BBH coalescence in the small mass ratio limit: Marrying black hole perturbation theory and PN knowledge

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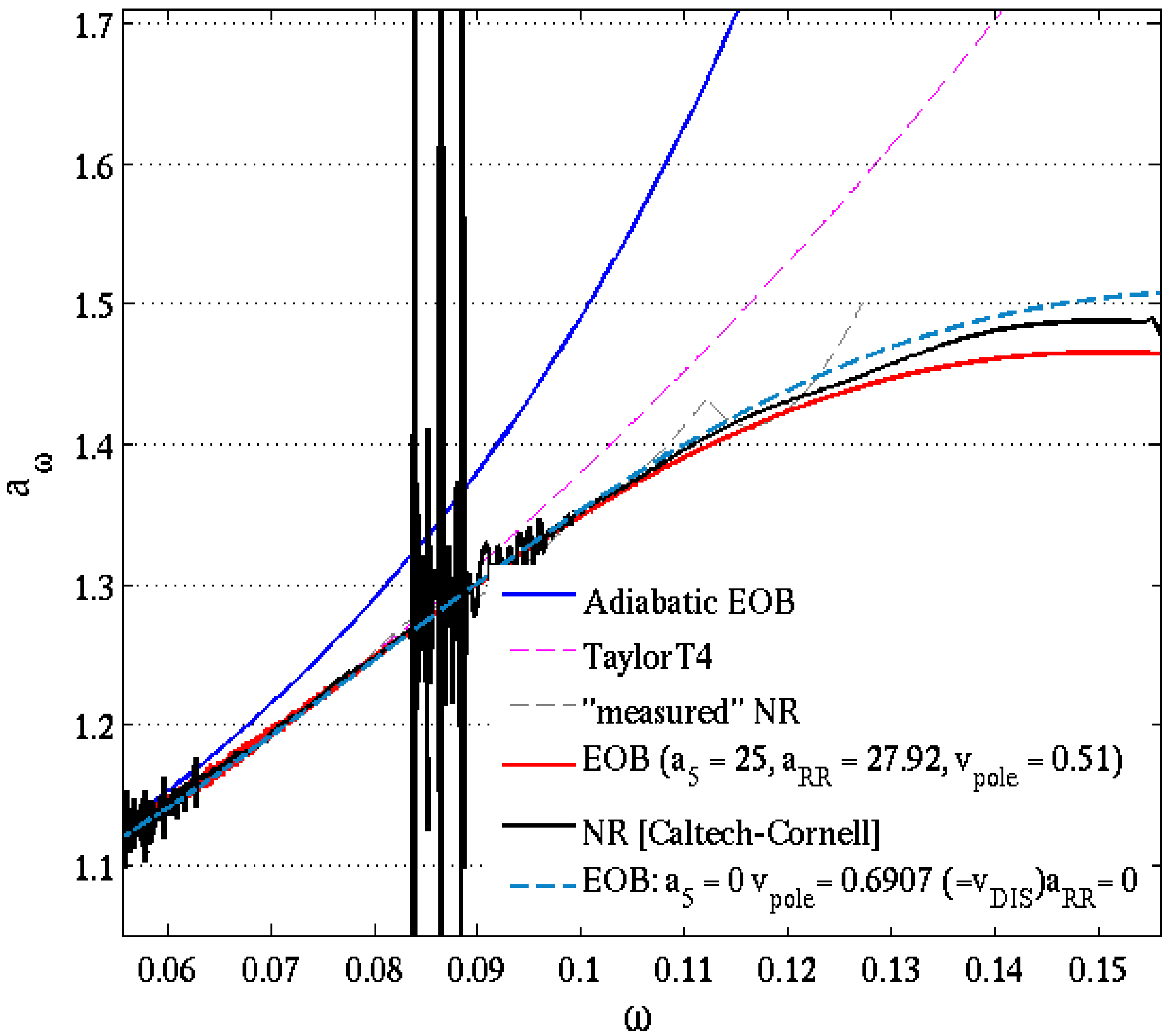
Small mass limit: Nagar Damour Tartaglia 2006
Damour Nagar 2006
Damour Nagar 2007a

Introduction

- The EOB approach works for *any value* of the mass-ratio of the bodies
- Thus, the EOB approach can deal naturally with EMRI (QC-orbits) $\nu = m_1 m_2/M^2 << 0.1$
- 3PN quadrupole-like (multipolar) resummed waveform for the inspiral
  + QNMs superposition to describe merger and ringdown: full (approximate) description!
- Simplify things in the extreme mass ratio limit: work only at linear order in ν.
  Neglect ν-dependent (EOB) corrections in the conservative part of the dynamics
  (the background is Schwarzschild) and consider only the *linear-in-ν-contribution* to
  radiation reaction (angular momentum loss).
- Thus: Schwarzschild black hole + point particle + nongeodesic motion driven by PN-based
  (but EOB-resummed) radiation reaction force

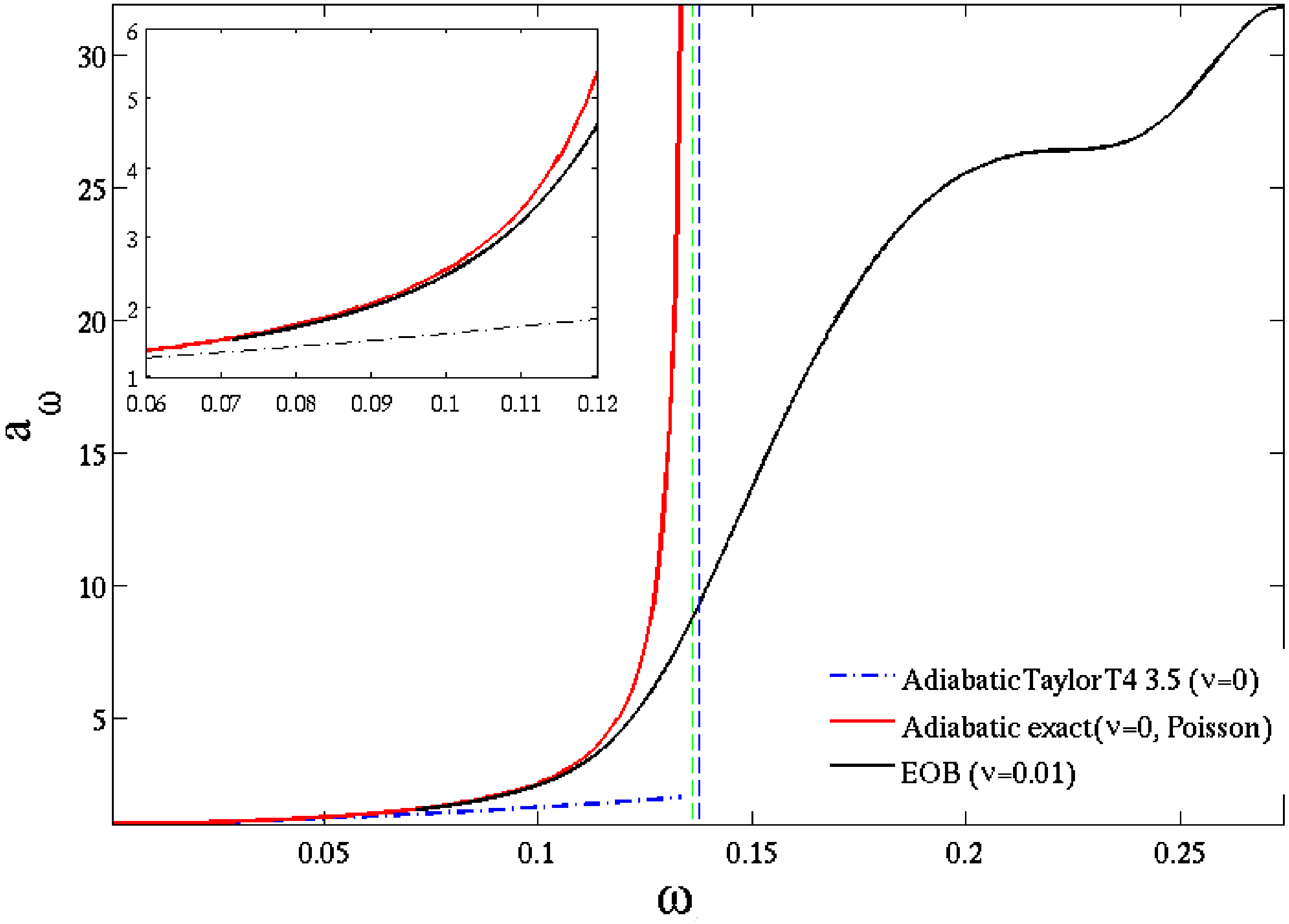
EOB reminder: comparing *curvature* phase acceleration curves: CC (actual data), TaylorT4, adiabatic, untuned and tuned EOB



- Most “intrinsic” curve (OK with DN07b, which used published data)
- Evident presence of nonadiabatic effects [DN07b]
- T4: strongly deviates from NR after GW frequency 0.1

Curvature acceleration curves in the small-ν case

- Note the influence of nonadiabatic effects!



“Low-order” EOB problem

Subject of this talk: a “low-order” EOB problem [linear in ν]:

- BBH merger in the small mass ratio limit, i.e. $m_1=\mu \ll m_2=M$ *(say* $\boldsymbol{\nu = m_1 m_2/M^2 << 0.1}$ *)*

- Gauge-invariant metric perturbation theory, i.e. solve the linearized Einstein’s equation around Schwarzschild background (Zerilli-Moncrief and Regge-Wheeler equations).
  Point-particle approximation for the BH of smaller mass.

- Radiation reaction: 2.5 Post-Newtonian Padé resummed expression of the radiation reaction (damping) force to regularize the badly behaved standard PN expansion [*Damour, Iyer&Sathyaprakash 1998, Buonanno&Damour 2000*]
  Possibility to accurately follow the sequence inspiral-plunge-ringdown.

- It is an “almost” analytical problem (ODEs and linear PDEs)!

Motivations and Overview

General motivations

- In the extreme mass ratio limit (*ν<0.1*), there are *no computations* available to date of the GWs from the plunge (from quasi-circular orbits) coming from the solution of Einstein's equation (in some approximation.)

- Gives complementary information to that given by full 3D numerical simulations ["contrasting" knowledge]

In addition: useful laboratory to learn the physics of the plunge [and of EOB]

- Understand here the main qualitative physical elements of the plunge

- Test resummation procedure

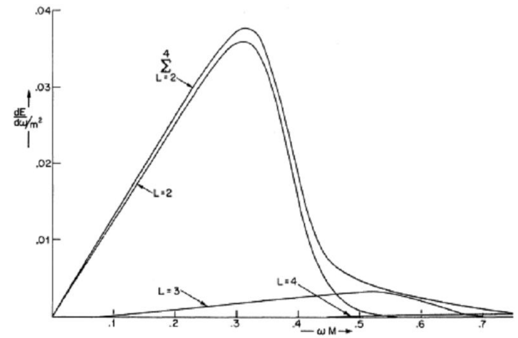
- Dynamics, matching procedure etc. learned here before the study of the comparable mass case. Learn here, try to apply there!

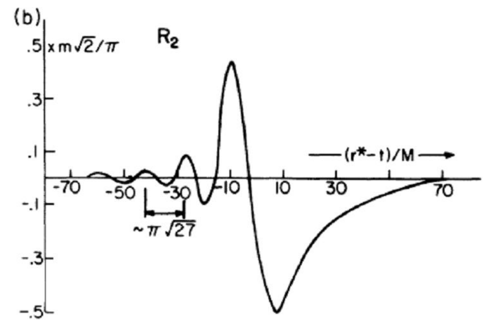
The extreme mass ratio: long (*perturbative*) history

Early GWs calculations: Radial plunge of a particle from infinity into a nonspinning BH

DRPP, *Phys. Rev. Lett.* ***27****, 1466 (1971).*
DRT, *Phys. Rev. D* ***5****, 2932 (1972).*

Precursor-Burst-Ringdown
ME/μ^2 = 0.0104.

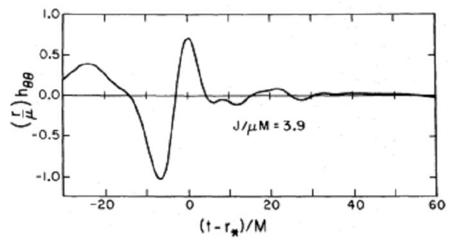


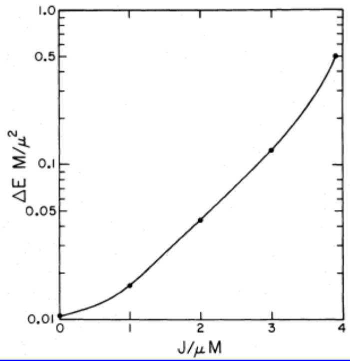


Particle plunging from infinity *with angular momentum.*

S. Detweiler and E. Szedenits, Astrophys. J. ***231****, 211 (1979).*
K.I. Oohara and T Nakamura, Prog. Theor. Phys. ***70****, 757 (1983)*

ME/μ^2 enhanced as much as a factor of 50.

