Gravitational radiation reaction in non-vacuum space-time

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World-line derivation of self-force in curved space

Within the linear theories (including the linearized General Relativity) one can use the standard formalism of distributions describing a point particle by the delta-function. This open a simple way to calculate the reaction force just substituting the retarded field in the Hadamard form directly into the particle EOM (Gal'tsov, Spirin and Staub 06').

The singular term is uniquely identified and removed by a renormalization, the remaining finite terms giving the regular self-force. This works for electric and scalar charges in an arbitrary curved space leading to DeWitt-Brehme-Hobbs and Quinn equations respectively as well as in linearized gravity on a vacuum background leading to the MiSaTaQuWa equation. In the latter case, since mass does not enter the geodesic equation, it is the one-bein on the world-line which undergoes renormalization.

The present approach has the following advantages:

- It is based on the rigorous definitions of the theory of distributions
- It avoids using any acausal potentials
- provides a uniquely defined regular self-force
- in the gravitational case ensures that the particle moves along the geodesic in the perturbed space-time
- allows to make a modest progress towards the non-vacuum background case

Scalar radiation

Consider a point particle with scalar charge q and bare mass m_0 moving along a world line $x^\alpha = z^\alpha(\tau)$ which ineracts with gravity and a linear scalar field:

$$S = -\int (m_0 + q\phi)\sqrt{-\dot{z}^2}d\tau - \frac{1}{8\pi}\int \partial_\mu\phi\,\partial^\mu\phi\sqrt{-g}d^4x.$$

The coupled EOM-s are:

$$\phi_{;\mu}{}^{;\mu}(x) = -4\pi\,\rho, \qquad \rho(x) = q\int \delta^4(x,z)d\tau.$$

$$m_0 + q\phi\ddot{z}^\mu = -q\Pi^{\mu\nu}\phi_{;\,\nu}, \qquad \Pi^{\mu\nu} = g^{\mu\nu} + \dot{z}^\mu\dot{z}^\nu$$

We substitute the retarded solution constructed with the Green's function

$$G^{\rm ret}(x,z) = \frac{\theta[\Sigma(x),z]}{8\pi}\left[u\delta(\sigma) - v\theta(-\sigma)\right]$$

into the charge EOM, obtaining

$$\phi_{;\,\nu}\big|_{x=z(\tau)} = q\int_{-\infty}^{\tau}\left[-u\delta'(\sigma)\sigma_\nu - u_{;\nu}\delta(\sigma) - v\delta(\sigma)\sigma_\nu + v_{;\,\nu}\right]d\tau'$$

where integration is performed over all the history of the particle motion. At the r.h.s. we have both the direct terms (δ) and tail. To calculate the direct part one has to expand the Singe function with both points on the world-line:

Expansions

$$\sigma(\tau,\tau') := \sigma\left[\left(z(\tau), z(\tau')\right)\right]$$

in terms of the difference of the proper time moments $s = \tau - \tau'$. Any biscalar function is simply Taylor expanded

$$\sigma(z(\tau), z(\tau')) = \sum_{k=0}^{\infty} \frac{1}{k!} D^k \sigma(\tau,\tau)(\tau - \tau')^k$$

where D will be further treated as covariant derivative along the world-line, e.g.

$$\dot\sigma = \sigma_\alpha \dot z^\alpha \quad , \quad \ddot\sigma = \sigma_{\alpha\beta}\dot z^\alpha \dot z^\beta + \sigma_\alpha \ddot z^\alpha \text{etc.}$$

