Self-force for a particle in circular orbit around Schwarzschild blackhole

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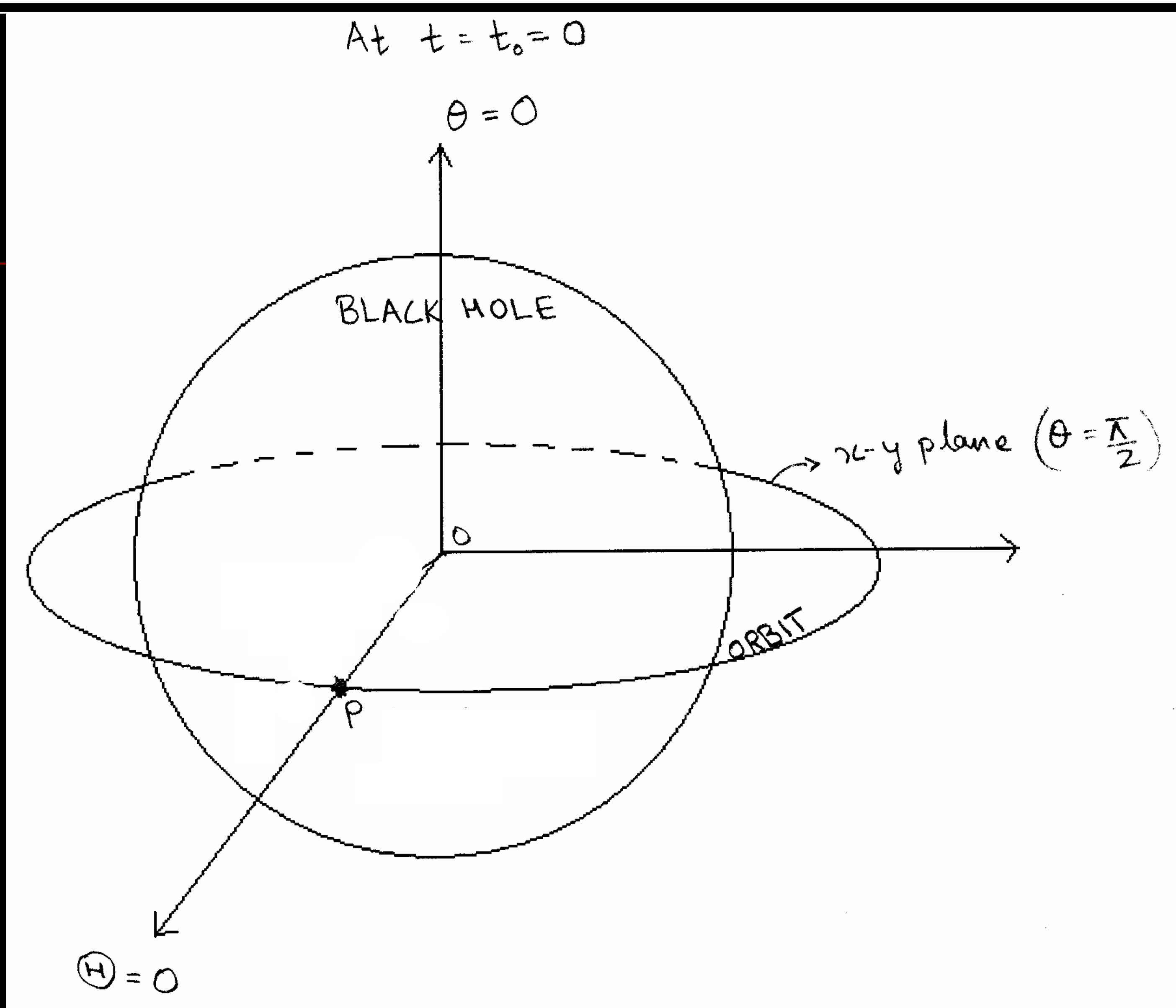
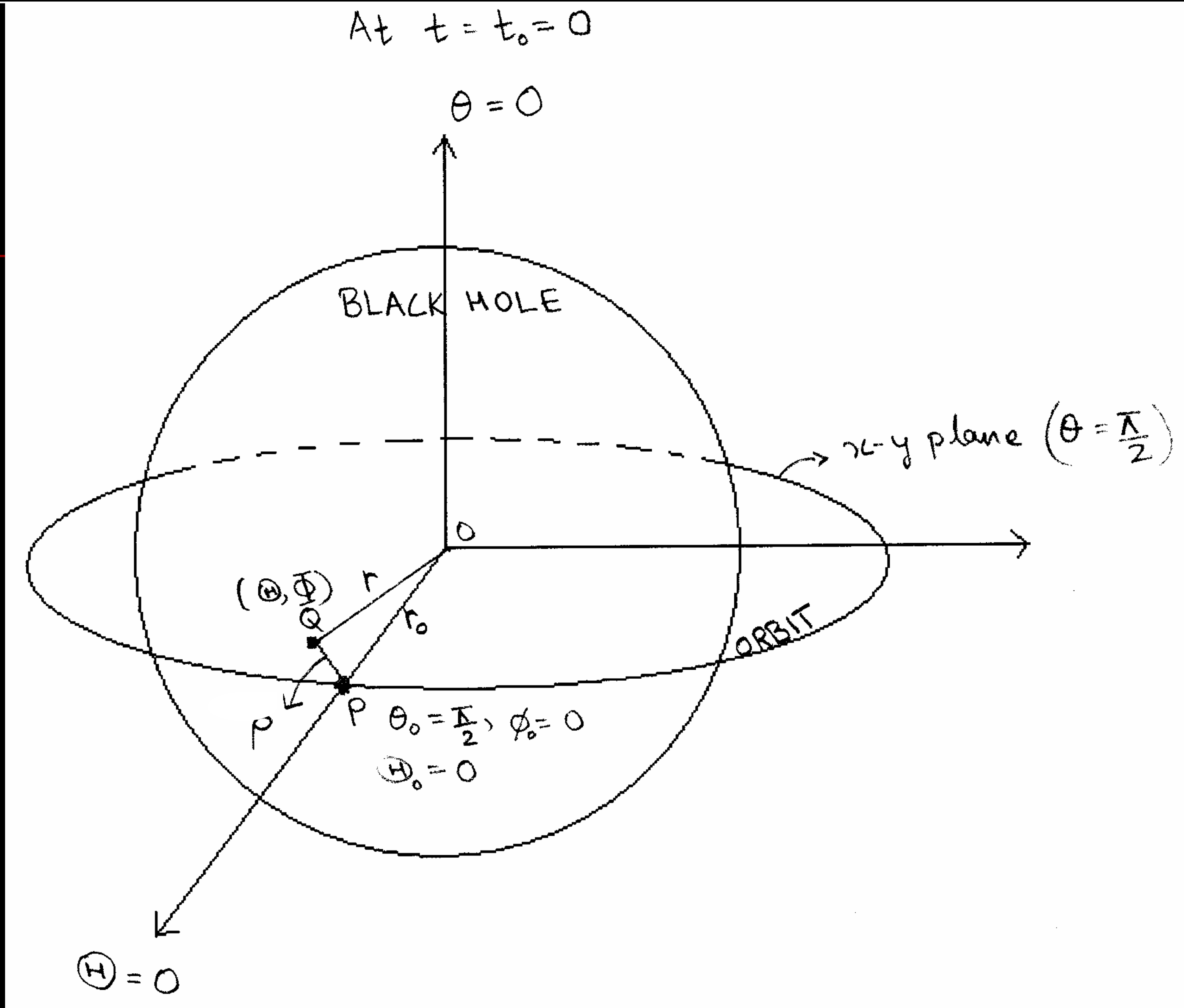


