Mathematical issues, post-Newtonian comparison, rotating spacetime “features”

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Overview

- Brief explanation of title
- Why discuss mathematical issues?
- Why is PN comparison of interest?
- What is a spacetime “feature?”
- Indications for future work

Brief explanation of title

- Speak to a larger audience than the school
  - Help make contact with mathematicians
    - Many questions of mathematical interest
  - Help make contact with PN community
    - New comparisons in the last 12 months
  - Explain interesting and exciting new topics
- Tie together different topics and approaches
- Communicate possibility for rich new input

Why discuss mathematical issues?

- They really crop up in many contexts
  - Regularization is a common headache
- May interest people in other fields
- They continually need addressing
  - What is gauge invariant, what to compare
- Foundation for a concerted focus
  - Regularize, l=0,1, mode sum methods
- Helps catalogue incremental progress

Examples

- Equations have explicit symmetries (eg NP)
  - Steep learning curve - good pay-off
- Stability of black holes
  - Schwarzschild (Kay and Wald)
  - Kerr (Add your name here)
- Relate radiation gauges to radiation reaction
  - Seems essential for progress in Kerr
- Role of Equivalence Principle (a posteriori)?

Killing vectors and type D spacetimes

- Killing vectors exist in all type D spacetimes

$$\xi^a = \psi_2^{-1/3}(-\rho' l^a + \rho n^a + \tau' m^a - \tau \bar{m}^a)$$

- They form an operator that commutes with all GHP derivatives (non-accelerating metrics)

$$\mathcal{V} = \psi_2^{-1/3}(\tau' \eth - \tau \eth' - \rho' \text{Þ} + \rho \text{Þ}' + \frac{1}{2}\psi_2 p + \frac{\rho}{2\bar{\rho}}\bar{\psi}_2 q)$$

- Special relations are required for commutation

Teukolsky-Starobinsky Identities

- Relies on a commutation relation:

$$\text{Þ}^4 \psi_2^{-4/3} \eth'^4 \bar{\Psi} = \eth'^4 \psi_2^{-4/3} \text{Þ}^4 \bar{\Psi}$$

- Not known to hold for accelerating metrics

$$\begin{aligned} \text{Þ}'^4 \bar{\psi}_2^{-4/3} \text{Þ}^4 \psi_2^{-4/3} \psi_4 &= \eth'^4 \bar{\psi}_2^{-4/3} \eth^4 \psi_2^{-4/3} \psi_4 - 9 \mathcal{V} \bar{\mathcal{V}} \psi_4 \\ \text{Þ}^4 \bar{\psi}_2^{-4/3} \text{Þ}'^4 \psi_2^{-4/3} \psi_0 &= \eth^4 \bar{\psi}_2^{-4/3} \eth'^4 \psi_2^{-4/3} \psi_0 - 9 \mathcal{V} \bar{\mathcal{V}} \psi_0 \end{aligned}$$

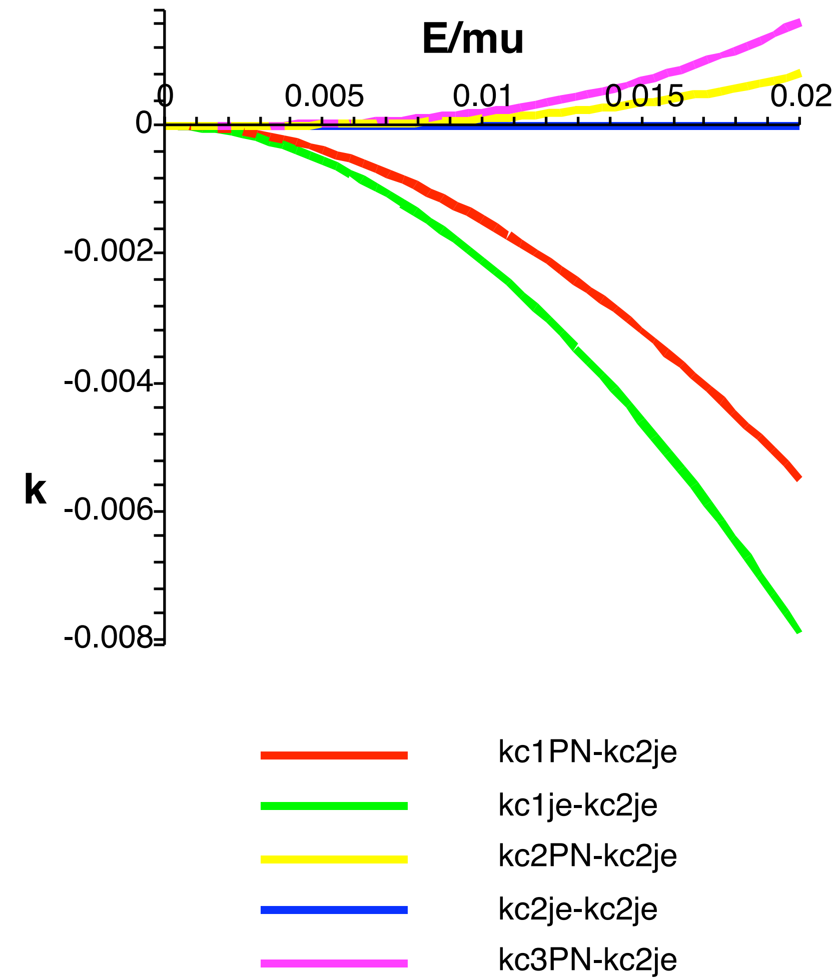
- Why not for all separable type D perturbations?

Why is PN comparison of interest?

