FINDING FIELDS AND SELF-FORCE IN A GAUGE APPROPRIATE TO SEPARABLE WAVE EQUATIONS: A PROGRESS REPORT

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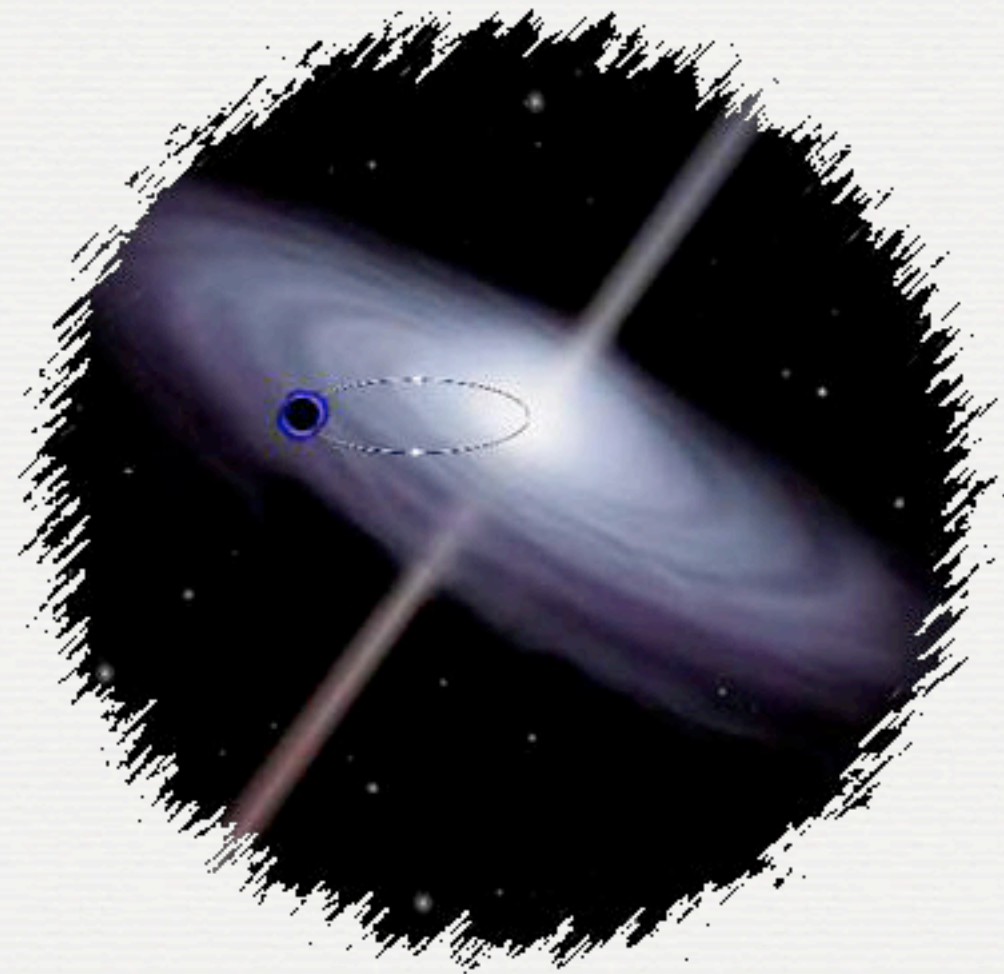
Overview

- Motivation
- Metric Perturbations from Weyl Scalars
- The UWM Method
- (A Couple of) Results

MOTIVATION I

- EMRIs are an important source for LISA.
- Need self-force to compute accurate waveforms for data analysis.
- Mass ratio of ~10^-6 means point particle approximation is good.



http://lisa.jpl.nasa.gov/gallery/stellar-mass-black-hole.html

MOTIVATION II

Want to exploit what we've already got (Teukolsky):

- Decoupling
- Separability
- Applicability to Kerr

http://lisa.jpl.nasa.gov/gallery/stellar-mass-black-hole.html

Hertz Potentials

Metric perturbations from Teukolsky:

Cohen & Kegeles (1975), Chrzanowski (1975), Wald (1978), Stewart (1979)

$$h_{ab} = D_{ab}\bar{\Psi} + \bar{D}_{ab}\Psi$$

Where:

- Ψ is a solution to Teukolsky.
- Each component of D_{ab} is a 2nd order diff. op.

Hertz Potentials

Metric exists in a Radiation Gauge:

$$l^a h_{ab} = 0 \quad \text{or} \quad n^a h_{ab} = 0$$

and

$$g^{ab} h_{ab} = 0$$

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Only exists in the absence of sources!

