Mass, inertia and gravitation

Marc-Thierry Jaekel

Laboratoire de Physique Théorique
Ecole Normale Supérieure, CNRS, UPMC
24, rue Lhomond, F-75231 Paris Cedex 05, France

School on Mass
Orleans, 23 June 2008

jaekel@lpt.ens.fr
reynaud@spectro.jussieu.fr

Outline

- Introduction
- Vacuum fluctuations and inertia
- Mass as a quantum observable
- Metric extensions of General Relativity
- Conclusion

Vacuum fluctuations and inertia
Motion in vacuum

A fluctuating environment leads to dissipative effects on motion

A. Einstein, Ann. Physik 17 (1905) 549; Phys. Z. 18 (1917) 121

Relations between fluctuations and dissipation hold at the quantum level

R. Kubo, Rep. Prog. Phys., 29 (1966) 255

In quantum field theory, they result from the evolution of interacting fields

$$\Phi(t) = S_t^{-1} \Phi_{in}(t) S_t, \qquad S_t = T exp \frac{-i}{\hbar} \int_{-\infty}^{t} H_I^{in}(t') dt', \quad H = H_o + H_I$$

In particular, output fields are determined by a scattering matrix:

$$\Phi_{out}(t) = S^{-1} \Phi_{in}(t) S, \qquad S = S_\infty$$

A perturbation $\delta H_I = -\delta\lambda(t) B(t)$ induces a perturbation of observables:

$$\delta A(t) = [A(t), S_t^{-1} \delta S_t] = \int_{-\infty}^{\infty} dt' \chi_{AB}(t - t') \delta\lambda(t')$$

The linear response to the perturbation is given by the generator $B(t)$:

$$\chi_{AB}(t - t') = \frac{i}{\hbar} \theta(t - t') [A(t), B(t')]$$

Displacements in space δq are generated by the total momentum P

$$\frac{i}{\hbar}[P, \quad]\delta q \quad \rightarrow \quad \delta H_I = \frac{i}{\hbar}[H, P]\delta q = -\delta q F$$

Motional susceptibility

Small motions induce Hamiltonian perturbations proportional to the force

$$\delta H_I(t) = -F(t)\delta q(t)$$

Quantum fluctuations of forces are characterized by their correlations in time

$$< F(t)F(0) > \equiv C_{FF}(t) \equiv \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} e^{-i\omega t} C_{FF}[\omega]$$

Linear response theory relates the mean motional force to force correlations

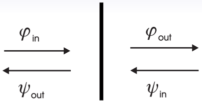
$$\langle \delta F(t) \rangle = \int_{-\infty}^{t} dt' \chi_{FF}(t-t')\delta q(t')$$

The dissipative part gives the radiative reaction force induced by motion.
The (causal) response function is the retarded part of the force commutator

$$\chi_{FF}(t) = 2i\theta(t)\xi_{FF}(t), \qquad \xi_{FF}(t) = \frac{1}{2\hbar} < [F(t), F(0)] > = \frac{C_{FF}(t) - C_{FF}(-t)}{2\hbar}$$

The response function χ_{FF} being analytic in ω, its dispersive part $Re(\chi_{FF})$ and its dissipative part $Im(\chi_{FF})$ are related by Kramers-König relations.

Relativity of motion



Vaccum fields exert a fluctuating force on a scatterer.
Energy-momenta are quadratic forms in quantum fields.
Balance of energy-momentum between scatterer and fields is expressed in terms of outcoming and incoming fields, hence in terms of the S-matrix and input field correlations

$$\Phi_{\text{out}}[\omega] = S[\omega]\Phi_{\text{in}}[\omega]$$

$$< \Phi_{\text{in}}[\omega]\Phi_{\text{in}}[\omega'] > \equiv C_{\text{in}}[\omega, \omega']$$

