Higher-order self force effects in a nonlinear scalar model of EMRIs: Lessons for gravity?

Chad Galley

Jet Propulsion Laboratory
and
Theoretical Astrophysics, California Institute of Technology

Capra 14
Southampton, UK
July 7, 2011

Overview

- Importance of high-accuracy gravitational wave source modeling

- Warm-up to GSF:

  A nonlinear scalar analog model of gravitational EMRIs

- Nonperturbative scalar self force effects

Why bother with higher order? (1)

- The more accurately the waveform can be calculated, the more accurately the parameters can be measured

$$\Phi \sim \frac{1}{\varepsilon} + O(\varepsilon^0)$$

Burko (2003), Detweiler (2005), Rosenthal (2006)

C. Evans' talk

Why bother with higher order? (1)

- The more accurately the waveform can be calculated, the more accurately the parameters can be measured

$$\Phi \sim \frac{1}{\varepsilon} + O(\varepsilon^0)$$

Burko (2003), Detweiler (2005), Rosenthal (2006)

C. Evans' talk

- Two-timescale dynamics: Time-averaged, dissipative part of 2nd order self force is just as important as fluctuations in conservative part of 1st order self force

Mino (2007), Hinderer & Flanagan (2008)

T. Hinderer's talk

Why bother with higher order? (1)

- The more accurately the waveform can be calculated, the more accurately the parameters can be measured

$$\Phi \sim \frac{1}{\varepsilon} + O(\varepsilon^0)$$

Burko (2003), Detweiler (2005), Rosenthal (2006)

C. Evans' talk

- Two-timescale dynamics: Time-averaged, dissipative part of 2nd order self force is just as important as fluctuations in conservative part of 1st order self force

Mino (2007), Hinderer & Flanagan (2008)

T. Hinderer's talk

- Transient resonances will effect parameter estimation and possibly even detectability

$$\Phi \sim \frac{1}{\varepsilon} + \frac{1}{\sqrt{\varepsilon}} + O(\varepsilon^0)$$

Flanagan & Hinderer (2010)

T. Hinderer's talk

- Change of phase by about 20 rad if waveform doesn't track resonance

Why bother with higher order? (2)

- Could describe binaries with intermediate mass ratios and possibly with comparable mass components

  T. Damour's talk

- Comparisons with post-Newtonian calculations

  A. Le Tiec's talk

- More accurate EOB models from higher-order SF data

  T. Damour's talk

- Make definite statements about errors in truncating the perturbation theory at lower orders (i.e., use 2nd order to put error bars on 1st order)

- Other (?)...

Why bother with scalar fields?

- Historically, scalar models offer a simpler framework

  - The most useful regularization scheme -- Detweiler & Whiting (2003) -- first developed and understood in a scalar model

  - Numerical self force computations first accomplished for linear scalar models

  - Conceptually cleaner because not a gauge theory

- As a result, the physics of higher-order self force corrections can be investigated more easily compared to the gravitational case

A nonlinear scalar model

for EMRIs

Galley, 1012.4488 and 1107.0766

A scalar analog of EMRIs [Galley, 1012.4488]

- In the Lorenz gauge, a vacuum background spacetime, and ignoring spin and finite size effects of the small body:

$$S[z^\mu, h_{\mu\nu}] = -\frac{1}{16\pi G} \sum_{n=2}^{\infty} \frac{1}{n!} \int_x a_n(g_{\alpha\beta}) \nabla h \nabla h \, h^{n-2} - m \sum_{n=0}^{\infty} \frac{1}{n!} \int d\tau \, b_n(u^\alpha) \, h^n(z^\alpha)$$

*Verified explicitly through sixth order

A scalar analog of EMRIs [Galley, 1012.4488]

- In the Lorenz gauge, a vacuum background spacetime, and ignoring spin and finite size effects of the small body:

$$S[z^\mu, h_{\mu\nu}] = -\frac{1}{16\pi G}\sum_{n=2}^{\infty}\frac{1}{n!}\int_x a_n(g_{\alpha\beta})\nabla h \nabla h\, h^{n-2} - m\sum_{n=0}^{\infty}\frac{1}{n!}\int d\tau\, b_n(u^\alpha)\, h^n(z^\alpha)$$

*Verified explicitly through sixth order

$$\Downarrow$$

$$\boxed{S[z^\mu, \phi] = -\sum_{n=2}^{\infty}\frac{a_n}{n!}\int_x \nabla_\alpha \phi \nabla^\alpha \phi\, \phi^{n-2} - m\sum_{n=0}^{\infty}\frac{b_n}{n!}\int d\tau\, \phi^n(z^\alpha)}$$

A scalar analog of EMRIs [Galley, 1012.4488]

- In the Lorenz gauge, a vacuum background spacetime, and ignoring spin and finite size effects of the small body:

$$S[z^\mu, h_{\mu\nu}] = -\frac{1}{16\pi G} \sum_{n=2}^{\infty} \frac{1}{n!} \int_x a_n(g_{\alpha\beta}) \nabla h \nabla h \, h^{n-2} - m \sum_{n=0}^{\infty} \frac{1}{n!} \int d\tau \, b_n(u^\alpha) \, h^n(z^\alpha)$$