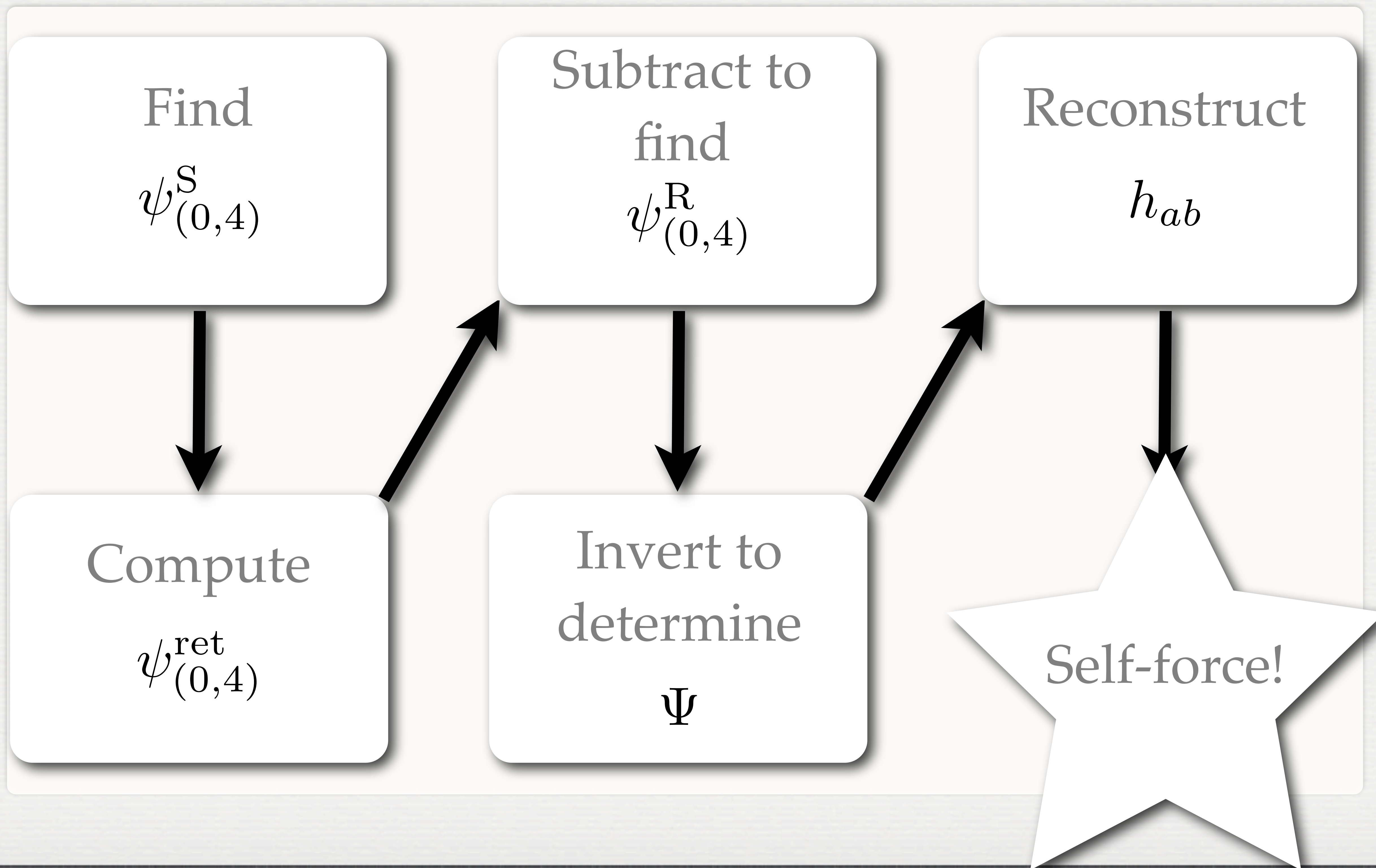
LP, K. Shankar & B. Whiting (2007)

The Solution

The Detweiler-Whiting decomposition (adapted to Teukolsky):

- $\Psi^{\mathrm{ret}} = \Psi^{\mathrm{S}} + \Psi^{\mathrm{R}}$
- Regular field is a **vacuum** solution to Teukolsky and contains all the information about the self-force.
- Singular field is the inhomogeneous solution and contains no information about self-force.

The Method



Find
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Compute
$\psi^{\mathrm{ret}}_{(0,4)}$
Subtract to find
$\psi^{\mathrm{R}}_{(0,4)}$
Invert to determine
Ψ
Reconstruct
h_{ab}
Self-force!