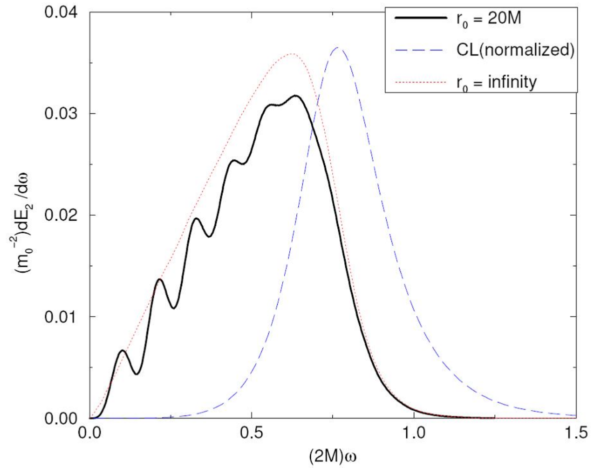




Most recent refinements: radial plunge from *finite distance.*

C.O. Lousto and R.H. Price, Phys. Rev. D ***56****, 6439 (1997).*
K. Martel and E. Poisson, Phys. Rev. D ***66****, 084001 (2002).*

Effect of initial data: interference bumps.



Metric perturbations of a Schwarzschild spacetime

Remark: Regge-Wheeler and Zerilli-Moncrief equations from the 10 Einstein equations.
Gauge-invariant and ***coordinate-independent*** (*in t,r*) formalism.

[*Regge&Wheeler1957, Zerilli1970, Moncrief1974, Gerlach&Sengupta1978, Gundlach&Martin-Garcia2000, Sarbach&Tiglio2001, Martel&Poisson2005, Nagar&Rezzolla 2005]*

In Schwarzschild coordinates:

$$\partial_t^2\Psi^{(\mathrm{o})}_{\ell m} - \partial^2_{r_*}\Psi^{(\mathrm{o})}_{\ell m} + V^{(\mathrm{o})}_{\ell}\Psi^{(\mathrm{o})}_{\ell m} = S^{(\mathrm{o})}_{\ell m}$$

odd-parity (Regge-Wheeler) $r_* = r + 2M\ln[r/(2M) - 1]$

$$\partial_t^2\Psi^{(\mathrm{e})}_{\ell m} - \partial^2_{r_*}\Psi^{(\mathrm{e})}_{\ell m} + V^{(\mathrm{e})}_{\ell}\Psi^{(\mathrm{e})}_{\ell m} = S^{(\mathrm{e})}_{\ell m}$$

even-parity (Zerilli-Moncrief) $\lambda = \ell(\ell+1)$

In the *wave zone*: GW amplitude, emitted power and angular momentum flux

$$h_+ - \mathrm{i}h_\times = \frac{1}{r}\sum_{\ell,m}\sqrt{\frac{(\ell+2)!}{(\ell-2)!}}\left(\Psi^{(\mathrm{e})}_{\ell m} + \mathrm{i}\Psi^{(\mathrm{o})}_{\ell m}\right)\ {}_{-2}Y^{\ell m}(\theta,\phi) + \mathcal{O}\left(\frac{1}{r^2}\right)$$

$$\frac{dE}{dt} = \frac{1}{16\pi}\sum_{\ell,m}\frac{(\ell+2)!}{(\ell-2)!}\left(\left|\frac{d\Psi^{(\mathrm{e})}_{\ell m}}{dt}\right|^2 + \left|\frac{d\Psi^{(\mathrm{o})}_{\ell m}}{dt}\right|^2\right)$$

$$\frac{dJ}{dt} = \frac{1}{32\pi}\sum_{\ell,\,m}\left\{\mathrm{i}m\frac{(\ell+2)!}{(\ell-2)!}\left[\dot{\Psi}^{(\mathrm{e})}_{\ell m}\bar{\Psi}^{(\mathrm{e})}_{\ell m} + \dot{\Psi}^{(\mathrm{o})}_{\ell m}\bar{\Psi}^{(\mathrm{o})}_{\ell m}\right] + c.c.\right\}$$

The particle dynamics

Hamiltonian formalism (conservative part of the dynamics)

$$\hat{H}_{\text{eff}} = \sqrt{A\left(1+\frac{p_\varphi^2}{\hat{r}^2}\right)+p_{r_*}^2}$$

$$A(\hat{r}) = B(\hat{r})^{-1} = 1 - 2/\hat{r}$$
$$p_r = \hat{P}_r = P_R/\mu$$
$$p_\varphi = \hat{P}_\varphi/M = P_\varphi/(\mu M)$$
$$p_r = p_{r_*} A^{-1}$$

$$\dot{\hat{r}} = \frac{A}{\hat{H}_{\text{eff}}} p_{r_*} \ ,$$
$$\dot{\varphi} = \frac{A}{\hat{H}_{\text{eff}}} \frac{p_\varphi}{\hat{r}^2} \ ,$$
$$\boxed{\dot{p}_\varphi = \hat{\mathcal{F}}_\varphi \ ,}$$
$$\dot{p}_{r_*} = -\frac{\hat{r}-2}{\hat{r}^3 \hat{H}_{\text{eff}}}\left[p_\varphi^2\left(\frac{3}{\hat{r}^2}-\frac{1}{\hat{r}}\right)+1\right]$$
$$\dot{\hat{r}}_* = \frac{p_{r_*}}{\hat{H}_{\text{eff}}} \ .$$

Non conservative part of the dynamics

$$\hat{\mathcal{F}}_\varphi = -\frac{32}{5}\mu\omega^5\hat{r}^4\frac{\hat{f}_{\text{DIS}}}{1-\sqrt{3}\omega\hat{r}}$$

Padé resummed estimate at *2.5 PN* of the angular momentum flux [***TD, BI & BS, PRD 57, 885 (1998) , Buonanno-Damour, PRD 62, 064015 (2000)***]
Consistent below LSO [***TD & AG, PRD 73, 124006 (2006)***]

Explicit evolution of R_* of the particle

The source terms

Even-parity

$$
\begin{aligned}
S^{(\mathrm{e})}_{\ell m} = & -\frac{16\pi\mu Y^*_{\ell m}}{r\hat{H}\lambda[(\lambda-2)r+6M]}\Bigg\{\left(1-\frac{2M}{r}\right)\left(\hat{P}^2_\varphi+r^2\right)\partial_{r_*}\delta(r_*-R_*(t)) \\
& + \Bigg\{-2\mathrm{i}m\left(1-\frac{2M}{r}\right)\hat{P}_{R_*}\hat{P}_\varphi+\left(1-\frac{2M}{r}\right)\Bigg[3M\left(1+\frac{4\hat{H}^2 r}{(\lambda-2)r+6M}\right) \\
& - \frac{r\lambda}{2}+\frac{\hat{P}^2_\varphi}{r^2(\lambda-2)}\left[r(\lambda-2)(m^2-\lambda-1)+2M(3m^2-\lambda-5)\right] \\
& + \left(\hat{P}^2_\varphi+r^2\right)\frac{2M}{r^2}\Bigg]\Bigg\}\delta(r_*-R_*(t))\Bigg\}
\end{aligned}
$$

Odd-parity

$$
\begin{aligned}
S^{(\mathrm{o})}_{\ell m} = & \frac{16\pi\mu\partial_\theta Y^*_{\ell m}}{r\lambda(\lambda-2)}\Bigg\{\left[\left(\frac{\hat{P}_{R_*}\hat{P}_\varphi}{\hat{H}}\right)_{,t}-2\hat{P}_\varphi\frac{r-2M}{r^2}-\mathrm{i}m\frac{r-2M}{r^3}\frac{\hat{P}_{R_*}\hat{P}^2_\varphi}{\hat{H}^2}\right]\delta(r_*-R_*(t)) \\
& + \left(1-\frac{P^2_{R_*}}{\hat{H}}\right)\hat{P}_\varphi\partial_{r_*}\delta(r_*-R_*(t))\Bigg\}, \qquad (22)
\end{aligned}
$$

$$\lambda = \ell(\ell+1)$$

Numerics and tests with geodesic motion

Numerics

✓Couple of wave-equations: standard numerical techniques (*Lax-Wendroff*)

✓Smoothing the delta-function (*$\sigma \ll M$*). Extensive testing (*$\sigma \approx \Delta r_*$ is ok*)
In practice, the *finite-size* effects are irrelevant (we shall see tests of this in next slides)

$$\delta(r_* - R_*(t)) \equiv \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(r_* - R_*(t))^2}{2\sigma^2}}$$

Tests: Geodesic motion

✓ Circular and radial orbits: comparison with literature [*waveforms and energy*]
[*KM, PRD **69**, 044025 (2004)*, *KM & EP, PRD **66**, 084001 (2002)*, *COL & RHP, PRD **55**, 2124 (1997)*]

✓*Circular orbits:* good agreement for energy and angular momentum fluxes.

Numerics and tests with geodesic motion

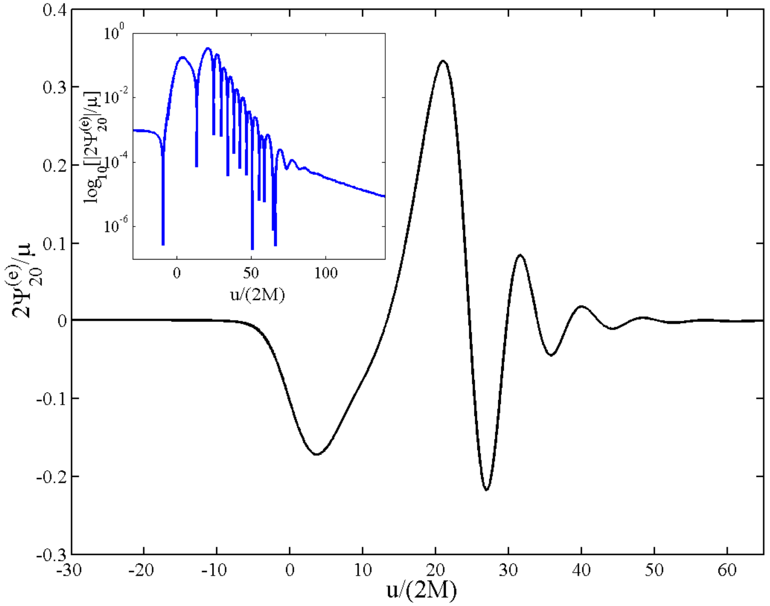
- **Circular orbits** [comparison with Martel 2004]

Table 1. Energy and angular momentum fluxes extracted at $r_{\rm obs} = 1000M$ for a particle orbiting the black hole on a circular orbit of radius $r = 7.9456$. Comparison with the results of Martel [29].

($\Delta r_* = 0.02M$)

ℓ	m	$(\dot{E}/\mu^2)_{\rm here}$	$(\dot{E}/\mu^2)_{\rm Martel}$	rel. diff.	$(\dot{J}/\mu^2)_{\rm here}$	$(\dot{J}/\mu^2)_{\rm Martel}$	rel. diff.
2	1	8.1998×10^{-7}	8.1623×10^{-7}	0.4%	1.8365×10^{-5}	1.8270×10^{-5}	0.5%
	2	1.7177×10^{-4}	1.7051×10^{-4}	0.7%	3.8471×10^{-3}	3.8164×10^{-3}	0.5%
3	1	2.1880×10^{-9}	2.1741×10^{-9}	0.6%	4.9022×10^{-4}	4.8684×10^{-8}	0.7%
	2	2.5439×10^{-7}	2.5164×10^{-7}	1.1%	5.6977×10^{-6}	5.6262×10^{-6}	1.2%
	3	2.5827×10^{-5}	2.5432×10^{-5}	1.5%	5.7846×10^{-4}	5.6878×10^{-4}	1.7%
4	1	8.4830×10^{-13}	8.3507×10^{-13}	1.6%	1.8999×10^{-11}	1.8692×10^{-11}	1.6%
	2	2.5405×10^{-9}	2.4986×10^{-9}	1.7%	5.6901×10^{-8}	5.5926×10^{-8}	1.7%
	3	5.8786×10^{-8}	5.7464×10^{-8}	2.3%	1.3166×10^{-6}	1.2933×10^{-6}	1.8%
	4	4.8394×10^{-6}	4.7080×10^{-6}	2.7%	1.0838×10^{-4}	1.0518×10^{-4}	3.0%

- **Radial plunge** [Checked with Lousto-Price 1997 and Martel-Poisson 2002]
  - Conformally flat initial data
  - Radial plunge along z-axis



Needs resummation of energy flux!

