Tests of basic laws of gravity

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Outline

1. Foundations of General Relativity

Outline

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1. Foundations of General Relativity
2. Experimental tests of foundations of GR
   - Universality of Free Fall
   - Universality of Gravitational Redshift
   - Tests of local Lorentz invariance
3. The consequences of the experiments
4. Solar system tests: The search for the gravitational field equations
5. The full theory

Outline

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Universality principles

- Gravity acts always and everywhere
- Gravity can be transformed away
- Gravity acts on all kinds of matter
- Gravity acts on all kinds of matter in the same way
- Gravity acts on all kinds of clocks
- Gravity acts on all kinds of clocks in the same way
- Gravity is created from all kinds of matter
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gravity $\Leftrightarrow$ universality principles

The present situation

All predictions of General Relativity are experimentally well tested and confirmed

Foundations

The Einstein Equivalence Principle

- Universality of Free Fall
- Universality of Gravitational Redshift
- Local Lorentz Invariance

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Gravity is a metrical theory

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Gravity is a metrical theory

$\Rightarrow$

Predictions for metrical theory

- Solar system effects
  - Perihelion shift
  - Gravitational redshift
  - Deflection of light
  - Gravitational time delay
  - Lense–Thirring effect
  - Schiff effect
- Strong gravitational fields
  - Binary systems
  - Black holes
- Gravitational waves

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$\Leftarrow$

Consequences

BH, binary systems, lensing, ...

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ZARM

Reasons for testing GR

Physical reason

Fundamental theories have to be tested as good as possible

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Practical reason

Metrology

Metrology $\Leftrightarrow$ highest precision preparation of time, length, ...

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Theoretical reason

Search for Quantum Gravity

Quantum Gravity $\Rightarrow$ modifications of GR

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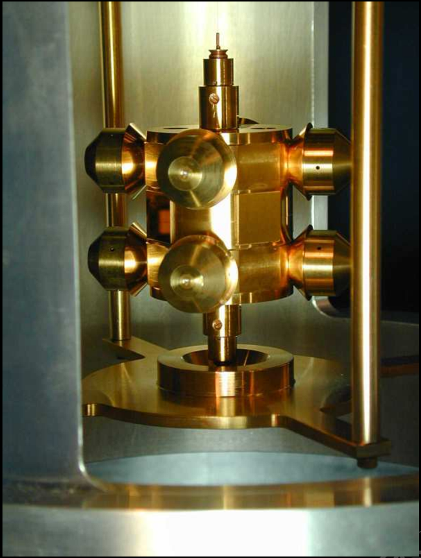
Tests of UFF

Test of UFF = test of identical behavior of matter in gravitational field

matter includes multipoles, coupling to curvature (gravity gradient),

→ motion in general non–geodesic

exact behavior numerically calculable



The free fall: The notions

Trajectories

- Trajectory of a particle $x = x(t)$
- The set of all trajectories is a path structure $\longrightarrow$ geodesic deviation, ...

Equations of notion

- Newton's setup: trajectory determined through initial position $x_0 = x(t_0)$ and inital velocity $v_0 = \dot{x}(t_0)$.

$\Rightarrow$ ordinary differential equations of second order: $\ddot{x}^\mu = f^\mu(p; x, \dot{x})$
$p =$ particle parameter (z.B. mass, charge, etc)

  - Consideration of radiation back reaction: differential equation of higher order
  - Extended bodies: differential equation of higher order

The free fall: The notions

Universality of Free Fall

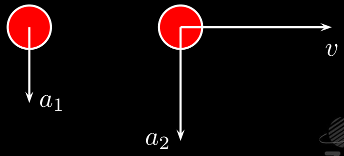
Universality of free fall: equation of motion $\ddot{x} = H(x, \dot{x})$
equation of motion does not depend on particle parameter p

Gravity can be transformed away

$\exists$ coordinate system $\forall$ particles : $\quad \ddot{x} = 0$
Then in an arbitrary coordinate system

$$\ddot{x}^\mu = -\Gamma^\mu_{\rho\sigma}(x)\dot{x}^\rho\dot{x}^\sigma \qquad \text{geodesic equation (without back reaction!)}$$

Condition to be able to transform away gravity is stronger then pure UFF.
It might be that the acceleration toward the Earth depends on the horizontal velocity.



Description of tests of the Universality of Free Fall

Haugan formalism

- Ansatz: Hamiltonian

$$E = mc^2 + \frac{1}{2} m \left(\delta_{ij} + \frac{\delta m_{\mathrm{i}ij}}{m} \right) v^i v^j + m \left(\delta_{ij} + \frac{\delta m_{\mathrm{g}ij}}{m} \right) U^{ij}(\vec{x})$$

- Canonical equations

$$a^i = \delta^{ij} \partial_j U + \frac{\delta m_{\mathrm{i}}^{ij}}{m} \partial_j U + \delta^{ij} \frac{\delta m_{\mathrm{g}kl}}{m} \partial_j U^{kl}(\vec{x}),$$

- For diagonal mass tensors $\delta m_{\mathrm{i}ij} = m_{\mathrm{i}} \delta_{ij}$, $\delta m_{\mathrm{g}ij} = m_{\mathrm{g}} \delta_{ij}$:

$$a^i = \delta^{ij} \frac{m_{\mathrm{g}}}{m_{\mathrm{i}}} \partial_j U$$

- Comparison of acceleration of two different particles: Eötvös coefficient

$$\eta = \frac{a_2 - a_1}{\frac{1}{2}(a_2 + a_1)} = \frac{(m_{\mathrm{g}}/m_{\mathrm{i}})_2 - (m_{\mathrm{g}}/m_{\mathrm{i}})_1}{\frac{1}{2}\left((m_{\mathrm{g}}/m_{\mathrm{i}})_2 + (m_{\mathrm{g}}/m_{\mathrm{i}})_1\right)}$$

UFF and charge

Standard theory

- In standard theory from ordinary coupling (deWitt & Brehme 1968) $a^\mu = \alpha \lambda_C R^\mu{}_\nu v^\nu \sim 10^{35}$ m/s^2

Anomalous coupling

- Anomalous coupling (Dittus, C.L., Selig, GRG 2004)

$$H = \frac{\boldsymbol{p}^2}{2m} + mU(\boldsymbol{x}) + \kappa e U(\boldsymbol{x}) = \frac{\boldsymbol{p}^2}{2m} + m\left(1 + \kappa \frac{e}{m}\right) U(\boldsymbol{x}) .$$

- Charge dependent anomalous gravitational mass tensor
- Can be generalized to charge dependent anomalous inertial mass tensor (e.g. Rohrlich 2000)

$\Rightarrow$ Charge dependent Eötvös factor

- It is possible to choose κ's such that for neutral composite matter UFF is fulfilled while for isolated charges UFF is violated

UFF and spin

Standard theory

- In standard theory from ordinary coupling: $a^\mu = \lambda_C R^\mu{}_{\nu\rho\sigma} v^\nu S^{\rho\sigma}$ $\Rightarrow$ violation of UFF at the order $10^{-20}\ \mathrm{m/s^2}$, beyond experiment

Anomalous coupling

- Speculations: violation P, C, and T symmetry in gravitational fields (Leitner & Okubo 1964, Moody & Wilczek 1974) suggest

$$V(r) = U(r)\left[1 + A_1(\boldsymbol{\sigma}_1 \pm \boldsymbol{\sigma}_2)\cdot\widehat{\boldsymbol{r}} + A_2(\boldsymbol{\sigma}_1 \times \boldsymbol{\sigma}_2)\cdot\widehat{\boldsymbol{r}}\right] ,$$

- One body (e.g., the Earth) is unpolarized $\rightarrow$

$$V(r) = U(r)\left(1 + A\boldsymbol{\sigma}\cdot\widehat{\boldsymbol{r}}\right) .$$

  Hyperfine splittings of H ground state: $A_p \leq 10^{-11}$, $A_e \leq 10^{-7}$
- Hari Dass 1976, 1977, includes velocity of the particles

$$V(r) = U_0(r)\left[1 + A_1\boldsymbol{\sigma}\cdot\widehat{\boldsymbol{r}} + A_2\boldsymbol{\sigma}\cdot\frac{\boldsymbol{v}}{c} + A_3\widehat{\boldsymbol{r}}\cdot\left(\boldsymbol{\sigma}\times\frac{\boldsymbol{v}}{c}\right)\right]$$

Tests of UFF

1 Tests with bulk matter

Method	Grav field	Accuracy	Experiment
Torsion pendulum	Sun	$\eta \leq 2 \cdot 10^{-13}$	Adelberger 2006

2 Tests with quantum matter

Method	Grav field	Accuracy	Experiment
Atom interferometery	Earth	$\eta \leq 10^{-9}$	Chu, Peters 1999

3 Gravitational self energy

Method	Grav field	Accuracy	Experiment
Torsion pendulum and LLR	Sun	$\eta \leq 1.3 \cdot 10^{-3}$	Baessler et al 1999

4 Charged particles

Method	Grav field	Accuracy	Experiment
Free fall of electron	Earth	$\eta \leq 10^{-1}$	Witteborn & Fairbank 1967

Tests of UFF

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5 Particles with spin

Method	Grav field	Accuracy	Experiment
Weighting polarized bodies	Earth	$\eta \leq 10^{-8}$	Hsie et al 1989

6 Anti–particles

Method	Grav field	Accuracy	Experiment
Free fall of anti–Hydrogen	Earth	$\eta \leq 10^{-3} - 10^{-5}$	(estimate)

Outline

Tests of the Universality of Gravitational Redshift

Description

- Gravitational redshift

$$\nu(x_1) = \left(1 - (1 + \alpha_{\rm clock}) \frac{U(x_1) - U(x_0)}{c^2}\right) \nu(x_0)$$

  may depend on used clock
- Comparison of two colocated clocks

$$\frac{\nu_1(x_1)}{\nu_2(x_1)} \approx \left(1 - (\alpha_{\rm clock2} - \alpha_{\rm clock1}) \frac{U(x_1) - U(x_0)}{c^2}\right) \frac{\nu_1(x_0)}{\nu_2(x_0)} .$$

- Null test
- Tested quantity: $\alpha_{\rm clock2} - \alpha_{\rm clock1}$
- Need of large differences in the gravitational potential

Tests of the Universality of Gravitational Redshift

Tests

Comparison	Accuracy	Experiment
Cs – Resonator	$2 \cdot 10^{-2}$	Turneaure & Stein 1987
Mg – Cs (fine structure)	$7 \cdot 10^{-4}$	Godone et al 1995
Resonator – I_2 (electronic)	$4 \cdot 10^{-2}$	Braxmaier et al 2002
Cs – H-Maser (hf)	$2.5 \cdot 10^{-5}$	Bauch et al 2002
Cs – Hg	$5 \cdot 10^{-6}$	Fortier et al 2007

Each time a better clock was available, immediately a redshift experiment has been carried through

Clocks

- Atomic clocks
  - Based on principal transitions
  - Based on fine structure
  - Based on hyperfine transitions
- Molecular clocks
  - Based on rotational transitions
  - Based in vibrational transitions
- Light clocks
- Gravitational clocks
  - Planetary motion
  - Binary systems
- Rotation
  - Earth
  - Pulsars
- Decay of particles

All based on different physical principles, laws.
Clocks of different nature exhibit a different dependence on fundamental constants

Outline

Postulates

Postulates of SR

Postulate 1: The velocity of light is constant.

Postulate 2: The relativity principle.

The meaning of the postulates

- c does not depend on
  - the velocity of the source $\Rightarrow$ **uniqueness** of c
  - the velocity of the observer
  - the direction of propagation
  - its frequency and polarization
- The meaning of c
  - For all particles the velocity of light is the limiting velocity

    $$c = c_+ = c_- = c_\nu = v_p^{\max} = v_e^{\max} = v_{\text{grav}}$$

  - c is universal and can, thus, be interpreted as geometry
- **All** physics is the same in **all** inertial systems
  - Experimental results do not depend on the orientation of the laboratory
  - Experimental results do not depend on the velocity of the laboratory

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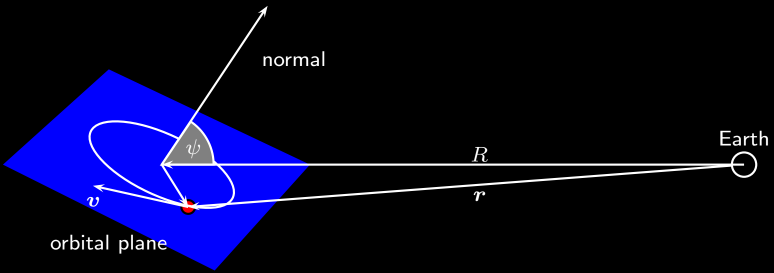
quite complex structure

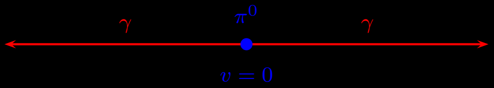
Independence of c from the velocity of the source



Description and result (Brecher 1977)

- Model $c' = c + \kappa v$
- Time of flight consideration: May happen
  - Reversal of chronological order (one light ray may overtake the other)
  - Multiple images
  - ...
- Result $|\kappa| \leq 10^{-11}$

Independence of c from the velocity of the source



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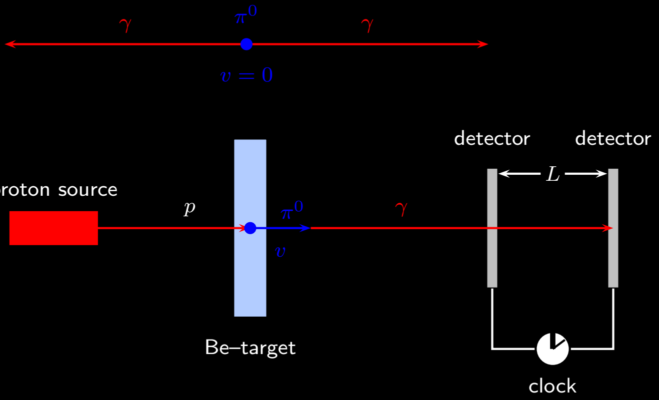

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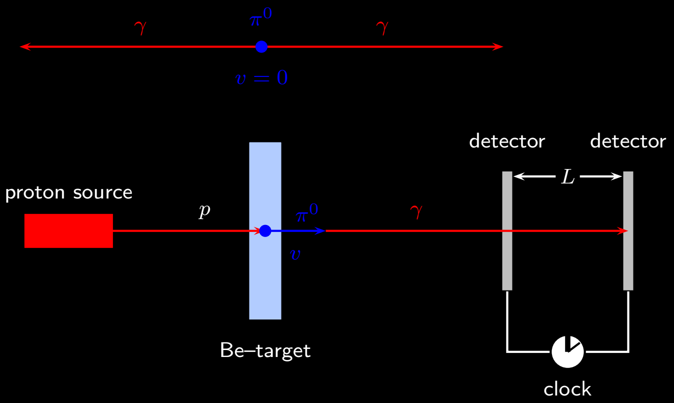
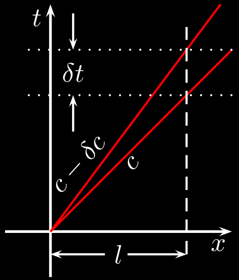


Description and result (Alväger et al 1964)

- Model $c' = c + \kappa v$
- Result $|\kappa| \leq 10^{-6}$ at $v = 0.99975\, c$

Tests of the universality of c



Polarization / dispersion of light

Method: Comparison of arrival times of light with different polarization / different frequency from distant galaxies

