Lorentz-Dirac force from quantum electrodynamics

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CAPRA Meeting, 27 June, 2008

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The infinite mass of the charged particle is subtracted by the mass counterterm in QED. (Not discussed.)

Outline

Motivation

Which “straightforward exercise” should we do?

Quantum position shift

Equality of classical and quantum position shifts

Summary and outlook

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- Classical electrodynamics is an approximation to QED. The radiation-reaction force should ultimately be of quantum origin.

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- Derivation of the Lorentz-Dirac force in perturbative QED will be another justification for the “reduction of order”.
- There might be a one-loop quantum correction bigger than the radiation-reaction force.

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How do we compare quantum and classical forces?

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Can we reproduce this position change in QED?

We first discuss the general formula for the position change due to a perturbative force.

Position shift by a perturbative force in general

Consider the motion of a particle described by a Hamiltonian $H(\mathbf{x}, \mathbf{p})$:

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Let $(X_i(t), P_i(t))$ be a solution.

Now add a perturbative force $\Delta F_i(t)$.

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Expression for the retarded solution $\Delta X_i(t)$ in terms of $\Delta F_i(t)$?

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Expression for the retarded solution $\Delta X_i(t)$ in terms of $\Delta F_i(t)$?

$H = \sqrt{(\mathbf{p} - e\mathbf{A}(\mathbf{x}, t)))^2 + m^2} + eA_0(\mathbf{x}, t)$ (the Lorentz force)
$\Delta F(t)$ (the Lorentz-Dirac force),
for example.

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Then the retarded position shift, $\Delta X_i(t)$, due to the perturbation $\Delta F_i(t)$ is

$$\Delta X_i(t) = \int_{-\infty}^{t} \Delta x_{i;k}(t;s) \Delta F_k(s)\, ds.$$

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or by letting $t = 0$

$$\Delta X_i(0) = -\int_{-\infty}^{0} \Delta F_k(t)\Delta x_{k;i}(t;0)\,dt.$$

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Interpretation of $\Delta x_{k;i}(t;0)$?

Position shift by a perturbative force in general

$$\Delta X_i(0) = -\int_{-\infty}^{0} \Delta F_k(t)\Delta x_{k;i}(t;0)\,dt\,.$$

Interpretation of $\Delta x_{k;i}(t;0)$? Suppose the unperturbed solution goes through the spacetime origin, i.e. $X_k(0) = 0$ and consider a set of solutions $(x_{k(\mathbf{p})}(t), p_{k(\mathbf{p})}(t))$ labelled by $\mathbf{p}$ such that

$$x_{k(\mathbf{p})}(0) = 0, \;\; p_{k(\mathbf{p})}(0) = p_k\,.$$

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Hence

$$\frac{\partial x_{k(\mathbf{p})}(t)}{\partial p_i} = \Delta x_{k;i}(t;0).$$

Position shift by a perturbative force in general

Therefore, the position shift at $t = 0$ due to a perturbative force $\Delta F_i(t)$ is

$$\Delta X_i(0) = -\int_{-\infty}^{0} \Delta F_k(t) \frac{\partial x_{k(\mathbf{p})}(t)}{\partial p_i}\, dt,$$

where $x_{k(\mathbf{p})}(t)$ is the position of the unperturbed particle which at $t = 0$ is at the orgin and has momentum **p**.

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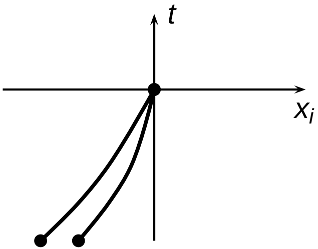

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Position expectation value of a wave packet

Aim: to reproduce the position-shift formula with $\Delta F_i(t)$ being the Lorentz-Dirac force in the $\hbar \to 0$ limit of QED.

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We calculate the shift in the position expectation value of a wave packet at $t = 0$ due to radiation reaction.

The position expectation value can be identified with the centre of the charge distribution: particle creation is absent to all orders in $\hbar$ if $\mathbf{A}(t)$ is smooth.

Position expectation value of a wave packet

The charge-density operator is $\rho(t,\mathbf{x}) = \frac{i}{\hbar} :\varphi^\dagger \overleftrightarrow{\partial_t} \varphi:$, where $\varphi(t,\mathbf{x})$ is the scalar field operator.