The PN expansions are non-uniformly and non-monotonically convergent in the strong-field regime.
One needs to "resum" them in some form in order to extend their validity during the late-inspiral and plunge

- Factorize a simple pole in the GW energy flux

- Resum using near-diagonal Padé approximants (DIS98)

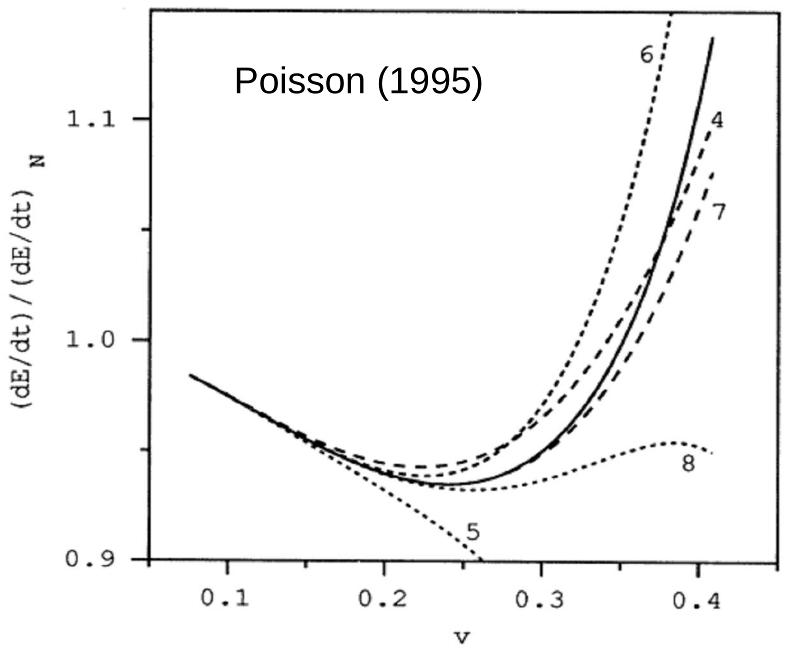


FIG. 1. Various representations of $(dE/dt)/(dE/dt)_N$ as a function of orbital velocity $v = (M/r)^{1/2} = (\pi M f)^{1/3}$. The solid curve represents the exact result $P(v)$, as calculated numerically. The various broken curves represent the post-Newtonian approximations $P_n(v)$, for $n = \{4, 5, 6, 7, 8\}$. The smallest value of v corresponds to an orbital radius r of $175M$; the largest value of v corresponds to $r = 6M$, the innermost stable circular orbit.

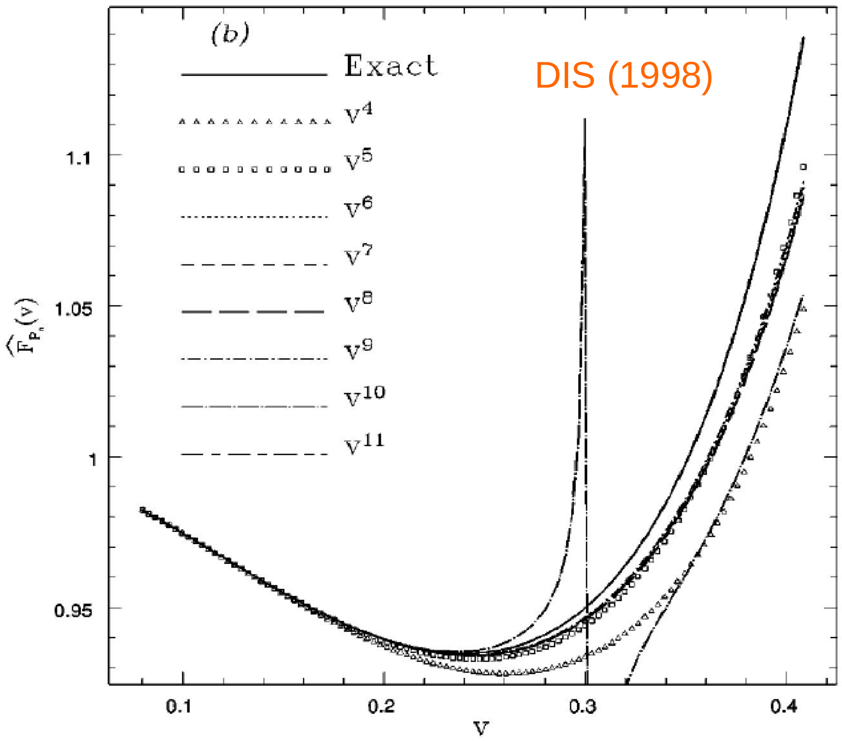


FIG. 3. Newton-normalized gravitational wave luminosity in the test particle limit: (a) T-approximants and (b) P-approximants.

Resumming radiation reaction

Padé resummation of $$\hat{\mathcal{F}}_{\varphi}^{\rm Taylor} = -\frac{32}{5}\,\nu\,\Omega^5\,r^4\,\hat{F}^{\rm Taylor}(v_\varphi)$$

Linear in ν

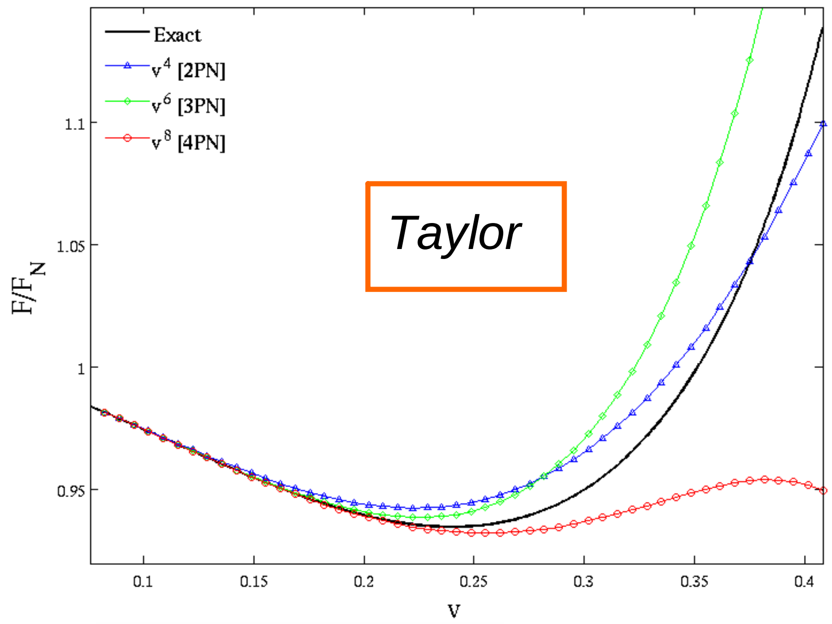
- factorize a pole parametrized by **v_{pole}=1/Sqrt[3]**
- consider logarithms as coefficients
- use small-mass 2.5 PN flux only [ν=0]
- choose P^2_3 which has no spurious poles

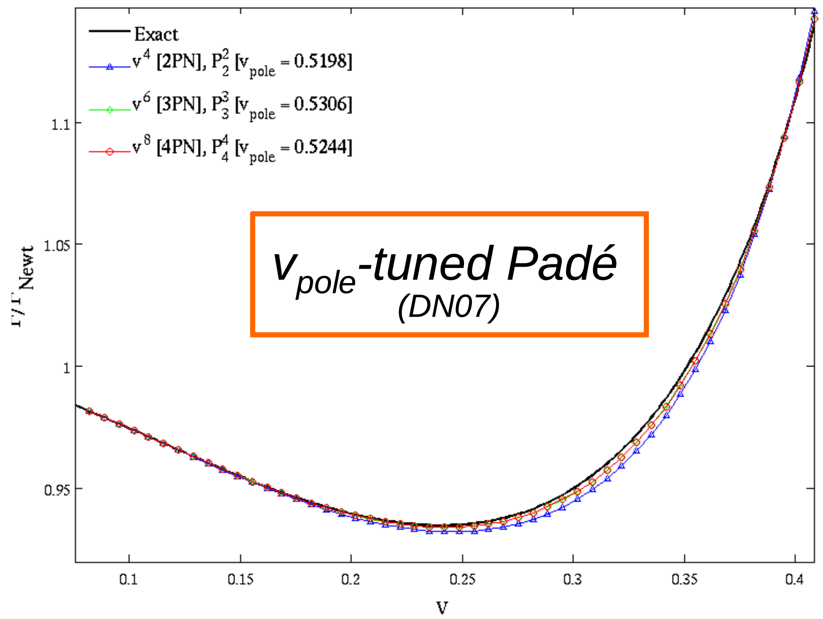
$$\boxed{\hat{F}^{\rm resummed}(v_\varphi) = \left(1-\frac{v_\varphi}{v_{\rm pole}}\right)^{-1} P^2_3\left[\left(1-\frac{v_\varphi}{v_{\rm pole}}\right)\hat{F}^{\rm Taylor}(v_\varphi)\right]}$$

$$\begin{aligned}\hat{F}^{\rm Taylor}(v) &= 1 + A_2(\nu)\,v^2 + A_3(\nu)\,v^3 + A_4(\nu)\,v^4 + A_5(\nu)\,v^5 \\ &\quad \cancel{+ A_6(\nu,\log v)\,v^6 + A_7(\nu)\,v^7 + A_8(\nu=0,\log v)\,v^8}\end{aligned}$$

ν=0

Evidently, one can improve flexing v_{pole} [next future]





- Maximum difference on interval v<0.4:

Taylor(2PN): 0.039	Padé(2PN): 0.0069
Taylor(3PN): 0.130	Padé(3PN): 0.0033
Taylor(4PN): 0.189	Padé(4PN): 0.0035

Henri Padé, 1863-1953

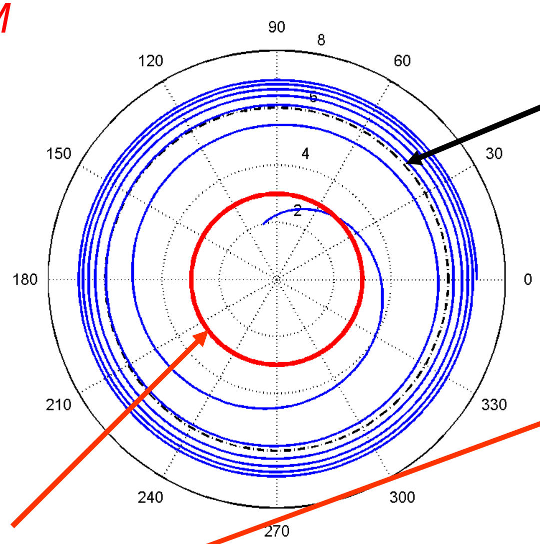
The orbit: transition from inspiral to plunge

Setting up initial data for particle dynamics

Solve the EoM in the adiabatic approximation to first order beyond the adiabatic approximation, i.e. $\boldsymbol{p_r \neq 0}$

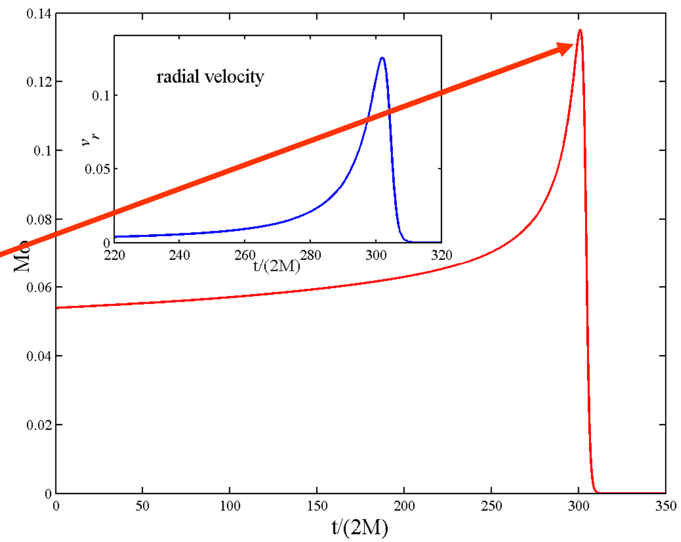
$\mu = 0.01M$

$r_0 = 7M$



Last Stable Orbit(LSO) r = 6M

($\approx$*1.5* orbits more before the merger)



Light Ring r = 3M

Setting up initial data for gravitational perturbation

Initially, no GW perturbation. Initial burst of unphysical radiation radiated away and causally disconnected from the rest of the dynamics (the system has the time to adjust itself to the correct configuration).