*Verified explicitly through sixth order

$$\Downarrow$$

$$\boxed{S[z^\mu, \phi] = -\sum_{n=2}^{\infty} \frac{a_n}{n!} \int_x \nabla_\alpha \phi \nabla^\alpha \phi \, \phi^{n-2} - m \sum_{n=0}^{\infty} \frac{b_n}{n!} \int d\tau \, \phi^n(z^\alpha)}$$

- However, one can make a field redefinition that removes all self-interaction terms in the bulk spacetime

$$\nabla_\alpha \psi = \nabla_\alpha \phi \left(1 + \sum_{n=1}^{\infty} \frac{2 a_{n+2}}{(n+2)!} \phi^n \right)^{1/2}$$

$$\boxed{S[z^\mu, \psi] = -\frac{1}{2} \int_x \nabla_\alpha \psi \nabla^\alpha \psi - m \sum_{n=0}^{\infty} \frac{c_n}{n!} \int d\tau \, \psi^n(z^\mu)}$$

Two roads to self force at any order (1)

- First road -- A variational principle of an effective action

Paper 1
Galley, 1012.4488

$$S[z^\mu, \psi]$$

$$S_{\rm eff}[z_1^\mu, z_2^\mu]$$

$$0 = \frac{\delta S_{\rm eff}[z_1^\mu, z_2^\mu]}{\delta z_1^\mu(\tau)}\bigg|_{z_1 = z_2 = z} \Longrightarrow m a^\mu = F_R^\mu(\tau)$$

Two roads to self force at any order (1)

- First road -- A variational principle of an effective action

Paper 1
Galley, 1012.4488

$$S[z^\mu, \psi]$$

$$\downarrow$$

$$S_{\rm eff}[z_1^\mu, z_2^\mu]$$

$$\downarrow$$

$$0 = \frac{\delta S_{\rm eff}[z_1^\mu, z_2^\mu]}{\delta z_1^\mu(\tau)}\bigg|_{z_1=z_2=z} \Longrightarrow ma^\mu = F_R^\mu(\tau)$$

$\longrightarrow$ Self-consistent formalism for open classical systems

[see Galley 1012.4488, Galley & Tiglio (2010), Galley & Hu (2010)]

Two roads to self force at any order (2)

- Second road -- The Detweiler-Whiting (DW) scheme [Detweiler & Whiting (2003)]

$$S[z^\mu, \psi]$$

Paper 1
Galley, 1012.4488

Paper 2
Galley, 1107.0766

$$S_{\rm eff}[z_1^\mu, z_2^\mu]$$

$$0 = \frac{\delta S[z^\mu]}{\delta z^\mu(\tau)} \Longrightarrow m a^\mu = F_R^\mu(\tau)$$

$$\Box \psi(x) = \cdots$$

$$0 = \frac{\delta S_{\rm eff}[z_1^\mu, z_2^\mu]}{\delta z_1^\mu(\tau)}\bigg|_{z_1 = z_2 = z} \Longrightarrow m a^\mu = F_R^\mu(\tau)$$

$$m a^\mu = F_R^\mu(\tau)$$

Two roads to self force at any order (2)

- Second road -- The Detweiler-Whiting (DW) scheme [Detweiler & Whiting (2003)]

$$S[z^\mu, \psi]$$

Paper 1
Galley, 1012.4488

Paper 2
Galley, 1107.0766

$$S_{\rm eff}[z_1^\mu, z_2^\mu]$$

$$0 = \frac{\delta S[z^\mu]}{\delta z^\mu(\tau)} \Longrightarrow m a^\mu = F_R^\mu(\tau)$$

$$\Box \psi(x) = \cdots$$

$$0 = \frac{\delta S_{\rm eff}[z_1^\mu, z_2^\mu]}{\delta z_1^\mu(\tau)}\bigg|_{z_1 = z_2 = z} \Longrightarrow m a^\mu = F_R^\mu(\tau)$$

$$m a^\mu = F_R^\mu(\tau)$$

- But, the DW scheme is implemented <u>only at first</u> order in perturbation theory

Two roads to self force at any order (2)

- Second road -- The Detweiler-Whiting (DW) scheme [Detweiler & Whiting (2003)]

$$S[z^\mu, \psi]$$

Paper 1
Galley, 1012.4488

Paper 2
Galley, 1107.0766

$$S_{\rm eff}[z_1^\mu, z_2^\mu]$$

$$0 = \frac{\delta S[z^\mu]}{\delta z^\mu(\tau)} \Longrightarrow m a^\mu = F_R^\mu(\tau)$$

$$\Box \psi(x) = \cdots$$

$$0 = \frac{\delta S_{\rm eff}[z_1^\mu, z_2^\mu]}{\delta z_1^\mu(\tau)}\bigg|_{z_1=z_2=z} \Longrightarrow m a^\mu = F_R^\mu(\tau) \longleftrightarrow m a^\mu = F_R^\mu(\tau)$$

Both methods give same
regular self force!