Result

Carrol, Field and Jackiw 1992, Kauffmann and Haugan 1995, Kostelecky and Mathews 2002

$$\left|\frac{c_+ - c_-}{c_+}\right| \leq 10^{-32}$$

Schafer 1999, Biller 1999

$$\left|\frac{c_\nu - c_{\nu'}}{c_\nu}\right| \leq 6 \cdot 10^{-21}$$

Tests of the universality of c

Velocity of neutrinos
Method: Comparison of arrival times of photons and neutrinos from supernova SN1987a

Result

Longo 1987, Stodolsky 1988

$$\left|\frac{c - v_\nu}{c}\right| \leq 10^{-8}$$

Limiting velocity of protons

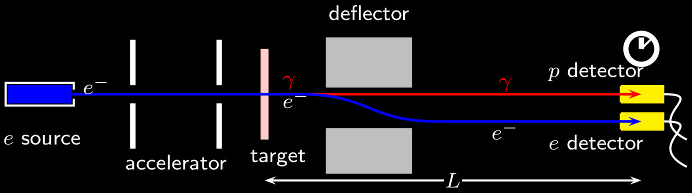
Result

Coleman and Glashow 1998

$$\left|\frac{v_p^{\max} - c}{c}\right| \leq 10^{-21}$$

Tests of the universality of c

Limiting velocity of electrons



Result

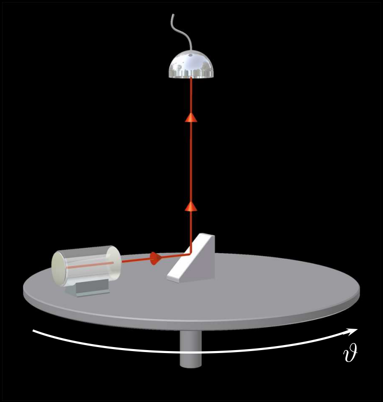
Giurogossian et al 1965

$$\left| \frac{v_e^{\max} - c}{c} \right| \leq 10^{-6}$$

Other tests by Brown et al 1963, Alspector et al 1976, Kalbfleisch et al 1976 with result of same order of accuracy

Test of isotropy of c

- Interference experiment – Michelson–Morley ....
- Measuring frequency of light in rotating resonator



Model independent description

- Light wave
  $\varphi = Ae^{i(k_+ \cdot x - \omega t)} + Be^{i(k_- \cdot x - \omega t)}$
- Dispersion relation $\omega = k_\pm c_\pm$
- Boundary conditions
- Effective: 2–way velocity of light

$$\frac{2}{c} = \frac{1}{c_+} + \frac{1}{c_-}$$

- Observable frequency

$$\nu = \frac{m}{2L} c$$

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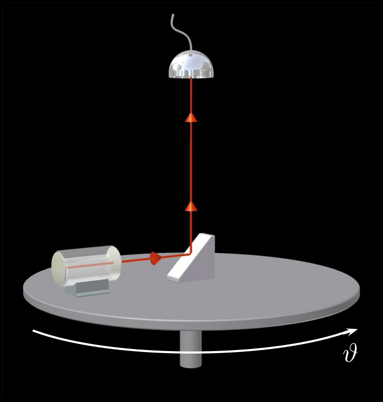

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$$\nu(\vartheta) = \frac{m}{2L} c(\vartheta)$$

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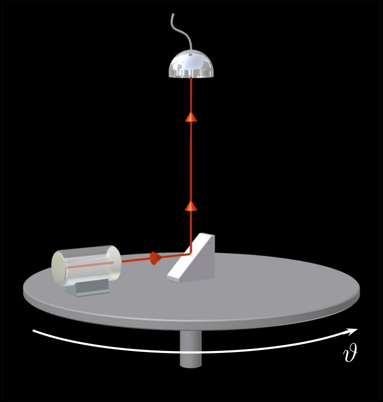


Result

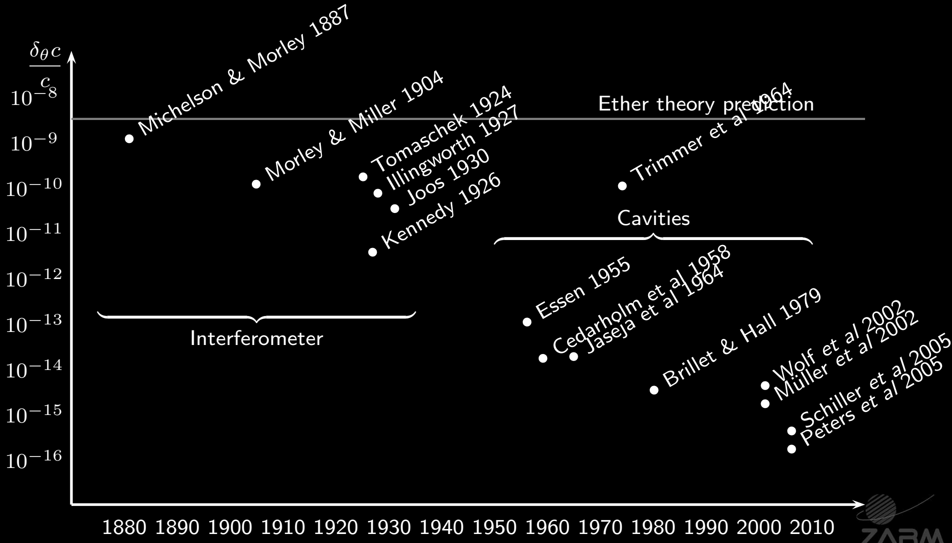
- Herrmann, Peters et al 2005
- Cryogenic resonators, one rotating, one fixed
- Result

$$\left|\frac{\Delta_\vartheta c}{c}\right| \leq (2.5 \pm 4) \cdot 10^{-16}$$

Interpretation (Müller et al 2004)

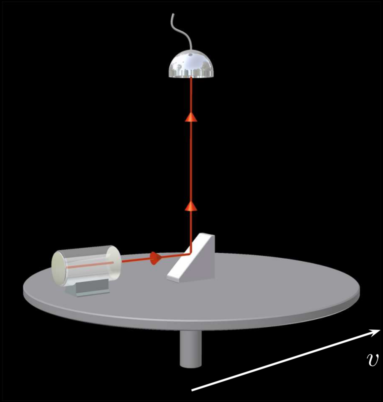
- Velocity of light
- Property of solid
- Electron properties (in L)

Summary: Test of isotropy of c



Test of independence of c from laboratory velocity

- Interference experiment – Kennedy–Thorndike ....
- Measuring frequency of light in moving resonator



Model independent description

- Same description as above
- Observable frequency

$$\nu = \frac{m}{2L} c$$

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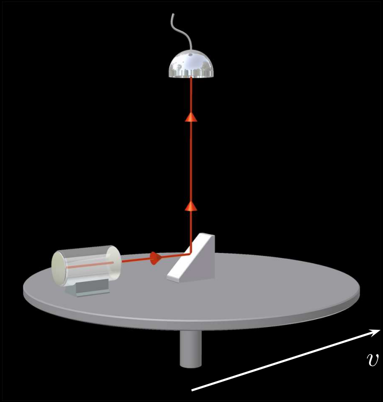


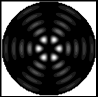
Model independent description

- Same description as above
- Observable frequency

$$\nu(v) = \frac{m}{2L} c(v)$$

Test of independence of c from laboratory velocity

Cavity–clock comparison



p = 5
m = 3

p = 5
m = 30

p = 5
m = 100

Whispering gallery modes

Method and result

- Wolf et al 2004
- Whispering gallery modes
- Comparison with H–maser
- Result

$$\left|\frac{\Delta_v c}{c}\right| \leq (4.5 \pm 4.5) \cdot 10^{-16}$$

- $\delta v \leftrightarrow$ rotation of Earth

Hughes–Drever experiments

The model

Modified Schrödinger equation

$$i\hbar\partial_t\psi = -\frac{\hbar^2}{2m}\left(\delta^{ij} + \alpha^{ij}\right)\partial_i\partial_j\psi$$

Leads to a splitting of the Zeeman singlett

Hughes–Drever experiments

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Modified Schrödinger equation

$$i\hbar\partial_t\psi = -\frac{\hbar^2}{2m}\left(\delta^{ij} + \alpha^{ij}\right)\partial_i\partial_j\psi$$

Leads to a splitting of the Zeeman singlett

Experiments

experiment	method	estimate
Hughes et al 1960	NMR with ^{7}Li	$\vert\alpha^{ij}\vert \leq 10^{-20}$
Drever 1961	NMR with ^{7}Li	$\vert\alpha^{ij}\vert \leq 2\times 10^{-23}$
Prestage et al. 1985	NMR with ^{9}Be$^+$	$\vert\alpha^{ij}\vert \leq$
Lamoreaux et al. 1986, 1989, 1990	NMR with ^{201}Hg	$\vert\alpha^{ij}\vert \leq 2\times 10^{-28}$
Chupp et al 1989	NMR with ^{21}Ne	$\vert\alpha^{ij}\vert \leq 5\times 10^{-30}$

Methods for testing time–dilation

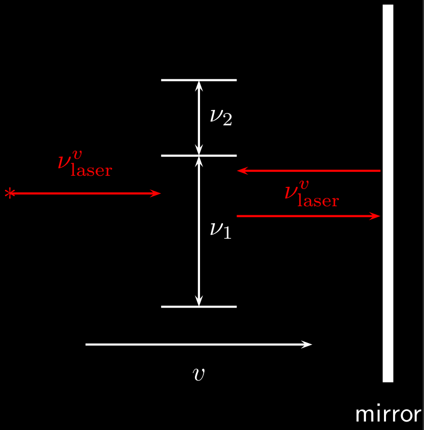
General: Comparison of identical cloks in different motion

- Transport of macrocopic clocks
- Photon absorption / emission
- Two–photon absorption
- Rotor experiments
- Saturation spectroscopy
- Particle decay

Test of time–dilation: Transport of clocks

The Experiment by Hafele and Keating 1968

Test of time–dilation: 2 Photon absorption



Description

- Resonance condition for $v = 0$

$$2v^{v=0}_{\text{laser}} = \nu_1 + \nu_2$$

- Resonance condition for $v \neq 0$

$$\begin{aligned} \nu_1 + \nu_2 &= \nu_+ + \nu_- \\ \nu_\pm &= \nu^v_{\text{laser}} (1 \pm v) \sqrt{1 - v^2} \end{aligned}$$

- Consequence

$$\nu^{v=0}_{\text{laser}} = \nu^v_{\text{laser}} \sqrt{1 - v^2}$$

with

$$v = \frac{\nu_+ - \nu_-}{\nu_+ + \nu_-}$$

- No need of synchronization

Further tests

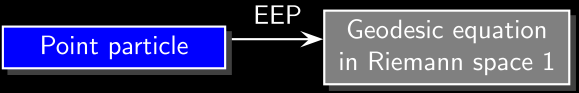
- Time dilation with saturation spectroscopy
- Rotor time dilation experiments
- Sagnac effect
- Experiments testing $E = mc^2$
- Test of dispersion
- ...

Outline

Remark: Implications of Einstein Equivalence Principle

Point particle

Remark: Implications of Einstein Equivalence Principle



Remark: Implications of Einstein Equivalence Principle

Point particle —EEP→ Geodesic equation in Riemann space 1

Spin–1/2 particle

Remark: Implications of Einstein Equivalence Principle

Point particle —EEP→ Geodesic equation in Riemann space 1

Spin–1/2 particle —EEP→ Dirac equation in Riemann space 2

Remark: Implications of Einstein Equivalence Principle

Point particle —EEP→ Geodesic equation in Riemann space 1

Spin–1/2 particle —EEP→ Dirac equation in Riemann space 2

Electromagn. field

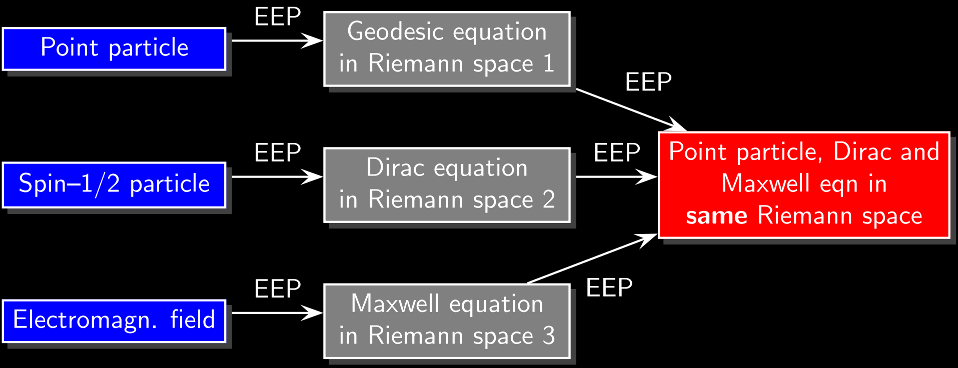
Remark: Implications of Einstein Equivalence Principle

Point particle —EEP→ Geodesic equation in Riemann space 1

Spin–1/2 particle —EEP→ Dirac equation in Riemann space 2

Electromagn. field —EEP→ Maxwell equation in Riemann space 3

Remark: Implications of Einstein Equivalence Principle



Ehlers, Pirani & Schild 1972; Audretsch & C.L. 1986; C.L., Macias & Müller 2005; C.L. & Hehl 2005.

Metric theory

Implication

Gravity = space–time metric

- time $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$
- paths $D_v v = 0 \quad \leftrightarrow \quad \frac{d^2 x^\mu}{ds^2} + \left\{ {}^{\;\mu}_{\rho\sigma} \right\} \frac{dx^\rho}{ds} \frac{dx^\sigma}{ds} = 0$
- Dirac, Maxwell, ...