To expand bitensors with indices relating to both unprimed and primed z one has first to parallel transport them in order to have indices associated with the fixed point $z(\tau)$, so they behave as scalars with respect to $z(\tau')$. Thus we obtain

$$\sigma(s) = -\frac{s^2}{2} - \ddot z^2(\tau)\frac{s^4}{24} + \mathcal{O}(s^5), \quad \sigma^\mu(s) = s\left(\dot z^\mu - \ddot z^\mu \frac{s}{2} + \dddot z^\mu \frac{s^2}{6}\right) + \mathcal{O}(s^4),$$

where the DeWitt-Brehme index convention is adopted (μ, ν associated with z and α, β with z'), in particular, $\sigma^\mu = \partial\sigma(z, z')/\partial z_\mu$. It can be easily checked that the normalization $g^{\mu\nu}\sigma_\mu\sigma_\nu = 2\sigma$ holds in all orders. For the delta-function one gets:

$$\delta(-\sigma) = \delta(s^2/2) + s^4 \frac{\ddot z^2(\tau)}{24}\delta'(s^2/2) + \ldots$$

where the derivative w.r.t. the argument is understood. The retardation condition adds a factor

$$\frac{1}{2}\left(1+\frac{s}{|s|}\right).$$

In expansions of u-terms one encounters Ricci-terms:

$$u = 1 + \frac{1}{12} R_{\sigma\tau} \dot{z}^\tau \dot{z}^\sigma s^2 + \ldots, \quad u_{;\nu} = \frac{1}{6} R_{\nu\tau} \dot{z}^\tau s + \ldots.$$

$$u\sigma_\nu = \dot{z}_\nu s - \ddot{z}_\nu \frac{s^2}{2} + \frac{s^3}{12} R_{\lambda\rho} \dot{z}^\lambda \dot{z}^\rho \dot{z}_\nu + \frac{s^3}{6} \dddot{z}_\nu + \ldots.$$

Combining the expansions one is led to two type of integrals:

$$A_l = \int_{-\infty}^{\infty} s^l \, \frac{d}{ds^2} \delta(s^2) \, ds, \quad B_l = \int_{-\infty}^{\infty} s^l \, \frac{d}{ds^2} \left(\frac{s}{|s|} \delta(s^2) \right) \, ds$$

with $l \geq 2$. By the parity argument, the A-type integrals vanish for odd l while B-type vanish for even l. The argument of the delta-function and its derivatives is quadratic in the integration variable, which requires suitable regularization to be done. The standard regularization (used by Rohrlich is the radiation reaction problem in CED) consists in the point-splitting with the small length parameter $\varepsilon > 0$:

$$\delta(s^2) = \lim_{\varepsilon \to 0} \delta(s^2 - \varepsilon^2) = \lim_{\varepsilon \to 0} \left(\frac{\delta(s-\varepsilon)}{2\varepsilon} + \frac{\delta(s+\varepsilon)}{2\varepsilon} \right)$$

Since the most singular is the term $u\delta'(s)\sigma_\nu$, the maximal order giving non-zero contribution in the limit $\varepsilon \to 0$ will be s^3, so we can set in the expansion of the delta function $\delta(-\sigma) = 2\delta(s^2)$. This means that all the integrals will fully reduce to those known in the flat space CED. Actually we obtain the only non-zero integrals

$$A_2 = -\frac{1}{2\epsilon}, \quad B_3 = -1.$$

Using this we obtain:

$$\phi_{;\,\nu} = q\left(\frac{1}{2\epsilon}\ddot{z}_\nu - \frac{1}{3}\dddot{z}_\nu - \frac{1}{6}R_{\nu\tau}\dot{z}^\tau - \frac{1}{6}R_{\gamma\delta}\dot{z}^\gamma\dot{z}^\delta\dot{z}_\nu + \frac{1}{12}R\dot{z}_\nu + \int\limits_{\infty}^{\tau} v_{;\,\nu}\,d\tau'\right)$$

The terms $-1/6\,R_{\lambda\rho}\dot{z}^\lambda\dot{z}^\rho\dot{z}_\nu + 1/12\,R\dot{z}_\nu$ are annihilated by the projector $\Pi^{\mu\nu}$, so we find

$$f^\mu_{\rm sc} = m^2 q^2\left[\Pi^{\mu\nu}\left(\frac{1}{3}\dddot{z}_\nu + \frac{1}{6}R_{\nu\tau}\dot{z}^\tau - \int\limits_{-\infty}^{\tau} v_{;\,\nu}\,d\tau'\right) + \ddot{z}^\mu\left(\frac{1}{2\epsilon} - \int\limits_{-\infty}^{\tau} v d\tau'\right)\right].$$

To eliminate a divergent term we set $\quad m = m_0 - \dfrac{q^2}{2\varepsilon},$

so finally

$$m(\tau)\ddot{z}^\mu = q^2\left[\Pi^{\mu\nu}\left(\frac{1}{3}\dddot{z}_\nu + \frac{1}{6}R_{\nu\alpha}\dot{z}^\alpha - \int\limits_{-\infty}^{\tau} v_{;\,\nu}\,d\tau'\right)\right]$$

where $m(\tau) = m + q\int\limits_{-\infty}^{\tau} v d\tau'$.