- But, the DW scheme is implemented <u>only at first</u> order in perturbation theory

Self force from the Detweiler-Whiting scheme: the Big Picture

- ☐ Calculate the (formally singular) perturbations of the field through a given order
- ☐ Decompose the Green's function (or "propagator") into the DW regular (R) and singular (S) functions
- ☐ Regularize divergent integrals

  Singular integrals of singular integrals of... -- New feature!

- ☐ Cancel divergences in the source of the perturbations by introducing counter terms into the action -- Resulting regular field is the radiative field

  Absent at first order -- New feature!

- ☐ Substitute in resulting radiative field into SF equations of motion

  Since counter terms were added to the action, they will also appear in the SF eom -- New feature!

- ☐ Renormalize the body's mass via a mass counter term

Feynman diagrams

- Field equation of motion

$$\Box\psi = m \int d\tau \, \frac{\delta^4(x-z)}{g^{1/2}} \left(c_1 + c_2 \psi(z^\mu) + \frac{1}{2} c_3 \psi^2(z^\mu) + \cdots \right)$$

Feynman diagrams

- Field equation of motion

$$\Box\psi = m \int d\tau \, \frac{\delta^4(x-z)}{g^{1/2}} \left(c_1 + c_2 \psi(z^\mu) + \frac{1}{2} c_3 \psi^2(z^\mu) + \cdots \right)$$

- Formally singular, perturbative solution

$$\psi = \int d\tau \, D(x, z^\mu)(-mc_1) + \int d\tau \, D(x, z^\mu)(-mc_2) \int d\tau' \, D(z^\mu, z^{\mu'})(-mc_1) + \cdots$$

Feynman diagrams

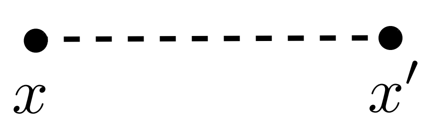
- Field equation of motion

$$\Box\psi = m\int d\tau\, \frac{\delta^4(x-z)}{g^{1/2}}\left(c_1 + c_2\psi(z^\mu) + \frac{1}{2}c_3\psi^2(z^\mu) + \cdots\right)$$

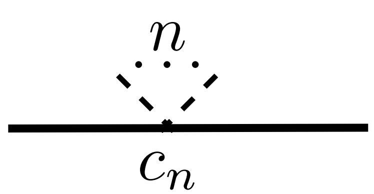
- Formally singular, perturbative solution

$$\psi = \int d\tau\, D(x,z^\mu)(-mc_1) + \int d\tau\, D(x,z^\mu)(-mc_2)\int d\tau'\, D(z^\mu, z^{\mu'})(-mc_1) + \cdots$$

- Feynman rules



$$\equiv D(x,x')$$



$$\equiv \frac{\delta^n S}{\delta\psi(x_1)\cdots\delta\psi(x_n)}\bigg|_{\psi=0} = -mc_n\int d\tau\, \frac{\delta^4(x_1-z)}{g^{1/2}}\cdots\frac{\delta^4(x_n-z)}{g^{1/2}}$$

Feynman diagrams

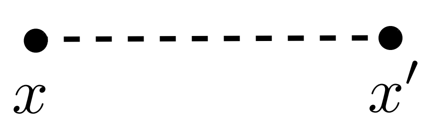
- Field equation of motion

$$\Box\psi = m\int d\tau\,\frac{\delta^4(x-z)}{g^{1/2}}\left(c_1 + c_2\psi(z^\mu) + \frac{1}{2}c_3\psi^2(z^\mu) + \cdots\right)$$

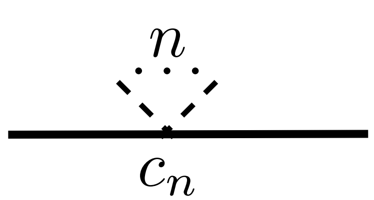
- Formally singular, perturbative solution

$$\psi = \int d\tau\, D(x,z^\mu)(-mc_1) + \int d\tau\, D(x,z^\mu)(-mc_2)\int d\tau'\, D(z^\mu, z^{\mu'})(-mc_1) + \cdots$$

