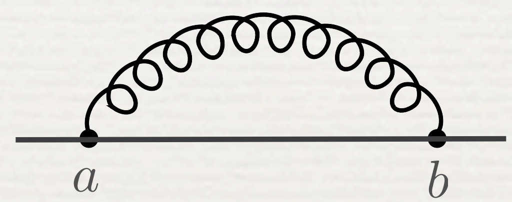
$$\sim \varepsilon \left(\frac{L}{\varepsilon}\right)^{1-\frac{n}{2}}$$

Feynman rules

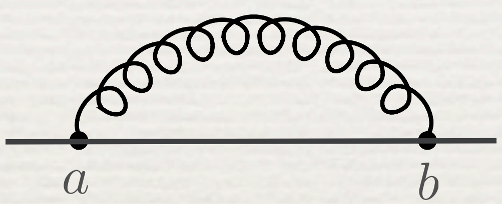
- **Translate diagrams into mathematics using Feynman rules:**
  - Include a factor of $iV_{pp}^{(n)}$ for a particle-field vertex and $iV^{(n)}$ for a field self-interaction vertex
  - Include a factor of $(-1)^{a+1}$ for each vertex with CTP label a
  - Insert a factor of $D_{ab}^{\alpha\beta\gamma'\delta'}(x,x')$ for each graviton line starting (ending) at a vertex labelled by the CTP index $a\,(b)$ at spacetime event x (x')
  - Sum over all CTP indices, integrate over proper time for each particle-field vertex and integrate over all spacetime for each graviton self-interaction vertex
  - Divide by the appropriate symmetry factor
  - For each external graviton, insert a factor of $1/2$ and sum over its CTP index

Example: MSTQW self-force

- **MSTQW self-force -- first order correction to LO geodesic**

$$i\Gamma[\langle \hat{z}^\mu_a \rangle] = -im \int d\tau +$$ 

$$+O(\varepsilon^2 L)$$



$$= \left(\frac{1}{2!}\right)\left(\frac{im}{2m_{pl}}\right)^2 \sum_{a,b=1}^{2} (-1)^{a+b} \int d\tau \int d\tau' \, \dot{z}^\alpha_a \dot{z}^\beta_a D^{ab}_{\alpha\beta\gamma'\delta'}(z^\mu_a, z^{\mu'}_b) \dot{z}^{\gamma'}_b \dot{z}^{\delta'}_b$$

- **Worldline fluctuations are small (decoherence) so expand to first order in coordinate difference** $z^\mu_- = z^\mu_1 - z^\mu_2$

$$i\Gamma[\langle \hat{z}^\mu_a \rangle] = -im \int d\tau \, z^\mu_- a_{+\mu} + \frac{im^2}{2m_{pl}^2} \int d\tau_+ \int d\tau'_+ \, z^\mu_- w_\mu{}^{\alpha\beta\nu} \nabla_\nu D^{ret}_{\alpha\beta\gamma'\delta'}(z^\mu_+, z^{\mu'}_+) \dot{z}^{\gamma'}_+ \dot{z}^{\delta'}_+ + O(z^2_-)$$

- **Integral of retarded propagator diverges at coincidence...**

Dimensional regularization

- **Many techniques developed to regularize UV divergences in QFT**
  - Pauli-Villars, cut-off, point-splitting, dimensional regularization,...
- **Dimensional regularization is particularly attractive**
  - Covariant
  - Preserves gauge symmetry
  - Mass-independent scheme
- **Mass-independent scheme**
  - Beta functions involve only logarithms of renormalization scale μ
  - Power divergences vanish in 4d for massless fields
  - Logarithmic divergences renormalize non-minimal couplings -- **RG flow**
  - Fewer Feynman diagrams to compute vs mass-dependent schemes

Regularizing the effective action

- **UV divergences are quasi-local but effective action is non-local**
- **Use Hadamard's *partie finie* method:** $D^{ren} \equiv D_{ret} - D^{div} = Pf(D_{ret})$
  - Renormalized propagator as pseudo-function
  - Integral of pseudo-function gives the finite part (by definition)

$$Fp \int_{-\infty}^{\infty} d\tau' \, D_{ret}(z^\alpha, z^{\alpha'}) j(\tau')$$
$$= \lim_{\epsilon \to 0} \left(\int_{-\infty}^{\tau-\epsilon} + \int_{\tau+\epsilon}^{\infty} \right) d\tau' \, D_{ret}(z^\alpha, z^{\alpha'}) j(\tau') - \int_{-\infty}^{\infty} d\tau' \, D^{div}(z^\alpha, z^{\alpha'}) j(\tau')$$

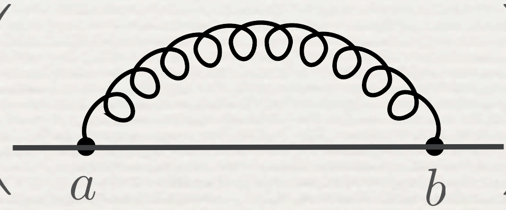
- **Divergent part is evaluated using momentum space techniques**
  - Familiar techniques and manipulations from flat spacetime QFT
  - Originally developed for scalar field by Bunch & Parker
  - Valid in any dimension & for massive, non-minimally coupled field, etc.

(Finite) MSTQW self-force

Divergence in first order effective action

Power divergence

Vanishes

Divergent part $\Bigg($ 

$\Bigg) \propto w^{mi} \int_{\mathbf{k}} \frac{k_i}{\mathbf{k}^2}$ and $\eta^{m(i}\eta_0^{a)} R_{aj0k} \int_{\mathbf{k}} \frac{k_i k^j k^k}{\mathbf{k}^6} = 0$

- **Finite & non-local remainder of effective action yields self-force equations of MSTQW**

$$\boxed{m a^\mu(\tau) = \frac{m^2}{2m_{pl}^2} w^{\mu\alpha\beta\nu}[z^\alpha] \lim_{\epsilon \to 0} \int_{-\infty}^{\tau-\epsilon} d\tau' \, \nabla_\nu D^{ret}_{\alpha\beta\gamma'\delta'}(z^\mu, z^{\mu'}) \dot{z}^{\gamma'} \dot{z}^{\delta'}}$$

Mino, Sasaki & Tanaka, PRD **55**, 3457 (1997)
Quinn & Wald, PRD **56**, 3381 (1997)

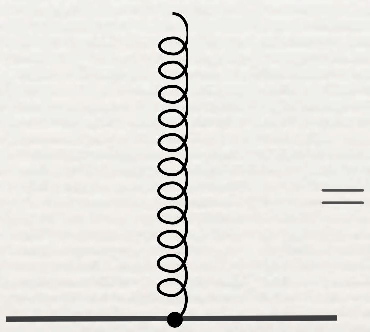
  - Real, causal and history-dependent -- guaranteed with CTP
  - Does not provide sufficiently accurate waveforms to precisely measure parameters of GW sources of LISA mission

Recipe for self-force

- **Follow recipe for calculating self-force at desired order:**
  - Use power counting rules to draw all connected tree-level Feynman diagrams at desired order
  - Use Feynman rules to translate diagrams into mathematical expressions
  - Expand the effective action to linear order in the worldline difference coordinate (assume astrophysical bodies are strongly decohered)
  - Isolate the quasi-local divergent part from the non-local finite part using Hadamard's *partie finie*
  - Use dimensional regularization to renormalize the mass and non-minimal couplings in the effective point particle action
  - Vary the resulting (finite) effective action with respect to the difference coordinate to find the self-force equation