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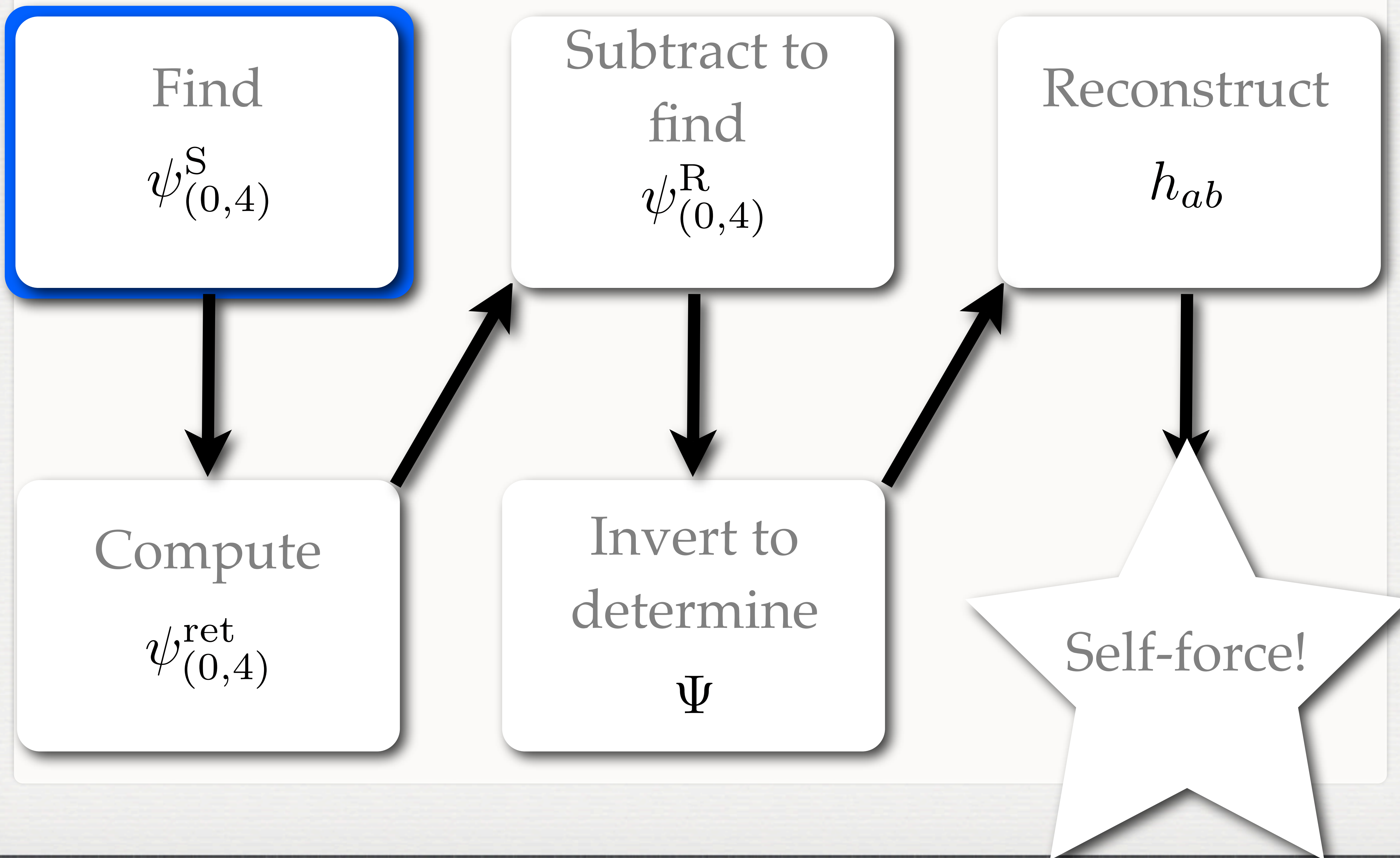
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FINDING $\psi^{\mathrm{S}}_{(0,4)}$