At t = t₀ = 0
θ = 0
BLACK HOLE
x-y plane (θ = π/2)
O
ORBIT
P
Θ = 0



At t = t₀ = 0
θ = 0
BLACK HOLE
x-y plane (θ = π/2)
O
(Θ, Φ)
r
Q
r₀
ORBIT
ρ
P
θ₀ = π/2, φ₀ = 0
Θ₀ = 0
Θ = 0

Conservative part of the self force

- Cons. part of S.F. is only axisymmetric.
- Only need to find the axisymmetric part of ψ_0^{REN} .
- $\psi_0^{RET} = \sum_{l,m} R_{lm} \, {}_2Y_{lm}(\theta, \phi)$
- Make a coordinate rotation ${}_2Y_{lm}(\theta, \phi) \rightarrow {}_2Y_{lm}(\Theta, \Phi)$.
- $\psi_0^{RET} = \sum_{l=2} X_l^{RET} \, {}_2Y_{l0}(\Theta, \Phi)$

Finding ψ_0^{REN}

- $\psi_0^{RET} = \psi_0^{REN} + \psi_0^{SING}$
- $\psi_0^{SING} = \sum_{l=2}^{\infty} \sum_{n=-\frac{3}{2}}^{\infty} C_n l^{-n} \, {}_2Y_{l0}(\Theta, \Phi)$
- $\psi_0^{REN} = \sum_l X_l^{REN} \, {}_2Y_{l0}(\Theta, \Phi)$

Where X_l^{REN} falls off faster than any power of l.

ψ_0^{RET}

- Integrate the Bardeen-Press equation (Teukolsky) to find R_{lm} of ψ_0^{RET} from the horizon ($\simeq$2M) and from infinity ($\simeq$1000M).
- 10^{th} order Runge-Kutta method.
- Accuracy is 1 in 10^{-12}.

ψ_0^{SING}

- $h_{\alpha\beta} = \frac{2\mu}{\rho}\delta_{\alpha\beta} \rightarrow \psi_0^{SING} = \frac{6\mu}{\rho^3}(l_T m_\rho - l_\rho m_T)^2$
- Construct locally inertial coordinates around the particle (t_0,r_0,θ_0,Φ_0).
- ψ_0^{SING} is expanded in powers of ρ.
- We get 12 terms out of which only one axisymmetric term survives in the limit $r \longrightarrow r_0$.

Terms in ψ_0^{SING}

$$\psi_0^{SING} = \left(\frac{c\sin^2\Theta}{\tilde{\rho}^5} + \frac{c\Delta^2 e^{2i\varphi}}{\tilde{\rho}^5} + \frac{ce^{-i\varphi}\Delta\sin\Theta}{\tilde{\rho}^5} \right) +$$
$$\left(\frac{ce^{-2i\varphi}\Delta^3}{\tilde{\rho}^5} + \frac{ce^{i\varphi}\sin^3\Theta}{\tilde{\rho}^5} + \frac{ce^{-i\varphi}\Delta^2\sin\Theta}{\tilde{\rho}^5} + \frac{c\Delta\sin^2\Theta}{\tilde{\rho}^5} \right.$$
$$\left. + \frac{cf(\varphi)\Delta^3\sin^2\Theta}{\tilde{\rho}^7} + \frac{ce^{-2i\varphi}\Delta^5}{\tilde{\rho}^7} + \frac{cf(\varphi)\Delta\sin^4\Theta}{\tilde{\rho}^7} + \frac{cf(\varphi)\Delta^2\sin^3\Theta}{\tilde{\rho}^7} + \frac{ce^{-i\varphi}\Delta^4\sin\Theta}{\tilde{\rho}^7} \right)$$

- $\Delta = r - r_0$
- c is a different $c(r_0)$ in each occurrence.
- f(φ) is a different f(φ) in each occurrence.