Example: GWs from a geodesic

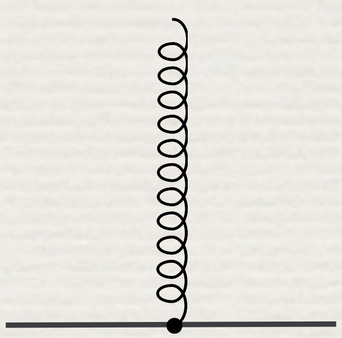
- Use Feynman rules to translate diagram



$$= \left(\frac{1}{1!}\right)\left(\frac{im}{2m_{pl}}\right)\sum_{m,a=1}^{2}(-1)^{a+1}\int d\tau'_a\, D_{ma\,\alpha\beta\gamma'\delta'}(x, z_a^{\mu'})u_a^{\gamma'}u_a^{\delta'}$$

- Sum over CTP indices and take limit that $z_2^\mu \to z_1^\mu = z^\mu$

$$\sum_{m,a=1}^{2}(-1)^{a+1}D_{ma\,\alpha\beta\gamma'\delta'}(x,x') = -2iD^{ret}_{\alpha\beta\gamma'\delta'}(x,x')$$



$$= \frac{m}{2m_{pl}}\int d\tau'\, D^{ret}_{\alpha\beta\gamma'\delta'}(x, z^{\mu'})u^{\gamma'}u^{\delta'}$$

Effacement Principle for EMRIs

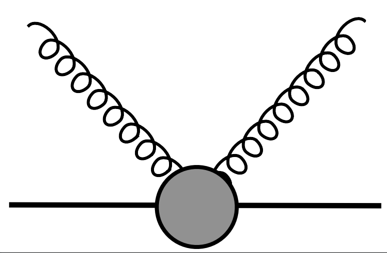
- **Action for effective point particle**

$$S_{pp}[z] = -m\int d\tau + c_E\int d\tau\,\mathcal{E}_{\alpha\beta}\mathcal{E}^{\alpha\beta} + c_B\int d\tau\,\mathcal{B}_{\alpha\beta}\mathcal{B}^{\alpha\beta} + \cdots$$

- **Non-minimal couplings determined through matching calculation**
  - Need to know the low-energy expansion of observables for full theory
  - To demonstrate matching procedure, an order of magnitude estimate suffices
- **Perturb geometry about a given vacuum background**

$$\mathcal{E}_{\alpha\beta} = \mathcal{E}^{(0)}_{\alpha\beta} + \mathcal{E}^{(1)}_{\alpha\beta} + \mathcal{E}^{(2)}_{\alpha\beta} + \cdots$$

- **Compute amplitude for graviton Compton scattering**



$$\sim \cdots \& \frac{c_{E,B}}{m_{pl}^2}\left(\frac{1}{\mathcal{R}^2}\right)^2 \& \cdots$$

Effacement Principle for EMRIs

- **Scattering cross-section in effective point particle theory**

$$\sigma_{pp} \sim \cdots \& \frac{c_{E,B}^2}{m_{pl}^4}\frac{1}{\mathcal{R}^8} \& \cdots$$

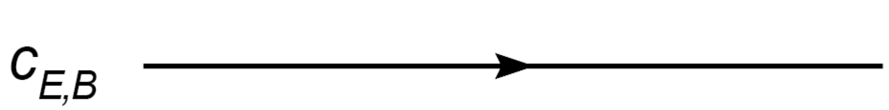
- **Scattering cross-section in full theory**

$$\sigma_{full} = r_m^2 f\left(\frac{r_m}{\mathcal{R}}\right) \sim \cdots \& \frac{r_m^{10}}{\mathcal{R}^8} \& \cdots$$

- **Match the cross-sections:**

$$c_{E,B} \sim m_{pl}^2 r_m^5 \sim \frac{m^5}{m_{pl}^8}$$

- **Leading order diagram due to finite size effects**



$$\sim c_{E,B}\int d\tau \left(\frac{1}{\mathcal{R}^2}\right)^2 \sim \varepsilon^4 L$$

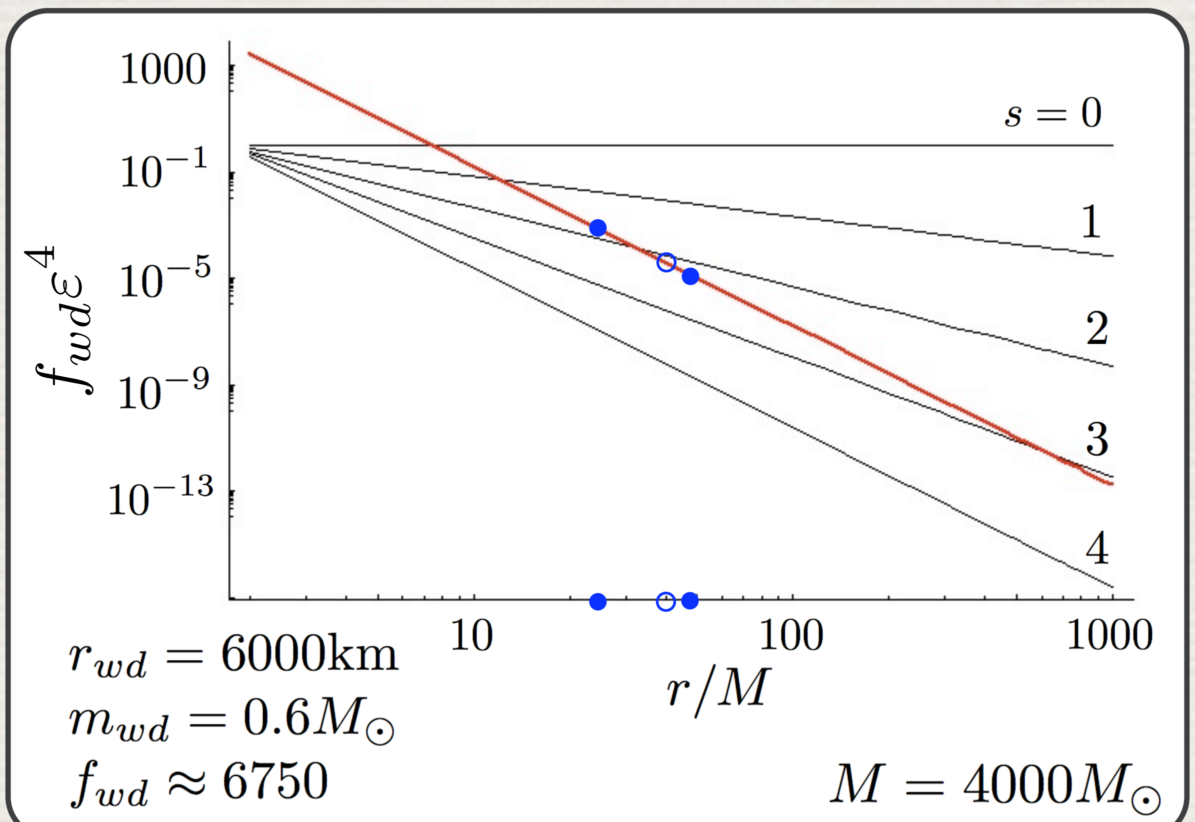
Effacement Principle for EMRIs

- **For a white dwarf (WD) finite size effects are enhanced due to stronger tidal interactions with companion SMBH**

Diagram with leading order induced tidal moments:

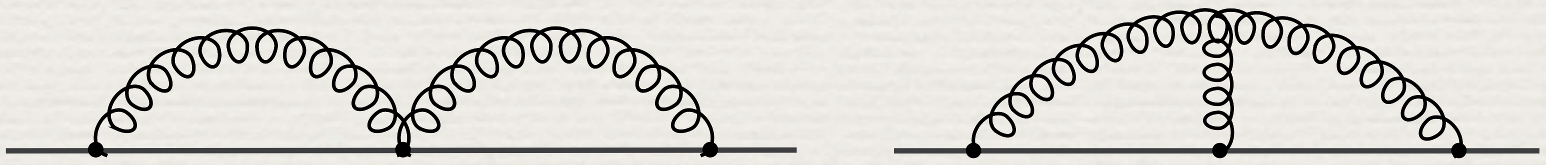
$r_m = f_{co} G m$ $\Rightarrow$ $c_{E,B} \longrightarrow \sim f_{co} \varepsilon^4 L \sim \varepsilon^s L$

For a Schwarzschild SMBH: $\varepsilon \equiv \dfrac{r_m}{\mathcal{R}} = 2^{3/4} 3^{1/4} f_{co} \dfrac{m}{M} \left(\dfrac{M}{r} \right)^{3/2}$

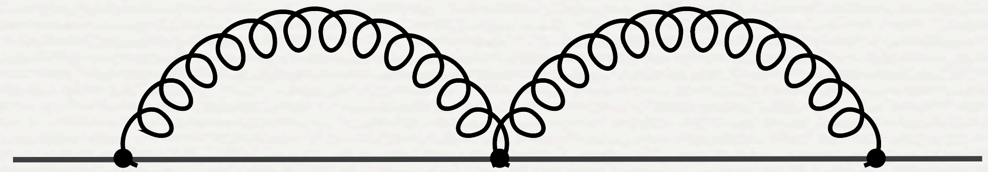


Second order self-force (in progress)

- **Previous work on second order self-force**
  - First formal expression given by Rosenthal Rosenthal, PRD **74**, 084018 (2006)
  - Expressed in a non-standard gauge
  - Potentially difficult to implement in practical calculations
- **Second order self-force calculations with EFT:**
  - All orders computed in same gauge throughout (Lorenz here)
  - Feynman diagrams:



  - First diagram represents first non-linear particle-field contribution
  - Second diagram represents first contribution from nonlinearities of GR



- **Contribution to second order effective action** $O(\varepsilon^2 L)$

$$= -\frac{im^3}{8m_{pl}^4}\int d\tau_+ z_-^{\mu}(\tau) w_{\mu}{}^{\alpha\beta\nu}\int d\tau' \nabla_\nu D^{ret}_{\alpha\beta\gamma'\delta'}(z_+^{\mu}, z_+^{\mu'}) u_+^{\gamma'} u_+^{\delta'} u_+^{\epsilon'} u_+^{\zeta'} \int d\tau''_+ \, D^{ret}_{\epsilon'\zeta'\eta''\theta''}(z_+^{\mu'}, z_+^{\mu''}) u_+^{\eta''} u_+^{\theta''}$$

$$-\frac{im^3}{4m_{pl}^4}\int d\tau \, z_-^{\mu} w_{\mu}{}^{\alpha\beta\gamma\delta\nu}\int d\tau' \nabla_\nu D^{ret}_{\alpha\beta\epsilon'\zeta'}(z_+^{\mu}, z_+^{\mu'}) u_+^{\epsilon'} u_+^{\zeta'} \int d\tau'' \, D^{ret}_{\gamma\delta\eta''\theta''}(z_+^{\mu}, z_+^{\mu'}) u_+^{\eta''} u_+^{\theta''}$$

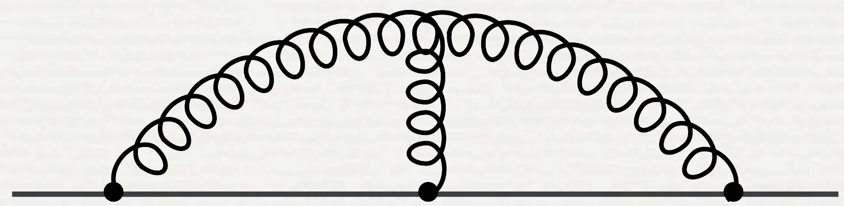
- **Regularize:** $D^{ren} = D^{ret} - D^{div}$

$$I_\mu^{(1)} = w_{\mu}{}^{\alpha\beta\nu}\int d\tau' \nabla_\nu D^{ren}_{\alpha\beta\gamma'\delta'} u_+^{\gamma'\delta'\epsilon'\zeta'} \int d\tau''_+ \, D^{div}_{\epsilon'\zeta'\eta''\theta''} u_+^{\eta''\theta''} = 0$$

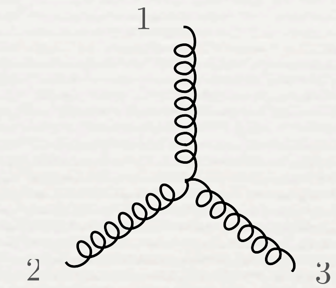
$$I_\mu^{(2)} = w_{\mu}{}^{\alpha\beta\nu}\int d\tau' \nabla_\nu D^{div}_{\alpha\beta\gamma'\delta'} u_+^{\gamma'\delta'\epsilon'\zeta'} \int d\tau''_+ \, D^{ren}_{\epsilon'\zeta'\eta''\theta''} u_+^{\eta''\theta''} \equiv 0$$

$$I_\mu^{(3)} = w_{\mu}{}^{\alpha\beta\nu}\int d\tau' \nabla_\nu D^{div}_{\alpha\beta\gamma'\delta'} u_+^{\gamma'\delta'\epsilon'\zeta'} \int d\tau''_+ \, D^{div}_{\epsilon'\zeta'\eta''\theta''} u_+^{\eta''\theta''} = 0$$