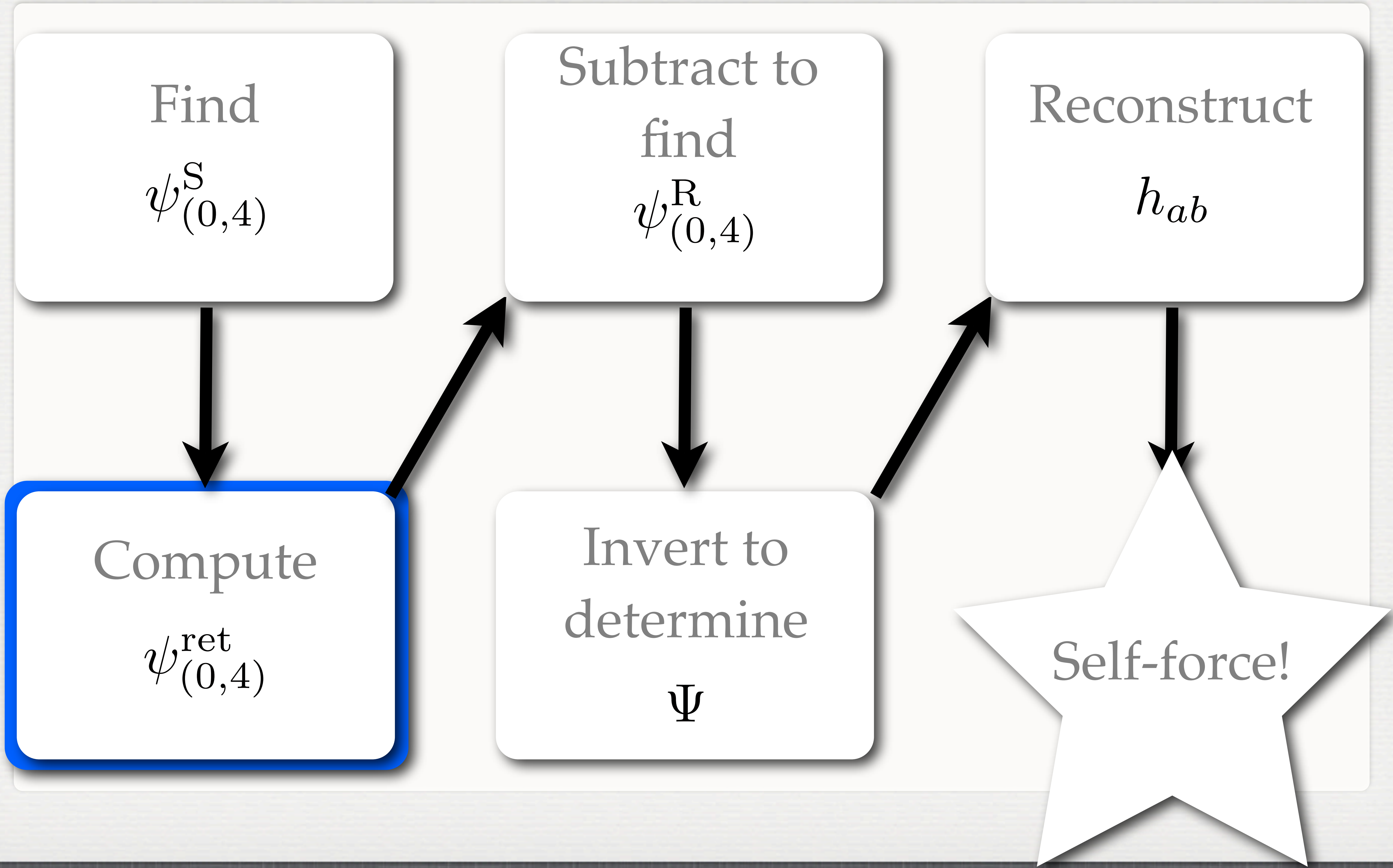
Following Detweiler-Whiting, construct locally inertial coordinates around the particle.

=> To subleading order, we can work in flatspace.

$$\psi^{\mathrm{S}}_0 = \frac{6\mu}{\rho^3}(l_t m_\rho - l_\rho m_t)^2$$

$$\psi^{\mathrm{S}}_4 = \frac{6\mu}{\rho^3}(n_t \bar{m}_\rho - n_\rho \bar{m}_t)^2$$

The Method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Constructed numericallyusing a Green's function expanded in solutions of Teukolsky*. (Toby Keidl)

Actual Teukolsky eqn, not Sasaki-Nakamura!