- Feynman rules

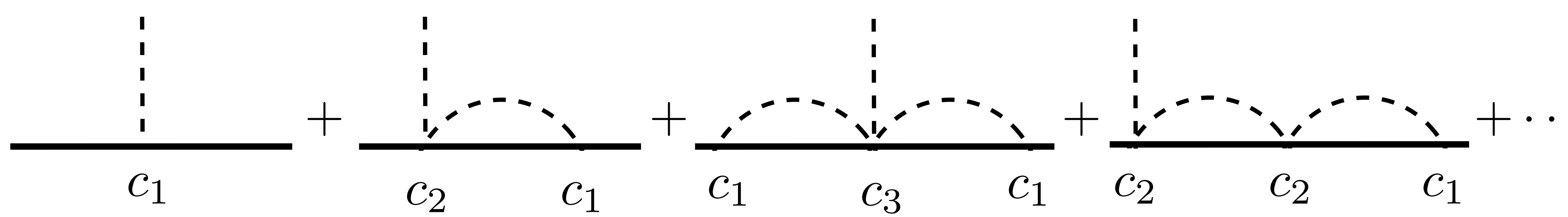


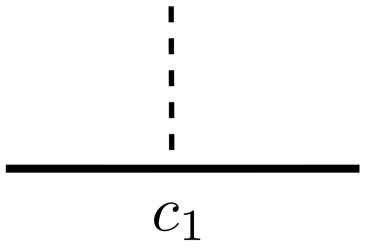
$\equiv D(x,x')$



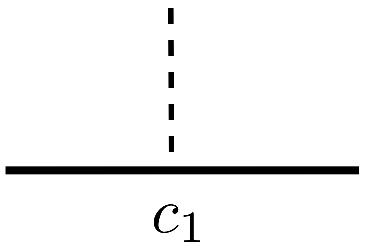
$$\equiv \left.\frac{\delta^n S}{\delta\psi(x_1)\cdots\delta\psi(x_n)}\right|_{\psi=0} = -mc_n\int d\tau\,\frac{\delta^4(x_1-z)}{g^{1/2}}\cdots\frac{\delta^4(x_n-z)}{g^{1/2}}$$

$\psi(x) =$



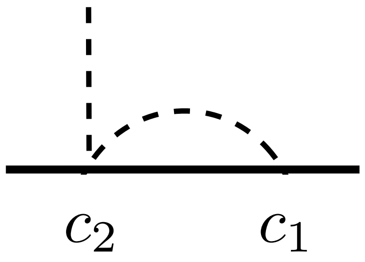
First order perturbations: 

- Trivial

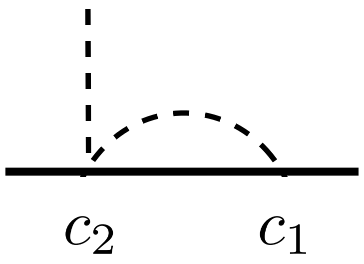


$$= -mc_1 \int d\tau \, D_{\mathrm{ret}}(x, z^\mu)$$

Second order perturbations (I):

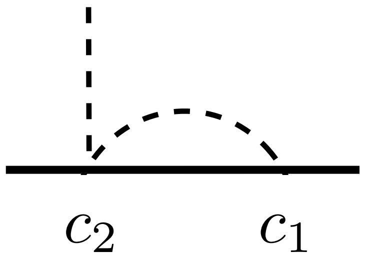


- Feynman rules give

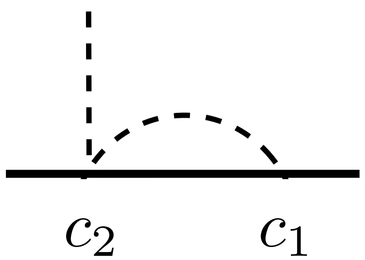


$$= m^2 c_1 c_2 \int d\tau \, D_{\mathrm{ret}}(x, z^\mu) \int d\tau' \, D_{\mathrm{ret}}(z^\mu, z^{\mu'})$$

Second order perturbations (I):



- Feynman rules give



$$= m^2 c_1 c_2 \int d\tau\, D_{\rm ret}(x, z^\mu) \int d\tau'\, D_{\rm ret}(z^\mu, z^{\mu'})$$

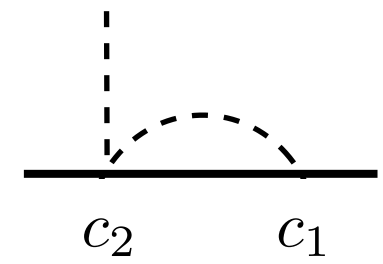
- Regularize the last integral by using the DW decomposition [Detweiler & Whiting (2003)]

$$D_{\rm ret}(x, x') = D_S(x, x') + D_R(x, x')$$

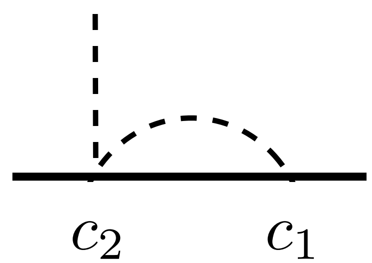
- Define

$$\frac{\Lambda}{4\pi} \equiv \int_{-\infty}^{\infty} d\tau'\, D_S(z^\mu, z^{\mu'}) = \frac{1}{2}\int_{-\infty}^{\infty} ds\, \frac{\delta(s)}{4\pi|s|} \qquad I_R(x) \equiv \int_{-\infty}^{\infty} d\tau'\, D_R(x, z^{\mu'})$$