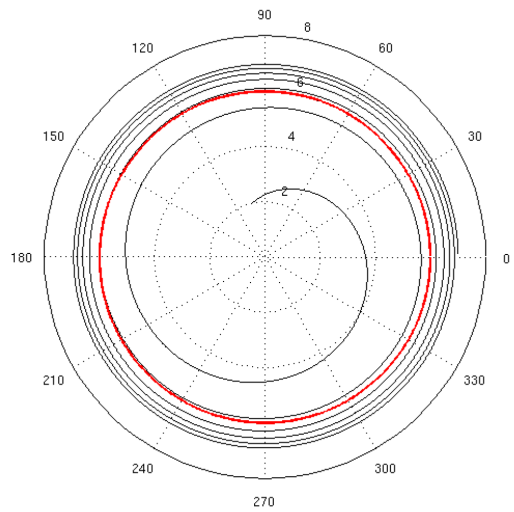
Transition from inspiral to plunge: $\nu \ll 1$

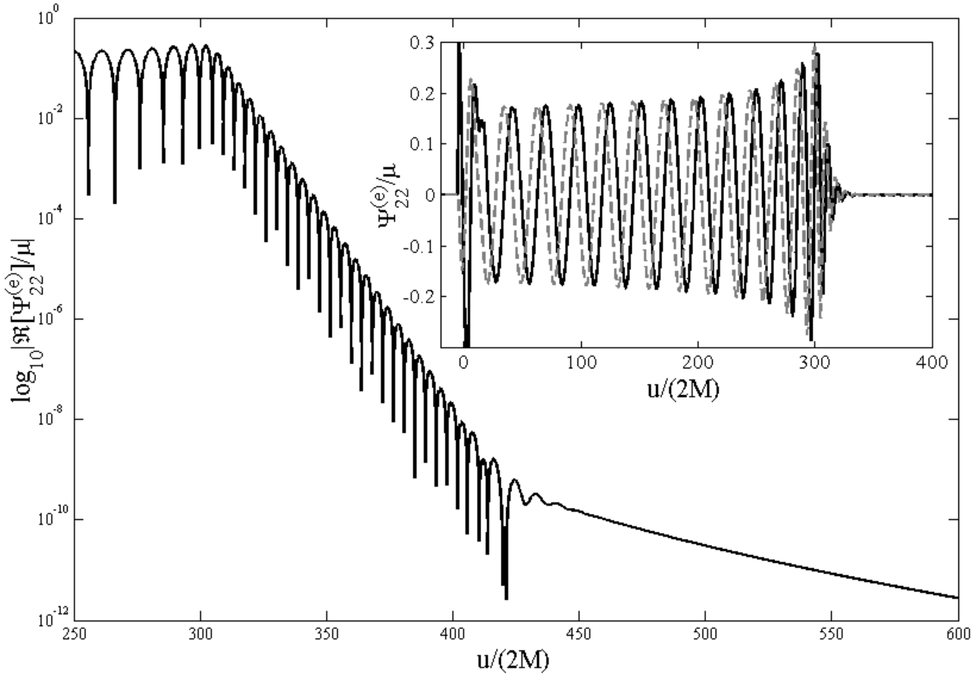
- EOB-inspired radiation reaction + Regge-Wheeler-Zerilli (RWZ) BH perturbation theory

- Schwarzschild black hole + test mass

- *EOB-type ν-dependent [2.5PN] radiation reaction force*

- Waveform: solution of the RWZ equation in the time-domain + a δ-function source term representing the particle



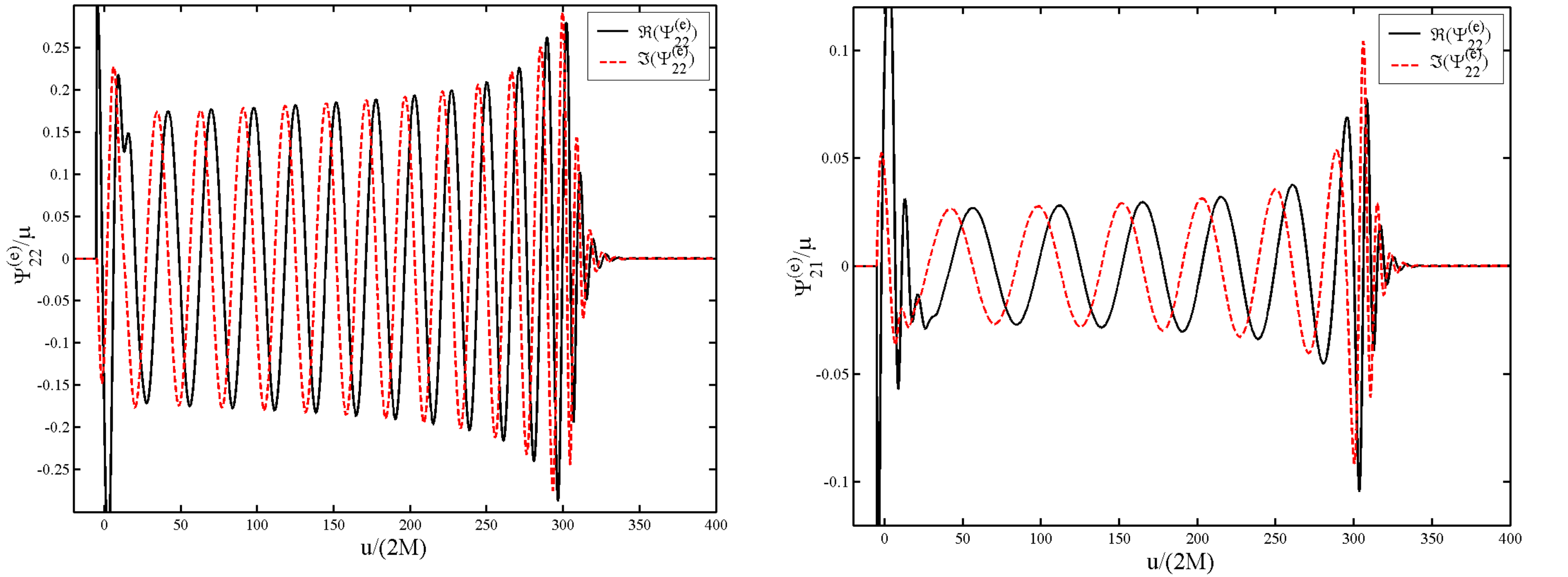


- Clean laboratory to experiment with the EOB matching procedure

- Useful to understand the essential physical elements entering in the plunge without the complications of 3D codes

- Gives complementary information to that available from NR simulations

Gravitational Waveforms: l=2



Smooth transition:
inspiral-plunge in the waveforms

Crossing the LSO: *$u/2M \approx 240$*

Crossing the Light-Ring: *$u/2M \approx 301$*

Energy and angular momentum released in GWs

Radiation during the plunge (high multipoles)

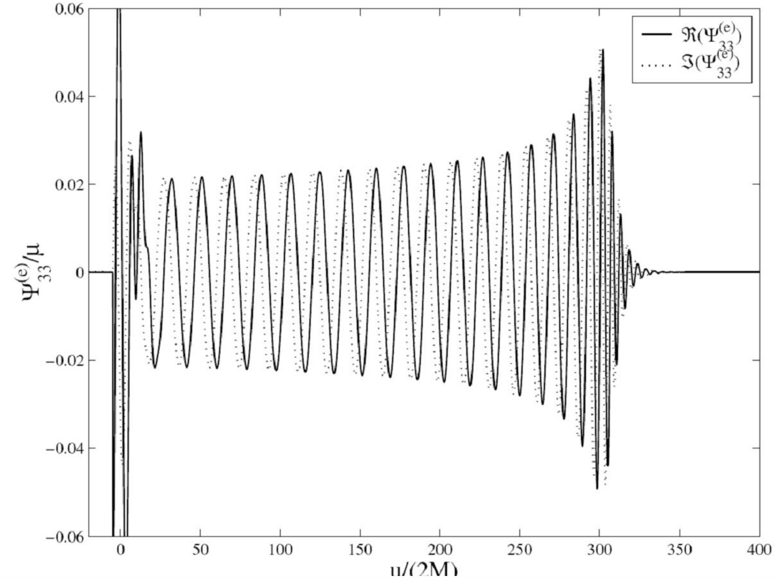
Table I: Energy and angular momentum emitted at infinity (observer at $r_{\text{obs}} = 250M$ by a particle with $\mu = 0.01M$ during the plunge phase only; the integrals are done from $r \simeq 5.9865M$, corresponding to retarded time $u/(2M) = 240$.

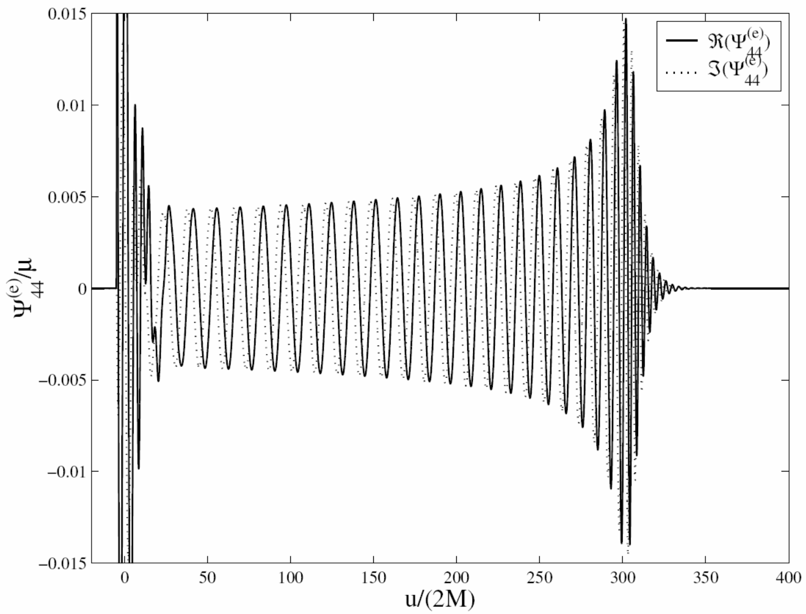
ℓ	m	$(M/\mu^2)E^\infty$	J^∞/μ^2
2	0	9.8×10^{-4}	0
	1	2.06×10^{-2}	0.084
	2	$\boxed{3.3 \times 10^{-1}}$	2.994
3	0	3.4×10^{-5}	0
	1	5.6×10^{-4}	1.2×10^{-3}
	2	8.1×10^{-3}	3.9×10^{-2}
	3	$\boxed{1.05 \times 10^{-1}}$	8.5×10^{-1}
4	0	1.7×10^{-6}	0
	1	2.4×10^{-5}	3.6×10^{-5}
	2	3.3×10^{-4}	1.1×10^{-3}
	3	3.5×10^{-3}	1.8×10^{-2}
	4	$\boxed{4.2 \times 10^{-2}}$	3.2×10^{-1}

Total Emission

$ME/\mu^2 \approx 0.5$

$J/(\mu M) \approx 0.04$





Finite-size (the Gaussian) effects on the source?

The δ*-function* is approximated by a finite-(tiny)size Gaussian. Is this allowed?

There are two (analytically equivalent) ways of writing the sources:

$$S_{\ell m}^{(\mathrm{e/o})} = G_{\ell m}^{(\mathrm{e/o})}(r,t)\delta(r_* - R_*(t)) + F_{\ell m}^{(\mathrm{e/o})}(r,t)\partial_{r_*}\delta(r_* - R_*(t))$$

standard

and (using integration by parts)

$$S_{\ell m}^{(\mathrm{e/o})} = \tilde{G}_{\ell m}^{(\mathrm{e/o})}(R_*(t))\delta(r_* - R_*(t)) + F_{\ell m}^{(\mathrm{e/o})}(R_*(t))\partial_{r_*}\delta(r_* - R_*(t))$$

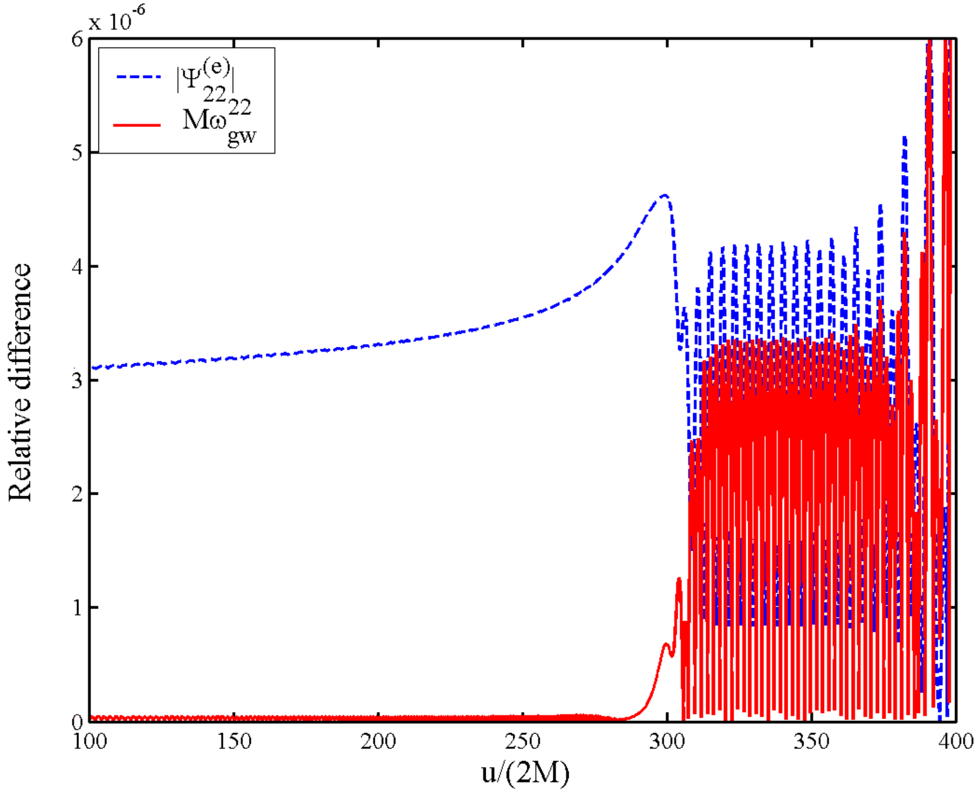
where

$$\tilde{G}_{\ell m}^{(\mathrm{e/o})}(R_*) = G_{\ell m}^{(\mathrm{e/o})}(R_*) - \left.\frac{\mathrm{d}F_{\ell m}^{(\mathrm{e/o})}}{\mathrm{d}r_*}\right|_{r_*=R_*}$$

One may be worried that, when going on a discrete grid, these two "numerically unequivalent" surces can give relevant differences

Finite-size (the Gaussian) effects on the source?