Classical motions can be seen either as perturbations of the scattering matrix (in the laboratory frame) or as perturbations of input field correlations (in the scatterer's frame).
Both computations of the motional force lead to the same result

$$\delta q \quad \longrightarrow \quad \delta S \quad \text{or} \quad \delta C_{\text{in}}$$

$$\delta S \quad \text{or} \quad \delta C_{\text{in}} \quad \longrightarrow \quad \chi \delta q$$

$$< \delta F[\omega] > = \chi[\omega]\delta q[\omega]$$

M.-T. Jaekel, S. Reynaud, Quantum Opt. 4 (1992) 39

Space-time symmetries

Quantum field fluctuations in vacuum are invariant under Lorentz (or conformal) transformations.
The dissipative part of the motional force, i.e. the radiation reaction force, vanishes for uniform (uniformly accelerated) motions. In particular:

- for scattering of a scalar field in 2-dimensional space-time

$$C_{FF}[\omega] \sim \theta(\omega)\omega^3$$

  the radiation reaction force is proportional to the third time derivative

$$< \delta F > \sim \dddot{q}$$

  S.A. Fulling, P.C.W. Davies, Proc. R. Soc.A348 (1976) 393

- for scattering of electromagnetic fields in 4-dimensional space-time

$$C_{A_\mu A_\nu}\left(x, x'\right) = \frac{\hbar}{\pi}\eta_{\mu\nu}\, c\left(x, x'\right), \quad c\left(x, x'\right) = \frac{1}{\left(x - x'\right)^2 - i\varepsilon\left(t - t'\right)}, \quad \varepsilon \to 0^+$$

  the radiation reaction force is proportional to Abraham-Lorentz vector

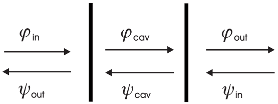
$$< \delta F^\mu > \sim \dddot{q}^\mu + (\ddot{q})^2 \dot{q}^\mu$$

  M.-T. Jaekel, S. Reynaud, Quantum Semiclass. Opt. 7 (1995) 499

Cavity in vacuum

Outcoming fields (and intracavity fields) are expressed in terms of input fields

$$\Phi_{\text{out}}[\omega] = S[\omega]\Phi_{\text{in}}[\omega]$$



For a cavity at rest, energy-momentum balance bewteen cavity and exterior (input and output) fields leads to a mean (Casimir) force F_C.
The Casimir energy E_C (and its derivative F_C) can be written in terms of time delays affecting field fluctuations inside the cavity

$$det S[\omega] = det S_1[\omega] det S_2[\omega] e^{i\Delta_q[\omega]}$$

$$E_C = \int_0^\infty \frac{\mathrm{d}\omega}{2\pi}\hbar\omega\tau[\omega], \qquad \tau[\omega] \equiv \frac{1}{2}\partial_\omega \Delta_q[\omega], \qquad F_C = \partial_q E_C$$

The Casimir force shows fluctuations related to those of the forces F_1 and F_2 acting upon the two mirrors

$$C_{ij}(t) = \langle F_i(t) F_j(0)\rangle - \langle F_i\rangle \langle F_j\rangle\,, \qquad F_C = F_1 - F_2$$

Cavity in motion

Motions of the two mirrors perturb the Hamiltonian

$$\delta H_I(t) = -\sum_j F_j(t)\delta q_j(t)$$

The mean force on each mirror is perturbed by both motions

$$\langle \delta F_i(t) \rangle = \sum_j \int_{-\infty}^{t} \mathrm{d}t' \; \chi_{ij}(t-t')\delta q_j(t')$$

Susceptibility functions χ_{ij} are the retarded parts of the mean values of the force commutators ξ_{ij}

$$\chi_{ij}(t) = 2i\theta(t)\xi_{ij}(t), \qquad \xi_{ij}(t) = \frac{[F_i(t), F_j(0)]}{2\hbar} = \frac{C_{ij}(t) - C_{ji}(-t)}{2\hbar}$$

At the quasistatic limit, corrections to Casimir force and forces depending on accelerations are obtained

$$\chi_{ij}[\omega] = -\kappa_{ij} + \omega^2 \mu_{ij} + \ldots$$

$$\delta F_i(t) = -\sum_j \left(\kappa_{ij}\delta q_j(t) + \mu_{ij}\delta q_j''(t) + \ldots \right)$$