Surviving term in ψ_0^{SING}

$$\psi_0^{SING} = \left(\frac{c\sin^2\Theta}{\tilde{\rho}^5} + \frac{ce^{2i\varphi}\Delta^2}{\tilde{\rho}^5} + \frac{ce^{-i\varphi}\Delta\sin\Theta}{\tilde{\rho}^5} \right) +$$

$$\left(\frac{c\Delta^3 e^{-2i\varphi}}{\tilde{\rho}^5} + \frac{ce^{i\varphi}\sin^3\Theta}{\tilde{\rho}^5} + \frac{ce^{-i\varphi}\Delta^2\sin\Theta}{\tilde{\rho}^5} + \frac{c\Delta\sin^2\Theta}{\tilde{\rho}^5} \right.$$

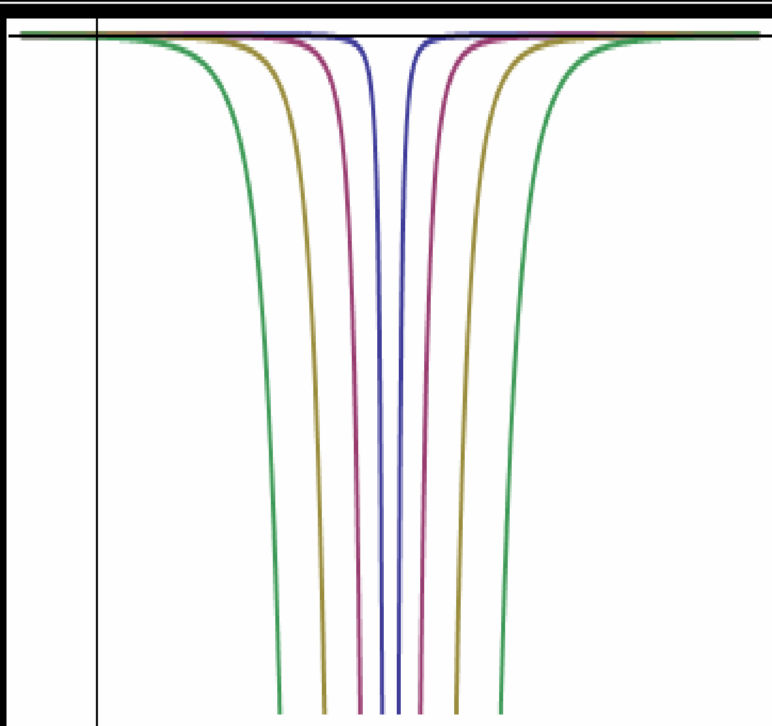
$$\left. + \frac{cf(\varphi)\Delta^3\sin^2\Theta}{\tilde{\rho}^7} + \frac{ce^{-2i\varphi}\Delta^5}{\tilde{\rho}^7} + \frac{cf(\varphi)\Delta\sin^4\Theta}{\tilde{\rho}^7} + \frac{cf(\varphi)\Delta^2\sin^3\Theta}{\tilde{\rho}^7} + \frac{ce^{-i\varphi}\Delta^4\sin\Theta}{\tilde{\rho}^7} \right)$$