In practice, they are equivalent!



In the simulation we use *$\sigma \approx \Delta r^* = 0.01M$* . Convergence as soon as $\sigma \ll M$

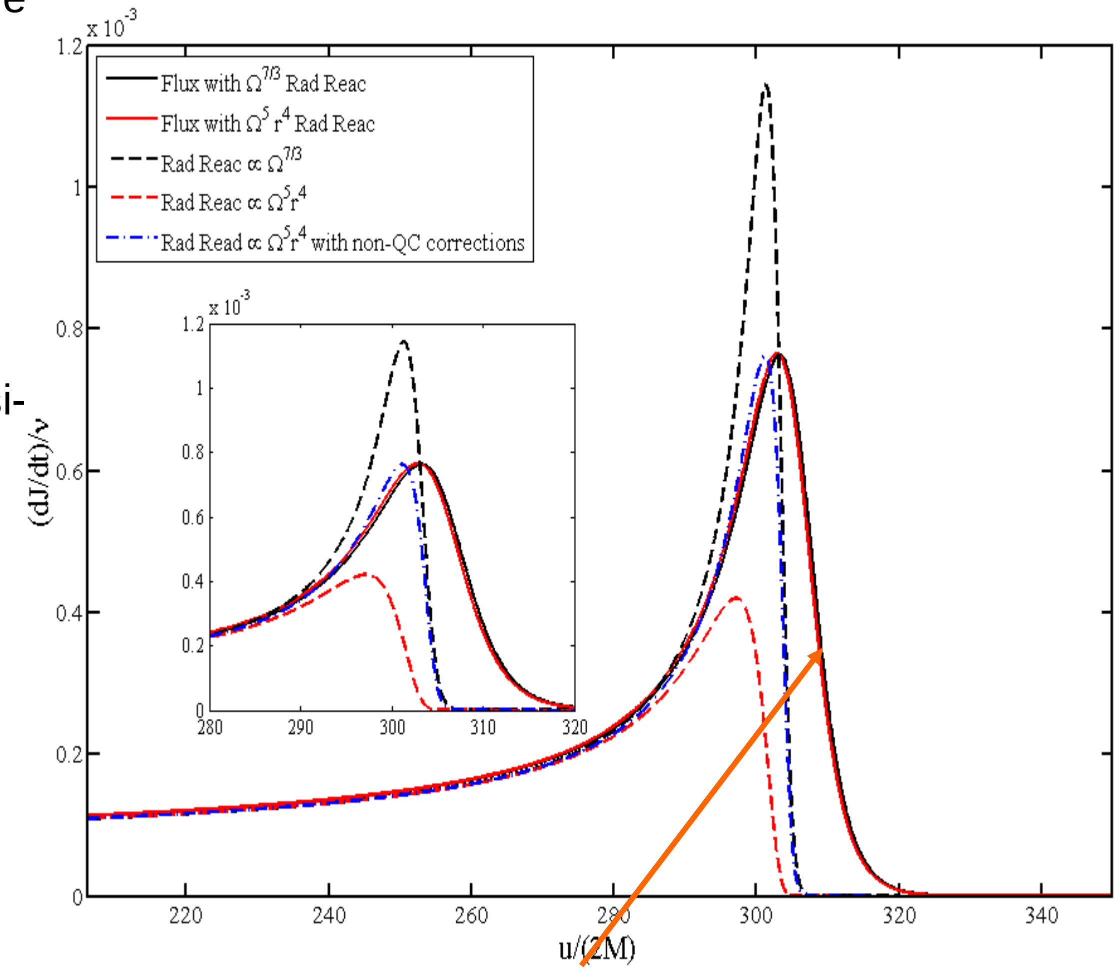
EMRL: consistency of radiation reaction

- Computation of the angular momentum flux dJ/dt from the multipoles of the RWZ function.
- Various EOB-radiation reaction
- Consistency of dJ/dt with RR
- Waveforms unaffected by RR
- Discover the need of non-quasi-circular corrections to the flux

$$\hat{\mathcal{F}}^{\rm BD} = -\frac{32}{5}\nu v_\omega^7 \frac{\hat{f}_{\rm DIS}(v_\omega)}{1-\sqrt{3}v_\omega}$$

$$v_\omega = \Omega^{1/3}$$

$$\boxed{\hat{\mathcal{F}}^{\rm DG} = -\frac{32}{5}\nu \Omega^5 r^4 \frac{\hat{f}_{\rm DIS}(v_\varphi)}{1-\sqrt{3}v_\varphi} \qquad v_\varphi = \Omega r}$$



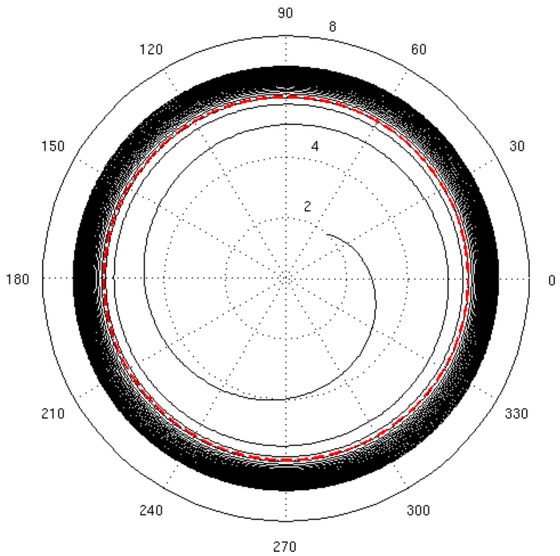
Regge-Wheeler-Zerilli fluxes

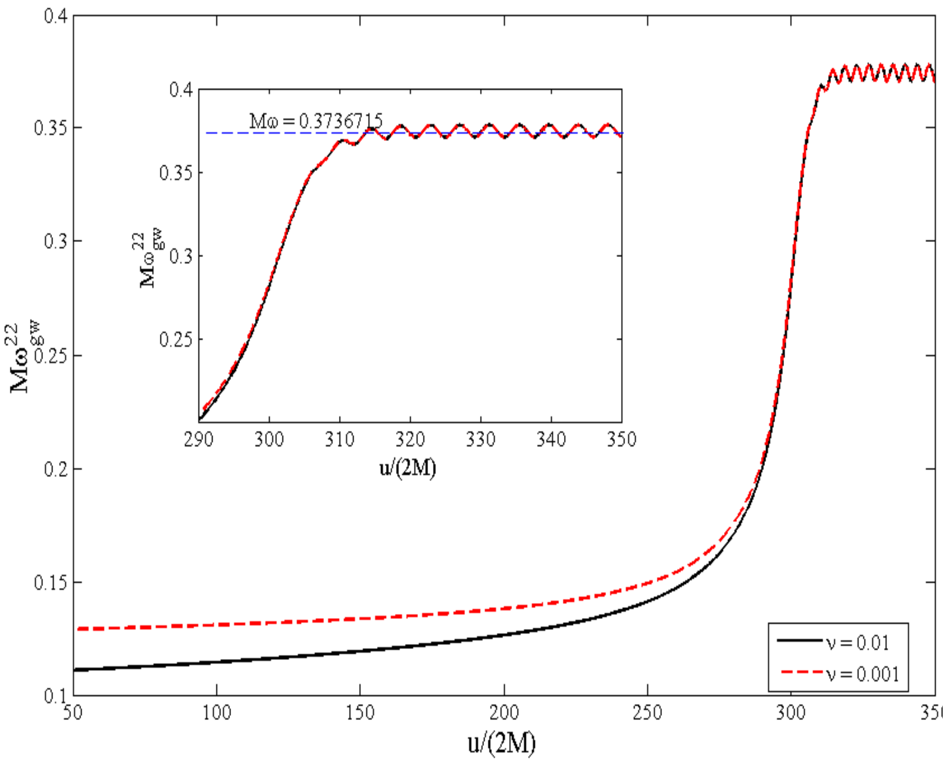
EMRL: quasi-geodesic plunge

- Small dependence of the dynamics on radiation reaction after crossing the LSO [u/(2M>280]

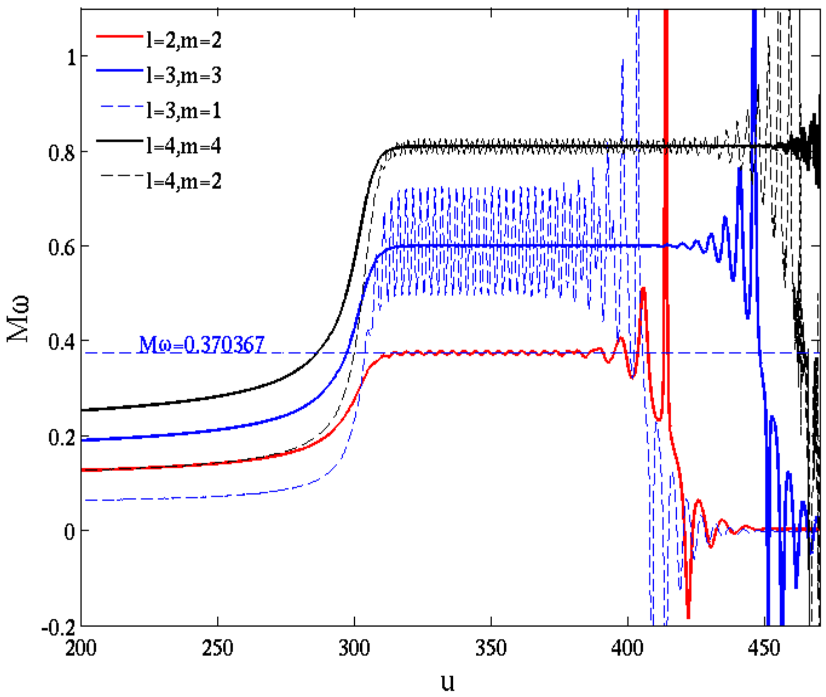
- Quasi-geodesic "universal" plunge phase (after the crossing of the LSO) when ν tends to zero.

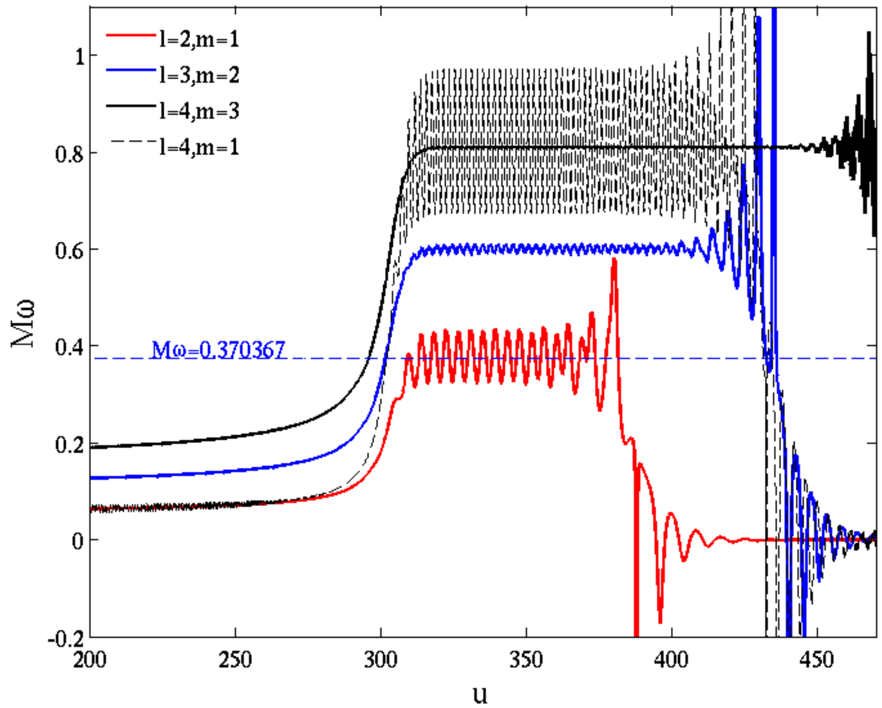
- *Comparison ν=0.01 and ν=0.001*





EMRL: frequency

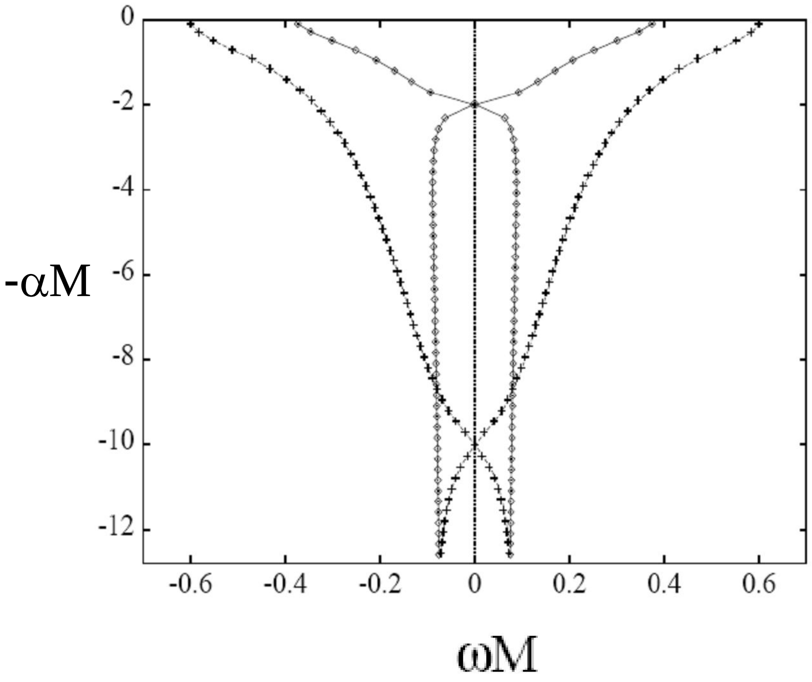




- Easy access to many gravitational wave multipoles
- Positive and negative frequency QNMs are both excited [DNT2006]
- *The excitation of the negative frequency modes depends systematically on m*
- Information that complements the one available via full GR simulations [see later]