M.-T. Jaekel, S. Reynaud, J. Phys. I France 2 (1992) 149

Inertia of Casimir energy

The total force induced by a global motion of the cavity provides the inertial force exerted by vacuum fields on the cavity

$$\delta F(t) = -\mu \delta \ddot{q}(t) \qquad \mu = \sum_i \sum_j \mu_{ij}$$

The corresponding mass correction may be expressed in terms of the Casimir energy E_C and the mean Casimir force F_C

$$\mu = \frac{E_C - F_C q}{c^2}$$

M.-T. Jaekel, S. Reynaud, J. Phys. I France 3 (1993) 1093

This expression identifies with the inertial mass of a stressed rigid body
A. Einstein, Jahr. Radioakt. Elektron., 4 (1907) 411, 5 (1908) 98

Inertial motion may generally be shown to express conservation of the symmetry generator associated with Lorentz boost.

Mass as a quantum observable
Mass fluctuations

Time delays affecting vacuum fields modify the mean value of the mass

$$< \mu >= \int_0^\infty \frac{d\omega}{2\pi} \hbar\omega\tau[\omega]$$

The induced mass also depends on the energy-density of vacuum fields inside the cavity.
The resulting mass fluctuations can be written in terms of time delays

$$C_{MM}[\omega] = 2\hbar^2\theta(\omega) \int_0^\omega \frac{d\omega'}{2\pi} \omega'(\omega - \omega')\tau[\omega']\tau[\omega - \omega']$$

A simple model for a partially transmitting mirror

$$2\tau[\omega] = \partial_\omega \Delta[\omega] = \frac{2\Omega}{\Omega^2 + \omega^2}$$

shows that mass can have important fluctuations on short time scales

$$< M^2 > - < M >^2 = 2 < M >^2$$

while remaining practically constant in the low frequency domain:

$$C_{MM}[\omega] \sim \frac{\hbar^2}{6\pi}\theta(\omega)\frac{\omega^3}{\Omega^2} \qquad \text{for} \qquad \omega \ll \Omega$$

M.-T. Jaekel, S. Reynaud, Phys. Lett. A 180 (1993) 9

Conformal algebra

Generators of spacetime symmetries correspond to quantities which are preserved by field propagation and also describe changes of reference frame. Relativistic transformations include translations P_μ and Lorentz transformations $J_{\mu\nu}$

$$[P_\mu, P_\nu] = 0$$

$$[J_{\mu\nu}, P_\rho] = i\hbar\,(\eta_{\nu\rho}P_\mu - \eta_{\mu\rho}P_\nu)$$

$$[J_{\mu\nu}, J_{\rho\sigma}] = i\hbar\,(\eta_{\nu\rho}J_{\mu\sigma} + \eta_{\mu\sigma}J_{\nu\rho} - \eta_{\mu\rho}J_{\nu\sigma} - \eta_{\nu\sigma}J_{\mu\rho})$$

For Maxwell fields in 4-dimension, symmetries also include dilatations D

$$[D, P_\mu] = i\hbar P_\mu \qquad\qquad [D, J_{\mu\nu}] = 0$$

and transformations C_μ to uniformly accelerated frames

$$[D, C_\mu] = -i\hbar C_\mu$$

$$[J_{\mu\nu}, C_\rho] = i\hbar\,(\eta_{\nu\rho}C_\mu - \eta_{\mu\rho}C_\nu)$$

$$[C_\mu, C_\nu] = 0$$

$$[P_\mu, C_\nu] = -2i\hbar\,(\eta_{\mu\nu}D - J_{\mu\nu})$$

Quantum positions

The relativistic definition of localization in space-time can be implemented using quantum fields.
Positions in space-time are defined as quantum observables, built from the generators of space-time symmetries

$$X_\mu = \frac{1}{P^2} \cdot \left(P^\lambda \cdot J_{\lambda\mu} + P_\mu \cdot D \right)$$

Quantum observables and relativistic frame transformations are then included in a unique algebra.
Positions include a time operator conjugate to energy

$$[P_\mu, X_\nu] = -i\hbar\eta_{\mu\nu}$$

and tranform according to classical rules under rotations and dilatation

$$\frac{i}{\hbar}[J_{\mu\nu}, X_\rho] = \eta_{\mu\rho}X_\nu - \eta_{\nu\rho}X_\mu, \qquad \frac{i}{\hbar}[D, X_\mu] = X_\mu$$

Transformations of positions between accelerated frames are given by the conformal generators C_μ.