- **All divergences vanish leaving a regular remainder (finite part)**

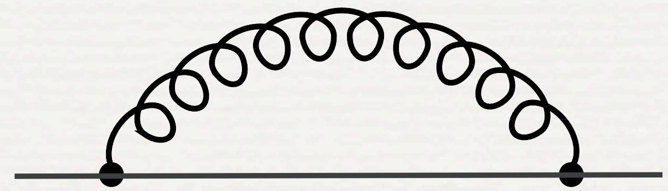


- **This diagram is built upon the 3-graviton vertex**



$$
\begin{aligned}
&V^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3}(g) = \\
&+\frac{1}{4}\, I^{\mu_1\nu_1\mu_2\nu_2}\, I^{\mu_3\nu_3}{}_{\alpha\beta}\, \nabla_1^\alpha \nabla_3^\beta + \frac{1}{4}\, I^{\mu_1\nu_1\mu_3\nu_3}\, I^{\mu_2\nu_2}{}_{\alpha\beta}\, \nabla_1^\alpha \nabla_3^\beta + \frac{1}{2}\, I^{\mu_2\nu_2\mu_3\nu_3}\, I^{\mu_1\nu_1}{}_{\alpha\beta}\, \nabla_1^\alpha \nabla_3^\beta \\
&-\frac{1}{2}\, I_{\alpha\gamma}{}^{\mu_3\nu_3}\, I_{\beta\delta}{}^{\mu_1\nu_1}\, I^{\gamma\delta\mu_2\nu_2}\, \nabla_1^\alpha \nabla_3^\beta - \frac{1}{2}\, I_{\alpha\gamma}{}^{\mu_2\nu_2}\, I_{\beta\delta}{}^{\mu_1\nu_1}\, I^{\gamma\delta\mu_3\nu_3}\, \nabla_1^\alpha \nabla_3^\beta - \frac{1}{2}\, I_{\alpha\gamma}{}^{\mu_3\nu_3}\, I_{\beta\delta}{}^{\mu_2\nu_2}\, I^{\gamma\delta\mu_1\nu_1}\, \nabla_1^\alpha \nabla_3^\beta \\
&-\frac{1}{4}\, I^{\mu_1\nu_1\mu_2\nu_2}\, I^{\mu_3\nu_3}{}_{\alpha\beta}\, \nabla_2^\alpha \nabla_3^\beta + \frac{1}{2}\, I_{\alpha\gamma}{}^{\mu_2\nu_2}\, I_\beta{}^{\gamma\mu_3\nu_3}\, I^{\alpha\beta\mu_1\nu_1}\, \nabla_1^\rho \nabla_{3\rho} - \frac{1}{2}\, g^{\mu_1\nu_1} I^{\mu_2\nu_2\mu_3\nu_3}\, \nabla_1^\rho \nabla_{3\rho} \\
&-\frac{1}{8}\, g^{\mu_1\nu_1} I^{\mu_2\nu_2\mu_3\nu_3}\, \nabla_2^\rho \nabla_{3\rho} + \frac{1}{2}\, g^{\mu_1\nu_1} I_\alpha{}^{\gamma\mu_3\nu_3}\, I_{\gamma\beta}{}^{\mu_2\nu_2}\, \nabla_1^\alpha \nabla_3^\beta + \frac{1}{4}\, g^{\mu_1\nu_1} I_\alpha{}^{\gamma\mu_3\nu_3}\, I_{\gamma\beta}{}^{\mu_2\nu_2}\, \nabla_2^\alpha \nabla_3^\beta \\
&+\frac{1}{4}\, g^{\mu_2\nu_2} I^{\mu_1\nu_1\mu_3\nu_3}\, \nabla_2^\rho \nabla_{3\rho} - \frac{1}{2}\, g^{\mu_2\nu_2} I_\alpha{}^{\gamma\mu_3\nu_3}\, I_{\gamma\beta}{}^{\mu_1\nu_1}\, \nabla_2^\alpha \nabla_3^\beta - \frac{1}{8}\, g^{\mu_1\nu_1} g^{\mu_2\nu_2} I^{\mu_3\nu_3}{}_{\alpha\beta}\, \nabla_1^\alpha \nabla_3^\beta \\
&+\frac{1}{8}\, g^{\mu_1\nu_1} g^{\mu_2\nu_2}\, I^{\mu_3\nu_3}{}_{\alpha\beta}\, \nabla_2^\alpha \nabla_3^\beta - \frac{1}{4}\, g^{\mu_3\nu_3} I^{\mu_1\nu_1\mu_2\nu_2}\, \nabla_1^\rho \nabla_{3\rho} + \frac{1}{2}\, g^{\mu_3\nu_3} I_\alpha{}^{\gamma\mu_2\nu_2}\, I_{\gamma\beta}{}^{\mu_1\nu_1}\, \nabla_1^\alpha \nabla_3^\beta \\
&+\frac{1}{2}\, g^{\mu_3\nu_3} I_\alpha{}^{\gamma\mu_2\nu_2}\, I_{\gamma\beta}{}^{\mu_1\nu_1}\, \nabla_2^\alpha \nabla_3^\beta - \frac{1}{4}\, g^{\mu_1\nu_1} g^{\mu_3\nu_3} I^{\mu_2\nu_2}{}_{\alpha\beta}\, \nabla_1^\alpha \nabla_3^\beta - \frac{1}{4}\, g^{\mu_1\nu_1} g^{\mu_3\nu_3} I^{\mu_2\nu_2}{}_{\alpha\beta}\, \nabla_2^\alpha \nabla_3^\beta \\
&+\frac{1}{8}\, g^{\mu_1\nu_1} g^{\mu_2\nu_2} g^{\mu_3\nu_3}\, \nabla_1^\rho \nabla_{3\rho} + \big(\text{permutations of } \{1,2,3\}\big)
\end{aligned}
$$

- **Calculation in progress but easily doable…**



- **Second order contribution from effective action at** $O(\varepsilon L)$

$$= \frac{im^2}{2m_{pl}^2}\int d\tau\, z_-^\mu(\tau)\, a_\mu{}^{\alpha\beta}\int d\tau'\, D^{ren}_{\alpha\beta\gamma'\delta'}(z_+^\mu, z_+^{\mu'})\, u_+^{\gamma'} u_+^{\delta'}$$

$$a_\mu{}^{\alpha\beta} = -\frac{1}{2} u^\alpha u^\beta a_\mu + w_\mu{}^{(\alpha} a^{\beta)}$$

- **Can interpret as a correction to CO's mass**

$$m^{eff}_{\mu\nu}(\tau) = m g_{\mu\nu} + \frac{m^2}{2m_{pl}^2}\left(\frac{1}{2} g_{\mu\nu} u^\alpha u^\beta - w_\mu{}^{(\alpha} g^{\beta)}_{\nu}\right)\int d\tau'\, D^{ren}_{\alpha\beta\gamma'\delta'} u^{\gamma'} u^{\delta'}$$

- **Can interpret as contribution to 2nd order self-force with constant mass**

$$= -\frac{im^3}{8m_{pl}^4}\int d\tau\, z_-^\mu \left[w_\mu{}^{\alpha\beta\nu} u_+^{\gamma\delta} + 2 w_\mu{}^{(\gamma} w^{\delta)\alpha\beta\nu}\right]\int d\tau' \nabla_\nu D^{ren}_{\alpha\beta\epsilon'\zeta'} u_+^{\epsilon'\zeta'}\int d\tau''\, D^{ren}_{\gamma\delta\eta''\theta''} u_+^{\eta''\theta''}$$

Second order self-force equations

- **Second order self-force so far...**

$$a_\mu = \mathrm{MSTQW} - \frac{m^3}{8m_{pl}^4} w_\mu{}^{\alpha\beta\nu} \int d\tau' \, \nabla_\nu D^{ren}_{\alpha\beta\gamma'\delta'}(z^\mu, z^{\mu'}) u^{\gamma'\delta'\epsilon'\zeta'} \int d\tau'' \, D^{ren}_{\epsilon'\zeta'\eta''\theta''}(z^{\mu'}, z^{\mu''}) u^{\eta''\theta''}$$