Second order perturbations (I):



- Feynman rules give



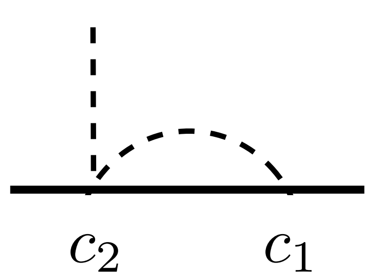
$$= m^2 c_1 c_2 \int d\tau \, D_{\rm ret}(x, z^\mu) \int d\tau' \, D_{\rm ret}(z^\mu, z^{\mu'})$$

- Regularize the last integral by using the DW decomposition [Detweiler & Whiting (2003)]

$$D_{\rm ret}(x, x') = D_S(x, x') + D_R(x, x')$$

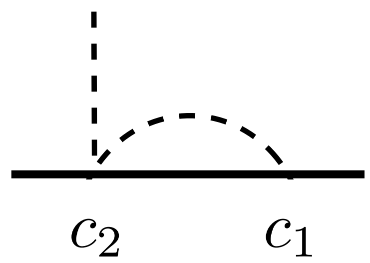
- Define

$$\frac{\Lambda}{4\pi} \equiv \int_{-\infty}^{\infty} d\tau' \, D_S(z^\mu, z^{\mu'}) = \frac{1}{2} \int_{-\infty}^{\infty} ds \, \frac{\delta(s)}{4\pi |s|} \qquad I_R(x) \equiv \int_{-\infty}^{\infty} d\tau' \, D_R(x, z^{\mu'})$$



$$= m^2 c_1 c_2 \left(\frac{\Lambda}{4\pi} \right) \int d\tau \, D_{\rm ret}(x, z^\mu) + m^2 c_1 c_2 \int d\tau \, D_{\rm ret}(x, z^\mu) I_R(z^\mu)$$

Second order perturbations (2):



$$= m^2 c_1 c_2 \left(\frac{\Lambda}{4\pi}\right) \int d\tau\, D_{\rm ret}(x, z^\mu) + m^2 c_1 c_2 \int d\tau\, D_{\rm ret}(x, z^\mu) I_R(z^\mu)$$

- The divergent first term is proportional to the first order perturbation and can be cancelled by adding a counter term to the action

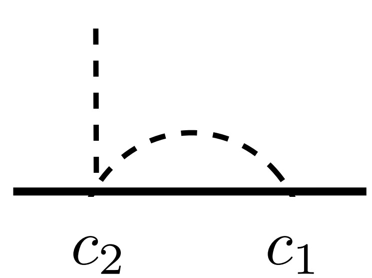
$$-\delta_1 \int d\tau\, \psi(z^\mu)$$

which sources the field equation but also the self force

$$\Box\psi = \cdots - \delta_1 \int d\tau\, \frac{\delta^4(x-z)}{g^{1/2}}$$

- Removing the singular field implies

$$\delta_1 = m^2 c_1 c_2 \left(\frac{\Lambda}{4\pi}\right)$$



$$\Big|_R = m^2 c_1 c_2 \int d\tau\, D_{\rm ret}(x, z^\mu) I_R(z^\mu)$$

Comparison with Rosenthal's expression (1)

E. Rosenthal CQG (2006)

$$\Box\phi = \phi_{,\alpha}\phi^{,\alpha} - q\int d\tau\,\frac{\delta^4(x-z)}{g^{1/2}}$$

- Rosenthal developed a somewhat complicated procedure to derive the regular 2nd order perturbations

  - Investigate behavior of wave equation when $q \to 0$

  - Make ansatz for particular solution

  - Use physical considerations to identify divergent b.c.'s for field as field point approaches worldline

  - Solve the wave equation with those divergent b.c.'s

$$\phi(x) = q\int d\tau\, D_{\rm ret}(x,z^\mu) + \frac{q^2}{2}\left[\int d\tau\, D_{\rm ret}(x,z^\mu)\right]^2 - q^2\int d\tau\, D_{\rm ret}(x,z^\mu) I_R(z^\mu) + \cdots$$

Comparison with Rosenthal's expression (2)

$$\Box\phi = \phi_{,\alpha}\phi^{,\alpha} - q\int d\tau\, \frac{\delta^4(x-z)}{g^{1/2}}$$

- Rosenthal's model is a member of our class of nonlinear theories:

$$b_1 = -\frac{q}{m},\;\; b_2 = +2\frac{q}{m},\;\; a_3 = -6$$

- Psi field:

$$\psi_{\rm rad}(x) = q\int d\tau\, D_{\rm ret}(x, z^\mu) - q^2\int d\tau\, D_{\rm ret}(x, z^\mu) I_R(z^\mu) + \cdots$$