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With $e = 0$, if the corresponding one-particle wave function is

$$\varphi(t,\mathbf{x}) = \int \frac{d^3\mathbf{p}}{(2\pi\hbar)^3\sqrt{2p_0}} f(\mathbf{p}) e^{-i(p_0 t - \mathbf{p}\cdot\mathbf{x})/\hbar},$$

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where $p_0 = \sqrt{\mathbf{p}^2 + m^2}$ ($c = 1$), we have

$$\langle x_i \rangle_0 = i\frac{\hbar}{2} \int \frac{d^3\mathbf{p}}{(2\pi\hbar)^3} f^*(\mathbf{p}) \overleftrightarrow{\partial_{p_i}} f(\mathbf{p}).$$

Quantum position shift

$$\langle x_i \rangle_0 = i\frac{\hbar}{2}\int \frac{d^3\mathbf{p}}{(2\pi\hbar)^3}\, f^*(\mathbf{p})\, \overleftrightarrow{\partial}_{p_i}\, f(\mathbf{p}),$$

where $f(\mathbf{p})$ is the momentum-space representation of the one-particle wave function.

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With $e \neq 0$ (neglecting terms which vanish as $\hbar \to 0$ in the end) we have (with $k \equiv \|\mathbf{k}\|$)

$$\begin{aligned} f(\mathbf{p}) \quad \to \quad & [1 + i\mathcal{F}(\mathbf{p})]\, f(\mathbf{p}) \\ & + \frac{i}{\hbar} f(\mathbf{p}) \otimes \int \frac{d^3\mathbf{k}}{(2\pi)^3 2k} \mathcal{A}^\mu(\mathbf{p},\mathbf{k}) a^\dagger_\mu(\mathbf{k}) |0\rangle, \end{aligned}$$

$\mathcal{F}(\mathbf{p})$: forward scattering amplitude of order e^2,
$\mathcal{A}^\mu(\mathbf{p},\mathbf{k})$: one-photon emission amplitude of order e.

Quantum position shift

With

$$F(\mathbf{p}) \equiv [1 + i\mathcal{F}(\mathbf{p})]\, f(\mathbf{p})$$
$$G^\mu(\mathbf{p}, \mathbf{k}) \equiv \mathcal{A}^\mu(\mathbf{p}, \mathbf{k}) f(\mathbf{p})\,.$$

We find the position expectation value to be

$$\begin{aligned}\langle x_i \rangle_e =& \frac{i\hbar}{2} \int \frac{d^3\mathbf{p}}{(2\pi\hbar)^3} F^*(\mathbf{p}) \overleftrightarrow{\partial}_{p_i} F(\mathbf{p}) \\ &- \frac{i}{2} \int \frac{d^3\mathbf{k}}{(2\pi)^3 2k} \int \frac{d^3\mathbf{p}}{(2\pi\hbar)^3} G^*_\mu(\mathbf{p}, \mathbf{k}) \overleftrightarrow{\partial}_{p_i} G^\mu(\mathbf{p}, \mathbf{k}).\end{aligned}$$

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We let $f(\mathbf{p})$ be sharply peaked with width of order $\hbar$.

Quantum position shift

Then, neglecting the terms which will vanish as $\hbar \to 0$ in the end,

$$\begin{aligned}
\Delta^Q X_i(0) &\equiv \langle x_i \rangle_e - \langle x_i \rangle_0 \\
&= -i\hbar \frac{\partial}{\partial p_i} \mathrm{Re}\, \mathcal{F}(\mathbf{p}) \\
&\quad - \frac{i}{2} \int \frac{d^3k}{(2\pi)^3 2k} \mathcal{A}^*_\mu(\mathbf{p},\mathbf{k}) \overleftrightarrow{\partial}_{p_i} \mathcal{A}^\mu(\mathbf{p},\mathbf{k}),
\end{aligned}$$

where $\mathbf{p}$ is now the expectation value of the momentum of the charged scalar particle at $t = 0$ (after the acceleration).

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The one-photon emission term?

Outline

One-photon emission amplitude

We now have

$$\Delta X_i(0) = -\int_{-\infty}^{0} \Delta F_k(t) \frac{\partial x_{k(\mathbf{p})}(t)}{\partial p_i}\, dt$$

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$\Delta^Q X_i(0) = \Delta X_i(0)$, with $\Delta F_i(t) =$ Lorentz-Dirac force?