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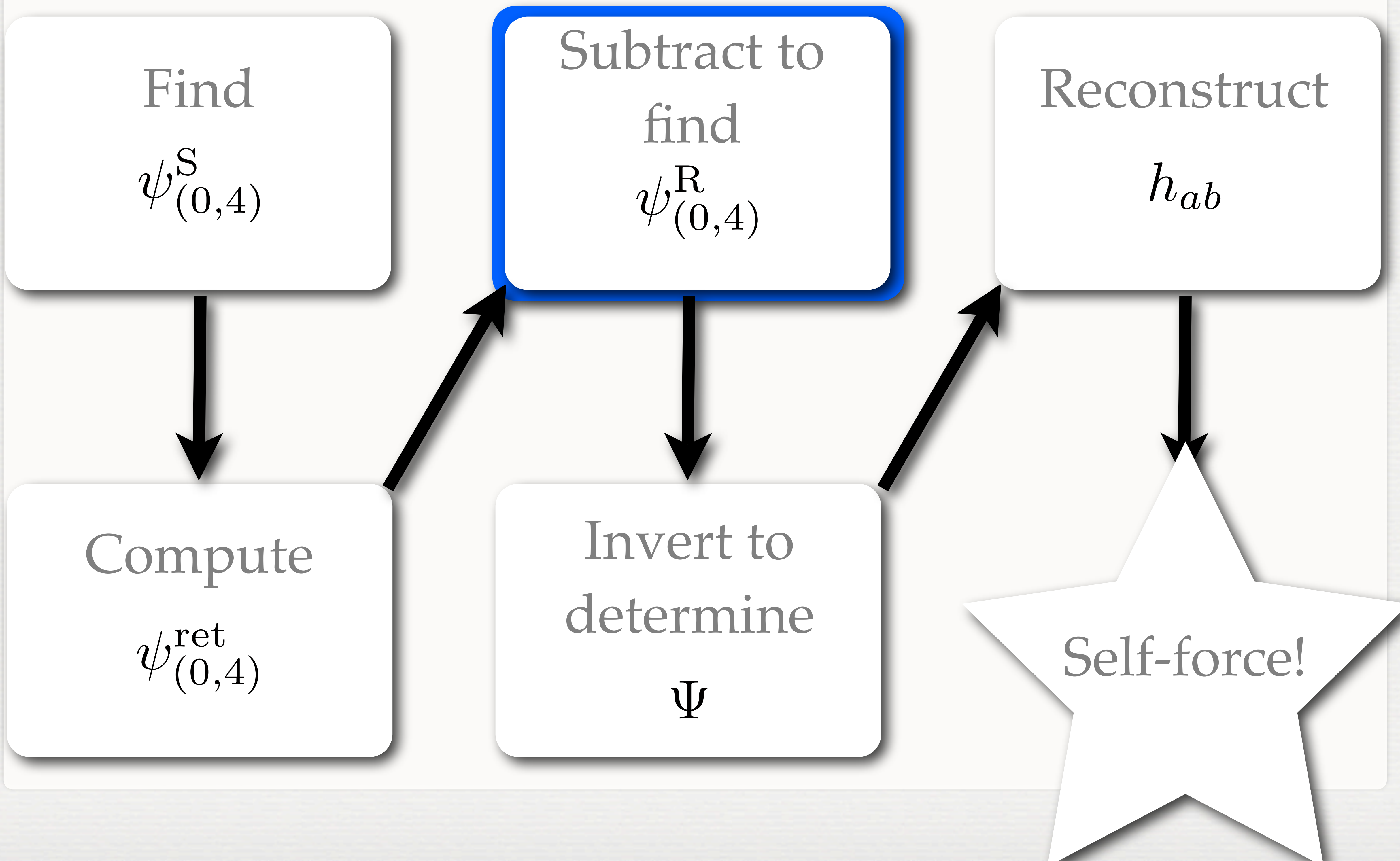
Teukolsky source looks like

$$(D_{\text{ang}}D_{\text{ang}} + D_{\text{ang}}D_{\text{radial}} + D_{\text{radial}}D_{\text{radial}})\delta^3(x - z(t))$$

leading sub-leading higher order

The Method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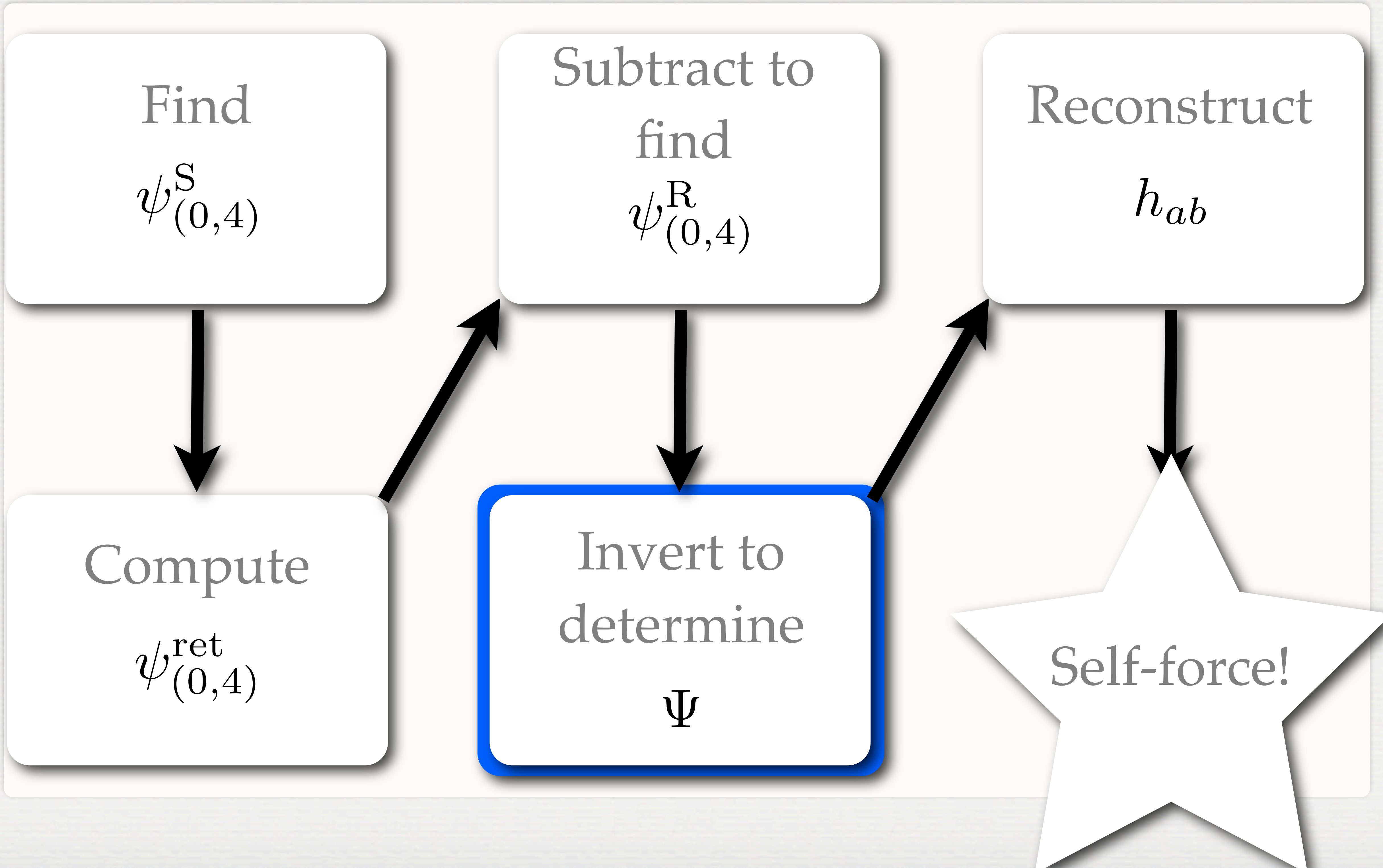
Compute