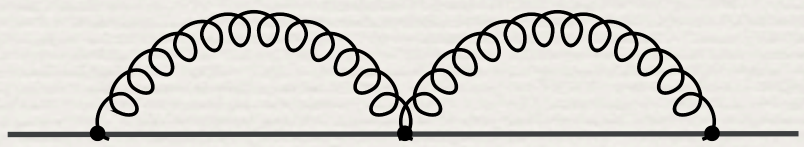
$$-\frac{m^3}{4m_{pl}^4}\left[w_\mu{}^{\alpha\beta\gamma\delta\nu} + \frac{1}{2} w_\mu{}^{\alpha\beta\nu} u^{\gamma\delta} + w_\mu{}^{(\gamma} w^{\delta)\alpha\beta\nu} \right]$$

$$\times \int d\tau' \, \nabla_\nu D^{ren}_{\alpha\beta\gamma'\delta'}(z^\mu, z^{\mu'}) u^{\epsilon'\zeta'} \int d\tau'' \, D^{ren}_{\gamma\delta\eta''\theta''}(z^{\mu'}, z^{\mu''}) u^{\eta''\theta''}$$

$$+\left(\text{diagram with } 3-\text{graviton vertex}\right)$$

Necessity of CTP formalism

- **Other approaches to potentially study EMRIs**
  - Goldberger and Rothstein's approach extended to curved spacetime (PN-EFT, in-out)
  - Kol and Smolkin's Classical EFT (ClEFT)
- **What is the problem with using these other formalisms?**
  - Non-causal self-force equations of motion
  - For example, one diagram of second order self-force in PN-EFT and ClEFT



$$\propto \int d\tau d\tau' d\tau''\, u^\alpha u^\beta D^F_{\alpha\beta\gamma'\delta'} u^{\gamma'} u^{\delta'} u^{\epsilon'} u^{\zeta'} D^F_{\epsilon'\zeta'\eta''\theta''} u^{\eta''} u^{\theta''}$$

$$D_F = -\frac{i}{2}\left(D_{ret} + D_{adv}\right) + \frac{1}{2} D_H \qquad D^H_{\alpha\beta\gamma'\delta'} = \langle \{ \hat{h}_{\alpha\beta}(x), \hat{h}_{\gamma'\delta'}(x') \} \rangle$$

  - Real part of variation gives equations of motion, which are not causal
  - Or, replace Feynman propagator with $(D_{ret} + D_{adv})/2$ but self-force then involves advanced propagator...

Nonlinear scalar gravity

- **May be useful as a toy model:**
  - Study practical methods for computing higher order self-force and radiation
  - Can check computations with known results at first order (more to say later...)
  - May help to resolve issues regarding relevance/importance of higher order perturbations (e.g., 2nd order self-force)
  - Derive higher order self-force and radiation as a model for IMRIs
  - Include finite size effects, which might be relevant for IMRIs

Nonlinear scalar gravity

- **A class of non-linear scalar models on a vacuum background:**

$$S[z,\phi] = -\frac{1}{2}\int d^4x\, g^{1/2}\phi_{,\alpha}\phi^{,\alpha} A^2\big(\phi/m_{pl}\big) - m\int d\tau\, B\big(\phi/m_{pl}\big)$$

- **Equations of motion:**

$$\Box\phi = -\frac{1}{m_{pl}}\,\frac{A'}{A}\,\phi_{,\alpha}\phi^{,\alpha} + \frac{m}{m_{pl}}\int d\tau\,\frac{\delta^4(x-z)}{g^{1/2}}\,\frac{B'}{A^2}$$

$$a^\mu = -w^{\mu\nu}\nabla_\nu \ln B$$

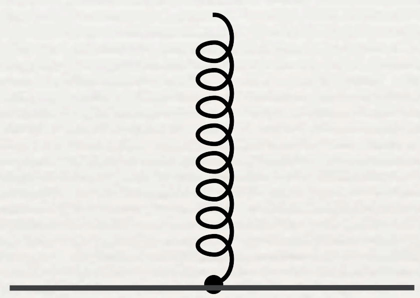
  - Leading order motion is geodesic
  - Expand A and B functions for small field

$$A^2\big(\phi/m_{pl}\big) = 1 + \sum_{n=1}^{\infty}\frac{a_n}{n!}\left(\frac{\phi}{m_{pl}}\right)^n \qquad B\big(\phi/m_{pl}\big) = 1 + \sum_{n=1}^{\infty}\frac{b_n}{n!}\left(\frac{\phi}{m_{pl}}\right)^n$$

- **Same power counting, same diagrammatic structure, same Effacement Principle**

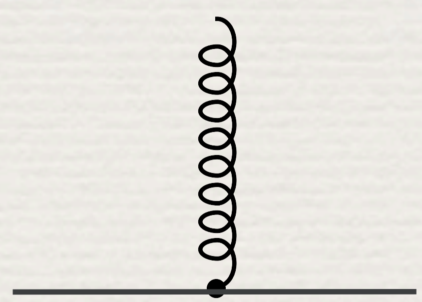
First order waves & self-force

- **Feynman diagram for leading order scalar perturbations**



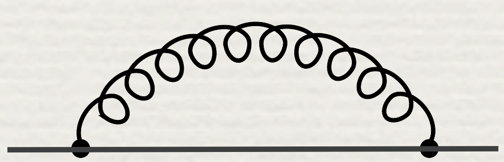
$$= \frac{1}{2}\sum_{a,b=1}^{2}(-1)^{a+1}\left(-\frac{imb_1}{m_{pl}}\right)\int d\tau\, D_{ab}(x, z_b^\alpha) \sim \varepsilon^0\sqrt{\varepsilon L}$$

- **Sum over CTP indices and set difference coordinate to zero**



$$= -\frac{mb_1}{m_{pl}}\int d\tau\, D_{ret}(x, z^\alpha)$$

- **First order self-force**

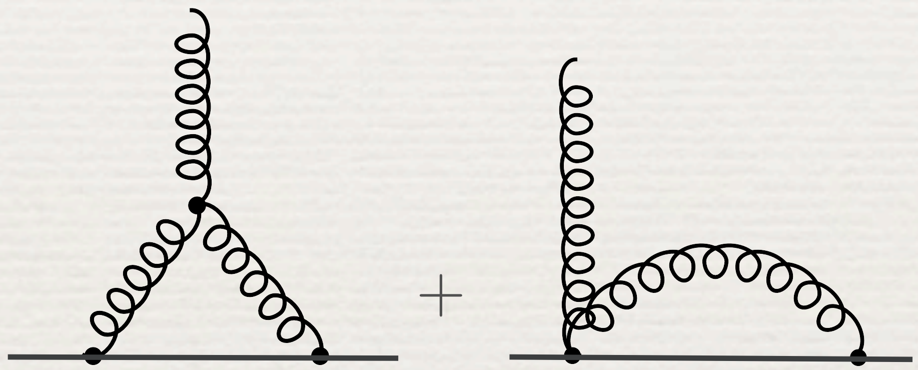


$$= \frac{im^2b_1^2}{m_{pl}^2}\int d\tau\, z_-^\mu\,(a_\mu + w_\mu{}^\nu\nabla_\nu)\,D_{ren}(z^\mu, z^{\mu'})$$

Second order waves

- **Corrections to geodesic motion from first order self-force create second order perturbations of scalar waves**

- **Feynman diagrams**



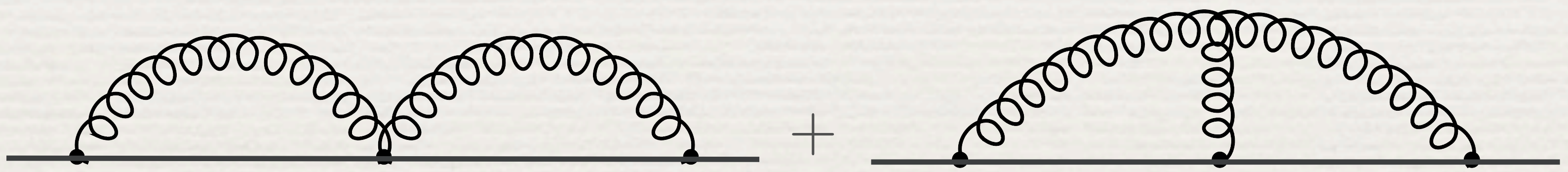
$$= -\frac{a_1}{4m_{pl}}\left(-\frac{mb_1}{m_{pl}}\int d\tau\, D_{ret}(x,z^\mu)\right)^2$$

$$+\frac{m^2}{m_{pl}^3}b_1^2\left(\frac{b_2}{b_1}-\frac{a_1}{2}\right)\int d\tau\, D_{ret}(x,z^\mu)\int d\tau'\, D_{ren}(z^\mu,z^{\mu'})$$