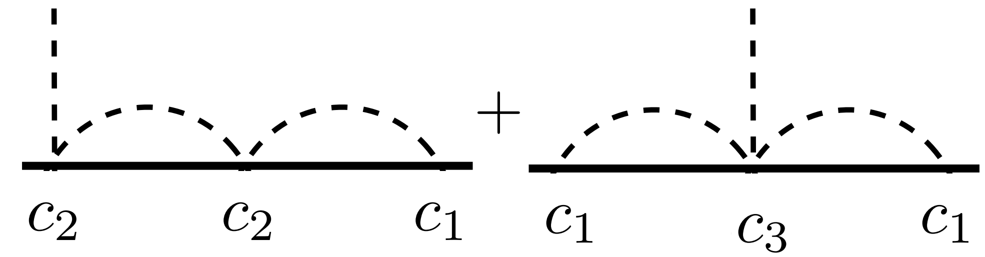
- Using the inverse of the field redefinition

$$\phi(x) = \psi(x) + \frac{1}{2}\psi^2(x) + \cdots$$

gives agreement with Rosenthal

$$\boxed{\phi(x) = q\int d\tau\, D_{\rm ret}(x, z^\mu) + \frac{q^2}{2}\left[\int d\tau\, D_{\rm ret}(x, z^\mu)\right]^2 - q^2\int d\tau\, D_{\rm ret}(x, z^\mu) I_R(z^\mu) + \cdots}$$

Third order perturbations (1):



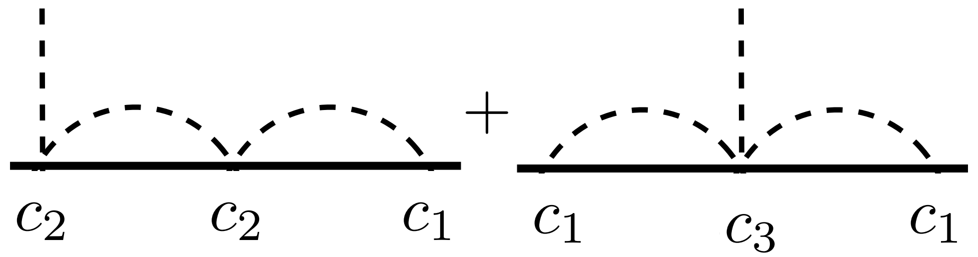
- Feynman rules give

$$= -m^3 c_1 c_2^2 \int d\tau'\, D_{\rm ret}(x, z^{\mu'}) \int d\tau'' d\tau'''\, D_{\rm ret}(z^{\mu'}, z^{\mu''}) D_{\rm ret}(z^{\mu''}, z^{\mu'''})$$

$$-\frac{1}{2} m^3 c_1^2 c_3 \int d\tau'\, D_{\rm ret}(x, z^{\mu'}) \left[\int d\tau''\, D_{\rm ret}(z^{\mu'}, z^{\mu''}) \right]^2$$

- First line contains a singular integral of a singular integral in DW decomposition

Third order perturbations (I):



- Feynman rules give

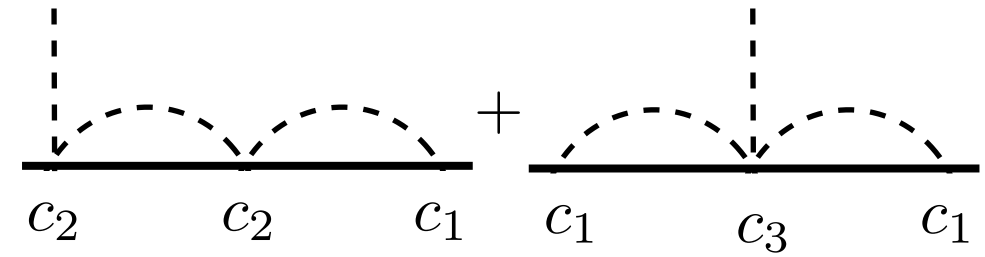
$$= -m^3 c_1 c_2^2 \int d\tau' \, D_{\rm ret}(x, z^{\mu'}) \int d\tau'' d\tau''' \, D_{\rm ret}(z^{\mu'}, z^{\mu''}) D_{\rm ret}(z^{\mu''}, z^{\mu'''})$$

$$-\frac{1}{2} m^3 c_1^2 c_3 \int d\tau' \, D_{\rm ret}(x, z^{\mu'}) \left[\int d\tau'' \, D_{\rm ret}(z^{\mu'}, z^{\mu''}) \right]^2$$

- First line contains a singular integral of a singular integral in DW decomposition

$$\int d\tau'' \, D_R(z^{\mu'}, z^{\mu''}) \int d\tau''' \, D_R(z^{\mu''}, z^{\mu'''}) = \int d\tau'' \, D_R(z^{\mu'}, z^{\mu''}) I_R(z^{\mu''})$$

Third order perturbations (I):



- Feynman rules give

$$= -m^3 c_1 c_2^2 \int d\tau' \, D_{\rm ret}(x, z^{\mu'}) \int d\tau'' d\tau''' \, D_{\rm ret}(z^{\mu'}, z^{\mu''}) D_{\rm ret}(z^{\mu''}, z^{\mu'''})$$