$\psi^{\mathrm{ret}}_{(0,4)}$

$$\psi^{\mathrm{R}}_{(0,4)} = \psi^{\mathrm{ret}}_{(0,4)} - \psi^{\mathrm{S}}_{(0,4)}$$

Is now a (local) regular, source-free solution of the Teukolsky equation.

The Method



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A choice about how to construct the Hertz potential:

$$\psi_0^{\mathrm{R}} = D^4_{\mathrm{radial}}\Psi$$

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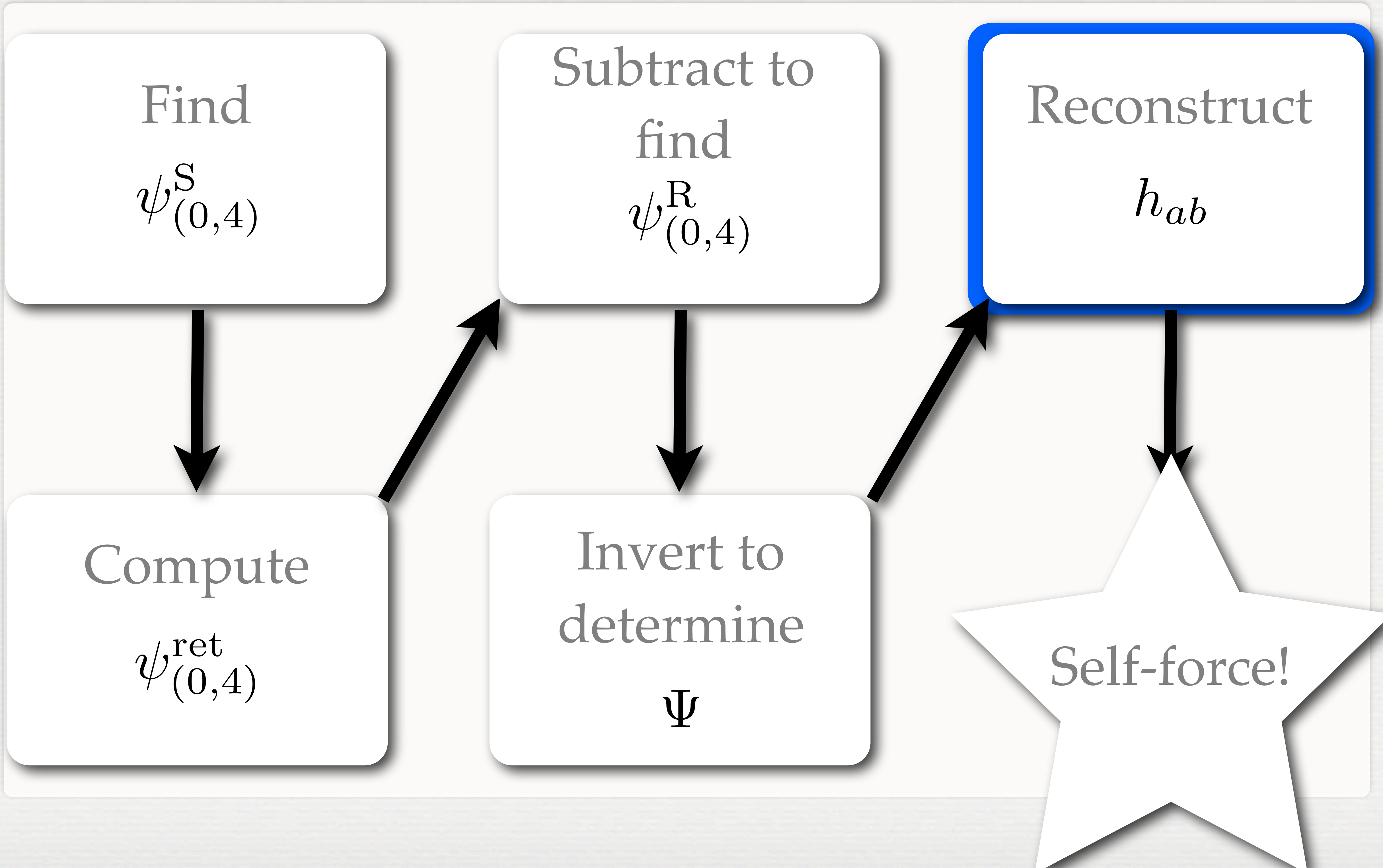
The angular inversion can be done algebraically!
(in the frequency domain)