Equivalence principle

The mass observable built on energy-momentum is a Lorentz invariant

$$M^2 = P^\mu P_\mu, \qquad \frac{i}{\hbar}[P_\mu, M^2] = \frac{i}{\hbar}[J_{\mu\nu}, M^2] = 0$$

but cannot be considered as a parameter

$$\frac{i}{\hbar}[D, M^2] = -2M^2, \qquad \frac{i}{\hbar}[C_\mu, M^2] = -4M^2 \cdot X_\mu$$

Mass transformations under uniform accelerations involve a position dependent conformal factor.
Equivalently, positions can be defined from the shift of the mass observable under transformations to accelerated frames.
The quantum red shift law takes the same form as the classical Einstein law, but in terms of quantum positions

$$\Delta = \frac{a^\mu}{2} C_\mu, \qquad \frac{i}{\hbar}[\Delta, M] = -M \cdot \Phi, \qquad \Phi = a^\mu X_\mu$$

Using conformal symmetry, the classical covariance rules may be implemented in the algebra of quantum observables.

M.-T. Jaekel, S. Reynaud, Europhys. Lett. 38 (1997) 1

Metric extensions of General Relativity
Quantum fluctuations of metric fields

Gravitation fields also have quantum fluctuations which may be represented as perturbations of Minkowski metric

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \qquad \eta_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1) \quad , \quad |h_{\mu\nu}| << 1$$

Metric fluctuations may equivalently be written as functions of position in spacetime or of a wavevector in Fourier space

$$h_{\mu\nu}(x) \equiv \int \frac{d^4k}{(2\pi)^4} e^{-ikx} h_{\mu\nu}[k]$$

Gauge invariant fields are provided by Riemann, Ricci, scalar and Einstein curvatures

$$R_{\lambda\mu\nu\rho} = \frac{1}{2}\{k_\lambda k_\nu h_{\mu\rho} - k_\lambda k_\rho h_{\mu\nu} - k_\mu k_\nu h_{\lambda\rho} + k_\mu k_\rho h_{\lambda\nu}\}$$

$$R_{\mu\nu} = R^{\lambda}{}_{\mu\lambda\nu} \quad , \quad R = R^{\mu}{}_{\mu} \quad , \quad E_{\mu\nu} = R_{\mu\nu} - \eta_{\mu\nu}\frac{R}{2}$$

Classically, metric fields are determined from energy-momentum sources by Einstein-Hilbert equations of General Relativity (GR)

$$E_{\mu\nu} = \frac{8\pi G_N}{c^4} T_{\mu\nu}$$

Radiative corrections

Quantum fluctuations of metric fields and stress tensors modify the effective coupling between gravitation and its sources.
Gravitation equations of GR are generalized as a linear response relation between Einstein curvature and the energy-momentum tensor

$$E_{\mu\nu}[k] = \chi_{\mu\nu}^{\lambda\rho}[k]\, T_{\lambda\rho}[k] = \{\frac{8\pi G_N}{c^4}\delta_\mu^\lambda\delta_\nu^\rho + \delta\chi_{\mu\nu}^{\lambda\rho}[k]\}T_{\lambda\rho}[k]$$