Analysis of QNMs signature: oscillations in ω_{gw}

Why oscillations in the GW frequency?

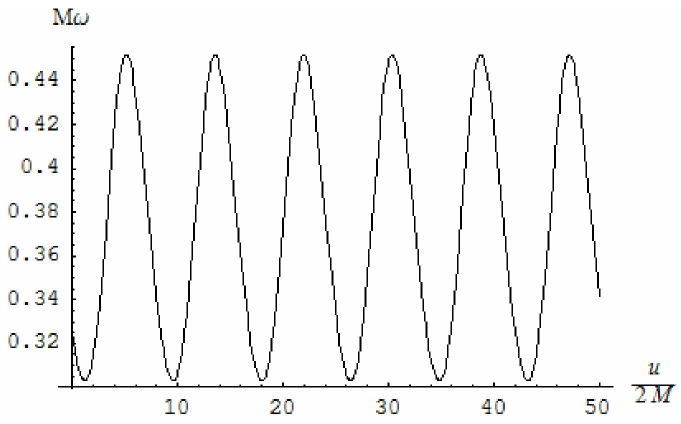


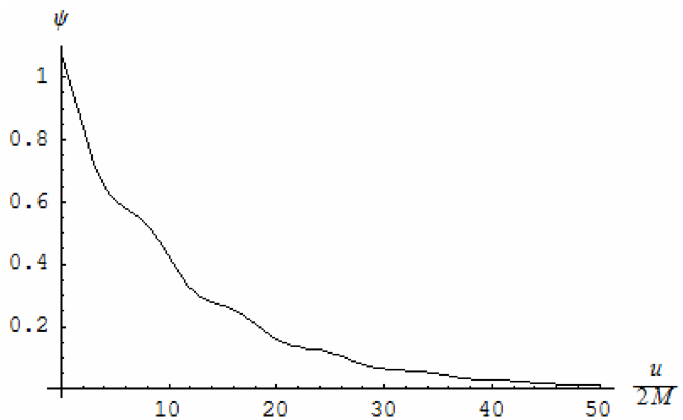
$$\Psi = e^{-\sigma_0^+ t} + \epsilon e^{-\sigma_0^- t}$$

$$\sigma_n^\pm = \alpha_n \pm i\omega_n$$

$$\omega_{GW} = \frac{1 - 2\epsilon \sin(2\omega_0 t) - \epsilon^2}{1 + 2\epsilon \cos(2\omega_0 t) + \epsilon^2}\ \omega_0$$

$$|\Psi| = \sqrt{1 + 2\epsilon \cos(2\omega_0 t) + \epsilon^2}\ e^{-\alpha_0 t}$$





Analysis of QNMs signature

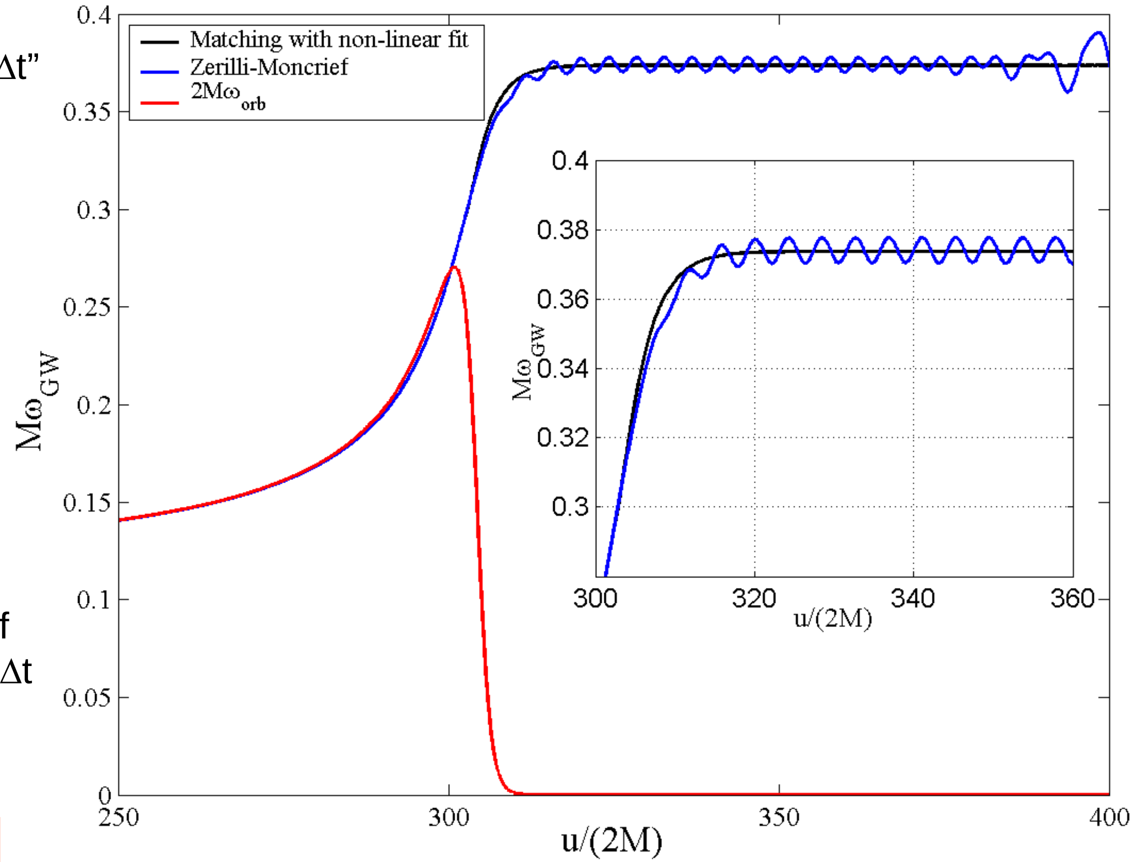
Choose a certain "time" and "interval Δt"

Assume the waveform is given by a superposition of QNMs as

$$\Psi_{22}^{QNM} = \sum_{n=0}^{N} C_n e^{-\sigma_n^+ t}$$

Determine the coefficients by means of a nonlinear fit to the real waveform on Δt

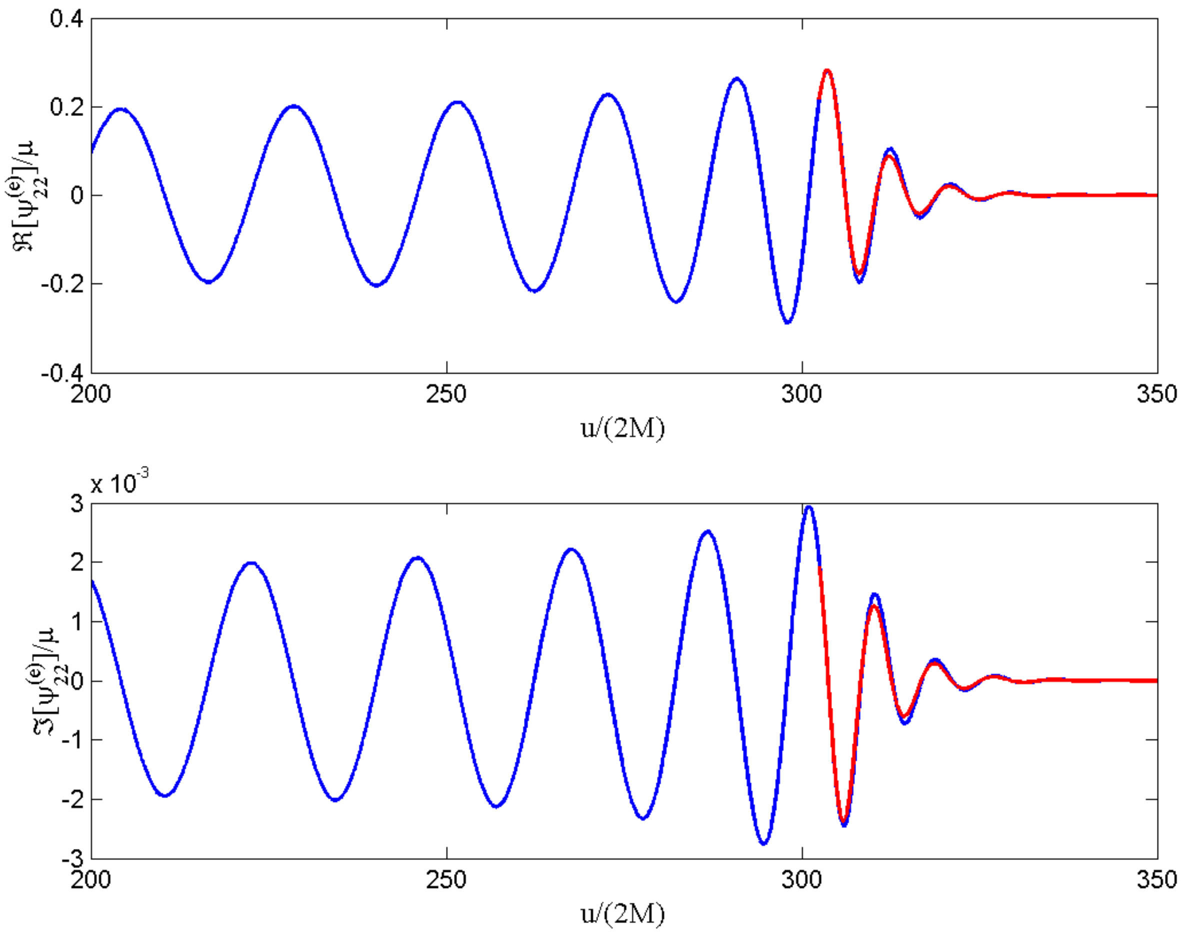
Need a finite interval: Δt ~ 8M



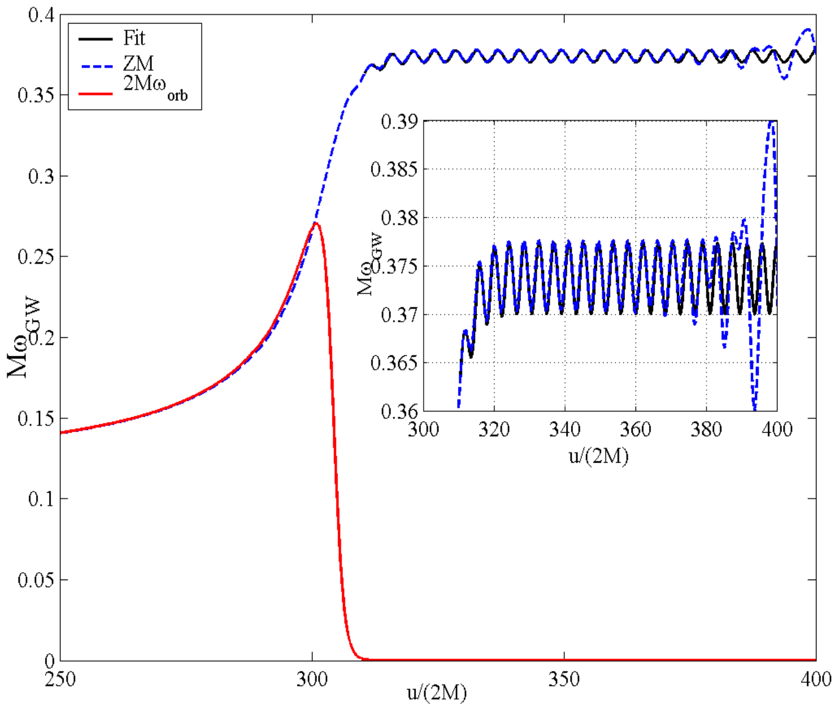
- Only positive frequency modes to start…
- Need of first five modes to have a good fit!