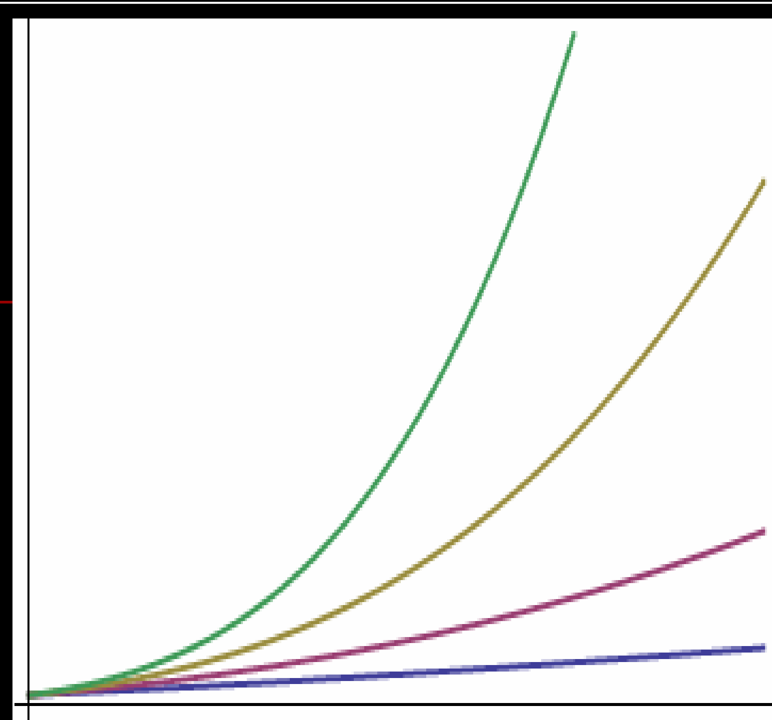
Decomposition

$$
\begin{aligned}
1^{st}\,term \quad &= \frac{\sin^2\Theta}{[a\Delta^2 + b\chi(\Phi)(1-\cos\Theta)]^{5/2}} \\
&\propto \frac{\sin^2\Theta}{(\delta^2 + 1 - \cos\Theta)^{5/2}}
\end{aligned}
$$

$$
\frac{1}{(e^T + e^{-T} - 2u)^{1/2}} = \sum_{l=0}^{\infty} e^{-(l+1/2)T} P_l(u)
$$

$$
(\delta^2 + 1 - u)^{p/2} = \sum_{l=0}^{\infty} A_l^{p/2}(\delta) P_l(u) \quad where\, A_l^{-k-1/2} = \frac{2l+1}{\delta^{2k-1}(2k-1)}[1 + O(l\delta)], \delta \to 0
$$





- Green $x^{-6} \leftrightarrow l^{9/2}$
- Golden $x^{-5} \leftrightarrow l^{7/2}$
- Magenta $x^{-4} \leftrightarrow l^{5/2}$
- Blue $x^{-3} \leftrightarrow l^{3/2}$

ψ_0^{REN}

- Hence we have both ψ_0^{RET} and ψ_0^{SING} as a sum over *l* in terms of $_2Y_{l,m}$s'.

$$\psi_o^{RET} = \sum_{l=2}^{\infty} X_l^{RET} \, {}_2Y_{l,0}$$

$$\psi_o^{SING} = \sum_{l=2}^{\infty} (X_l^{SING(1)} + X_l^{SING(2)} + ...)\, {}_2Y_{l,0}$$

- Each term in ψ_0^{SING} removes a power of *l* as shown next.