Electromagnetic case

Now we start with the bare-mass EOM

$$m_o \ddot{z}^\mu = eF^\mu{}_\nu \dot{z}^\nu$$

and substitute the retarded field to the right hand side. The expansions are similar e.g.

$$u_{\nu\alpha}\sigma^{;\mu}\dot{z}^\nu(\tau)\dot{z}^\alpha(\tau-s) = -s\dot{z}^\mu + \frac{s^2}{2}\ddot{z}^\mu + s^3\left(-\frac{1}{6}\dddot{z}^\mu - \frac{1}{12}R_{\lambda\nu}\dot{z}^\lambda\dot{z}^\nu\dot{z}^\mu - \frac{1}{2}\ddot{z}^2\dot{z}^\mu\right) + O(s^4),$$

$$v_{\nu\alpha}\sigma^{\;\mu}\dot{z}^\nu\dot{z}^\alpha = \frac{s}{2}\dot{z}^\mu R_{\alpha\nu}\dot{z}^\alpha\dot{z}^\nu + \frac{s}{12}R\dot{z}^\mu + O(s^2), \quad v^\mu{}_\alpha\sigma_{;\nu}\dot{z}^\nu\dot{z}^\alpha = -\frac{s}{2}R^\mu_\alpha\dot{z}^\alpha + \frac{s}{12}R\dot{z}^\mu + O(s^2)$$

Finally we obtain the self-force

$$f^\mu_{\rm em} = e^2\left[-\frac{1}{2\epsilon}\ddot{z}^\mu + \Pi^{\mu\nu}\left(\frac{2}{3}\dddot{z}_\nu + \frac{1}{3}R_{\nu\alpha}\dot{z}^\alpha\right) + \dot{z}^\nu(\tau)\int_{-\infty}^{\tau}(v^\mu{}_{\alpha;\nu} - v_{\nu\alpha}{}^{;\mu})\dot{z}^\alpha(\tau')d\tau'\right]$$

The mass is renormalized according to

$$m = m_0 + \frac{e^2}{2\varepsilon},$$

and we note that signs of divergent terms in the scalar and vector cases are different, so for a particle endowed both with the scalar and the electric charges related by the "BPS" condition $e^2 = q^2$, the self-force does not contain divergent terms. The final result coincides with the DeWitt-Brehme-Hobbs equation

$$m\ddot{z}^\mu = e^2\Pi^{\mu\nu}\left(\frac{2}{3}\dddot{z}_\nu + \frac{1}{3}R_{\nu\alpha}\dot{z}^\alpha\right) + e^2\dot{z}^\nu(\tau)\int_{-\infty}^{\tau}(v^\mu{}_{\alpha;\nu} - v_{\nu\alpha}{}^{;\mu})\dot{z}^\alpha(\tau')d\tau'$$

Linearized gravity

Now we would like to substitute the self-field $h_{\mu\nu}$ of the particle into the geodesic equations for the perturbed field

$$g_{\mu\nu} = g^B_{\mu\nu} + h_{\mu\nu}. \qquad (1)$$

In the original Mino et. derivation it was suggested that the divergent term be eliminated by the mass renormalization. But since the mass does not enter the geodesic equation, this is problematic. We suggest that actually it is the affine parameter which has to be renormalized. So we start with the manifestly reparametrization invariant formulation choosing the particle action

$$S[z^\mu, e] = -\frac{1}{2}\int \left[e(\tau)\, g_{\mu\nu}\dot{z}^\mu \dot{z}^\nu + \frac{m^2}{e(\tau)}\right] d\tau.,$$

Variation over the ein-bein $e(\tau)$ gives the constraint equation

$$g_{\mu\nu}\dot{z}^\mu \dot{z}^\nu = \frac{m^2}{e^2}0,$$

while the geodesic equation will read

$$\frac{D}{d\tau} e(\tau)\dot{z}^\mu = 0.$$

Passing to the perturbed metric (1) we expand correspondingly

$$e(\tau) = e_B + \delta e$$

and without loss of generality we can choose $e_B = \mathrm{const}$. It is this quantity which can be used to absorb the divergence.