Predictions

All metric theories imply

- gravitational redshift
- light deflection
- perihelion shift
- gravitational time delay
- Lense–Thirring effect
- Schiff effect
- geodetic precession

Einstein's theory is singled out by certain values for these effects

How to find Einstein's theory? $\longrightarrow$ PPN

PPN formalism

Physical situation

Spherically symmetric metric $\Rightarrow$

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = g_{tt} dt^2 - g_{rr} dr^2 - r^2 d\vartheta^2 - r^2 \sin^2 \vartheta d\varphi^2$$

g_{tt}, g_{rr} $\leftrightarrow$ Gravitational field equations: not known

Parametrization for

- asymptotically flat
- weak fields: $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad h_{\mu\nu} \ll 1$

$$\begin{aligned} g_{00} &= -1 + 2\alpha \frac{U}{c^2} - 2\beta \frac{U^2}{c^4}, \qquad U = \text{ Newton potential} \\ g_{0i} &= 0 \\ g_{ij} &= (1 + 2\gamma) \frac{U}{c^2} \end{aligned}$$

PPN formalism

Physical situation

Axially symmetric metric $\Rightarrow$

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = g_{tt} dt^2 - g_{rr} dr^2 - r^2 d\vartheta^2 - r^2 \sin^2 \vartheta d\varphi^2 - g_{t3} dt d\varphi$$

$g_{tt}, g_{rr}, g_{ti} \leftrightarrow$ Gravitational field equations: not known

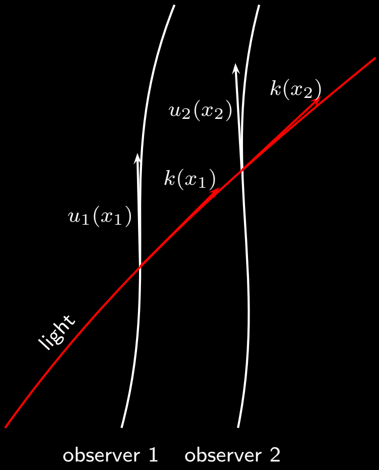
Parametrization for

- asymptotically flat
- weak fields: $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad h_{\mu\nu} \ll 1$

$$\begin{aligned} g_{00} &= -1 + 2\alpha \frac{U}{c^2} - 2\beta \frac{U^2}{c^4}, \qquad U = \text{Newton potential} \\ g_{0i} &= 4\mu \frac{(\boldsymbol{J} \times \boldsymbol{r})_i}{c^3 r^3} \\ g_{ij} &= (1 + 2\gamma) \frac{U}{c^2} \end{aligned}$$

Outline

Gravitational redshift



gravitational redshift for stationary observer
measured frequency $\nu = g(u, k)$

$$\Rightarrow \qquad \frac{\nu_2}{\nu_1} \approx 1 - (U(r_2) - U(r_1))$$

Pound & Rebka, PRL 1960: confirmation $\sim 1\%$
Vessot, Levine et al (GP-A), GRG 1978, PRL 1980: confirmation $\sim 10^{-4}$

Post–Newton

Equations of motion

Equation of motion

$$D_v v = 0 \qquad \leftrightarrow \qquad \frac{d^2 x^\mu}{ds^2} + \left\{ {}^{\mu}_{\rho\sigma} \right\} \frac{dx^\rho}{ds} \frac{dx^\sigma}{ds} = 0$$

PPN equation of motion

$$\ddot{\boldsymbol{x}} = -\frac{GM}{r^2}\boldsymbol{e}_r + 2(\gamma+\beta)\frac{(GM)^2}{c^2 r^3}\boldsymbol{e}_r - \gamma\frac{GM}{r^2}\frac{v^2}{c^2}\boldsymbol{e}_r + 2(\gamma+1)\frac{GM}{r^2}\frac{(\boldsymbol{e}_r\cdot\boldsymbol{v})\,\boldsymbol{v}}{c^2}$$

In spherical coordinates

$$\ddot{r} - r\dot{\varphi}^2 = -\frac{GM}{r^2} + 2(\gamma+\beta)\frac{(GM)^2}{c^2 r^3} - \gamma\frac{GM}{c^2 r^2}\left(\dot{r}^2 + r^2\dot{\varphi}^2\right) + 2(\gamma+1)\frac{GM}{c^2 r^2}\dot{r}^2$$

$$r\ddot{\varphi} + 2\dot{r}\dot{\varphi} = 2(\gamma+1)\frac{GM}{r^2}\frac{\dot{r}r\dot{\varphi}}{c^2}$$

Post–Newton

Conserved energy

$$\begin{aligned} E &= \frac{1}{2}\left(\dot{r}^2 + \frac{L^2}{r^2}\right) - \frac{L^2}{2r^2} + \frac{L^2}{2r^2}\left(1 - 4(\gamma+1)\frac{GM}{c^2 r}\right) \\ &\quad + \frac{2\gamma+1}{4c^2}\left(\dot{r}^2 + \frac{L^2}{r^2}\right)^2 - \frac{GM}{r} + (\gamma+\beta)\frac{(GM)^2}{c^2 r^2} + \mathcal{O}(2) \end{aligned}$$

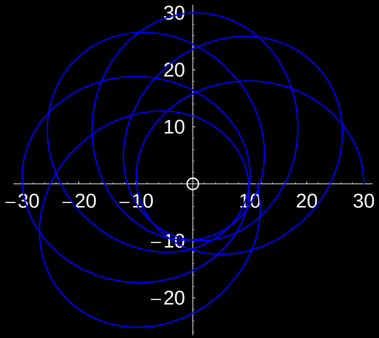
This gives

$$\frac{1}{2}\dot{r}^2 = E\left(1 - \frac{2\gamma+1}{c^2}E\right) - U_{\text{eff}}(r; E, L)$$

with

$$\begin{aligned} U_{\text{eff}}(r; E, L) &= -\frac{GM}{r}\left(1 - 2\frac{2\gamma+1}{c^2}E\right) + \frac{L^2}{2m^2 r^2} \\ &\quad + (3\gamma+\beta+1)\frac{(GM)^2}{c^2 r^2} - 2(\gamma+1)\frac{GML^2 m}{c^2 r^3} \end{aligned}$$

Perihelion shift



Description

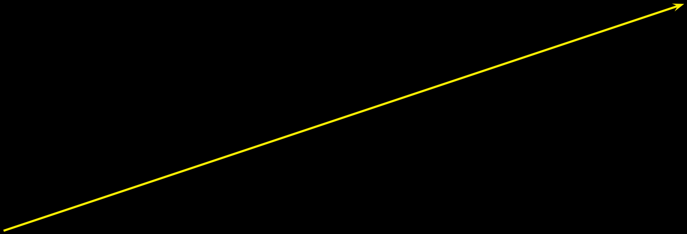
- Geodesic equation in Schwarzschild space–time
- The shift

$$\delta\varphi \quad \approx \frac{2(1+\gamma)-\beta}{3} \frac{6\pi GM}{c^2 a^2 (1-e^2)}$$

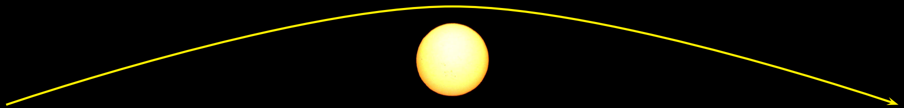
- Competing effects: Sun's quadrupole moment, other planets
- Measurement:

$$\left| \frac{2(1+\gamma)-\beta}{3} - 1 \right| \leq 10^{-4}$$

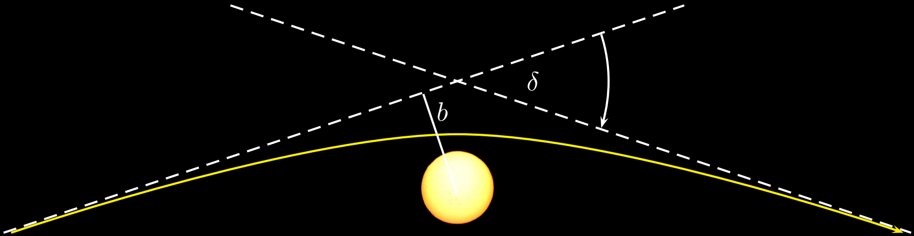
Light deflection



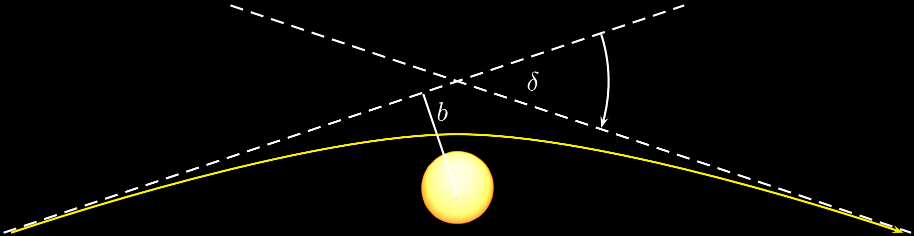
Light deflection



Light deflection



Light deflection



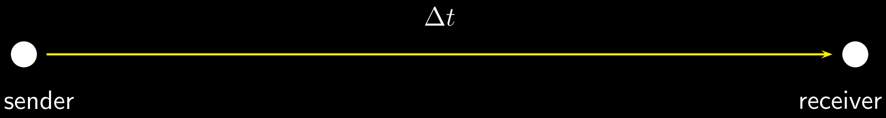
Measurement

- Deflection angle

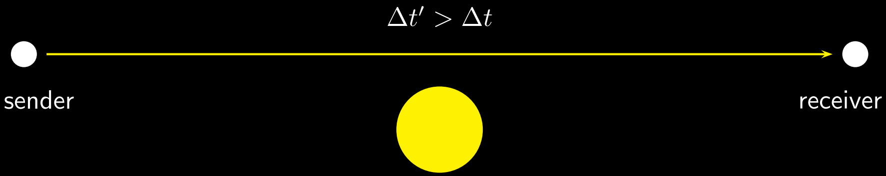
$$\Delta\varphi \approx \frac{\alpha+\gamma}{2}\frac{4GM}{c^2 b}$$

- Best observations with VLBI
- Eubanks et al 2001: $|\gamma - 1| \leq 10^{-4}$

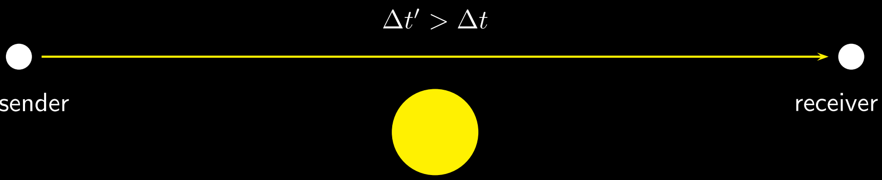
Gravitational time delay



Gravitational time delay



Gravitational time delay



Direct measurement

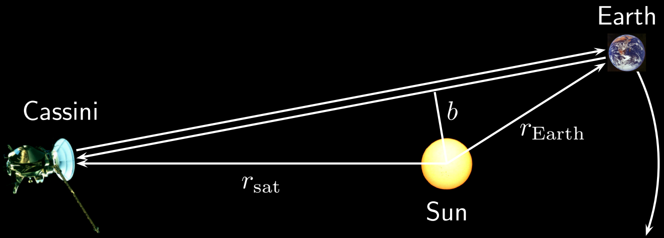
- Time delay

$$\delta t = 2(\alpha + \gamma)\frac{GM_\odot}{c^3} \ln \frac{4 x_{\mathrm{Sat}} x_{\mathrm{Earth}}}{b^2} \sim 10^{-4}\ \mathrm{s}$$

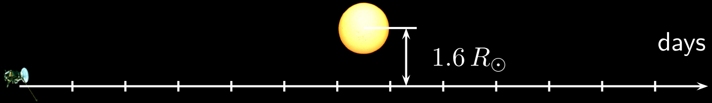
- Reflection of radar signals on surface of Venus (Shapiro 1968)
- Mars ranging, Viking Mars mission (Reasenberg 1979)
- Result: $|\gamma - 1| \leq 10^{-4}$

Gravitational time delay

Mission scenario



View from Earth



Gravitational time delay

Measurement of frequency change

- Change in signal time ⇒ change in frequency
- Emission of first wave crest at $t_{\mathrm{s}1}$
  Reception of first wave crest at $t_{\mathrm{r}1} = t_{\mathrm{s}1} + \Delta t(t_{\mathrm{s}1})$
- Emission of second wave crest at $t_{\mathrm{s}2} = t_{\mathrm{s}1} + \frac{1}{\nu_0}$
  Reception of second wave crest at $t_{\mathrm{r}2} = t_{\mathrm{s}1} + \Delta t(t_{\mathrm{s}2})$
- Measured frequency

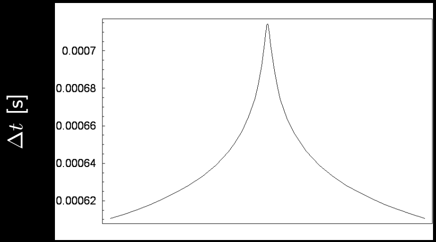
$$\nu = \frac{1}{t_{\mathrm{r}2} - t_{\mathrm{r}1}}$$

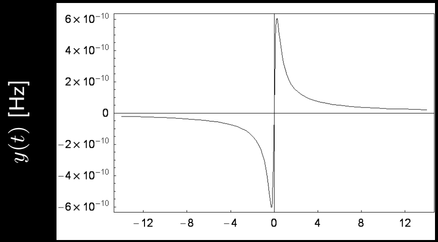
- Frequency shift

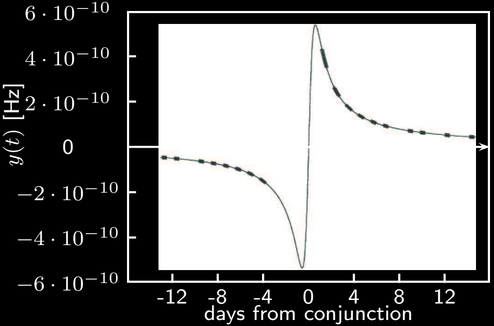
$$y(t) = \frac{\nu - \nu_0}{\nu_0} = 2(\alpha + \gamma)\frac{GM_\odot}{c^3}\frac{1}{b(t)}\frac{db(t)}{dt}$$

Use of three microwave bands in order to eliminate dispersion in Solar environment

Gravitational time delay



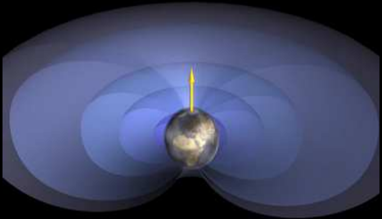




Result:

$$|\gamma - 1| \leq 2 \cdot 10^{-5}$$

The gravitomagnetic field



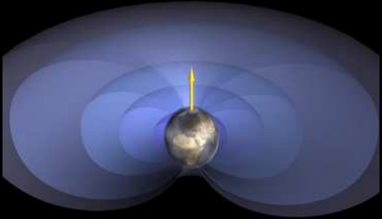
The gravitomagnetic field

- Space–time component of metric

$$g_{0i} = -\frac{G}{2} \frac{(\boldsymbol{r} \times \boldsymbol{J})^i}{r^3}$$

- Genuine post–Newtonian gravitational field
- Analogue of magnetic field

The gravitomagnetic field



Notions

- Angular momentum

$$\boldsymbol{L} = \boldsymbol{r} \times \boldsymbol{p}$$

- Runge–Lenz vector

$$\boldsymbol{A} = \boldsymbol{L} \times \dot{\boldsymbol{r}} + Gm\frac{\boldsymbol{r}}{r}$$

The gravitomagnetic field

- Space–time component of metric

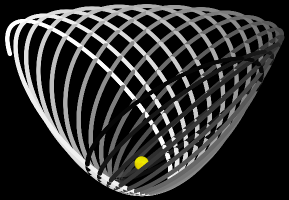
$$g_{0i} = -\frac{G}{2}\frac{(\boldsymbol{r} \times \boldsymbol{J})^i}{r^3}$$

- Genuine post–Newtonian gravitational field
- Analogue of magnetic field

Main consequences

- $\frac{d}{dt}\boldsymbol{L} \neq 0$ (Newton: $= 0$)
- $\frac{d}{dt}\boldsymbol{A} \neq 0$ (Newton: $= 0$)

Lense–Thirring effect



A particular orbit showing the Lense–Thirring effect

Observations

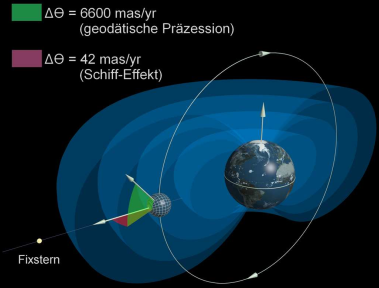
- exact solution: Weierstrass elliptic functions
- Observed quantities

$$\dot{\Omega} = \frac{2GJ}{c^2 a^3 (1-e^2)^{3/2}}$$

$$\dot{\omega} = -\frac{6GJ\cos i}{c^2 a^3 (1-e^2)^{3/2}}$$

- Measurement with LAGEOS satellites, together with data from CHAMP and GRACE
- Result: confirmation with approx 10 % error

Schiff effect



GP-B

Description

- Dynamics of direction of spin $D_v S = 0$
- Compared with direction given by distant stars
- Effective dynamics

$$\dot{\boldsymbol{S}} = \boldsymbol{\Omega} \times \boldsymbol{S}$$

with

$$\boldsymbol{\Omega} = \boldsymbol{\nabla} \times \boldsymbol{g}$$

- Ongoing data analysis
- Originally aimed accuracy: 0.1 mas/a

Result

Result

Within the range of experimental accuracy, all tests are compatible with

$$\beta = 1\,, \qquad \gamma = 1\,, \qquad \ldots$$

All measured values of PPN parameters are compatible with Einsteins General Relativity.