The Method



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METRIC RECONSTRUCTION

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The construction does not account for the non-radiated part of the perturbation. (in progress)

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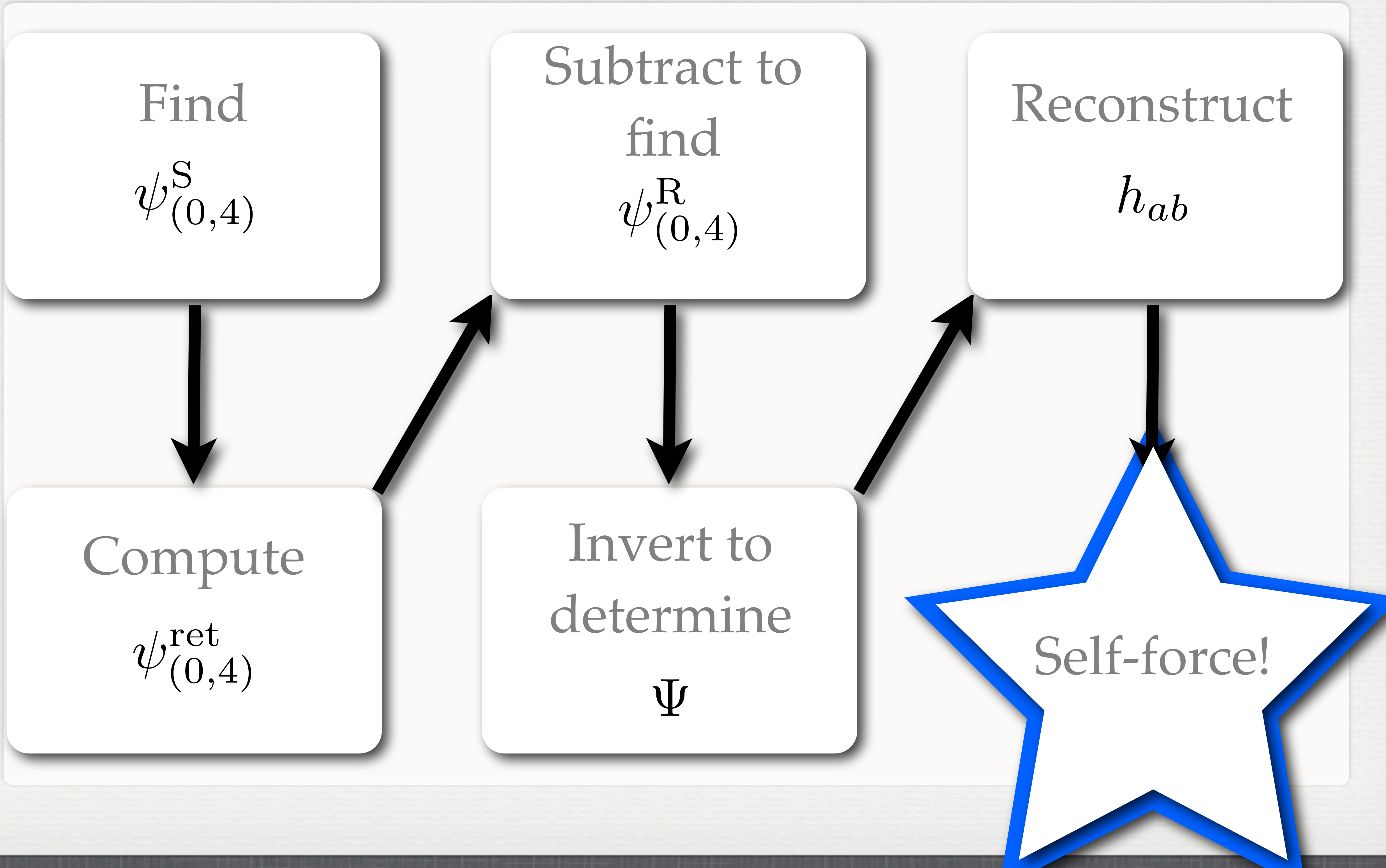
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SOME RESULTS