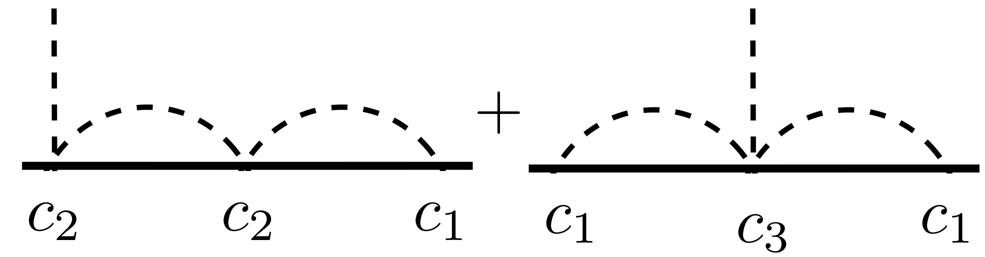
$$-\frac{1}{2} m^3 c_1^2 c_3 \int d\tau' \, D_{\rm ret}(x, z^{\mu'}) \left[\int d\tau'' \, D_{\rm ret}(z^{\mu'}, z^{\mu''}) \right]^2$$

- First line contains a singular integral of a singular integral in DW decomposition

$$\int d\tau'' \, D_R(z^{\mu'}, z^{\mu''}) \int d\tau''' \, D_R(z^{\mu''}, z^{\mu'''}) = \int d\tau'' \, D_R(z^{\mu'}, z^{\mu''}) I_R(z^{\mu''})$$

$$\int d\tau'' \, D_R(z^{\mu'}, z^{\mu''}) \int d\tau''' \, D_S(z^{\mu''}, z^{\mu'''}) = \frac{\Lambda}{4\pi} \int d\tau'' \, D_R(z^{\mu'}, z^{\mu''})$$

Third order perturbations (I):



- Feynman rules give

$$= -m^3 c_1 c_2^2 \int d\tau' \, D_{\text{ret}}(x, z^{\mu'}) \int d\tau'' d\tau''' \, D_{\text{ret}}(z^{\mu'}, z^{\mu''}) D_{\text{ret}}(z^{\mu''}, z^{\mu'''})$$

$$-\frac{1}{2} m^3 c_1^2 c_3 \int d\tau' \, D_{\text{ret}}(x, z^{\mu'}) \left[\int d\tau'' \, D_{\text{ret}}(z^{\mu'}, z^{\mu''}) \right]^2$$

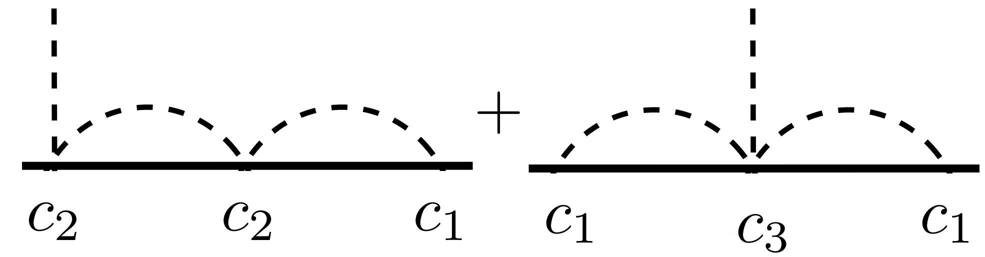
- First line contains a singular integral of a singular integral in DW decomposition

$$\int d\tau'' \, D_R(z^{\mu'}, z^{\mu''}) \int d\tau''' \, D_R(z^{\mu''}, z^{\mu'''}) = \int d\tau'' \, D_R(z^{\mu'}, z^{\mu''}) I_R(z^{\mu''})$$

$$\int d\tau'' \, D_R(z^{\mu'}, z^{\mu''}) \int d\tau''' \, D_S(z^{\mu''}, z^{\mu'''}) = \frac{\Lambda}{4\pi} \int d\tau'' \, D_R(z^{\mu'}, z^{\mu''})$$

$$\int d\tau'' \, D_S(z^{\mu'}, z^{\mu''}) \int d\tau''' \, D_R(z^{\mu''}, z^{\mu'''}) = \frac{1}{8\pi} \int_{-\infty}^{\infty} ds \frac{\delta(s)}{|s|} I_R(\tau'' + s) = \frac{\Lambda}{4\pi} I_R(z^{\mu''})$$

Third order perturbations (I):



- Feynman rules give

$$= -m^3 c_1 c_2^2 \int d\tau' \, D_{\rm ret}(x, z^{\mu'}) \int d\tau'' d\tau''' \, D_{\rm ret}(z^{\mu'}, z^{\mu''}) D_{\rm ret}(z^{\mu''}, z^{\mu'''})$$

$$-\frac{1}{2} m^3 c_1^2 c_3 \int d\tau' \, D_{\rm ret}(x, z^{\mu'}) \left[\int d\tau'' \, D_{\rm ret}(z^{\mu'}, z^{\mu''}) \right]^2$$