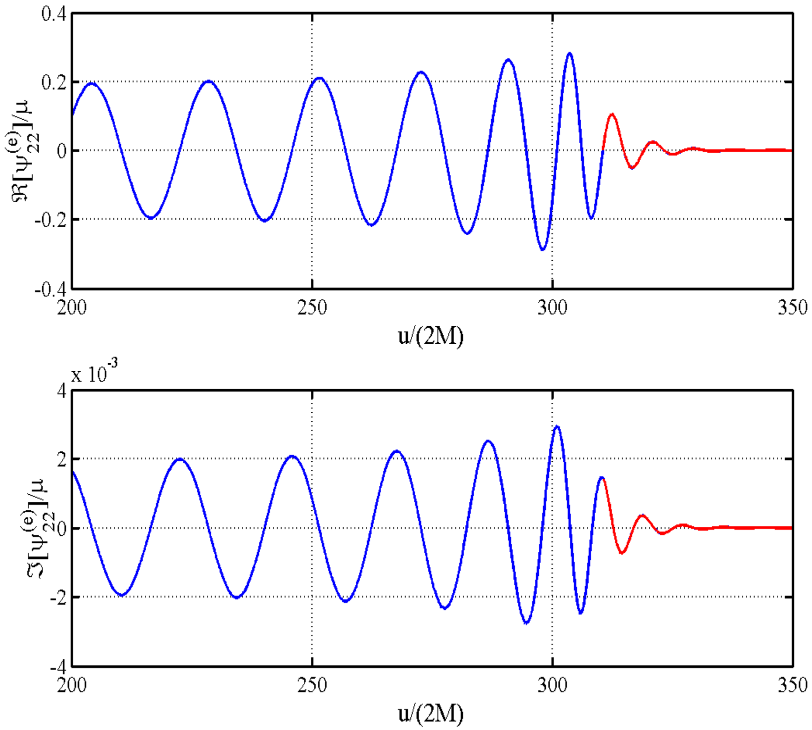
Analysis of QNMs signature

"Reconstructed" waveforms



Analysis of QNMs signature





- Fit adding also negative-frequency modes
- 3positive + 2 negative frequency modes
- Reproduction (and explanation) of the oscillation in the frequency!

Resummed EOB *metric* gravitational waveform: inspiral+plunge

- Zerilli-Moncrief normalized (even-parity) waveform (*Real part gives h_+ & imaginary part gives $h_\times$*).
- Multipolar decomposition (expansion on spin-weighted spherical harmonics) here, *l=m=2*.

$$\boxed{\left(\frac{c^2}{GM}\right)\Psi_{22}^{\rm insplunge}(t) = -4\sqrt{\frac{\pi}{30}}\nu(r_\omega\Omega)^2 f_{22}^{\rm NQC} F_{22} e^{-2i\Phi}}$$

New *PN-resummed (3^{+2}PN) correction factor* (*DN07a, 07b*): 3PN comparable mass + up to 5PN test-mass

$$\boxed{F_{22}(t) = \hat{H}_{\rm eff} T_{22} f_{22}(x(t)) e^{i\delta_{22}(t)}}$$

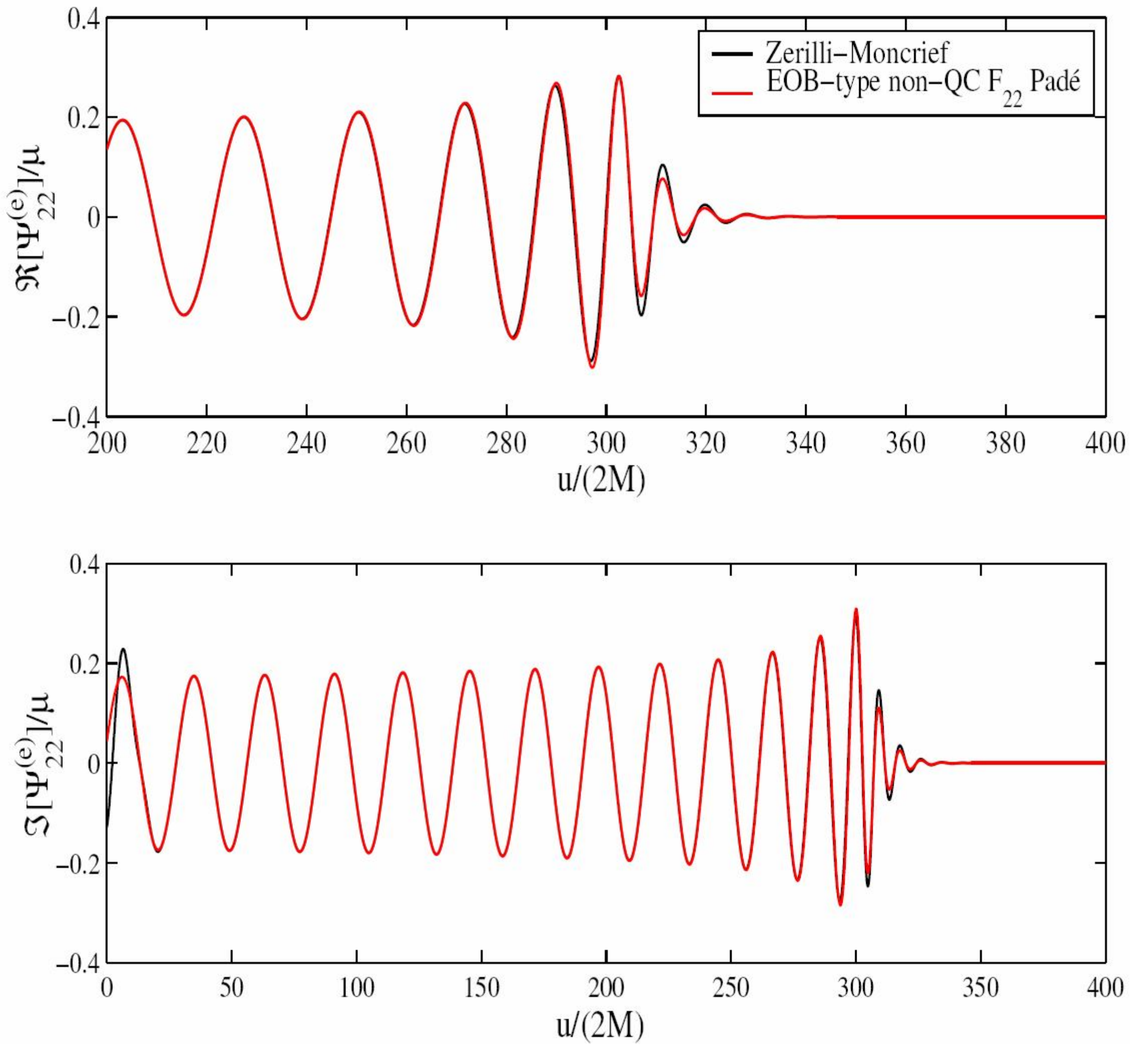
- H_{eff}: resums an infinite number of binding energy contributions
- $$\boxed{T_{\ell m} = \frac{\Gamma(\ell+1-2i\hat{\hat{k}})}{\Gamma(\ell+1)} e^{\pi\hat{\hat{k}}} e^{2i\hat{\hat{k}}\log(2kr_0)}}$$ *resums* an infinite number of leading logarithms in tail effects (both amplitude and phase) obtained from exact solution of Coulomb wave problem
- $$\boxed{f_{22}(x;\,\nu) = P_2^3\left[f_{22}^{\rm Taylor}(x;\,\nu)\right]}$$ Padé-resummed remaining PN-corrected amplitude [flexibility in choice of argument *x(t)*]
- δ_{22}: computed at 3.5PN

Non-quasi-circular corrections to waveform amplitude and phase:

$$\boxed{f_{22}^{\rm NQC} = \left[1 + a\frac{p_{r_*}^2}{(r\Omega)^2+\epsilon}\right]\exp\left(+ib\frac{p_{r_*}}{r\Omega}\right)}$$

b=0; a is fixed by requiring that the maximum of the modulus of the waveform coincides with the maximum of the orbital frequency

EMRL: EOB comparison [2.5PN]

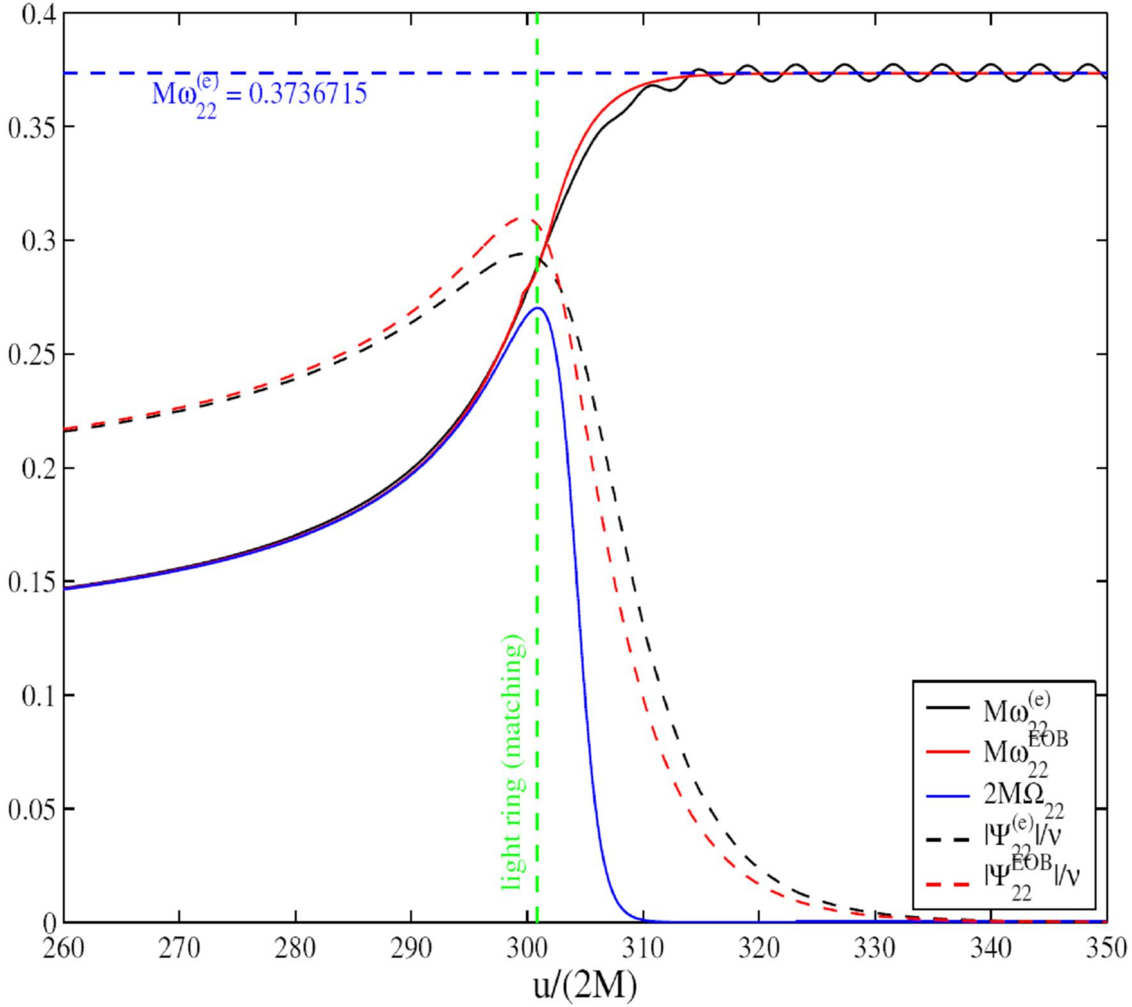


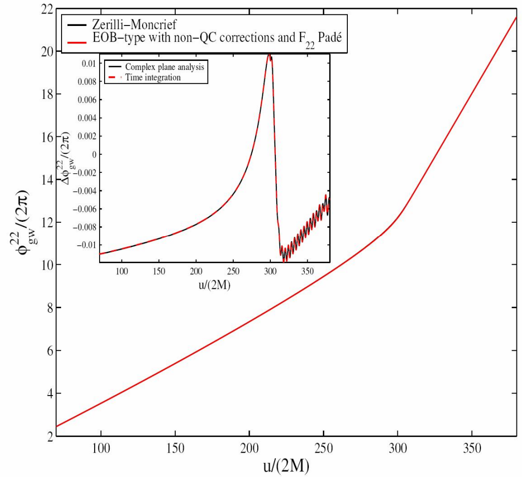
Match (around the light-ring) a
EOB-type quadrupole formula
+ PN (resummed) corrections
+ non-quasi-circular contributions
to a superposition of 5 BH QNMs