Unfortunately, no complete operational foundation of Einstein's equation is known

Outline

General Relativity

The space–time manifold

Set of points characterized by 4 numbers, topology, differentiability, ...

The space–time metric

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = g_{00} dt^2 + 2 g_{0i} dt dx^i + g_{ij} dx^i dx^j$$

Gravitational field equations

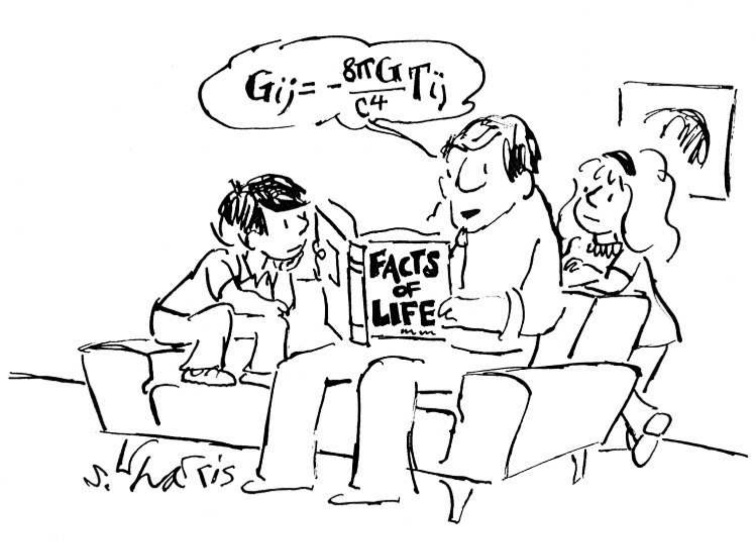
$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Equations of motion for matter

Follow from gravitational field equations $\Rightarrow$

- Point particles $D_v v = 0$, $g(v, v) = 1$
- Light rays $D_l l = 0$, $g(l, l) = 0$
- Electromagnetic field $* d * F = j$, $dF = 0$
- Spin–$\frac{1}{2}$–particle $(i\gamma D - m)\psi = 0$

General Relativity

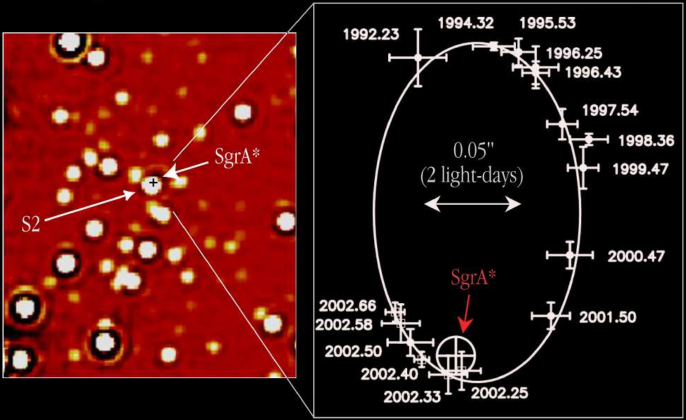


Outline

Outline

Black Hole at the Center of the Milky Way

Most important prediction: Black Holes $\longrightarrow$ Search for Black Holes



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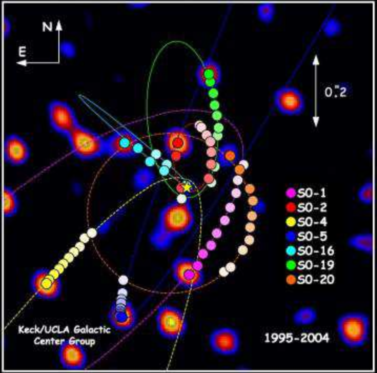
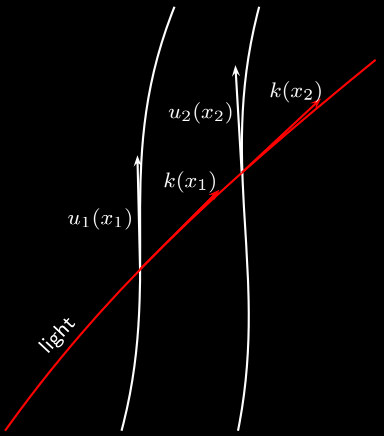


Black Hole: mass $3.7 \cdot 10^6\ M_\odot$ (Yusuf–Zasdeh *et al. Astrophys. J.* **644**, 198 (2006))
angular velocity $\sim 1/17\,\mathrm{min}$
R. Genzel (1995 – 2006)

Gravitational redshift



stationary gravitational field

- $\Rightarrow$ $k(\xi) = const$, ξ Killing vector
- $\Rightarrow$ gravitational redshift for stationary observer $u \sim \xi$

$$\frac{\nu_2}{\nu_1} = \frac{k(u_2)}{k(u_1)} = \sqrt{\frac{g_{tt}(r_2)}{g_{tt}(r_1)}} = \sqrt{\frac{1 - \frac{2M}{r_2}}{1 - \frac{2M}{r_1}}}$$

may approach ∞

Orbits in Schwarzschild space–time

Equations of motion

Equation of motion

$$D_v v = 0 \qquad \leftrightarrow \qquad \frac{d^2 x^\mu}{ds^2} + \left\{ {\mu \atop \rho\sigma} \right\} \frac{dx^\rho}{ds}\frac{dx^\sigma}{ds} = 0$$

Conservation laws

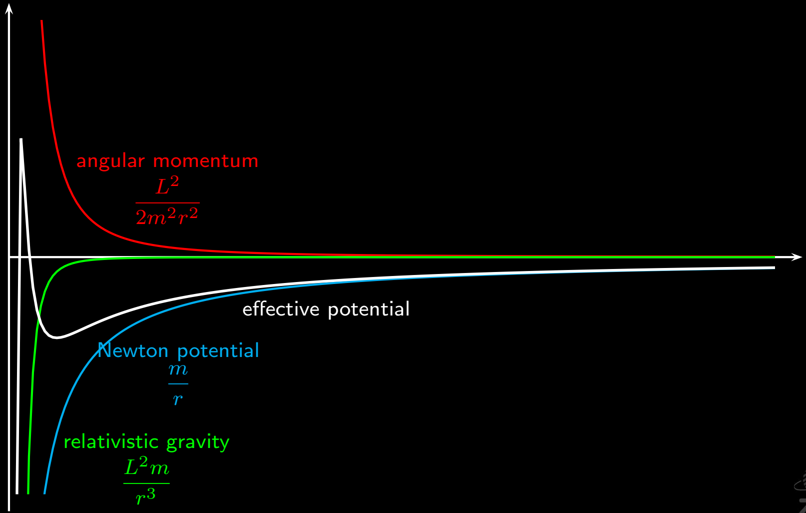
$$E = g_{00}\frac{dt}{ds}\,, \qquad L = r^2\frac{d\varphi}{ds}$$

Yields three equations

$$\begin{aligned}
\left(\frac{dr}{d\varphi}\right)^2 &= \frac{r^4}{L^2}\left(E^2 - \left(1 - \frac{2m}{r}\right)\left(\epsilon + \frac{L^2}{r^2}\right)\right) \\
\left(\frac{dr}{ds}\right)^2 &= E^2 - \left(1 - \frac{2m}{r}\right)\left(\epsilon + \frac{L^2}{r^2}\right) = E^2 - V_{\text{eff}} \\
\left(\frac{dr}{dt}\right)^2 &= \frac{1}{E^2}\left(1 - \frac{2m}{r}\right)^2\left(E^2 - \left(1 - \frac{2m}{r}\right)\left(\epsilon + \frac{L^2}{r^2}\right)\right).
\end{aligned}$$

Orbits in Schwarzschild space–time

Effective potential for $\epsilon = 1$



Orbits in Schwarzschild space–time

Final equations

With substitutions, e.g., $u = 2m/r$

$$
\begin{aligned}
d\frac{\varphi}{2} &= \frac{du}{\sqrt{4u^3 - g_2 u - g_3}} \\
d\frac{s}{2} &= \lambda L \frac{du}{\left(u + \frac{1}{3}\right)^2 \sqrt{4u^3 - g_2 u - g_3}} \\
d\frac{t}{2} &= \lambda L E \frac{du}{\left(u + \frac{1}{3}\right)^2 \left(\frac{2}{3} - u\right) \sqrt{4u^3 - g_2 u - g_3}} ,
\end{aligned}
$$

with

$$
g_2, g_3 \qquad \leftrightarrow \qquad L, E, M
$$

Orbits in Schwarzschild space–time

- Separation of variables

$$\varphi - \varphi_0 = \int_{u_0}^{u} \frac{du'}{\sqrt{P_3(u')}}$$

- Uniqueness of integration: u is function with 2 periods

$$u(\varphi + 2n\omega_1 + 2m\omega_2) = u(\varphi)$$

with half periods

$$\omega_1 = \int_{\mathrm{zero}_1}^{\mathrm{zero}_2} \frac{du}{\sqrt{P_3(u)}}\,, \qquad \omega_2 = \int_{\mathrm{zero}_3}^{\mathrm{zero}_4} \frac{du}{\sqrt{P_3(u)}}$$

Solution for orbit (Hagihara 1931)

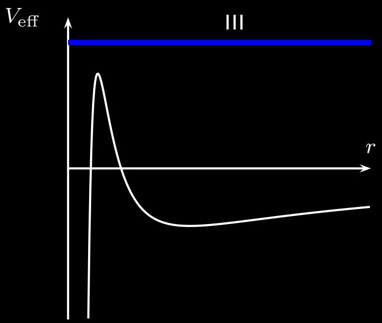
$$r(\varphi) = \frac{2m}{\frac{1}{3} + \wp\left(\frac{\varphi}{2}; g_2, g_3\right)}\,, \qquad r(\varphi) = \frac{2m}{\frac{1}{3} + \wp\left(\frac{\varphi}{2} + i\omega_2; g_2, g_3\right)}$$

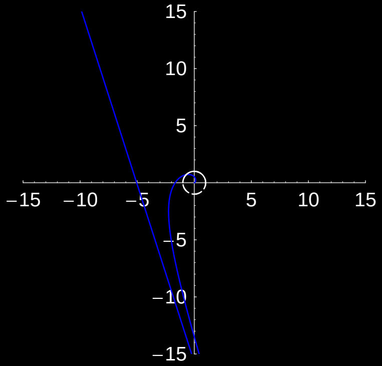
Orbits in Schwarzschild space–time

Solution for proper and coordinate time

$$
\begin{aligned}
s(\varphi) &= \frac{L}{2(\mu-1)}\left(-\zeta(\tfrac{\varphi}{2}+\varphi_1) - \zeta(\tfrac{\varphi}{2}-\varphi_1) - \varphi\wp(\varphi_1)\right. \\
&\qquad \left. + \sqrt{\frac{\lambda}{\mu-1}}\left(\ln\frac{\sigma(\frac{\varphi}{2}+\varphi_1)}{\sigma(\frac{\varphi}{2}-\varphi_1)} - \varphi\zeta(\varphi_1)\right)\right) \\
t(\varphi) &= -LE\left(\sqrt{\frac{\lambda}{\mu}}\left(\varphi\zeta(\varphi_2) - \ln\frac{\sigma(\frac{\varphi}{2}+\varphi_2)}{\sigma(\frac{\varphi}{2}-\varphi_2)}\right)\right. \\
&\qquad - \sqrt{\frac{\lambda}{\mu-1}}\left(1-\frac{1}{2(\mu-1)}\right)\left(\varphi\zeta(\varphi_1) - \ln\frac{\sigma(\frac{\varphi}{2}+\varphi_1)}{\sigma(\frac{\varphi}{2}-\varphi_1)}\right) \\
&\qquad \left. + \frac{1}{2(\mu-1)}\left(\zeta(\tfrac{\varphi}{2}+\varphi_1) + \zeta(\tfrac{\varphi}{2}-\varphi_1) - \frac{\varphi}{3}\right)\right) .
\end{aligned}
$$