Vacuum background

In the vacuum case the equation for the linearized trace-reversed perturbation in the Lorentz gauge

$$\psi_{\mu\nu} = h_{\mu\nu} - \frac{1}{2} g_{\mu\nu} h, \qquad \psi^{\mu\nu}{}_{;\nu} = 0,$$

is the Lichnerowicz equation

$$\psi^{\mu\nu;\xi}{}_{;\xi} + 2R^{\mu}{}_{\xi}{}^{\nu}{}_{\rho}\,\psi^{\xi\rho} = -2\kappa \overset{(P)}{T^{\mu\nu}},$$

where the particle stress tensor

$$T_P^{\mu\nu} = -m \int \bar{g}^{\mu}{}_{\alpha}\bar{g}^{\nu}{}_{\beta}\dot{z}^{\alpha}\dot{z}^{\beta}\frac{\delta^{(4)}(x - z(s))}{|g_B|^{1/2}}\,ds$$

is covariantly conserved with respect to the background metric (implying geodesic unperturbed motion) so that this equation preserves the Lorentz gauge condition. The derivative of the retarded solution in terms of the Hadamard four-index two-point functions $u^{\mu\nu\alpha\beta}(z(\tau),\, z'(\tau')),\; v^{\mu\nu\alpha\beta}(z(\tau),\, z'(\tau'))$ reads

$$\psi^{\mu\nu;\,\lambda}_{\rm ret}(\tau) = m\kappa \int\limits_{-\infty}^{\tau} (u^{\mu\nu\alpha\beta;\,\lambda}\delta(-\sigma) + u^{\mu\nu\alpha\beta}\sigma^{;\,\lambda}\delta'(-\sigma) + v^{\mu\nu\alpha\beta}\sigma^{;\,\lambda}\delta(-\sigma) + v^{\mu\nu\alpha\beta;\,\lambda})\dot{z}^{\alpha}\dot{z}^{\beta}d\tau'$$

The direct part containing delta-functions is calculated using the expansions in $s = \tau - \tau'$.

Substituting in the perturbed geodesic equation

$$\ddot{z}^{\mu} = -\frac{\kappa}{2}(g^{\mu\nu} + \frac{\dot{z}^{\mu}\dot{z}^{\nu}}{c^2})(2h_{\nu\lambda;\rho} - h_{\lambda\rho;\nu})\dot{z}^{\lambda}\dot{z}^{\rho},$$

we find

$$\ddot{z}^{\mu} = \kappa^2 e_0 \left[\frac{7}{2\epsilon}\ddot{z}^{\mu} + \left(g^{\mu\nu} + \frac{\dot{z}^{\mu}\dot{z}^{\nu}}{c^2} \right) \left(-\frac{11}{3}\dddot{z}_{\nu} + \mathcal{F}^{\mu \text{ tail}} \right) \right],$$

where e_0 is the bare value of e_B and $\mathcal{F}^{\mu \text{ tail}}$ is the MiSaTaQuWa tail. Performing the renormalization

$$\left(\frac{1}{e_0} - \frac{7\kappa^2}{2\epsilon} \right) \ddot{z}^{\mu} = \frac{1}{e_B}\ddot{z}^{\mu},$$

taking into account that the formally obtained antidamping term is zero within the adopted accuracy ($\ddot{z} = O(\kappa^2)$), and fixing the gauge $e_B = m$, finally we obtain:

$$\ddot{z}^{\mu} = -m\kappa^2 \left(g^{\mu\nu} + \dot{z}^{\mu}\dot{z}^{\nu} \right) \int_{-\infty}^{\tau} \Big\{ 2\big(v_{\nu\lambda\alpha\beta;\rho} - \frac{1}{2} g_{\nu\lambda} g^{\sigma\tau} v_{\sigma\tau\alpha\beta;\rho} \big)$$
$$- \big(v_{\lambda\rho\alpha\beta;\nu} - \frac{1}{2} g_{\lambda\rho} g^{\sigma\tau} v_{\sigma\tau\alpha\beta;\nu} \big) \Big\} \dot{z}^{\alpha}(\tau')\dot{z}^{\beta}(\tau')\dot{z}^{\lambda}(\tau)\dot{z}^{\rho}(\tau) d\tau' + O(\kappa^3).$$

Non-vacuum background

We consider now an arbitrary background space-time which satisfies non-vacuum Einstein's field equations

$$G^B_{\mu\nu} = \kappa^2 \, T^B_{\mu\nu},$$

The linearization of the field equations in the case of a non-vacuum space time is not a simple task, if one wants to assure the representation of the gravitational potential in terms of Green's function. First, only varying the mixed form of the Einstein equations one can obtain a linear homogeneous operator acting on the perturbation $h^{\mu\nu}$:

$$\left(g^{\mu\nu} - \kappa h^{\mu\nu}\right)\left(R_{\nu\lambda} + \delta R_{\nu\lambda}\right) = \kappa^2 \left(T^{\mu}{}_{\lambda} + \delta T^{\mu}{}_{\lambda} - \frac{1}{2}(T^{\nu}{}_{\nu} + \delta T^{\nu}{}_{\nu})\delta^{\mu}{}_{\lambda} \right),$$

where the perturbation with mixed indices is given by

$$\delta T^{\mu}{}_{\lambda} = -\kappa h^{\mu\nu} \overset{(B)}{T}{}_{\lambda\nu} + \overset{(P)}{T}{}^{\mu}_{\lambda}.$$

Finally one obtain the following operator acting on the trace-reversed potential:

$$\psi^{\mu\nu;\xi}{}_{;\xi} + 2R^{\mu}{}_{\xi}{}^{\nu}{}_{\rho}\,\psi^{\xi\rho} - 2\psi^{(\mu}{}_{\sigma}R^{\nu)\sigma} + \psi^{\mu\nu}R - g^{\mu\nu}R_{\alpha\beta}\psi^{\alpha\beta} = -2\kappa^2 \overset{(P)}{T}{}^{\mu\nu}.$$

It is supposed that the source term is given by the usual stress tensor of the point particle which is covariantly conserved w.r.t. the background metric $\nabla_{\mu} T^{\mu\nu}_{P} = 0$.

Bianchi identities

But calculating the divergence of this operator one obtains

$$\psi^{\mu\nu}{}_{;\nu;\xi}{}^{;\xi} + \psi^{\mu\nu}{}_{;\nu} R - \psi^{\nu\sigma}{}_{;\nu} R^{\mu\sigma} - 2G^{\sigma\mu}{}_{;\rho}\psi^{\rho\sigma} - R_{\alpha\beta}\psi^{\alpha\beta;\mu} = -2\kappa^2 \overset{(P)}{T^{\mu\nu}}{}_{;\nu}.$$

In general this is incompatible with the harmonic gauge condition, which is desired in order to be able to have Green's function in the Hadamard form. Assuming that $\psi^{\mu\nu}{}_{;\nu} = 0$ we are let with

$$-2G^{\sigma\mu}{}_{;\rho}\psi^{\rho\sigma} - R_{\alpha\beta}\psi^{\alpha\beta;\mu} = -2\kappa^2 \overset{(P)}{T^{\mu\nu}}{}_{;\nu}.$$

So, either you have to modify the source term $\overset{(P)}{T^{\mu\nu}}{}_{;\nu} \rightarrow \overset{(1)}{T^{\mu\nu}}{}_{;\nu}$ or to impose some condition on the background.

