Field regularization and smeared-out sources for self-force calculations

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Outline

- The goal
- Our prescription
- Implementation in (1+1)
- Some results
- Current work

Self-consistent particle motion

Goal: Incorporate the self-force in the motion of a particle and retrieve the resulting waveforms. (i.e. Develop a practical scheme for solving the MiSaTaQuWa equation.)

Traditional obstacles:

1. It is difficult to model a delta-function in time-domain codes.
2. It is computationally expensive to compute the local self-force.

Our prescription

1. Identify the part of the retarded field (ie. singular part) that doesn't contribute to the self-force.
2. Rearrange the field equation, bringing the singular part to the source side.
3. Solve the new field equation for the regular field.
4. Use regular field to compute self-force and *to update the source location*.

Our prescription

- Identify the part of the retarded field (ie. singular part) that doesn't contribute to the self-force.

Our prescription

Detweiler-Whiting decomposition

- *Local* decomposition of the retarded field.
- 2 pieces: one that has nothing to do with the motion of the charge, ψ^S, and another that does, ψ^R.

$$\psi^{\rm ret} = \psi^R + \psi^S$$
$$\mathcal{F}_a = \nabla_a \psi^R$$

- $\psi^R \longrightarrow$ smooth; homogeneous solution to the wave equation.
- $\psi^S \longrightarrow$ contains deviations from smoothness; satisfies

$$\Box \psi^S = -4\pi\delta(x)$$

- Approximate expressions for ψ^S can be computed.

Our prescription

- Identify the part of the retarded field (ie. singular part) that doesn't contribute to the self-force.

Let $\psi^{\text{ret}} = \psi^R + W\tilde{\psi}^S$, where $\tilde{\psi}^S$ is an approximation of ψ^S and W is an artificial "window function" to be defined later.

Our prescription

- Rearrange the field equation.

$$\nabla^a \nabla_a \psi^{\text{ret}} = -4\pi q \int_\gamma \delta^{(4)}(x - z(\tau)) d\tau$$

$$\Longrightarrow \nabla^a \nabla_a \psi^R = -\nabla^a \nabla_a (W \tilde{\psi}^S) - 4\pi q \int_\gamma \delta^{(4)}(x - z(\tau)) d\tau$$

Our prescription

- Integrate the equation for the regular field.

$$\nabla^a \nabla_a \psi^R = S_{\text{eff}}$$

where

$$S_{\text{eff}} \equiv -\nabla^a \nabla_a (W \tilde{\psi}^S) - 4\pi q \int_\gamma \delta^{(4)}(x - z(\tau)) d\tau$$

This part should be relatively easy for those who do numerical relativity!

Some details

- $\tilde{\psi}^S$ is not the exact D-W singular field. Therefore, S_{eff} is only of finite differentiability at the location of the charge.
- The window function W is an arbitrary construct. We choose that

  1. $W \to 1$ sufficiently fast as one approaches the particle,
  2. $\nabla_a W \to 0$ sufficiently fast as one approaches the particle, and
  3. $W \approx 0$ outside a compact region R surrounding the particle.

  These conditions guarantee that
  (a) $\nabla_a \psi^R|_{\text{point charge}} = F_a$
  (b) $\psi^R \approx \psi^{\text{ret}}$ outside R (e.g., wavezone)

  For our case we picked the simple function:
  $W(r) = \exp\left[-\frac{(r-R)^N}{\sigma^N}\right]$.

Constructing the DW singular field

Our prescription relies crucially on the calculation of the Detweiler-Whiting singular field.

This singular field is convenient to write down in so-called Thorne-Hartle-Zhang coordinates $\{t, x, y, z\}$, which are locally-inertial and harmonic coordinates associated with a given worldline.

In THZ coordinates, the singular field simply looks like a Coulomb field:

$$\psi^S = \frac{q}{\rho} + O(\rho^3/R^4)$$

This must be expressed in the background coordinates.

Constructing the DW singular field

To express the singular field in terms of the background coordinates, we need:

- the four-velocity of the geodesic trajectory at that point.
- the coordinate location of that point

These allow the explicit transformation from the background coordinates to the Thorne-Hartle-Zhang coordinates for general geodesics (ongoing project).

Once done, this will allow (3+1) evolution of a scalar charge in a Schwarzschild black hole.

Advantages of our prescription

- Generality.
  - Can be implemented in (3+1), (2+1), or (1+1).
- No delta functions.
  - The source is regular.
- ψ^R gives the retarded field in the wavezone.
  - Important for fluxes.
- The self-force is easy to compute from ψ^R.
  - It's just the gradient of ψ^R.