- Proof of concept: T. Kiedl, J. Friedman, A. Wiseman(2007)
- Circular orbits in Schwarzschild (almost)

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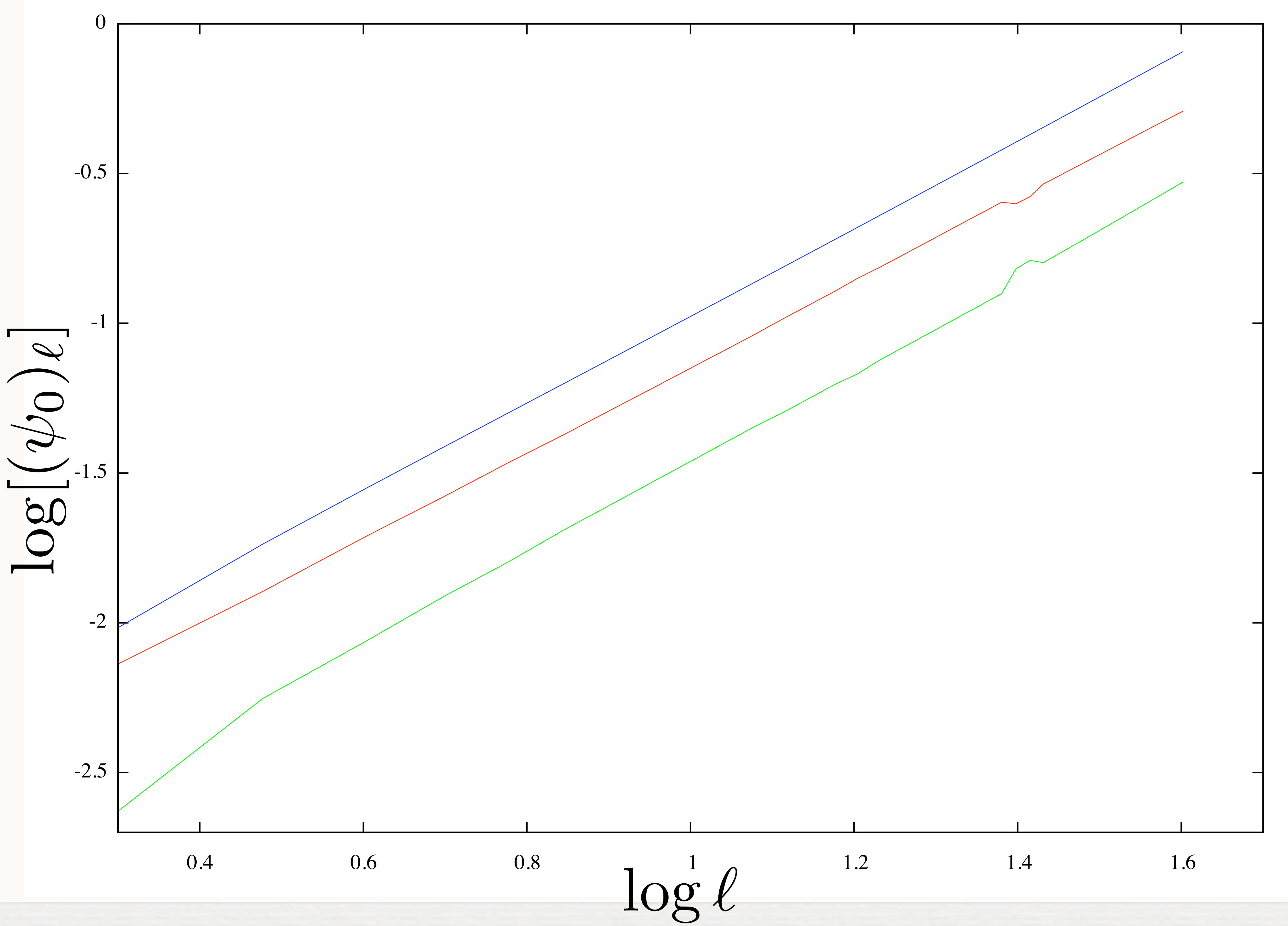

WHAT'S NEXT?

- Fix the falloff?
- Construct the metric.
- Get a self force.
- Evaluating the accuracy of adiabatic waveforms.