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$\Delta^Q X_i(0) = \Delta X_i(0)$, with $\Delta F_i(t) =$ Lorentz-Dirac force?

To lowest order in $\hbar$ in the WKB approximation, one finds

$$\mathcal{A}_\mu(\mathbf{p},\mathbf{k}) = -e \int d^4x e^{ik\cdot x} j^\mu_{(\mathbf{p})}(x),$$

where $j_{(\mathbf{p})}(x)$ is the 4-current of the classical particle accelerated by $\mathbf{A}(t)$, passing through the origin with momentum $\mathbf{p}$ at $t = 0$.

$\Delta^Q X_i(0)$ in terms of classical field

Remark: The current $j^\mu_{(\mathbf{p})}(x)$ is smoothly cut off for large $|t|$ as $j^\mu_{(\mathbf{p})}(x)\chi(t)$, where $\chi(t) = 0$ for large enough $|t|$.

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The retarded field $A^\mu_{-(\mathbf{p})}(x)$ from the current $ej^\mu_{(\mathbf{p})}(x)$ is

$$A^\mu_{-(\mathbf{p})}(x) = -\int \frac{d^3\mathbf{k}}{(2\pi)^3 2k} \times \left[\mathcal{A}^\mu(\mathbf{p},\mathbf{k})e^{-ik\cdot x} - \mathcal{A}^{\mu *}(\mathbf{p},\mathbf{k})e^{ik\cdot x}\right],$$

for large t such that $j^\mu_{(\mathbf{p})}(x)$ (which is smoothly cut off) vanishes there.

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for large t such that $j^\mu_{(\mathbf{p})}(x)$ (which is smoothly cut off) vanishes there. This equation allows us to write $\Delta^Q X_i(0)$ in terms of the retarded field $A_{-(\mathbf{p})}(x)$.

$\Delta^Q X_i(0)$ in terms of classical field

The result is

$$\Delta^Q X_i(0) = -\frac{1}{2}\int_{t=T} d^3\mathbf{x}(\partial_{p_i} A^{\mu}_{-(\mathbf{p})}) \overleftrightarrow{\partial}_t A_{-(\mathbf{p})\,\mu},$$

where $j^{\mu}_{-(\mathbf{p})}(x) = 0$ at $t = T$ due to the cut off.

$\Delta^Q X_i(0)$ in terms of classical field

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Then using

$$\int_{t=T} d^3\mathbf{x}\, G_-{}^{\mu}{}_{\alpha}(x-y) \overleftrightarrow{\partial}_t G_{-\mu\beta}(x-z) = -2G_{\mathrm{R}\alpha\beta}(y-z)\,,$$

where $G_R = G_- - G_+$, if $y^0, z^0 < T$,

$\Delta^Q X_i(0)$ in terms of classical field

The result is

$$\Delta^Q X_i(0) = -\frac{1}{2}\int_{t=T} d^3\mathbf{x}(\partial_{p_i} A^{\mu}_{-(\mathbf{p})}) \overleftrightarrow{\partial}_t A_{-(\mathbf{p})\,\mu},$$

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where $G_R = G_- - G_+$, if $y^0, z^0 < T$, we obtain

$$\begin{aligned}\Delta^Q X_i(0) &= e^2 \int d^4y d^4z\, \partial_{p^i} j_{(\mathbf{p})\mu}(y) G_R^{\mu\nu}(y-z) j_{(\mathbf{p})\nu}(z)\\ &= e\int d^4y\, \partial_{p^i} j^{\mu}_{(\mathbf{p})}(y) A_{\mathrm{R}(\mathbf{y})\,\mu}(y)\,.\end{aligned}$$

Equality of classical and quantum position shifts

$$\Delta^Q X_i(0) = e \int d^4x \, \partial_{p^i} j^{\mu}_{(\mathbf{p})}(x) A_{\mathrm{R}(\mathbf{p})\,\mu}(x).$$

Equality of classical and quantum position shifts

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This agrees with

$$\Delta X_i(0) = -\int_{-\infty}^{0} \Delta F_k(t) \frac{\partial x_k(t;\mathbf{p})}{\partial p_i}\, dt$$

with $\Delta F_k(t) = -eF_{\mathrm{R}k\nu}\frac{dx^{\nu}}{dt}$, which is the Lorentz-Dirac force. (The minus sign is due to index lowering.)

Outline

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See, Giles Martin, arXiv:0805.0666.