  - First term is proportional to square of first order field
  - Second term is regulated because of UV power divergence on worldline

Second order self-force

- **Second order self-force caused by CO's interaction with first & second order field perturbations**
- **Feynman diagrams (use recipe to evaluate)**



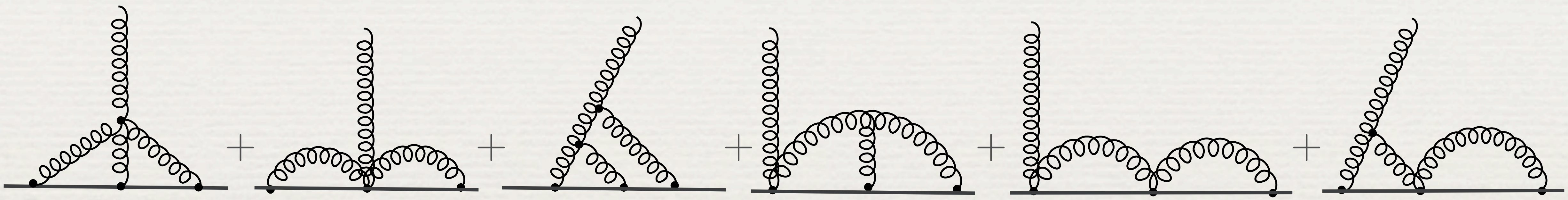
$$= -\frac{im^3 b_1^4}{m_{pl}^4}\left(\frac{b_2}{b_1} - \frac{a_1}{2}\right)\int d\tau\, z_-^\mu(\tau)\big(a_\mu + w_\mu{}^\nu \nabla_\nu\big)$$
$$\times \int d\tau' \int d\tau'' \left[D_{ren}(z^\mu, z^{\mu'}) D_{ren}(z^{\mu'}, z^{\mu''}) + \frac{1}{2} D_{ren}(z^\mu, z^{\mu'}) D_{ren}(z^\mu, z^{\mu''})\right]$$

- Second term proportional to square of first order self-force

Third order waves

- **Third order scalar perturbations created by second order corrections to CO's leading order geodesic motion**

- **Feynman diagrams**



$$
\begin{aligned}
&= -\frac{m^3}{2m_{pl}^6}\left[\left(-\frac{a_2}{2}+a_1^2\right)b_1^3-\frac{3}{2}a_1 b_1^2 b_2+b_1^2 b_3\right]\int d\tau\, D_{ret}(x,z^{\mu})\left(\int d\tau'\, D_{ren}(z^{\mu},z^{\mu'})\right)^2\\
&\quad -\frac{m^3}{m_{pl}^6}b_1^3\left(\frac{b_2}{b_1}-\frac{a_1}{2}\right)^2\int d\tau\, D_{ret}(x,z^{\mu})\int d\tau'\, D_{ren}(z^{\mu},z^{\mu'})\int d\tau''\, D_{ren}(z^{\mu'},z^{\mu''})\\
&\quad +\frac{m^3}{2m_{pl}^6}a_1 b_1^3\left(\frac{b_2}{b_1}-\frac{a_1}{2}\right)\int d\tau\, D_{ret}(x,z^{\mu})\int d\tau''\, D_{ret}(x,z^{\mu'})\int d\tau''\, D_{ren}(z^{\mu'},z^{\mu''})\\
&\quad -\frac{m^3}{6m_{pl}^6}b_1^3\left(-\frac{a_2}{2}+a_1^2\right)\int d\tau\, D_{ret}(x,z^{\mu})\int d\tau'\, D_{ret}(x,z^{\mu'})\int d\tau''\, D_{ret}(x,z^{\mu''})
\end{aligned}
$$

Rosenthal's scalar model

- **"Monopole" source:** $A = e^{-\phi/m_{pl}}$ $\quad B = \frac{1}{2} + \frac{1}{2}e^{-\phi/m_{pl}}$

$$\Box\phi = \frac{1}{m_{pl}}\phi_{,\alpha}\phi^{,\alpha} - \frac{m}{2m_{pl}}\int d\tau\, \frac{\delta^4(x-z)}{g^{1/2}}\, e^{+\phi/m_{pl}}$$

  - Second order scalar perturbations agrees with Rosenthal

$$\phi^{rad}_{(2)}(x) = \frac{1}{2m_{pl}}\left(\frac{m}{m_{pl}}\int d\tau\, D_{ret}(x, z^\mu)\right)^2 - \frac{m^2}{m^3_{pl}}\int d\tau\, D_{ret}(x, z^\mu)\int d\tau' D_{ren}(z^\mu, z^{\mu'})$$

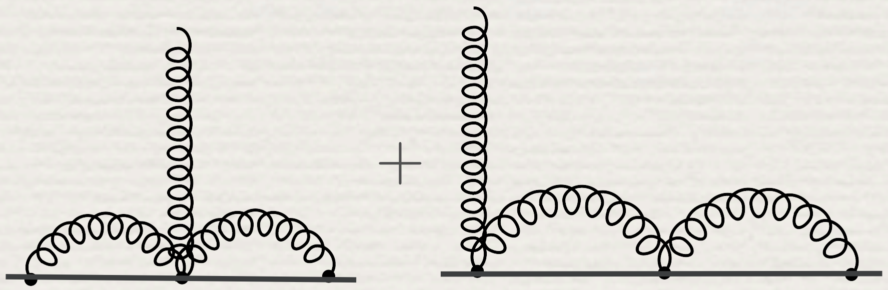
- **A model where:** $A = B = e^{-\phi/m_{pl}}$
  - Only first order self-force (linear in propagator) is non-zero
  - *n*th order radiation is proportional to *n*th power of first order waves

Choosing "nice" variables

- **The number of diagrams can be reduced by considering a field redefinition** $\phi_{,\alpha}(x)A = \sigma_{,\alpha}(x)$ $\quad C(\sigma/m_{pl}) = B(\phi/m_{pl})$

$$S[z,\sigma] = -\frac{1}{2}\int d^4x\, g^{1/2}\sigma_{,\alpha}\sigma^{,\alpha} - m\int d\tau\, C(\sigma/m_{pl})$$

  - No field self-interactions
  - For example, third order radiation diagrams number from 6 to 2



- **Does a similar thing happen in the gravitational case?**
  - Equivalent to a different gauge choice -- could ease self-force computations
  - Full metric is gauge dependent and probably does not help ease waveforms

Relevance of 2nd order self-force?