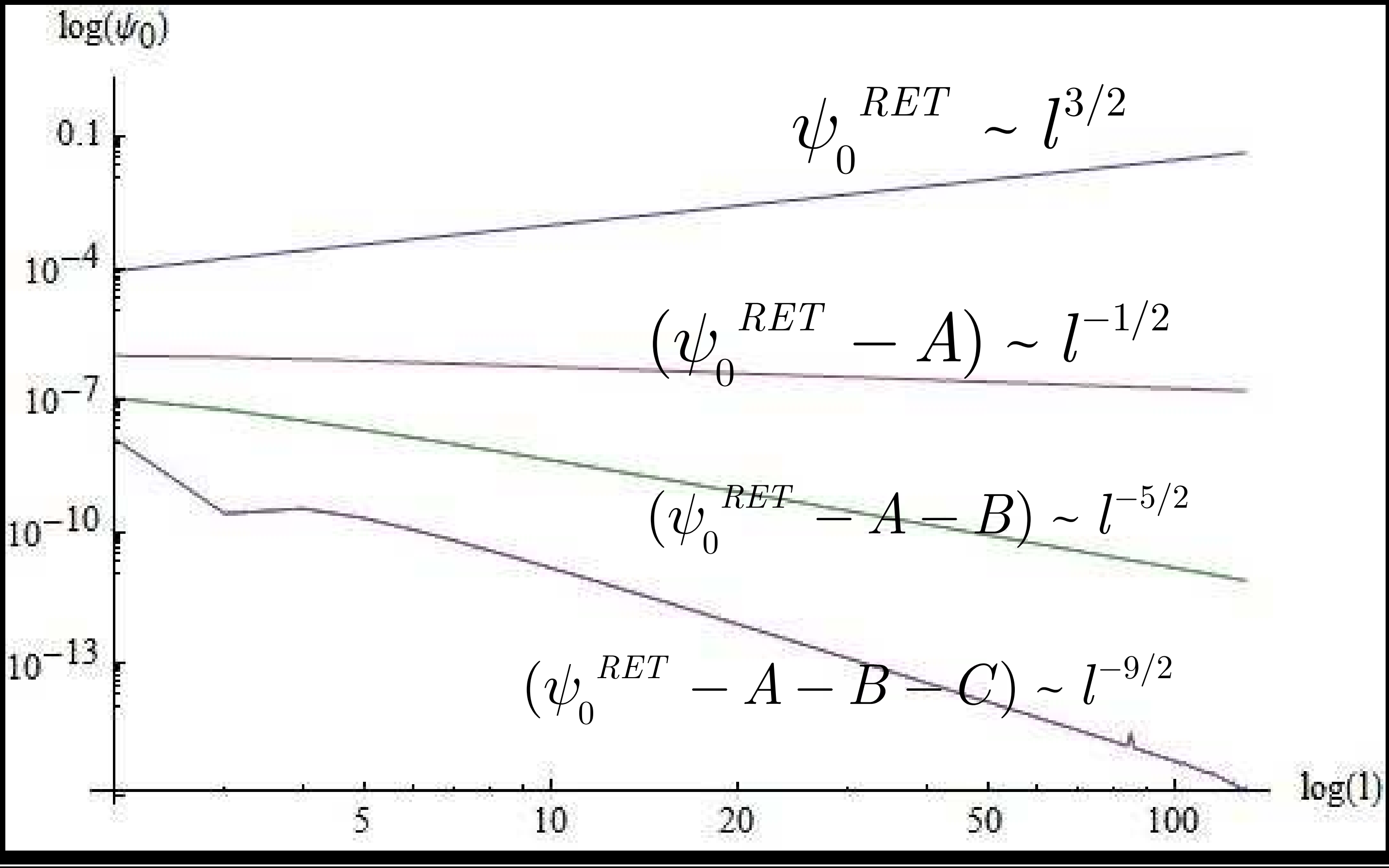
Subtracting higher powers of *l*

We use the following *ansatz* :