$$\Psi_{22}^{QNM} = \sum_{n=0}^{N} C_n e^{-\sigma_n^+ t}$$

$$\sigma_n^{\pm} = \alpha_n \pm \mathrm{i}\omega_n$$

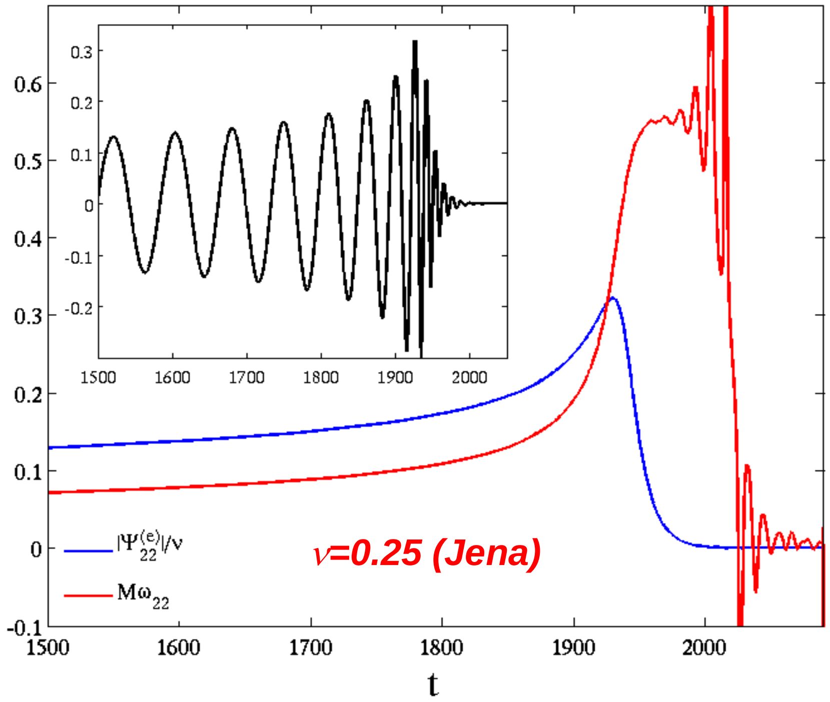
GW frequency: EMRL results

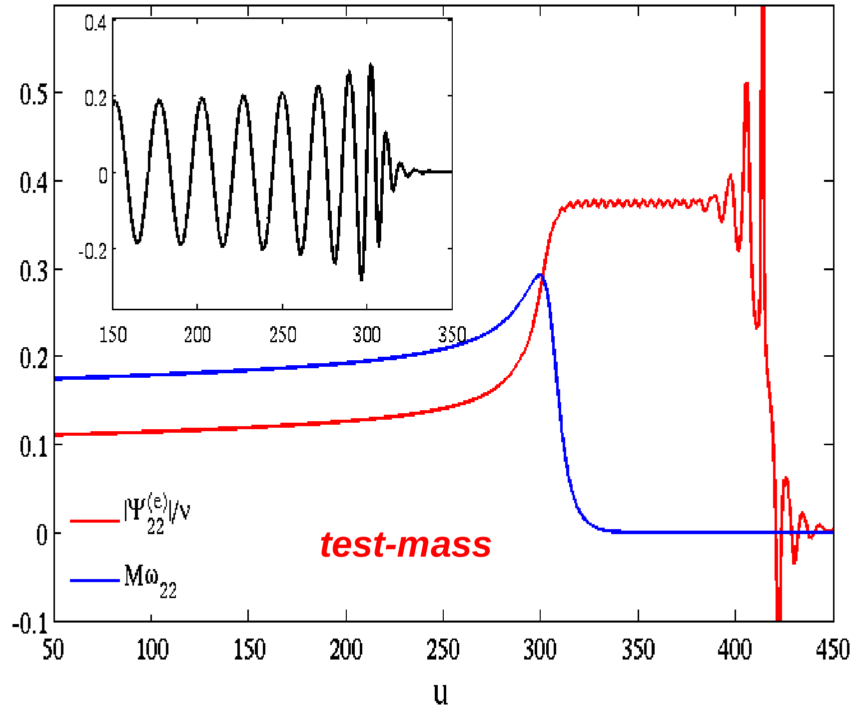




- Phase different of the order of 1% of a GW cycle

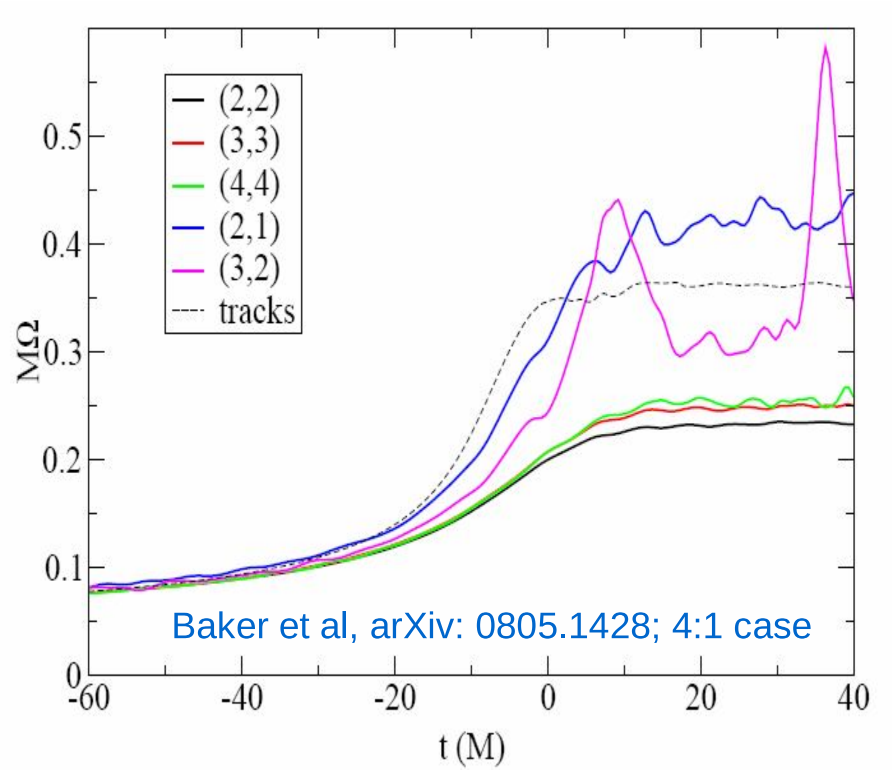
Comparison: test-mass vs Equal-mass

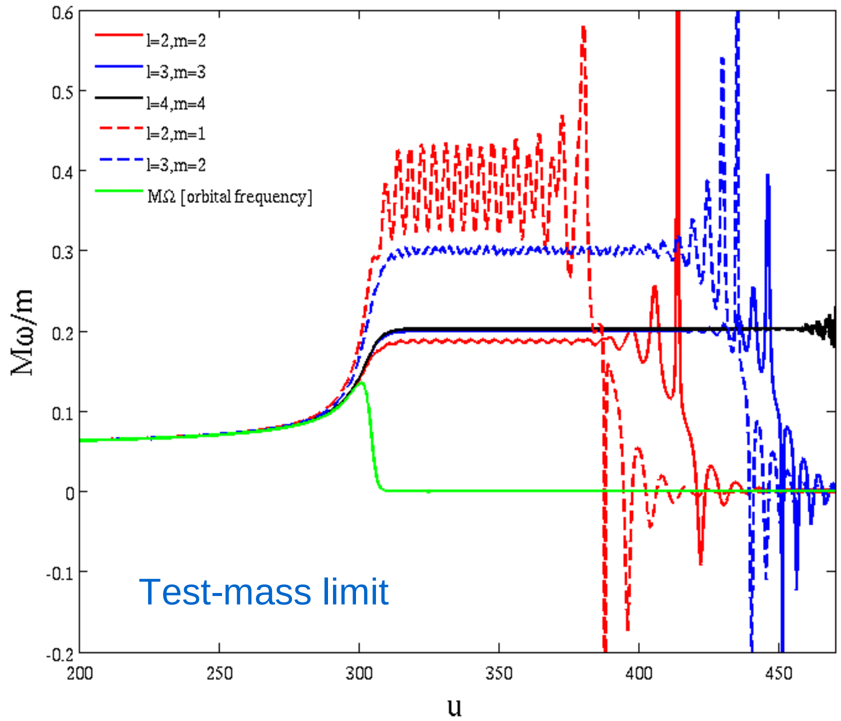




- Evident visual similarities between the test-mass and the equal mass case
- Same kind of qualitative transition inspiral-plunge ringdown
- Test mass: smaller final frequency

Exact results: visual analogies





- The l=m modes are nearly equal after division by m
- The “order” between the frequencies (after division by m) is independent of ν

Preliminary (ld) results on recoil in the EMR limit

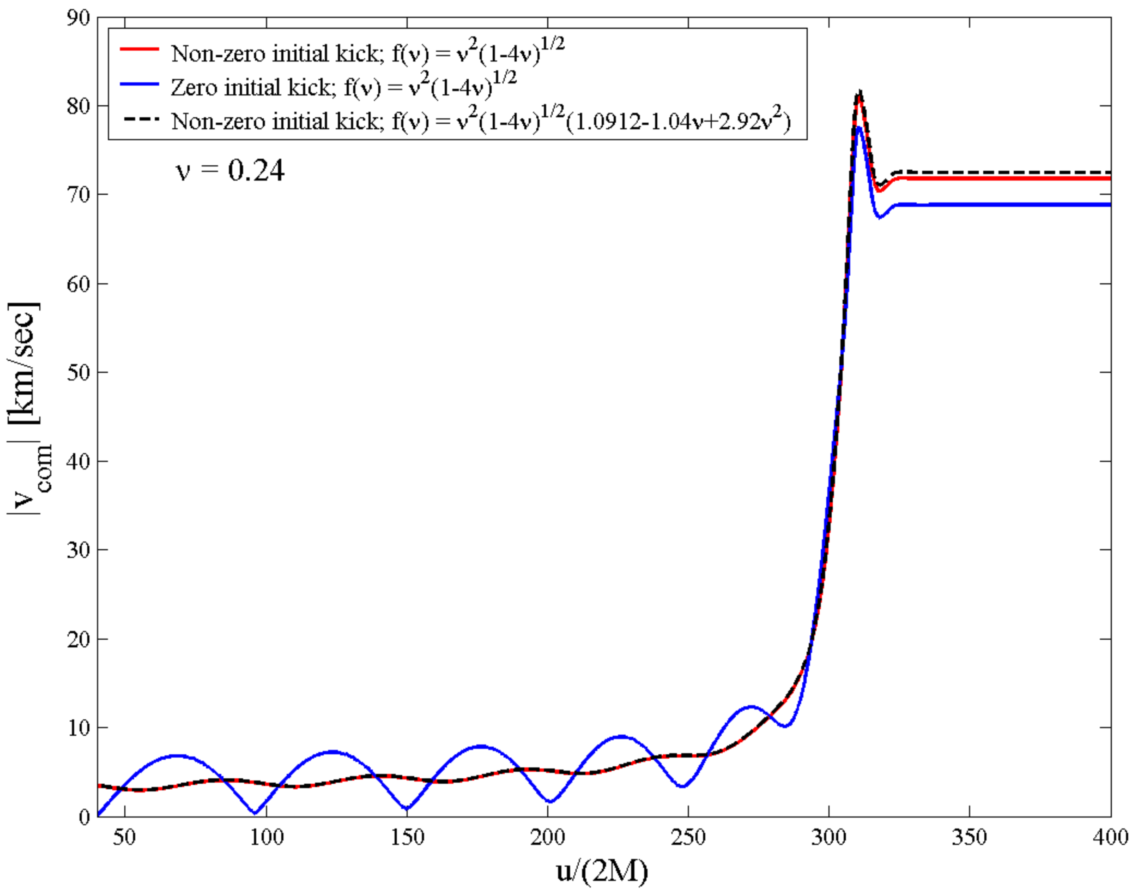
Compute recoil in the EMR limit

Rescale the number for comparing with NR result [à la Fitchett]

Preliminary: l=2 (all); l=3, m=±3 only (the l=3, m=0, ±2, ±1 still missing as well as higher multipoles)

v = 72.49 km/sec

for ν = 0.24 (Baker et al. 2006)



Conclusions

- In the EMR limit it is possible to couple Regge-Wheeler-Zerilli perturbation theory and EOB-inspired radiation reaction force to (approximately) solve the problem and to compute full numerical waveforms

- Useful laboratory to understand the physics and to test EOB ideas

- Approximate way to complete NR knowledge from the “bottom”.

- Our current results in the *EOB-linear-in-ν-limit* can be improved a lot by including higher order PN corrections, wiser ways of doing resummation etc.