- **Choose A and B such that**

$$A = e^{\phi/m_{pl}} \qquad B = 2 - e^{\phi/m_{pl}} = 1 - \frac{\phi}{m_{pl}} + \cdots$$

- **Nonlinear scalar theory:**

$$S[z,\phi] = -\frac{1}{2}\int d^4x\, g^{1/2}\phi_{,\alpha}\phi^{,\alpha}e^{2\phi/m_{pl}} - m\int d\tau\left(2 - e^{\phi/m_{pl}}\right)$$

- **Field redefinition gives a linear scalar theory**

$$\sigma_{,\alpha} = \phi_{,\alpha}A(\phi/m_{pl}) \Rightarrow \frac{\sigma}{m_{pl}} = e^{\phi/m_{pl}} - 1$$

  - Can easily calculate self-force and radiation in linear theory, transform back to nonlinear field and determine if second order self-force is actually relevant

$$S[z,\sigma] = -\frac{1}{2}\int d^4x\, g^{1/2}\sigma_{,\alpha}\sigma^{,\alpha} - m\int d\tau + \frac{m}{m_{pl}}\int d\tau\,\sigma$$

Self-force on spinning CO's

- **Introduce a tetrad on worldline that rotates with the CO**
  - Angular velocity $\Omega_{IJ} = e^{\mu}_{J} \dfrac{De_{\mu I}}{d\lambda}$
  - Spin angular momentum -- conjugate to angular velocity $S^{IJ} = -\dfrac{\delta S}{\delta \Omega_{IJ}}$
- **Effective point particle with spin**

$$S_{pp}[z,g] = -m \int d\tau + \frac{1}{2} \int d\tau \, S^{IJ} \Omega_{IJ} + \cdots$$

- **Power counting**

$$S \approx I\Omega \sim \frac{m^3}{m_{pl}^4} \frac{v_{rot}}{r_m} \Rightarrow \boxed{S \sim \mu^s L}$$

  - Maximally rotating CO: $v_{rot} \sim 1, \;\; s = 1$
  - Corotating CO: $\dfrac{v_{rot}}{r_m} \sim \dfrac{v}{r}, \;\; s = 2$
- **Spin Supplementary Condition (SSC) fixes center of center of mass**

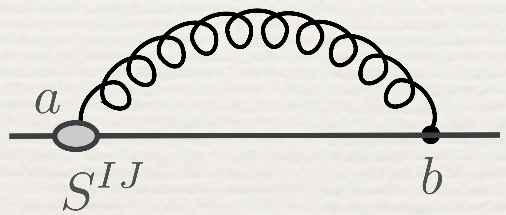
$$S^{\mu\nu} p_{\nu} = 0$$

Maximally rotating compact object

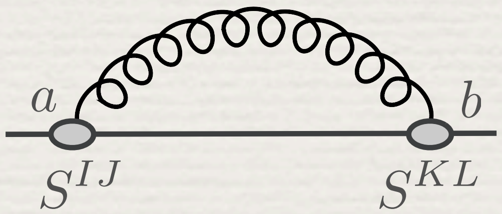
- **First order describes spin precession & MSTQW**

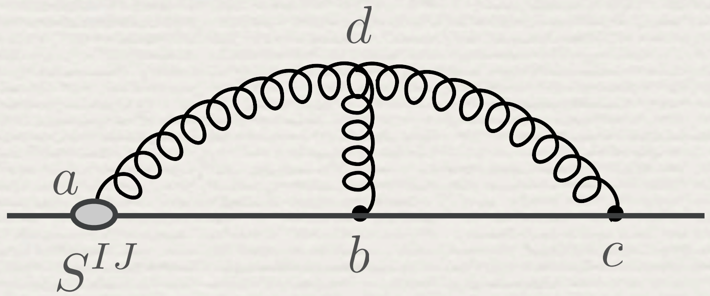
$$\frac{DS^{\mu\nu}}{d\tau} = p^\mu u^\nu - p^\nu u^\mu \qquad m\, a^\mu = -\frac{1}{2} R^\mu{}_{\alpha\beta\gamma} \dot{z}^\alpha S^{\beta\gamma} + \mathrm{MSTQW}$$

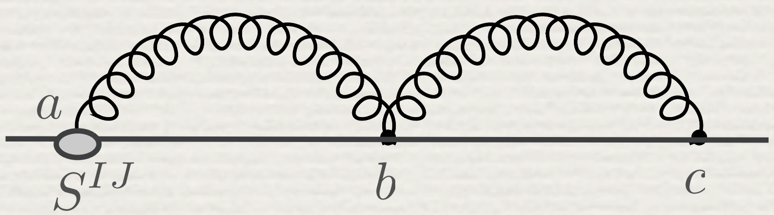
- **Second order contains leading order (LO) spin-orbit interaction**

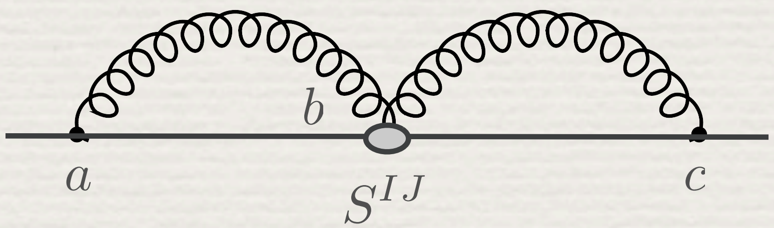


- **Third order contains LO spin-spin and NLO spin-orbit interactions**









Co-rotating compact object

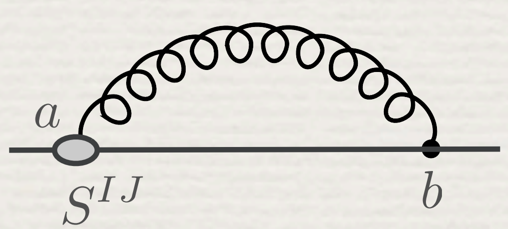
- **First order describes MSTQW self-force alone**

$$m\,a^\mu = MSTQW$$

- **Second order describes spin precession**

$$\frac{DS^{\mu\nu}}{d\tau} = p^\mu u^\nu - p^\nu u^\mu \qquad\qquad m\,a^\mu = -\frac{1}{2}R^\mu{}_{\alpha\beta\gamma}\dot{z}^\alpha S^{\beta\gamma} + \mathrm{MSTQW}$$

- **Third order contains LO spin-orbit interaction**



Conclusions

- **Compact object as an effective point particle**
  - Describes tidally induced moments, spin and intrinsic moments
- **EFT provides a systematic & efficient method for higher order self-force and radiation calculations**
- **Divergences are efficiently and unambiguously regularized**
  - Power divergences vanish & log divergences renormalize non-minimal couplings
- **An Effacement Principle for EMRIs**
  - Internal structure of COs affect motion at fourth order
  - WD tidal disruption a second order process?
- **Computed 2/3 of second order self-force**
- **Nonlinear scalar gravity model -- resolve relevance of 2nd order self-force?**
- **Leading order spin-spin and spin-orbit diagrams for rotating COs**

Future directions

- **Gravitational perturbations**
  - GWs resulting from second order self-force corrections
  - Better choice of field variables via a gauge transformation/field redefinition?
- **Use nonlinear scalar gravity model to:**
  - Determine if higher order effects are really relevant for LISA
  - Study and implement practical computations at higher orders
- **Spinning compact objects**
  - Precisely, how much does the spin of a CO affect the waveforms?
- **Can QFT techniques provide new methods for practical computations?**
  - P. Anderson, Hu & Eftekharzadeh; W. Anderson, Flannagan, Ottewill, Wardell
- **Continue to higher orders for LISA (and LIGO?) IMRIs?**
- **Include dissipative degrees of freedom to describe GW absorption**