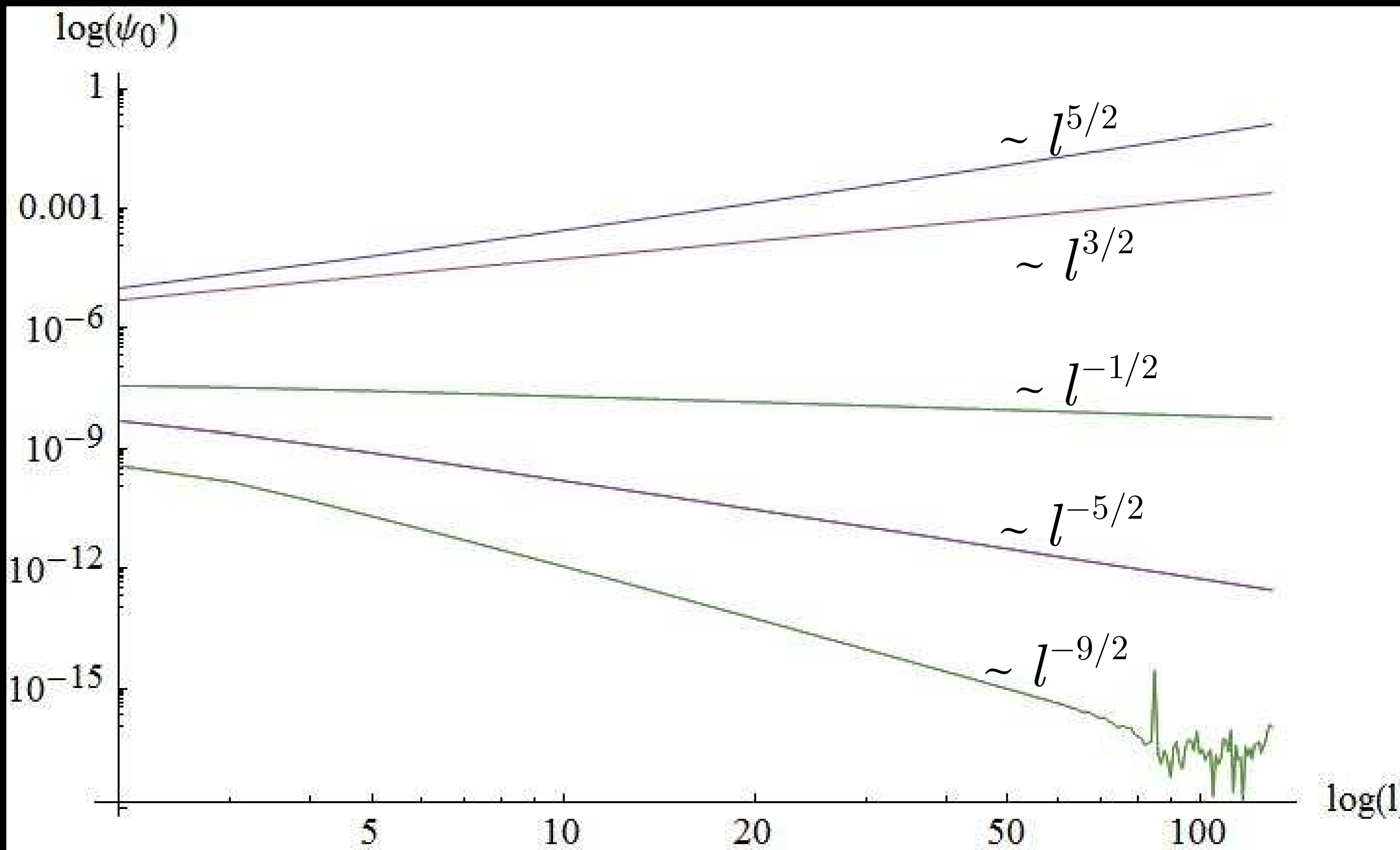
$$\psi_0^{\mathrm{SING}} = \sqrt{\frac{4\pi(l+2)!}{(l-2)!(2l+1)}}\left(A + \frac{B}{(2l+1)^2} + \frac{C}{(2l+1)^4} + \ldots\right)$$

$$\psi_0^{\mathrm{SING}\prime} = \sqrt{\frac{4\pi(l+2)!}{(l-2)!(2l+1)}}\left(A'(2l+1) + B' + \frac{C'}{(2l+1)^2} + \frac{D'}{(2l+1)^4} + \ldots\right)$$

log-log plot of ψ_0 vs. l with subsequent subtractions



log-log plot of ψ_0' vs. l



After ψ_0^{REN} ?

- $\psi_0^{REN} \longrightarrow \Psi^{REN} \longrightarrow h_{11}^{REN} \longrightarrow Self-force$

$$F^r = \frac{\mu}{8}\frac{(1-2M/r)^2}{(1-3M/r)}\left(2\partial_t - (1-\frac{2M}{r})\partial_r - 6\frac{M}{r^2}\right)h_{11}^{REN}$$

- Only one component of $h_{\alpha\beta}^{REN}$ contributes to F^r which is h_{11}^{REN} .

Comparison

- Expected value (ExVa) of h_{11} ~ 20% of $1/r_0$.
- At r_0 = 45M, ExVa = 0.0044
  - After subtracting A ~10^{-7}
  - After subtracting A,B ~ 10^{-8}
  - After subtracting A,B,C ~ 10^{-9}
- Similar thing happens for its prime (∂_r)
- With the numerical accuracy we conjecture that the perturbed axisymmetric Weyl scalar's contribution to the S.F. is negligible.
- We see a similar behavior in DMW.