M.-T. Jaekel, S. Reynaud, Ann. Physik (1995) 68

Radiative corrections differ in two sectors of different conformal weights, corresponding to trace and traceless parts.
The two sectors correspond to two different running coupling constants.
In the linearized approximation and for a static pointlike source:

$$T_{\mu\nu} = \delta_{\mu 0}\delta_{\nu 0}T_{00}, \qquad T_{00} = Mc^2\delta(k_0)$$

$$E_{\mu\nu} = E_{\mu\nu}^{(0)} + E_{\mu\nu}^{(1)}, \qquad \pi_{\mu\nu} \equiv \eta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}$$

$$E_{\mu\nu}^{(0)} = \{\pi_\mu^0\pi_\nu^0 - \frac{\pi_{\mu\nu}\pi^{00}}{3}\}\frac{8\pi G^{(0)}}{c^4}T_{00}, \qquad E_{\mu\nu}^{(1)} = \frac{\pi_{\mu\nu}\pi^{00}}{3}\frac{8\pi G^{(1)}}{c^4}T_{00}$$

$$G^{(0)} = G_N + \delta G^{(0)}, \qquad G^{(1)} = G_N + \delta G^{(1)}$$

Anomalous curvatures

General solutions remain in the vicinity of GR metric (written in Scwartzschild coordinates)

$$\left[E_\mu^\nu\right]_{\text{st}} = 8\pi\kappa\delta_\mu^{\text{o}}\delta_{\text{o}}^\nu\delta^{(3)}(x), \quad \kappa \equiv \frac{G_N M}{c^2}$$

$$[g_{\text{oo}}]_{\text{st}} = 1 - 2\kappa u = -\frac{1}{[g_{rr}]_{\text{st}}}, \qquad u \equiv \frac{1}{r}$$

Solutions to the generalized equations may be characterized by anomalous Ricci curvatures (which do not vanish outside sources)

$$E_\nu^\mu \equiv [E_\nu^\mu]_{\text{st}} + \delta E_\nu^\mu\,, \qquad \delta E_\nu^\mu(x) \equiv \int d^4x'\,\delta\chi_{\nu\lambda}^{\mu\rho}(x,x')T_\rho^\lambda(x')$$

The two running coupling constants $G^{(0)}$ and $G^{(1)}$ replacing Newton gravitation constant G_N are equivalent to two anomalous Ricci curvatures. The two sectors of anomalous curvatures are equivalent to anomalous parts in the two metric components describing isotropic solutions

$$g_{\text{oo}} = [g_{\text{oo}}]_{\text{st}} + \delta g_{\text{oo}} \quad , \quad g_{rr} = [g_{rr}]_{\text{st}} + \delta g_{rr}$$

$$\frac{\delta g_{\text{oo}}}{[g_{\text{oo}}]_{\text{st}}} = \int \frac{du}{[g_{\text{oo}}]_{\text{st}}^2}\int^u \frac{\delta E_{\text{o}}^{\text{o}}}{u^4}du' + \int \frac{\delta E_r^r}{u^3}\frac{du}{[g_{\text{oo}}]_{\text{st}}}, \quad \frac{\delta g_{rr}}{[g_{rr}]_{\text{st}}} = -\frac{u}{[g_{\text{oo}}]_{\text{st}}}\int \frac{\delta E_{\text{o}}^{\text{o}}}{u^4}du$$

Gravitation potentials

Solutions of Einstein-Hilbert equations lead to vanishing Ricci curvatures in empty space and may be described in terms of Newton gravitation potential.

The two independent components of anomalous curvatures are themselves equivalent to two gravitation potentials $\Phi_N + \delta\Phi_N$ and $\delta\Phi_P$, replacing Newton gravitation potential Φ_N

$$\delta E_0^0 \equiv 2u^4(\delta\Phi_N - \delta\Phi_P)'', \quad \delta E_r^r \equiv 2u^3 \delta\Phi_P' \qquad ()' \equiv \partial_u$$

The two gravitation potentials may be used to describe the two metric components in the isotropic case (with their anomalous parts)

$$\delta g_{rr} = \frac{2u}{(1-2\kappa u)^2}(\delta\Phi_N - \delta\Phi_P)'$$

$$\delta g_{00} = 2\delta\Phi_N + 4\kappa(1-2\kappa u)\int \frac{u(\delta\Phi_N - \delta\Phi_P)' - \delta\Phi_N}{(1-2\kappa u)^2} du$$

The two gravitational potentials provide metric extensions which remain close to GR but account for non linearities in the metric.