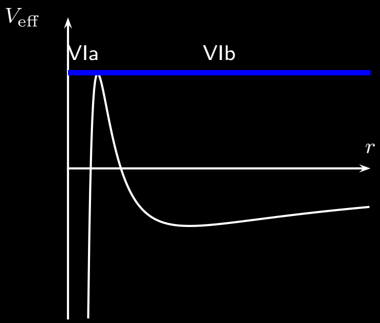
Orbits in Schwarzschild space–time

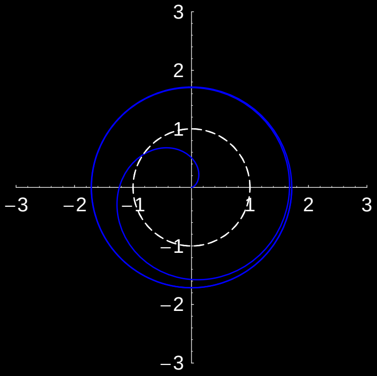




pseudo–hyperbolic

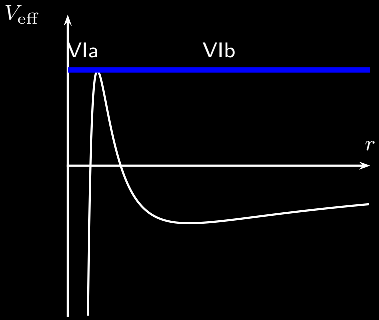
Orbits in Schwarzschild space–time

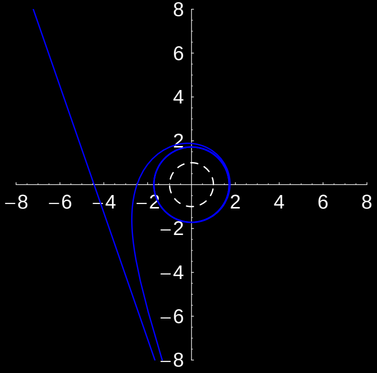




pseudo–hyperbolic spiral — hyperbolic spiral

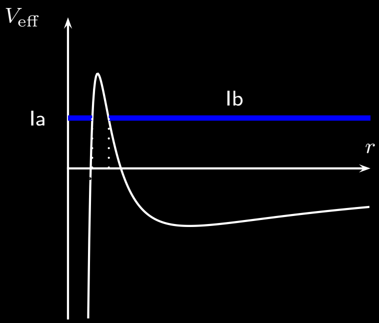
Orbits in Schwarzschild space–time

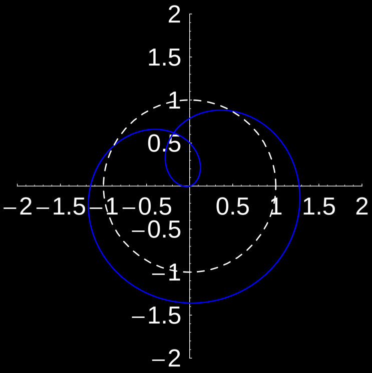




pseudo–hyperbolic spiral — hyperbolic spiral

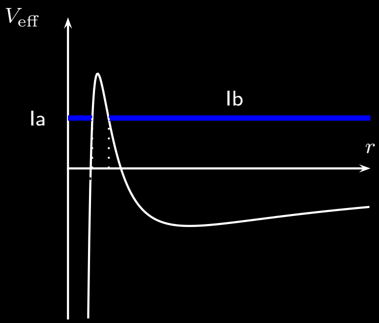
Orbits in Schwarzschild space–time

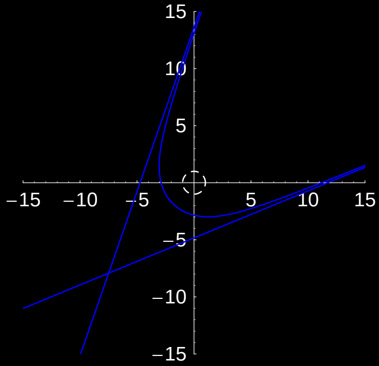




pseudo–hyperbolic — quasi hyperbolic

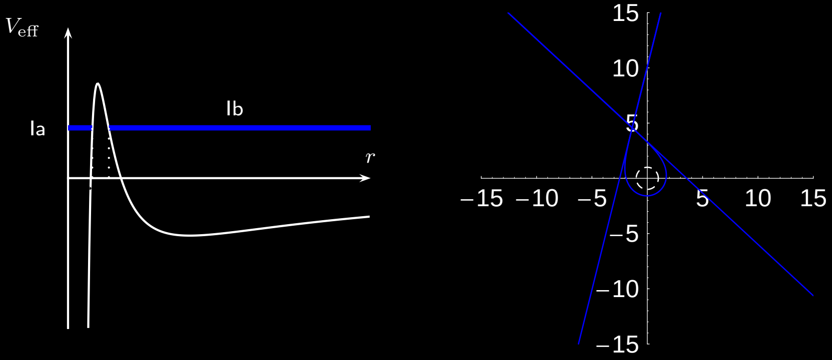
Orbits in Schwarzschild space–time





pseudo–hyperbolic — quasi hyperbolic

Orbits in Schwarzschild space–time



pseudo–hyperbolic — quasi hyperbolic

Particle deflection

In the case Ib the solution is given by

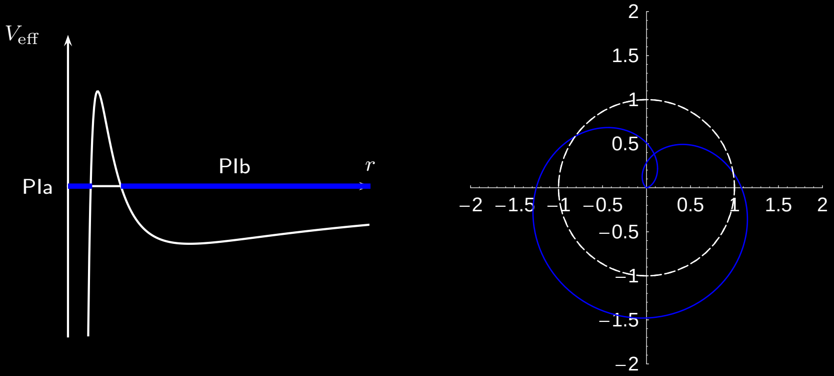
$$r(\varphi) = \frac{m}{\frac{1}{3} + \wp(\frac{\varphi}{2}; g_2, g_3)} = \frac{m}{e_3 + \frac{1}{3} + (e_2 - e_3)\mathrm{sn}^2(\sqrt{e_1 - e_3}\frac{\varphi}{2}, k)}$$

Deflection angles are given by vanishing of denumerator: Yields

$$\varphi_{1,2} = \frac{2}{\sqrt{e_1 - e_3}} F(\alpha, k)\,, \qquad \alpha = \arcsin\sqrt{-\frac{e_3 + \frac{1}{3}}{e_2 - e_3}}\,, \quad k = \sqrt{\frac{e_2 - e_3}{e_1 - e_3}}$$

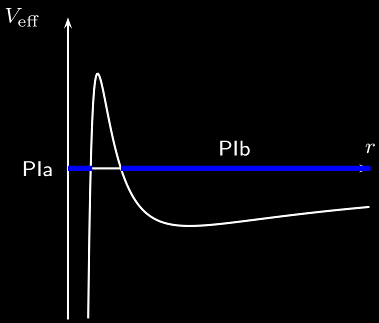
- Can be represented as function of r_{min} (impact parameter) and E.
- This can be used to perform a well–defined approximation for small $r_{\mathrm{S}}/r_{\mathrm{min}}$.

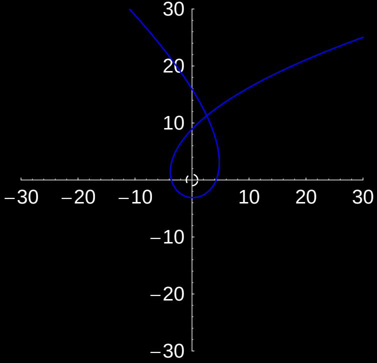
Orbits in Schwarzschild space–time



pseudo parabolic — quasi parabolic

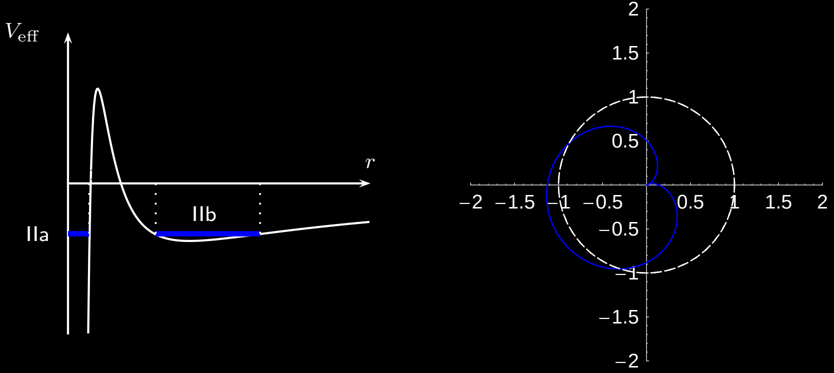
Orbits in Schwarzschild space–time





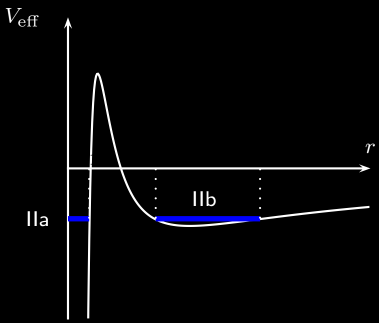
pseudo parabolic — quasi parabolic

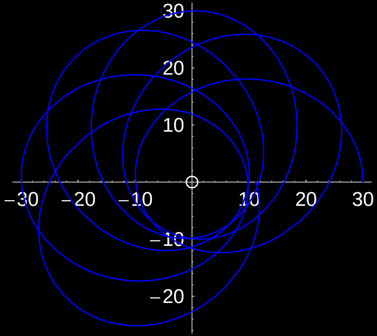
Orbits in Schwarzschild space–time



pseudo–elliptic — quasi elliptic: perihelion shift: $\delta\varphi = \omega_1 - 2\pi$

Orbits in Schwarzschild space–time





pseudo–elliptic — quasi elliptic: perihelion shift: $\delta\varphi = \omega_1 - 2\pi$

Perihelion shift

$$\omega_1 = \int_{r_{\min}}^{r_{\max}} \frac{d\varphi}{dr} dr = \int_{e_2}^{e_3} \frac{d\varphi}{dx} dx = \int_{e_2}^{e_3} \frac{dx}{\sqrt{\left(\frac{dx}{d\varphi}\right)^2}} = \int_{e_2}^{e_3} \frac{dx}{\sqrt{4x^3 - g_2 x - g_3}}$$

Exact perihelion shift

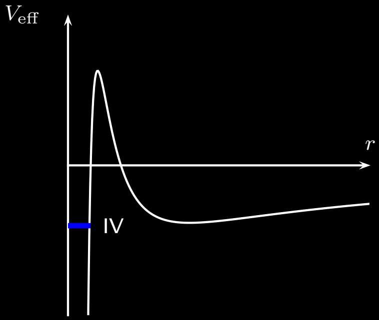
$$\delta\varphi = 2\omega_1 - 2\pi = \frac{4}{\sqrt{-e_2 - 2e_3}} \int_0^{\frac{\pi}{2}} \frac{dx}{\sqrt{1 - \frac{e_2 - e_3}{-e_2 - 2e_3} \sin^2 x}} - 2\pi\,.$$

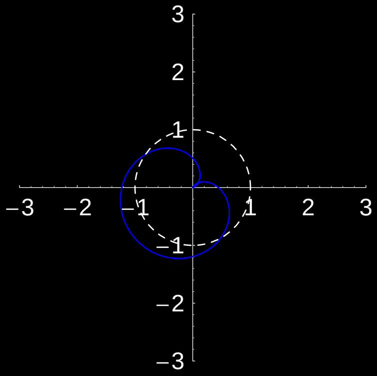
One can identify

$$e_2 = \frac{2m}{r_{\min}} - \frac{1}{3}\,, \qquad e_3 = \frac{2m}{r_{\max}} - \frac{1}{3}\,.$$

- Can be used for approximation
- Can be used for representation in terms of semi–major axis and eccentricity
- Application: Quasar QJ287: $\delta\varphi \approx 39^\circ$ per revolution (Valtonen et al 2008)

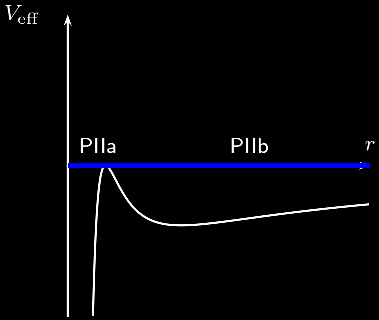
Orbits in Schwarzschild space–time

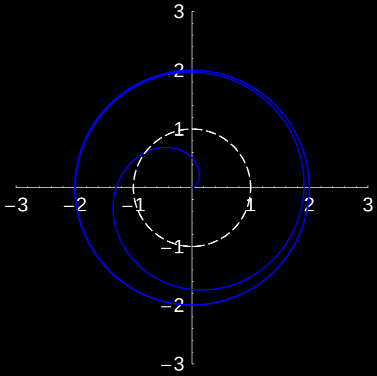




no name

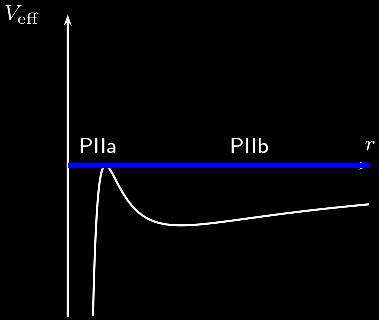
Orbits in Schwarzschild space–time

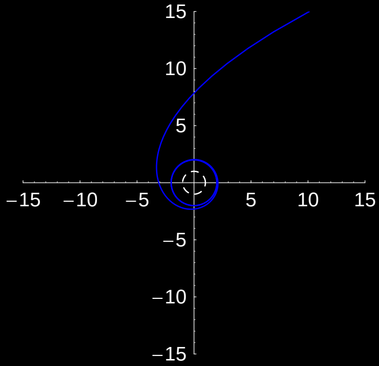




finite parabolic spiral — infinite parabolic spiral

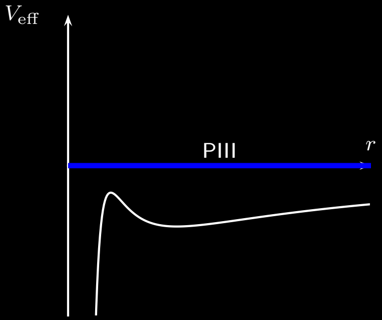
Orbits in Schwarzschild space–time

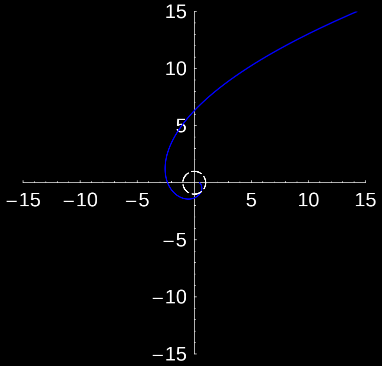




finite parabolic spiral — infinite parabolic spiral

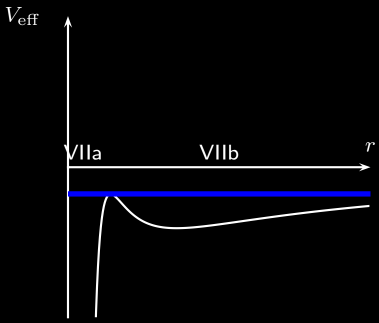
Orbits in Schwarzschild space–time

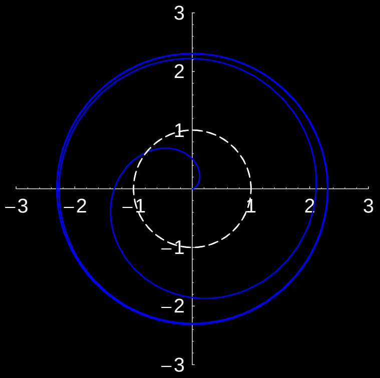




pseudo parabolic

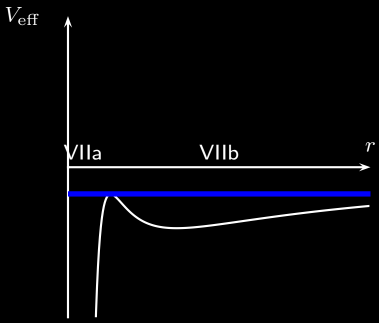
Orbits in Schwarzschild space–time

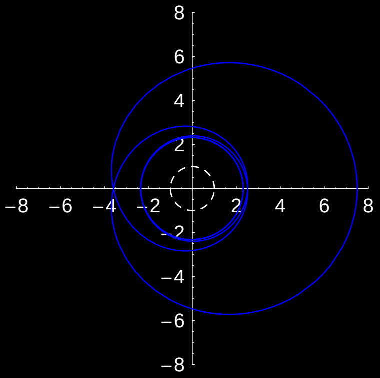




spiral — double spiral (Poincarés double circle limit)

Orbits in Schwarzschild space–time





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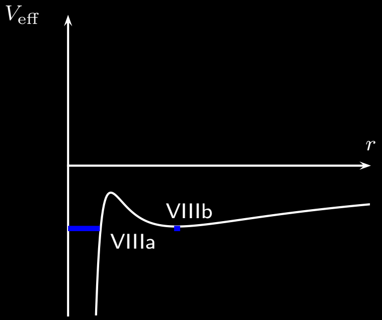
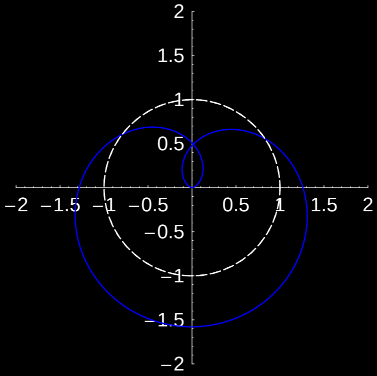
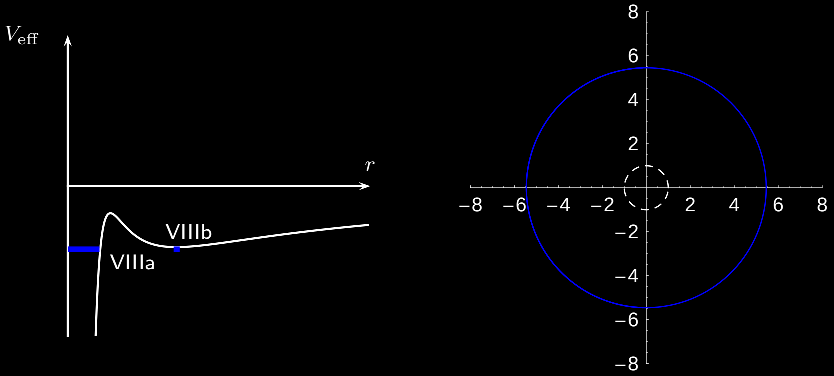
Orbits in Schwarzschild space–time