Extra slides

Sources of gravitational waves

- **Galactic binaries composed of ordinary stars, WDs, NSs, BHs**
  - Confusion noise at lower frequencies -- significant data analysis challenge
- **Massive BH mergers from colliding galaxies**
  - Possibly detectable at cosmological distances
- **Extreme mass ratio inspirals (EMRIs)**
  - Most promising sources for detection and parameter estimation
  - Clean tests of GR ("No Hair" theorem, direct proof of BH's existence,...)
- **Formation of supermassive black holes (SMBHs)**
- **Cosmic gravitational wave backgrounds**
- **Certain dark matter candidates**

www.cco.caltech.edu/~esp/lisa/lisatab.html

EMRIs and self-force

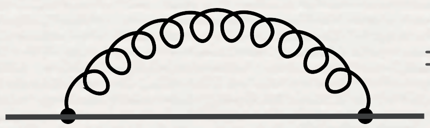
- **Rich diversity of orbits**
  - Circular inspiral
  - Periastron precession
  - Zoom-whirl orbits
  - Spin-orbit coupling
  - Evolving inclination angle
- **Self-force drives the inspiral due to emission of GWs**
  - Entails interactions with back-scattered waves emitted in the past
  - Intrinsically non-local and history-dependent force
  - Gauge-dependent

Gravitational self-force with EFT

- Self-force is computed from the effective action

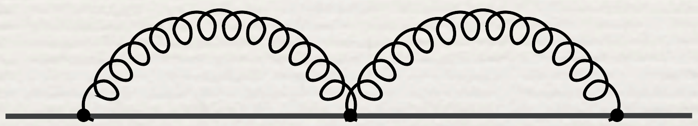
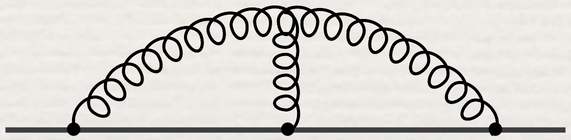
$$\Gamma[\langle \hat{z}^\mu \rangle, \langle \hat{h}_{\mu\nu} \rangle] = -m \int d\tau + \left(\begin{array}{c} \text{Sum of all tree level} \\ \text{connected diagrams} \end{array} \right) + \cdots$$

- First order -- MST-QW self-force equation (in Lorenz gauge)



$$= \frac{m}{2m_{pl}^2} w^{\mu\alpha\beta\nu} Fp \int d\tau' \, \nabla_\nu D^{ret}_{\alpha\beta\gamma'\delta'} u^{\gamma'} u^{\delta'}$$

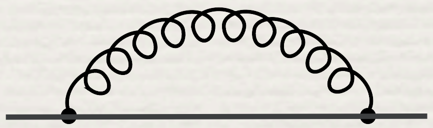
- Second order -- Two diagrams -- Preliminary results!

 $+$ 

$$= -\frac{m^2}{16 m_{pl}^4} w^{\mu\alpha\beta\nu} Fp \int d\tau' \, \nabla_\nu D^{ret}_{\alpha\beta\gamma'\delta'} u^{\gamma'} u^{\delta'} u^{\epsilon'} u^{\zeta'} Fp \int d\tau'' \, D^{ret}_{\epsilon'\zeta'\eta''\theta''} u^{\eta''} u^{\theta''}$$

$$+ \frac{m^2}{8 m_{pl}^4} w^{\mu\alpha\beta\gamma\delta\nu} Fp \int d\tau' \, \nabla_\nu D^{ret}_{\alpha\beta\epsilon'\zeta'} u^{\epsilon'} u^{\zeta'} Fp \int d\tau'' \, D^{ret}_{\gamma\delta\eta''\theta''} u^{\eta''} u^{\theta''} + \cdots$$

- First order diagram also gives a contribution at second order



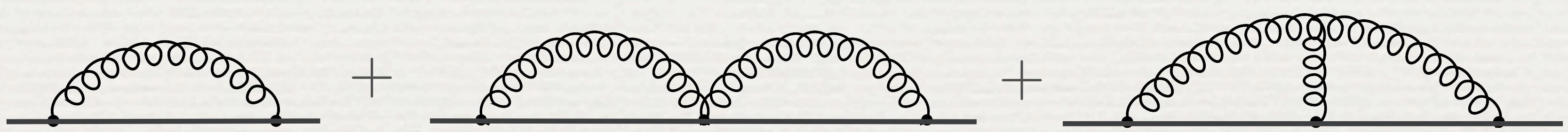
$$= \text{MSTQW} + \frac{m}{2m_{pl}^2} \left(-w^{\mu\alpha\beta\nu} a_\nu + w^{\mu(\alpha} a^{\beta)} \right) Fp \int d\tau' \, D^{ret}_{\alpha\beta\gamma'\delta'} u^{\gamma'} u^{\delta'}$$

A nonlinear scalar model

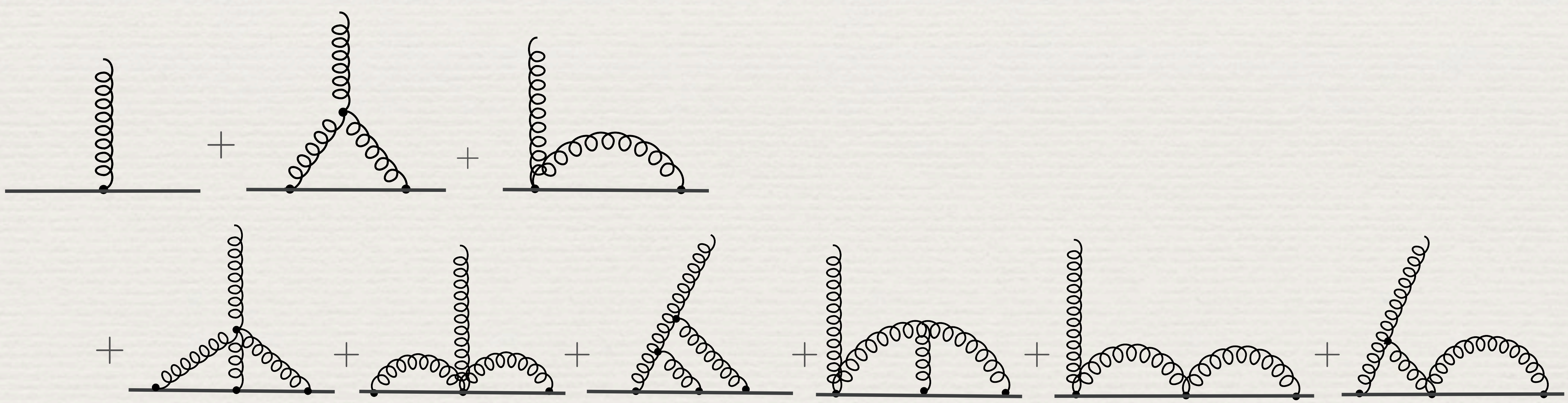
- **A class of non-linear scalar models on a vacuum background:**

$$S[z,\phi] = -\frac{1}{2}\int d^4x\, g^{1/2}\phi_{,\alpha}\phi^{,\alpha}A^2(\phi/m_{pl}) - m\int d\tau\, B(\phi/m_{pl})$$

- **Self-force through second order**



- **Scalar radiation through third order (from log of partition fn)**



- **Field redefinition simplifies the calculation greatly** $\phi_{,\alpha}(x)A = \sigma_{,\alpha}(x)$
  - No field self-interactions -- Does a similar thing occur with gravity?

Extra slides

- **Is Kol's ClEFT more efficient than using CSEFT (w/ CTP)?**
  - Potentially yes, but it is not clear what propagator should be associated with graviton line.
    - Using a ret propagator gives an advanced part in the EOM, which if dropped gives MSTQW but with an extra factor of 1/2
    - Using a Feynman propagator gives same problem as with retarded propagator
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