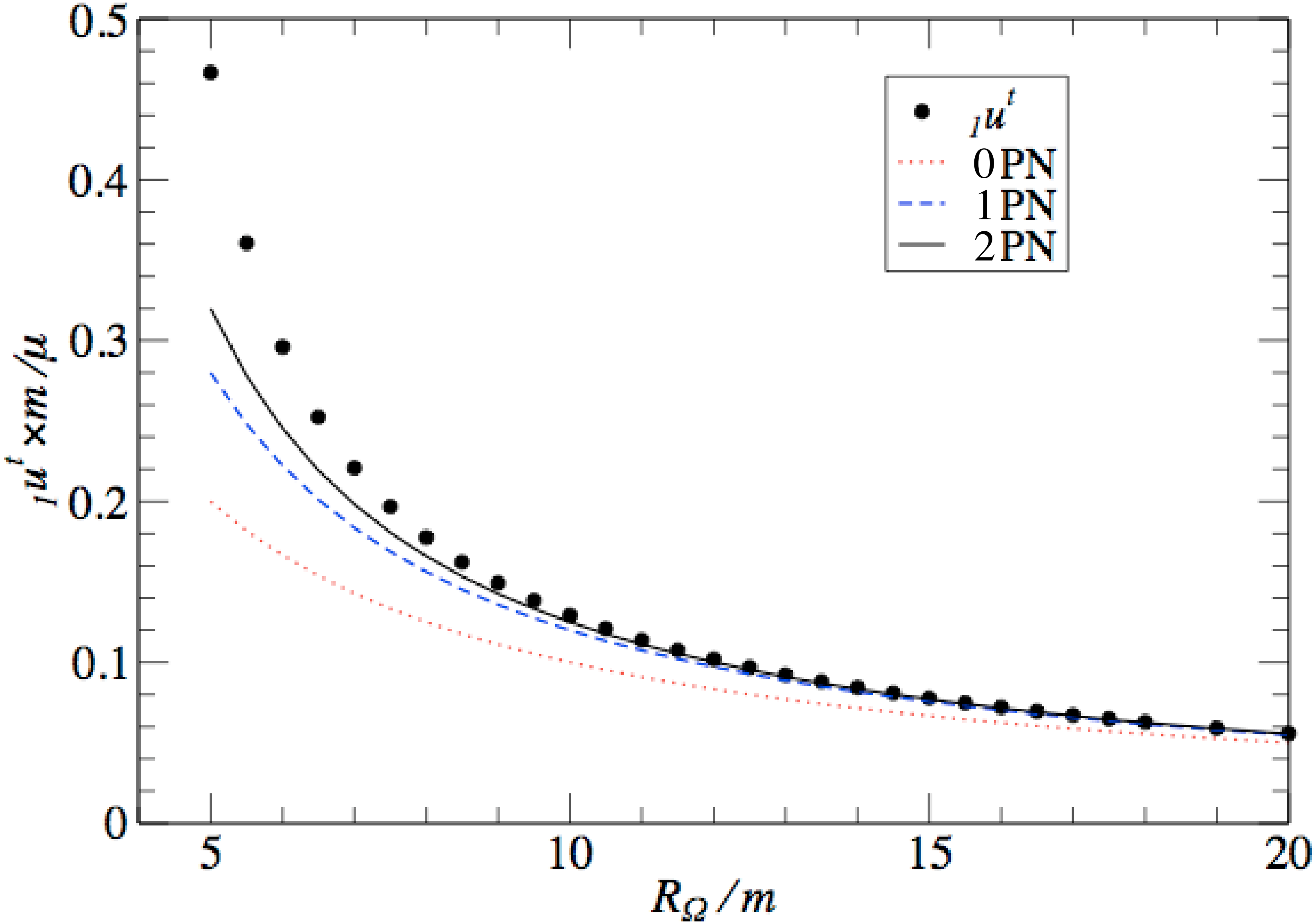
- Check accuracy, verify domains of validity
- PN often not used to qualify itself
- NR not able to reach extreme mass ratio
- PN provides the only available EMRI check
- NR not being used to qualify PN
- Helps formulate gauge invariant list
- Push PN beyond its current bounds
- Quantify structure of composite waveforms

What does PN give?

- Solve $er^2=0$ for $j(E)$
- Substitute in $k(j,E)$
- Subtract reference:
  - 1PN (strict) r
  - 2PN (strict) y
  - 3PN (strict) p
  - 1PN(exact) g
  - 2PN(exact) b



Self-force vs PN results (from Detweiler)



What is a spacetime "feature"

- Schwarzschild is spherically symmetric
  - This is a significant feature (E,J defined)
- Kerr is only axially symmetric
  - This is a significant "feature" (E defined)
- Radiation reaction spacetime is neither
  - Not even a significant "feature" (? defined)
- MUST approximate to make progress
  - Comparison helps validate approximation

Low multipole moments

- Give mass and angular momentum perturbations

$$\delta M = 2\int_\Sigma (\mathcal{T}_{ab} - \frac{1}{2}\mathcal{T} g_{ab}) n^a t^b \sqrt{h} d^3x$$

$$\delta J = -\int_\Sigma (\mathcal{T}_{ab} - \frac{1}{2}\mathcal{T} g_{ab}) n^a \phi^b \sqrt{h} d^3x$$

- Dipole component locally pure gauge
- Not given by metric reconstruction: $\ell \geq 2$
- Needed for regularization of singular field
- Are they needed for gauge invariant physics?

Indications for future work

- Time domain codes is a must
  - Symmetries less of an issue
- Should evolve orbits in real time
  - Need “self-force” for arbitrary geodesic
- Waveforms will bypass PN metric
- Perhaps the self-force will disappear
- Want to maximize use of symmetries
- Make 2^{nd} order a routine treatment