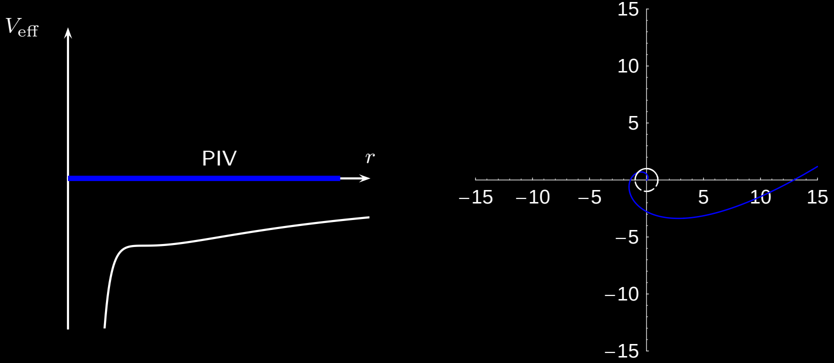
spiral — circle

Orbits in Schwarzschild space–time



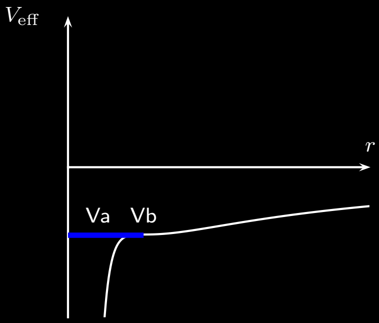
spiral — circle

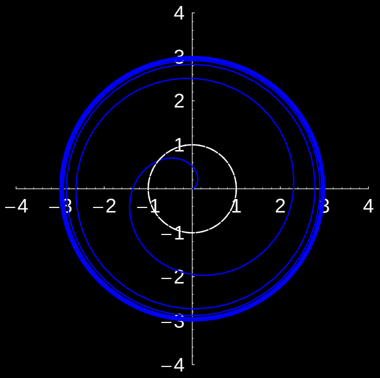
Orbits in Schwarzschild space–time



pseudo parabolic

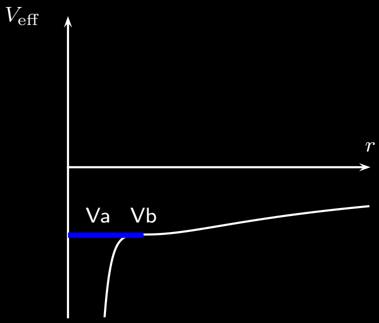
Orbits in Schwarzschild space–time

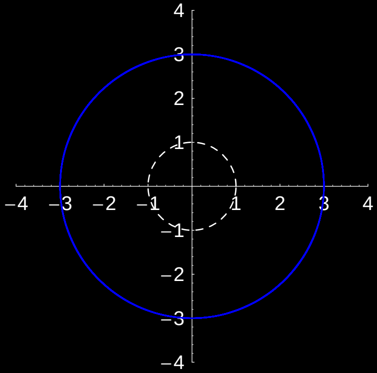




spiral — circle (ISCO)

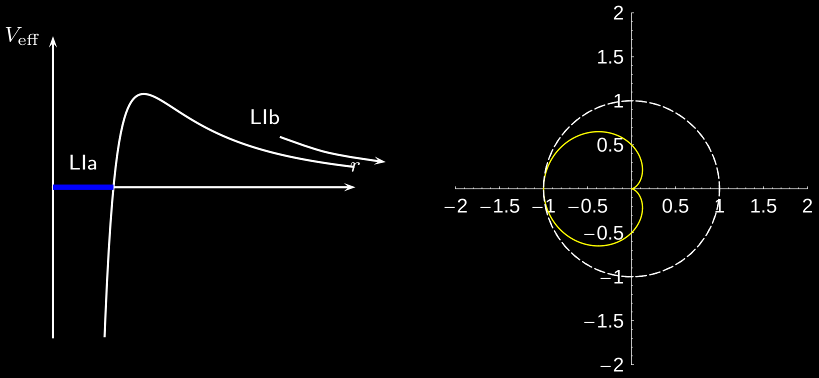
Orbits in Schwarzschild space–time





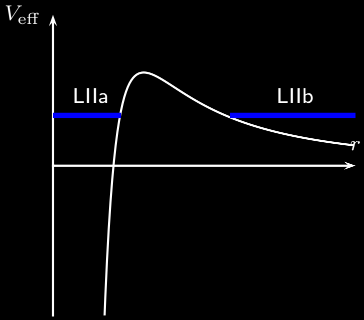
spiral — circle (ISCO)

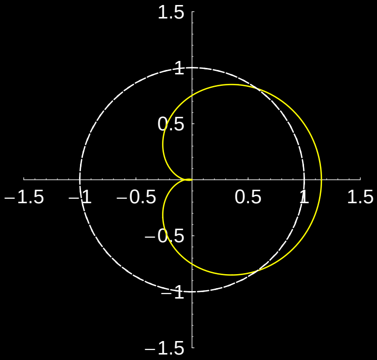
Light rays in Schwarzschild space–time



spiral — circle (ISCO)

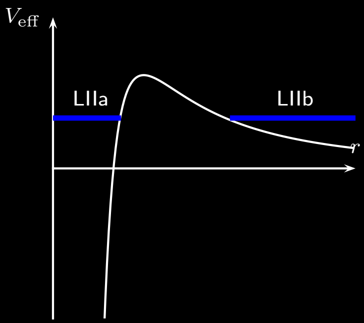
Light rays in Schwarzschild space–time

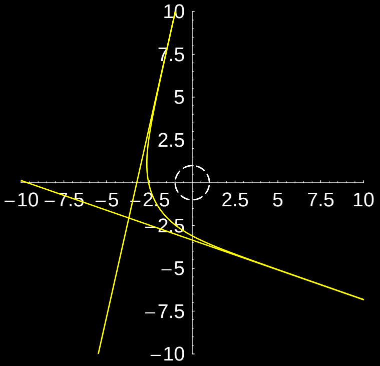




pseudo hyperbolic — quasi–hyperbolic

Light rays in Schwarzschild space–time





pseudo hyperbolic — quasi–hyperbolic

Light deflection

Solution in case LIIb given by

$$r(\varphi) = \frac{m}{\frac{1}{3} + \wp(\frac{\varphi}{2}; g_2, g_3)} = \frac{m}{e_3 + \frac{1}{3} + (e_2 - e_3)\mathrm{sn}^2\left(\sqrt{e_1 - e_3}\frac{\varphi}{2}, k\right)}$$

Deflection angle $\leftrightarrow$ vanishing of denumerator $\Rightarrow$

$$\Delta\varphi = \frac{4}{\sqrt{e_1 - e_3}} F(\alpha, k)\,, \qquad \sin\alpha = \sqrt{-\frac{e_3 + \frac{1}{3}}{e_2 - e_3}}\,,$$

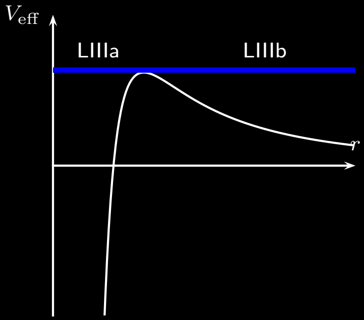
where

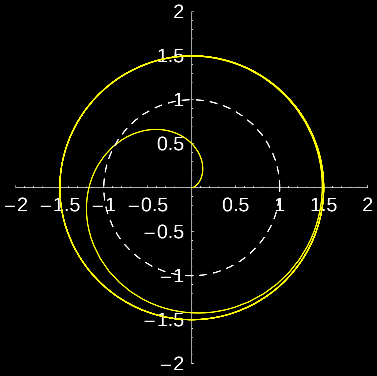
$$k^2 = \frac{e_2 - e_3}{e_1 - e_3} = \frac{\frac{3m}{r_{\min}} - \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{m}{r_{\min}} - \frac{3m^2}{r_{\min}^2}}}{2\sqrt{\frac{1}{4} + \frac{m}{r_{\min}} - \frac{3m^2}{r_{\min}^2}}}$$

$$\frac{e_3 + \frac{1}{3}}{e_2 - e_3} = \frac{-\frac{m}{r_{\min}} + \frac{1}{2} - \sqrt{\frac{1}{4} + \frac{m}{r_{\min}} - \frac{3m^2}{r_{\min}^2}}}{\frac{3m}{r_{\min}} - \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{m}{r_{\min}} - \frac{3m^2}{r_{\min}^2}}}$$

Can be used for approximation for $\frac{r_S}{r_{\min}} \ll 1$: $\Delta\varphi = \frac{4m}{r_{\min}} - (4 + 15\pi)\frac{m^2}{r_{\min}^2} + \ldots$

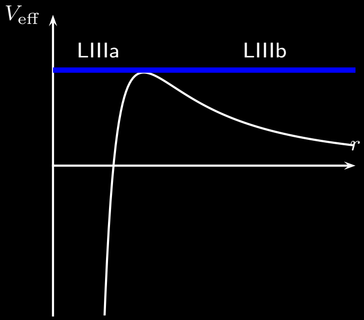
Light rays in Schwarzschild space–time

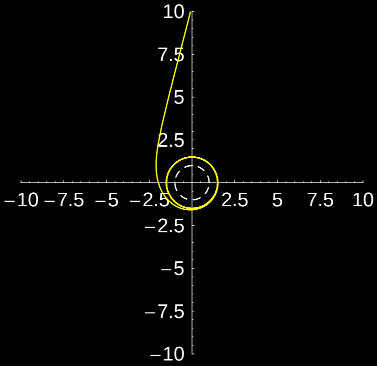




finite pseudo–hyperbolic spiral — infinite quasi-hyperbolic spiral — ISCO

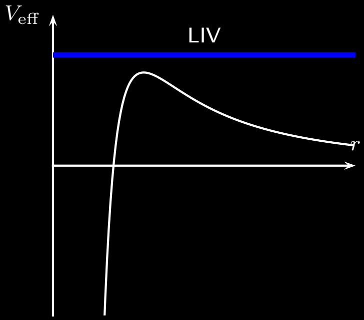
Light rays in Schwarzschild space–time

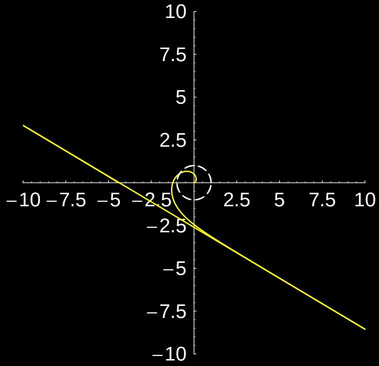




finite pseudo–hyperbolic spiral — infinite quasi-hyperbolic spiral — ISCO

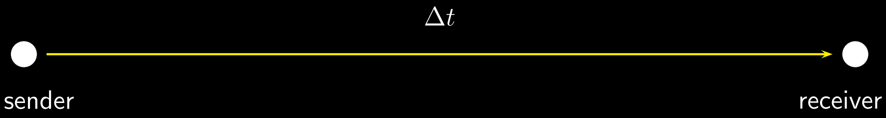
Light rays in Schwarzschild space–time



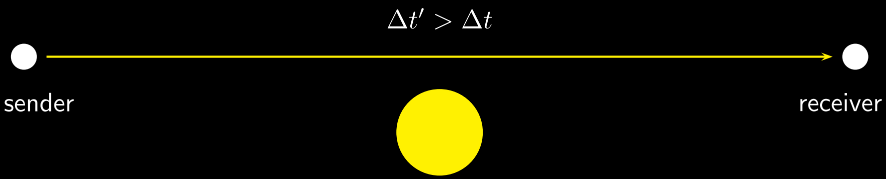


pseudo–hyperbolic

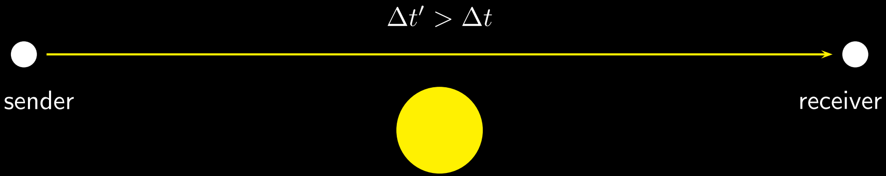
Gravitational time delay



Gravitational time delay



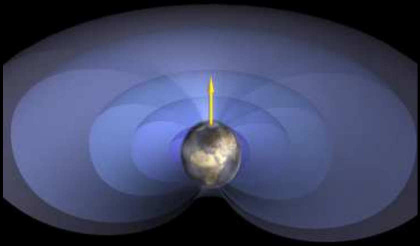
Gravitational time delay



Caution

- Within an exact framework for gravitational effects there is no definition or identification of points with and without gravitational field
  $\Rightarrow$ there is no notion of a gravitational time delay
- Within exact treatment there is only a combined effect due to gravitational time delay, redshift, kinematical time delay (Doppler effect) and light bending
- There is no way to isolate a gravitational time delay – only possible asymptotically, in weak field approximation

The gravitomagnetic field



The gravitomagnetic field

- Kerr metric
- Orbits in Kerr metric
- Perihelion shift
- Lense–Thirring effect
- combined effects
- ...

Gravitomagnetic clock effect

- rotating gravitating mass $\Rightarrow$ Kerr solution

$$ds^2 = \ldots + \frac{2Mr}{r^2 + a^2 \cos^2\vartheta} a \sin^2\vartheta \, d\varphi \, dt + \ldots$$

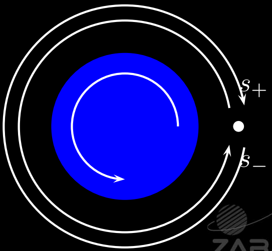
- geodesic equation for circular orbits in equatorial plane

$$\frac{d\varphi}{dt} = \pm\Omega_0 + \Omega_{\text{Lense-Thirring}}$$

- proper time difference of two counterpropagating clocks

$$s_+ - s_- = 4\pi \frac{J}{M} \sim 10^{-7}\ \text{s}$$

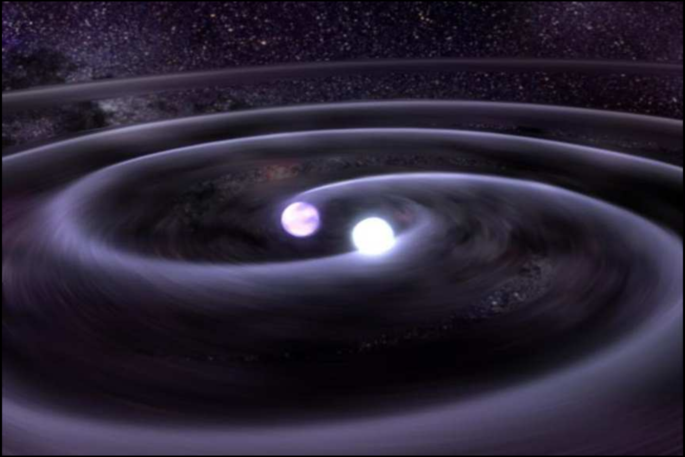
does not depend on G and on r
decreases with inclination, vanishes for polar orbits



Outline

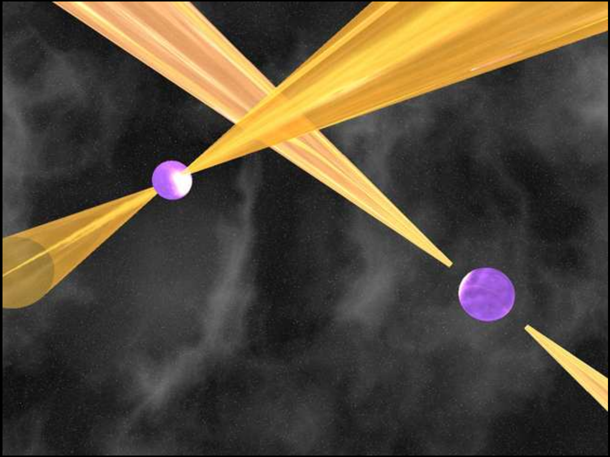
Binary systems

Strong field regime $\longrightarrow$ access to more PPN parameters, nonlinearities, ...