This situation can be clarified somehow using the perturbed Bianchi identities:

$$\nabla_\mu \mathsf{T}^\mu{}_\nu = 0 \Longleftrightarrow \begin{cases} \overset{(B)}{\nabla}_\mu \overset{(B)}{T^\mu}{}_\nu = 0 & \text{at zeroth order} \\ \overset{(B)}{\nabla}_\mu \overset{(1)}{T^\mu}{}_\nu + \overset{(1)}{\nabla}_\mu \overset{(B)}{T^\mu}{}_\nu = 0 & \text{at first order} \end{cases}.$$

At zero order the Bianchi identities are satisfied. But in the first perturbed order there is an extra term

$$\overset{(1)}{\nabla}_\mu \overset{(B)}{T^\mu}{}_\nu = \frac{1}{2\kappa}\left(h_{;\rho}G^\rho{}_\nu - h^\rho{}_{\mu;\nu}G^\mu{}_\rho\right).$$

So it is not possible to find general expression for the perturbed stress-energy tensor $\overset{(1)}{T}{}^{\mu}{}_{\nu}$ which would be conserved w.r.t. to background. However one can find some particular backgrounds for which the harmonic gauge will be compatible with the perturbed equations. One simple solution is the case of Einstein spaces $R_{\mu\nu} = \lambda g_{\mu\nu}$, where λ is arbitrary. One can easily see that for such a space

$$\left(h_{;\rho} G^{\rho}{}_{\nu} - h^{\rho}{}_{\mu;\nu} G^{\mu}{}_{\rho}\right) = 0$$

so that $\overset{(B)}{\nabla}_{\mu} \overset{(1)}{T}{}^{\mu}{}_{\nu} = 0$, and therefore the Lorentz gauge condition can be imposed. Assuming the Lorentz gauge and performing the point-splitting analysis for the generalized Lichnerowicz operator we obtain

$$\ddot{z}^{\mu} = \kappa^2 e_0 \left[\frac{7}{2\epsilon} \ddot{z}^{\mu} + \left(g^{\mu\nu} + \frac{\dot{z}^{\mu} \dot{z}^{\nu}}{c^2} \right) \left(-\frac{11}{3} \dddot{z}_{\nu} - \frac{11}{6} R_{\nu\lambda} \dot{z}^{\lambda} + \mathcal{F}^{\mu \text{ tail}} \right) \right],$$

with the extra Ricci term. Renormalization is the same as before, so we are left with an extra Ricci term in the MiSaTaQuWa equation. But if the Lorentz gauge is ensured imposing on the background the Einstein space condition, the Ricci tensor is proportional to the metric and it thus vanishes by virtue of the projector.

Therefore the MiSaTaQuWa equation holds without any modification in any Einstein space.

Schwarzschild and Kerr are "non-vacuum"

- Physical origin of the problems arising in attempts to find an equation with the gravitational self-force as a one-body equation lies in our desire to avoid taking into account perturbation of the background caused by the particle.
- The situation is well clarified in the perturbative treatment of the two-body (μ, M) problem in the post-linear theory Gal'tsov 79',82'. Expanding the gravitational field on the Minkowski background up to the second order, one calculates the non-radiative first order gravitational field. Then one can proceed either by calculating the second order radiative perturbation and the corresponding reaction force acting on each particle, or (if $M \gg \mu$)) treat the first order field one the heavier particle as the background in which moves the light particle.
- But from the actual calculation it follows that only in the slow-motion Newtonian case the two-body result will differ form the one-body result by the terms of the order $O(\mu/M)$. For the relativistic velocities (the relativistic gravitational bremsstrahlung Gal'tsov, Grats, Matiukhin 80', 84' two results are essentially different, the one-body one being incorrect.
- Clearly, the situation must be the same for particles moving in the Schwarzschild or Kerr backgrounds. Using one-body approxiamation one neglects the parr of gravitational radiation resulting from the motion of the Schwarzschild (Kerr) black hole under the influence of the small body. In the realtivistic regime both contributions are of the same order. In this sense Schwarzschild and Kerr are "non-vacuum" space-times.

Conclusions

No one-body equation for gravitational radiation and self-force

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