Scalar charge in a circular orbit of Schwarzschild

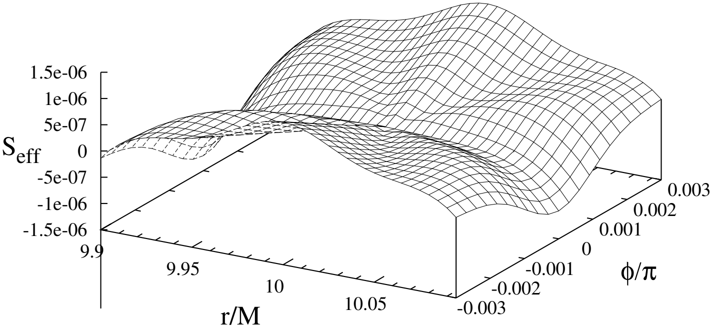
- Simplicity allows for comparisons.
- Spherical symmetry allows us to deal with a (1+1) problem:

$$\psi^R = \sum_{lm} \frac{1}{r} f_{lm}(t,r) Y_{lm}(\theta,\phi)$$

$$S_{lm}(t,r_*) = (r-2M) \int S_{\text{eff}}(x') Y_{lm}(\theta',\phi') d\Omega'.$$

$$-\frac{\partial^2 f_{lm}}{\partial t^2} + \frac{\partial^2 f_{lm}}{\partial r_*^2} - V(r_*) f_{lm} = S_{lm}(t,r_*)$$

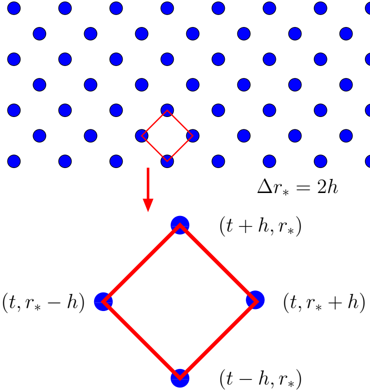
Effective source



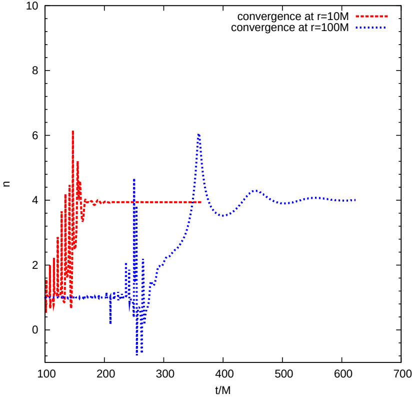
Effective source along the equatorial plane $\theta = \pi/2$.

Numerics

- time-domain
- characteristic grid in (t, r_*)-plane
- 4th-order convergent algorithm is derived
- initial data: unspecified
- boundary conditions: none, computational domain is made large



Checks: Convergence



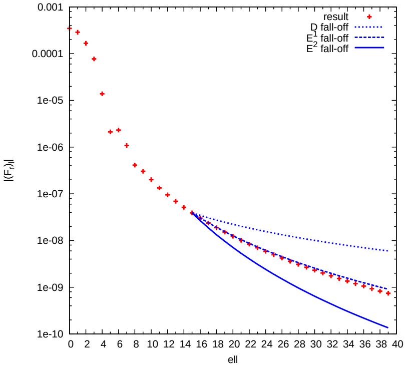
$$n(r,t) = \log\left|\frac{f_{4h}(r,t)-f_{2h}(r,t)}{f_{2h}(r,t)-f_h(r,t)}\right| / \log(2)$$

Convergence factor, n, computed at $r = 100M$ and $r = 10M$.

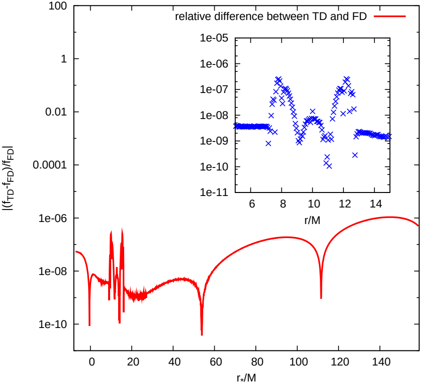
Checks: self-force, high-l convergence

The high-l fall-off in $\nabla_r \psi^{\mathrm{R}}$ must be dominated by the highest unremoved singular piece.

$$\lim_{r\to R}(\nabla_r\psi^S)_l = \left(l+\frac{1}{2}\right)A_r + B_r - \frac{2\sqrt{2}D_r}{(2l-1)(2l+3)} + \frac{(2l+1)E_r^{(1)}\mathcal{P}_{3/2}}{(2l-3)\ldots(2l+5)} + \frac{(2l+1)E_r^{(2)}\mathcal{P}_{5/2}}{(2l-5)\ldots(2l+7)} + \ldots$$



Results: wavezone



We reconstruct

$$\psi_{22}^{\text{ret}} = \psi_{22}^{R} + (W\psi^{S})_{22}$$

and compare with results from a frequency domain calculation.

Result: at worst 10^{-6} error.

Results: self-force

	R	Time-domain	Frequency-domain	error
$\partial_t \psi^R$	$10M$	3.750211×10^{-5}	3.750227×10^{-5}	0.000431%
$\partial_r \psi^R$	$10M$	1.380612×10^{-5}	1.378448×10^{-5}	0.157%
$\partial_t \psi^R$	$12M$	1.747278×10^{-5}	1.747254×10^{-5}	0.00139%
$\partial_r \psi^R$	$12M$	5.715982×10^{-6}	5.710205×10^{-6}	0.101%

We compare our results with that achieved previously by a very accurate frequency-domain calculation.

Current work

We need to assess the viability of our prescription in (3+1) implementations.

In collaboration with:
W. Tichy (FAU) - pseudospectral code
P. Diener (CCT/LSU) - multi-patch code
L. Lehner (LSU) - finite-difference code with AMR

Next goal: Get self-consistent motion for scalar charges using existing (3+1) numerical relativity infrastructure.

Next Capra!