Binary systems

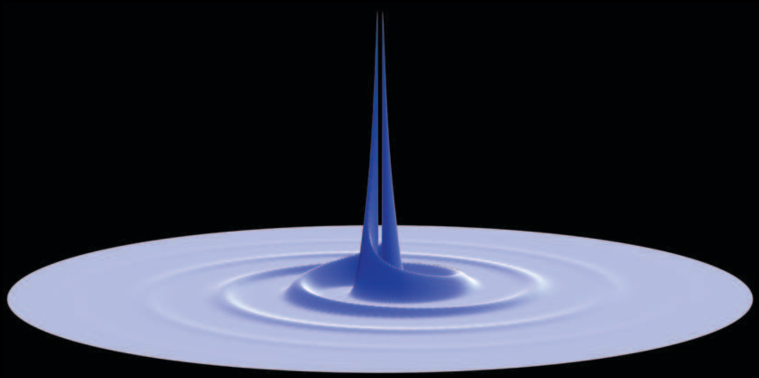
Strong field regime $\longrightarrow$ access to more PPN parameters, nonlinearities, ...



Outline

Gravitational waves

Radiation properties $\longrightarrow$ mass of graviton, ...

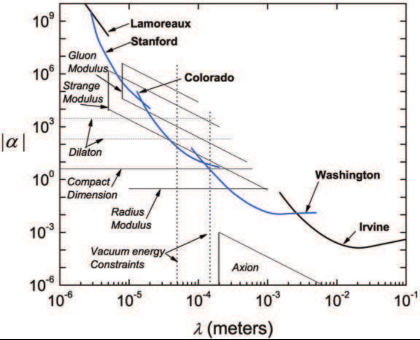


Outline

Outline

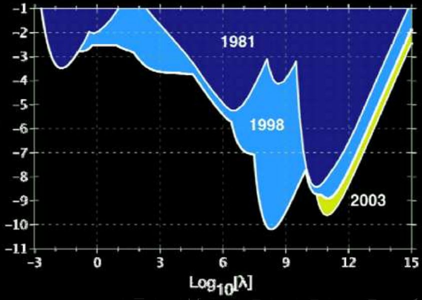
Test of Newton potential I

$$U = \frac{GM}{r}\left(1 + \alpha e^{-r/\lambda}\right)$$



short range limits

long range limits



Test of Newton potential I

Kostelecky framework (Kostelecky, PRD 2005): anisotropy of Newtonian potential

$$U = \frac{MG}{r}\left(1 + \frac{r^i c_{ij} r^j}{r^2}\right)$$

Experiments

- Atomic interferometry (Müller et al, PRL 2007)
- LLR (Battat, Chandler & Stubbs, PRL 2007)

Result

$$|c_{ij}| \leq 10^{-5} \ldots 10^{-9}$$

Outline

Active and passive mass

Gravitationally bound two–body system (Bondi, RMP 1957)

$$m_{1\mathrm{i}}\ddot{\boldsymbol{x}}_1 = m_{1\mathrm{p}} m_{2\mathrm{a}} \frac{\boldsymbol{x}_2 - \boldsymbol{x}_1}{|\boldsymbol{x}_2 - \boldsymbol{x}_1|^3}\,, \qquad m_{2\mathrm{i}}\ddot{\boldsymbol{x}}_2 = m_{2\mathrm{p}} m_{1\mathrm{a}} \frac{\boldsymbol{x}_1 - \boldsymbol{x}_2}{|\boldsymbol{x}_1 - \boldsymbol{x}_2|^3}$$

Unequal active and passive masses $\Rightarrow$ self–acceleration of center of mass

$$\ddot{\boldsymbol{X}} = \frac{m_{1\mathrm{p}} m_{2\mathrm{p}}}{M_\mathrm{i}} \bar{C}_{21} \frac{\boldsymbol{x}}{|\boldsymbol{x}|^3}\,, \qquad \bar{C}_{21} = \frac{m_{2\mathrm{a}}}{m_{2\mathrm{p}}} - \frac{m_{1\mathrm{a}}}{m_{1\mathrm{p}}}\,.$$

$\bar{C}_{21} = 0$: ratio of the active and passive masses are equal for both particles.
Dynamics of relative coordinate

$$\ddot{\boldsymbol{x}} = -\frac{m_{1\mathrm{p}} m_{2\mathrm{p}}}{m_{1\mathrm{i}} m_{2\mathrm{i}}} \left(m_{1\mathrm{i}} \frac{m_{1\mathrm{a}}}{m_{1\mathrm{p}}} + m_{2\mathrm{i}} \frac{m_{2\mathrm{a}}}{m_{2\mathrm{p}}} \right) \frac{\boldsymbol{x}}{|\boldsymbol{x}|^3}\,.$$

Interpretation

$\ddot{\boldsymbol{X}} \neq 0 \quad \Leftrightarrow \quad C_{12} \neq 0 \quad \Leftrightarrow$ violation of *actio* = *reactio* for gravity

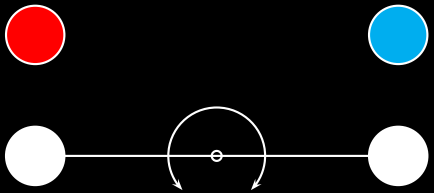
Experiment testing $m_{\rm ga} = m_{\rm gp}$

Measurement of relative acceleration

Step 1: Take two masses with $m_{\rm pg1} = m_{\rm pg2}$ (equal weight)

Step 2: Test active equality of these two masses with torsion balance

Experimental setup: Torsion balance with equal passive masses reacting on $m_{\rm ag1}$ and $m_{\rm ag2}$



No effect has been seen: $C_{12} \leq 5 \cdot 10^{-5}$
Kreuzer, PR 1868

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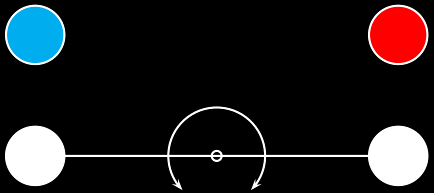

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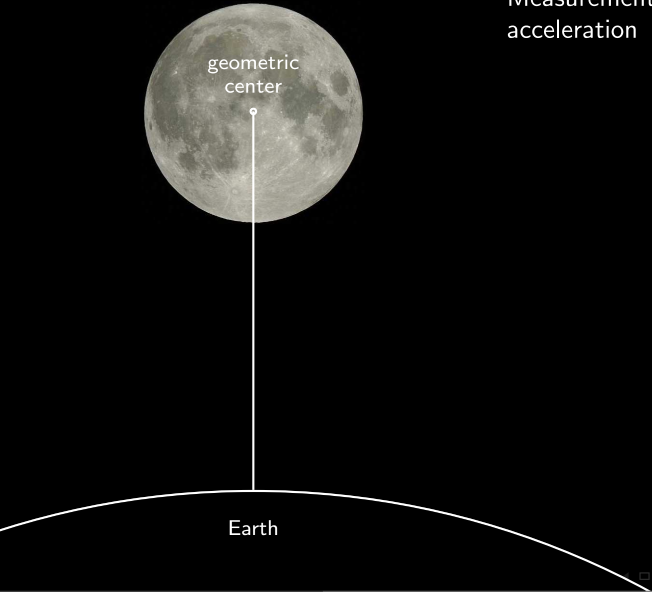
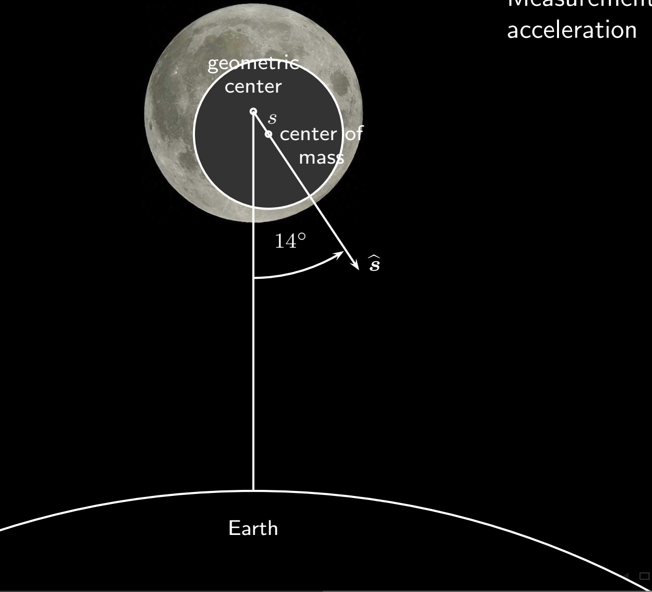


Measurement of center–of–mass acceleration

Experiment testing $m_{\mathrm{ga}} = m_{\mathrm{gp}}$



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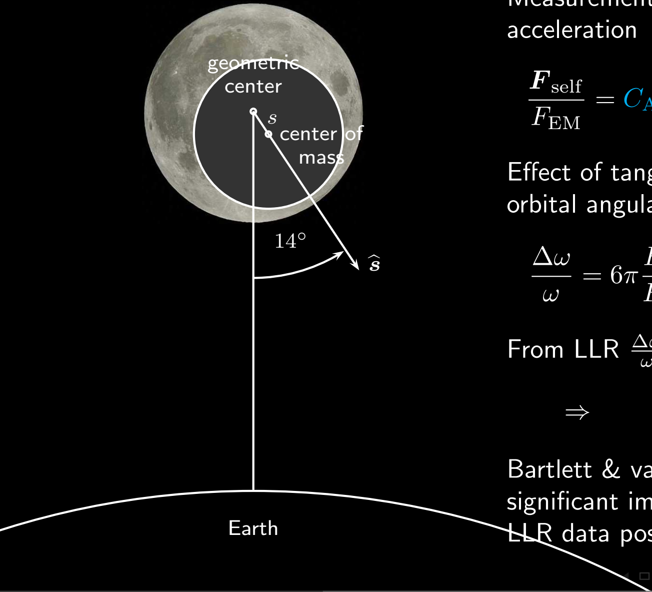

Measurement of center–of–mass acceleration

$$\frac{\boldsymbol{F}_{\rm self}}{F_{\rm EM}} = C_{\rm Al-Fe} \frac{M_{\rm M}}{M_\oplus} \frac{r_{\rm EM}^2}{r_{\rm M}^2} \frac{s}{r_{\rm M}} \frac{\rho}{\Delta\rho} \hat{\boldsymbol{s}}$$

Effect of tangential force: increase of orbital angular velocity

$$\frac{\Delta\omega}{\omega} = 6\pi \frac{F_{\rm self}}{F_{\rm EM}} \sin 14° \text{ per month}$$

From LLR $\frac{\Delta\omega}{\omega} \leq 10^{-12}$ per month

$$\Rightarrow \qquad C_{\rm Al-Fe} \leq 7 \cdot 10^{-13}$$

Bartlett & van Buren PRL 1986
significant improvement with new LLR data possible

Outline

Active and passive charges: Dynamics

Dynamics of two electrically bound particles ($\boldsymbol{E}$ = external electric field)

$$
\begin{aligned}
m_{1\mathrm{i}}\ddot{\boldsymbol{x}}_1 &= q_{1\mathrm{p}}q_{2\mathrm{a}}\frac{\boldsymbol{x}_2-\boldsymbol{x}_1}{|\boldsymbol{x}_2-\boldsymbol{x}_1|^3}+q_{1\mathrm{p}}\boldsymbol{E}(\boldsymbol{x}_1)\,,\\
m_{2\mathrm{i}}\ddot{\boldsymbol{x}}_2 &= q_{2\mathrm{p}}q_{1\mathrm{a}}\frac{\boldsymbol{x}_1-\boldsymbol{x}_2}{|\boldsymbol{x}_1-\boldsymbol{x}_2|^3}+q_{2\mathrm{p}}\boldsymbol{E}(\boldsymbol{x}_2)\,,
\end{aligned}
$$

center–of–mass and relative coordinate

$$
\boldsymbol{X} := \frac{m_{1\mathrm{i}}}{M_\mathrm{i}}\boldsymbol{x}_1+\frac{m_{2\mathrm{i}}}{M_\mathrm{i}}\boldsymbol{x}_2\,,\qquad \boldsymbol{x} := \boldsymbol{x}_2-\boldsymbol{x}_1\,,
$$

$M_\mathrm{i} = m_{1\mathrm{i}} + m_{2\mathrm{i}}$ = total inertial mass. Then

$$
\ddot{\boldsymbol{X}} = \frac{q_{1\mathrm{p}}q_{2\mathrm{p}}}{M_\mathrm{i}}C_{21}\frac{\boldsymbol{x}}{|\boldsymbol{x}|^3}+\frac{1}{M_\mathrm{i}}\left(q_{1\mathrm{p}}+q_{2\mathrm{p}}\right)\boldsymbol{E}\qquad C_{21} := \frac{q_{2\mathrm{a}}}{q_{2\mathrm{p}}}-\frac{q_{1\mathrm{a}}}{q_{1\mathrm{p}}}\,.
$$

$C_{21} = 0$: ratio between the active and passive charge is the same for both particles
$C_{21} \neq 0 \Rightarrow$ self–acceleration of center of mass along $\boldsymbol{x}$

Active and passive charges: Dynamics

Dynamics of relative coordinate

$$\ddot{\boldsymbol{x}} = -\frac{1}{m_{\rm red}} q_{1\rm p} q_{2\rm p} D_{21} \frac{\boldsymbol{x}}{|\boldsymbol{x}|^3}\,, \qquad (1)$$

where

$$D_{21} = \frac{m_{1\rm i}}{M_{\rm i}} \frac{q_{1\rm a}}{q_{1\rm p}} + \frac{m_{2\rm i}}{M_{\rm i}} \frac{q_{2\rm a}}{q_{2\rm p}} = \frac{q_{1\rm a}}{q_{1\rm p}} + \frac{m_{2\rm i}}{M_{\rm i}} C_{21}$$

In the standard framework, $D_{21} = 1$.
Solutions of equation of motion (1) are ellipses, circles.
The center of mass oscillates at a frequency ω, which is related to the energy of the system.
The acceleration of the center of mass vanishes on average, $\langle \ddot{\boldsymbol{X}} \rangle = 0$. Thus, not observable for atoms.
Extends to many particle systems, e.g., to atoms having many electrons.

Interpretation

$\ddot{\boldsymbol{X}} \neq 0 \quad \Leftrightarrow \quad C_{12} \neq 0 \quad \Leftrightarrow$ violation of *actio* = *reactio* for electromagnetism

Comparison: charge vs mass

Differences

- If UFF holds, $m_{\rm i} = m_{\rm p}$, then paths of particles depend on the active gravitational mass only.
- Since the timescale of electric phenomena is much shorter than that of gravitational ones, the motion of the center of mass cannot be monitored. This kind of test is therefore not at our disposal.
- Contrary to masses, electric charges can have different signs. Therefore, we can define

  active neutrality $q_{1\rm a} + q_{2\rm a} = 0$
  passive neutrality $q_{1\rm p} + q_{2\rm p} = 0$

  $\rightarrow$ alternative tests of the equality of active and passive charges: An actively neutral system may not be passively neutral and vice versa.
  active and passive neutrality $\Leftrightarrow C_{21} = 0$
  $\Rightarrow$ self–acceleration of the center of mass occurs if and only if the system possesses a nonzero total active or passive charge.