M.-T. Jaekel, S. Reynaud, Class. Quantum Grav. 23 (2006) 777

Phenomenology in the solar system

Tests in the solar system are usually performed by comparing observations with the predictions obtained from a family of parametrized post-Newtonian (PPN) metrics.
In the approximation of a pointlike gravitational source (and ignoring effects due to its rotation) PPN metrics may be written (using isotropic coordinates)

$$g_{00} = 1 + 2\phi + 2\beta\phi^2 + \ldots, \qquad \phi = -\frac{G_N M}{c^2 r}$$

$$g_{rr} = -1 + 2\gamma\phi + \ldots$$

Eddington parameters γ and β respectively describe effects on light deflection and on perihelia of planets.
PPN metrics appear as particular cases of the more general metric extensions of GR

$$\delta\Phi_N = (\beta - 1)\phi^2 + O(\phi^3), \qquad \delta\Phi_P = -(\gamma - 1)\phi + O(\phi^2)$$

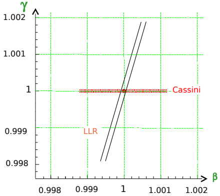
$$\delta E_0^0 = \frac{1}{r^2} O(\phi^2), \qquad \delta E_r^r = \frac{1}{r^2}\left(2(\gamma - 1)\phi + O(\phi^2)\right) \qquad \text{[PPN]}$$

Classical tests of gravitation

The equivalence principle is tested at the 10^{-13} level.

Tests in the solar system confirm GR

- Ranging on planets
- Astrometry and VLBI
- Lunar laser ranging
- Doppler velocimetry on probes
- Light deflection

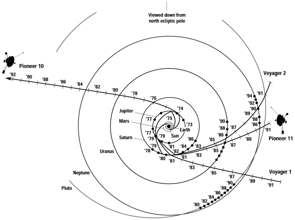


C.F. Will Living Reviews in Relativity, 9 (2006) 3

Tests are consistent with GR and provide bounds on potential deviations

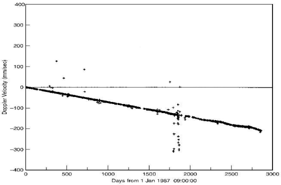
$$|\alpha - 1| < 3 \times 10^{-5}, \qquad |\beta - 1| < 1 \times 10^{-4}$$

Pioneer anomaly



Trajectories followed by the Pioneer 10/11 probes have shown anomalies

Comparison of navigation data (Doppler) with the modelized velocity (GR) shows a discrepancy (residuals) with linear time dependence



$$v_{obs} - v_{model} \simeq -a_P(t - t_{in}), \qquad a_P \simeq 0.9 \text{ nm s}^{-2}$$

J. Anderson et al., Phys. Rev. D 65 (2002) 082004

Modified gravitation in the solar system

Using a metric extension of GR, modified solutions for light-like propagation and massive probe geodesics are obtained.
One computes, in the extended framework, the time delay between emission of the uplink and reception of the transponded downlink.
Comparing with GR, the second time derivative (or Doppler time derivative) shows an anomalous acceleration $\delta a = -a_P \equiv -\frac{c^2}{l_H}$ for

- a metric anomaly linear in r in the first sector: $\delta\Phi_N \simeq \frac{r}{l_H}$
- a metric anomaly quadratic in r in the second sector: $\delta\Phi_P \simeq -\frac{c^2}{3G_N M}\frac{r^2}{l_H}$
- or a superposition of these two anomalies.

While producing Pioneer-like anomalies, the extended framework allows to preserve the agreement with classical tests.

Further related anomalies may also be looked for in:

- data analysis (annual, semi-annual, diurnal anomalies, ...),
- future more precise experiments (anomalous light deflection, GAIA, ...)
- future dedicated missions (metric character of the anomaly, ...).

M.-T. Jaekel, S. Reynaud, Class. Quantum Grav. 23 (2006) 7561

Conclusion

- Vacuum fluctuations do contribute to inertia.
  Inertial mass fluctuates.
- Mass can be treated as a quantum observable consistently with the equivalence principle.
- Quantum fluctuations modify the gravitational coupling and may produce observable effects at large length scale.
  Gravitation may already be modified at the solar system scale