Experiments

tests of neutrality of atoms and molecules = tests of the equality of active and passive charge

- a passively neutral system may still generate an electric field according to

$$\phi(\boldsymbol{x}) = \frac{q_{1\mathrm{a}}}{|\boldsymbol{x} - \boldsymbol{x}_1|} + \frac{q_{2\mathrm{a}}}{|\boldsymbol{x} - \boldsymbol{x}_2|} = \frac{q_{1\mathrm{a}} + q_{2\mathrm{a}}}{|\boldsymbol{x}|} + \ldots \approx C_{21} \frac{q_{2\mathrm{p}}}{|\boldsymbol{x}|}$$

- an actively neutral atom in an external electric field may feel a force

$$M_{\mathrm{i}} \ddot{\boldsymbol{X}} = (q_{\mathrm{p}1} + q_{\mathrm{p}2}) \boldsymbol{E} = \frac{q_{2\mathrm{p}}}{q_{2\mathrm{a}}} q_{1\mathrm{a}} C_{12} \boldsymbol{E}$$

  vanishes if ratios of active and passive charges are the same for all bodies

we can distinguish two types of tests of neutrality:

- Tests of active neutrality, which measure the electric monopole field created by a passively neutral system, and
- tests of passive neutrality, which measure the force imposed by an external field onto an actively neutral system.

Experiments: Neutrality of atoms

Table: Various tests of the neutrality of atoms. If no particle is specified, $q_{\rm p}$ refers to the passive charge of the atoms or molecules used in the experiment, divided by the charge number of that particle (and analogous for $q_{\rm a}$). See Unnikrishnan & Gillies 2004 for a review.

Method	Limit $/(10^{-20} e)$
Gas efflux (350 g CO_2) [Piccard & Kessler 1925]	$q_{p,\rm a}/q_{e,\rm a} = 0.1(5)$
Gas efflux (Ar/N) [Hillas & Cranshaw 1960]	$q_{\rm H,a} = 1(3)$; $q_{n,\rm a} = -1(3)$
Gas efflux [King 1960]	$q_{\rm He,a} = -4(2)$
Superfluid He [Classen et al 1998]	$q_{\rm He,a} = -0.22(15)$
Levitator [Marinelli & Morpurgo 1982]	$\lvert q_{\rm p}\rvert \lesssim 1000$
Acoustic resonator (SF_6) [Dylla & King 1973]	$\lvert q_{\rm p}\rvert \leq 0.13$
Cs beam [Hughes 1957]	$q_{\rm p} = 90(20)$
Neutron beam [Baumann et al 1988]	$q_{n,\rm p} = -0.4(1.1)$

limits go down to $10^{-21}\, e$ for active and passive charge of various combinations of electrons, protons, and neutrons.

$\Rightarrow \lvert C_{21}\rvert \leq 10^{-21}$.

Experiments: atomic spectroscopy

Center–of–mass motion of the two–particle system cannot be quantized.
Relative motion quantizable.
Hamiltonian

$$H = \frac{\boldsymbol{p}^2}{2m_{\rm red}} + D_{21}\frac{q_{1\rm p}q_{2\rm p}}{|\boldsymbol{x}|}\,.$$

Energy levels are proportional to modified fine structure constant

$$\alpha_{12} = \frac{q_{1\rm p}q_{2\rm p}D_{12}}{\hbar c} = \frac{1}{\hbar c}q_{1\rm p}q_{2\rm p}\left(\frac{q_{1\rm a}}{q_{1\rm p}} + \frac{m_{2\rm i}}{M_{\rm i}}C_{21}\right)\,.$$

Spacing between energy levels will deviate from the ordinary scaling with the charges ⇒ comparison of energy levels in different atoms yields test of C_{21}.
E.g., Hydrogen (one proton $q_1 = q_p$ and one electron $q_2 = q_e$) and ionized Helium He$^+$ ($q_1 = 2q_p$ and $q_2 = q_e$). Then

$$\alpha_{12}(\mathrm{He}^+) - 2\alpha_{12}(\mathrm{H}) \approx -\frac{q_{pp}q_{ep}}{\hbar c}\frac{m_{ei}}{m_{pi}}C_{21}$$

can deduce limit

$$|C_{21}| \leq \frac{\delta\alpha}{\alpha}\frac{m_{ei}}{m_{pi}} \approx 7\times10^{-13}\frac{m_{ei}}{m_{pi}} \approx 4\times10^{-16}\,.$$

Active and passive magnetic moment

Force between two magnetic moments

$$\begin{aligned} m_{1\mathrm{i}}\ddot{\boldsymbol{x}}_1 &= \boldsymbol{\nabla}_1 \left(\boldsymbol{\mu}_{1\mathrm{p}} \cdot \boldsymbol{B}_2(\boldsymbol{x}_1)\right) \, , \\ m_{2\mathrm{i}}\ddot{\boldsymbol{x}}_2 &= \boldsymbol{\nabla}_2 \left(\boldsymbol{\mu}_{2\mathrm{p}} \cdot \boldsymbol{B}_1(\boldsymbol{x}_2)\right) \, , \end{aligned}$$

where

$$\boldsymbol{B}_j(\boldsymbol{x}_k) = \frac{3((\boldsymbol{x}_k - \boldsymbol{x}_j) \cdot \boldsymbol{\mu}_{j\mathrm{a}})(\boldsymbol{x}_k - \boldsymbol{x}_j) - \boldsymbol{\mu}_{j\mathrm{a}}|\boldsymbol{x}_k - \boldsymbol{x}_j|^2}{|\boldsymbol{x}_k - \boldsymbol{x}_j|^5} \, .$$

Assume $\boldsymbol{\mu}_{1,2\mathrm{a,p}} = \mu_{1,2\mathrm{a,p}}\widehat{\boldsymbol{\mu}}_{1,2}$. We introduce

$$\begin{aligned} \widetilde{C}_{21} &= \frac{\mu_{2\mathrm{a}}}{\mu_{2\mathrm{p}}} - \frac{\mu_{1\mathrm{a}}}{\mu_{1\mathrm{p}}} \, , \\ \widetilde{D}_{21} &= \frac{m_{1\mathrm{i}}}{M_\mathrm{i}}\frac{\mu_{1\mathrm{a}}}{\mu_{1\mathrm{p}}} + \frac{m_{2\mathrm{i}}}{M_\mathrm{i}}\frac{\mu_{2\mathrm{a}}}{\mu_{2\mathrm{p}}} = \frac{\mu_{1\mathrm{a}}}{\mu_{1\mathrm{p}}} + \frac{m_{2\mathrm{i}}}{M_\mathrm{i}}\widetilde{C}_{21} \end{aligned}$$

then we obtain for the center–of–mass and relative coordinates

$$\ddot{\boldsymbol{X}} = -\frac{\mu_{2\mathrm{p}}\mu_{1\mathrm{p}}}{M_\mathrm{i}}\widetilde{C}_{21}\boldsymbol{\nabla}\frac{3(\boldsymbol{x}\cdot\widehat{\boldsymbol{\mu}}_2)(\widehat{\boldsymbol{\mu}}_1\cdot\boldsymbol{x}) - \widehat{\boldsymbol{\mu}}_1\cdot\widehat{\boldsymbol{\mu}}_2|\boldsymbol{x}|^2}{|\boldsymbol{x}|^5} \, ,$$

$$\ddot{\boldsymbol{x}} = \frac{\mu_{1\mathrm{p}}\mu_{2\mathrm{p}}}{m_\mathrm{red}}\widetilde{D}_{21}\boldsymbol{\nabla}\frac{3(\boldsymbol{x}\cdot\widehat{\boldsymbol{\mu}}_2)(\widehat{\boldsymbol{\mu}}_1\cdot\boldsymbol{x}) - \widehat{\boldsymbol{\mu}}_1\cdot\widehat{\boldsymbol{\mu}}_2|\boldsymbol{x}|^2}{|\boldsymbol{x}|^5} \, .$$

Active and passive magnetic moment

Relative motion can contribute to energy of Hydrogen atom: Hamiltonian for hyperfine interaction

$$H_{\mathrm{hf}} = -\frac{\mu_{1\mathrm{p}}\mu_{2\mathrm{p}}}{m_{\mathrm{red}}}\widetilde{D}_{21}\left(\frac{8\pi}{3}\delta(x)\widehat{\boldsymbol{\mu}}_1\cdot\widehat{\boldsymbol{\mu}}_2 + \frac{3(\boldsymbol{x}\cdot\widehat{\boldsymbol{\mu}}_2)(\widehat{\boldsymbol{\mu}}_1\cdot\boldsymbol{x}) - \widehat{\boldsymbol{\mu}}_1\cdot\widehat{\boldsymbol{\mu}}_2|\boldsymbol{x}|^2}{|\boldsymbol{x}|^5}\right) \quad (2)$$

The $\widehat{\boldsymbol{\mu}}_2 =$ total angular momentum operators. To obtain experimental limits, we can compare the hyperfine splitting of atoms having different nuclei.

Experiment

Comparison of hyperfine splitting of atoms having different nuclei:

- muonium, accuracy $1.1 \cdot 10^{-8}$. compatible with theory with accuracy $1.2 \cdot 10^{-7}$
- positronium: two precision measurements of 1S hyperfine splitting (Mills et al. 1975, Ritter et al. 1985); agree within the experimental error but deviate from the theoretical prediction.

Deviation can be modeled by $\widetilde{C}_{ee^+} \approx -1.5 \cdot 10^{-5}$
Thus,

$$|\widetilde{D}_{21}| \lesssim 10^{-4}$$

Summary

Active and passive charges

- Self–acceleration of center–of–mass
- Active and passive neutrality → experiments
- Theory cannot be described by an action principle
- Hamiltonian for relative motion
- Active and passive magnetic moments: similar features

Experiments

- Classical neutrality experiments $|C_{21}| \lesssim 10^{-21}$
- Spectroscopy $|C_{21}| \lesssim 10^{-16}$
- Active and passive magnetic moment $|\widetilde{D}_{21}| \lesssim 10^{-4}$

(CL, Macias & Müller, PRA 2007)

Summary in between

- Questioning Newton's first law: Testing Lorentz invariance

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- Questioning Newton's first law: Testing Lorentz invariance
- Questioning Newton's third law, questioning actio = reactio: Testing equality of active and passive mass, charge, ...
- Questioning Newton's second law:
  - Why the acceleration is linear to the force?
  - Why equations of motion are of second order in the time derivative?

Outline

MOdified Newtonian Dynamics

MOND ansatz

Modification of Newton's second law = modification of relation between applied force and resulting acceleration (Milgrom)

$$m\ddot{\boldsymbol{x}} = \boldsymbol{F} \qquad \longrightarrow \qquad m\ddot{\boldsymbol{x}}\mu(|\ddot{\boldsymbol{x}}|/a_0) = \boldsymbol{F}$$

with function

$$\mu(x) = \begin{cases} 1 & \text{for} \quad |\ddot{\boldsymbol{x}}| \gg a_0 \\ x & \text{for} \quad |\ddot{\boldsymbol{x}}| \ll a_0 \end{cases}$$

For Newtonian gravity

Newtonian force $\boldsymbol{F} = m\boldsymbol{\nabla}U$

- large accelerations / large forces $\ddot{\boldsymbol{x}} = \boldsymbol{\nabla}U$
- small accelerations / small forces $\ddot{\boldsymbol{x}}|\ddot{\boldsymbol{x}}| = a_0\boldsymbol{\nabla}U \rightarrow |\ddot{\boldsymbol{x}}| = \sqrt{a_0|\boldsymbol{\nabla}U|}$

reproduces many galactic rotation curves for $a_0 \sim 10^{-9}\ \mathrm{m/s}^2$ – may also reproduce dynamics of galactic clusters

Testing MOND

Torsion balance

Gundlach et al PRL 2007

- transscribes acceleration into torque (for hollow cylinder with radius R): $\tau = \frac{I}{R} a\mu(a/a_0)$
- takes $r\ddot{\theta} = a$ as acceleration
- no deviation from Newton's second law down to $a \sim 10^{-14}\ \mathrm{m/s}^2$

My problem: is it allowed to separate components?
Earlier experiment by Abramovici and Vager PRD 86

Free fall experiment

Ignatiev PRL 2007
Free fall with respect to galaxy:
MOND–situation possible on Earth once a year for 0.1 s within 10 l volume

Test of order of inertial law

Question: Why equations of motion are of second order in the time derivative?
Model: what happens if equations of motion contain a small third order term

$$m\ddot{\boldsymbol{x}} + \epsilon \dddot{\boldsymbol{x}} = \boldsymbol{F}(\boldsymbol{x}, \dot{\boldsymbol{x}})$$

Initial value problem:

If $\epsilon = 0$: need to know $\boldsymbol{x}(t_0)$ and $\dot{\boldsymbol{x}}(t_0)$

If $\epsilon \neq 0$: need to know $\boldsymbol{x}(t_0)$, $\dot{\boldsymbol{x}}(t_0)$, and $\ddot{\boldsymbol{x}}(t_0)$

In general: equation of motion with memory

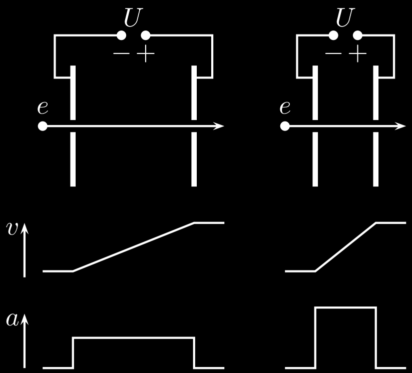
$$m\ddot{\boldsymbol{x}} + \sum_i \epsilon_i \frac{d^i \boldsymbol{x}}{dt^i}(t_0) = \boldsymbol{F}(\boldsymbol{x}, \dot{\boldsymbol{x}}) \qquad \Leftrightarrow \qquad m\ddot{\boldsymbol{x}} = \boldsymbol{F} + \int^t \boldsymbol{f}(t - t')dt'$$

solution depends on the path in the past.

Possible experiment for testing this $\rightarrow$

Test of the inertial law

Possible experiment: acceleration of charged particle



Is test of

$$\frac{1}{2}mv^2 = e\Delta V$$

Is this true for all accelerations?

Detailed analysis needs Noether theorem for higher order Lagrangians (introduction of interactions by means of gauge principle) and, based on that, integration of higher order equation of motion

ongoing work ...

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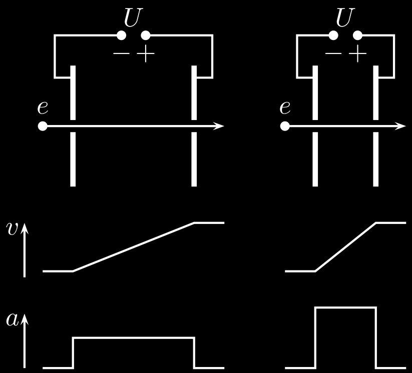

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General question

Which kind of interactions, which kind of geometry can be explored with a modified equation of motion?

Outline

But: Dark clouds over General Relativity?

Unexplained observations

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Is the gravitational physics in the Solar system really well understood?
Gravity at large distances? Weak gravity?

Fate of Einstein Equations?

Fate of Einstein Equations?

very likely to be modified due to Quantum Gravity.

... all this would need another two or three or four talks ....

Thank you!