- First line contains a singular integral of a singular integral in DW decomposition

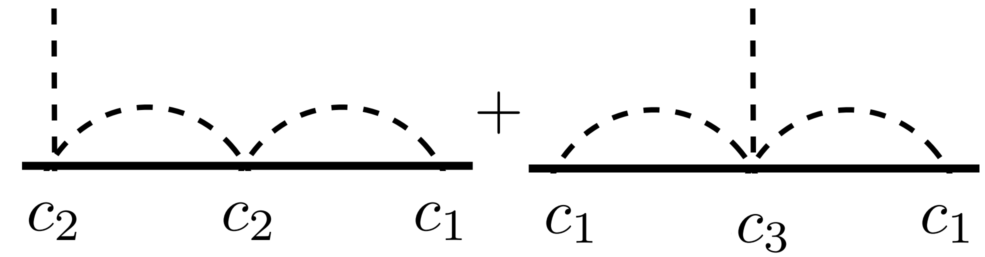
$$\int d\tau'' \, D_R(z^{\mu'}, z^{\mu''}) \int d\tau''' \, D_R(z^{\mu''}, z^{\mu'''}) = \int d\tau'' \, D_R(z^{\mu'}, z^{\mu''}) I_R(z^{\mu''})$$

$$\int d\tau'' \, D_R(z^{\mu'}, z^{\mu''}) \int d\tau''' \, D_S(z^{\mu''}, z^{\mu'''}) = \frac{\Lambda}{4\pi} \int d\tau'' \, D_R(z^{\mu'}, z^{\mu''})$$

$$\int d\tau'' \, D_S(z^{\mu'}, z^{\mu''}) \int d\tau''' \, D_R(z^{\mu''}, z^{\mu'''}) = \frac{1}{8\pi} \int_{-\infty}^{\infty} ds \frac{\delta(s)}{|s|} I_R(\tau'' + s) = \frac{\Lambda}{4\pi} I_R(z^{\mu''})$$

$$\int d\tau'' \, D_S(z^{\mu'}, z^{\mu''}) \int d\tau''' \, D_S(z^{\mu''}, z^{\mu'''}) = \int d\tau'' \, D_S(z^{\mu'}, z^{\mu''}) \frac{\Lambda}{4\pi} = \left(\frac{\Lambda}{4\pi} \right)^2$$

Third order perturbations (2):



- Putting together gives a divergent contribution

$$\text{divergent piece} = -m^3 c_1 \left(c_2^2 + \frac{1}{2} c_1 c_3 \right) \int d\tau' \, D_{\rm ret}(x, z^{\mu'}) \left[\left(\frac{\Lambda}{4\pi} \right)^2 + 2 \left(\frac{\Lambda}{4\pi} \right) I_R(z^{\mu'}) \right]$$

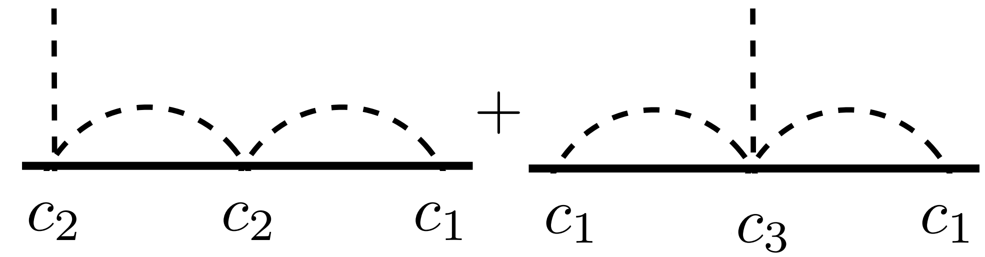
and a regular piece

$$\text{regular piece} = -m^3 c_1 c_2^2 \int d\tau' \, D_{\rm ret}(x, z^{\mu'}) \int d\tau'' \, D_R(z^{\mu'}, z^{\mu''}) I_R(z^{\mu''})$$
$$-\frac{1}{2} m^3 c_1^2 c_3 \int d\tau' \, D_{\rm ret}(x, z^{\mu'}) I_R^2(z^{\mu'})$$

- Introducing another counter term into the action allows for the divergent piece to be subtracted

$$-\frac{1}{2} \delta_2 \int d\tau \, \psi^2(z^\mu)$$

Third order perturbations (2):



- Putting together gives a divergent contribution

$$\text{divergent piece} \quad = -m^3 c_1 \left(c_2^2 + \frac{1}{2} c_1 c_3 \right) \int d\tau' \, D_{\text{ret}}(x, z^{\mu'}) \left[\left(\frac{\Lambda}{4\pi} \right)^2 + 2 \left(\frac{\Lambda}{4\pi} \right) I_R(z^{\mu'}) \right]$$

and a regular piece

$$\boxed{\begin{aligned} \text{regular piece} \quad &= -m^3 c_1 c_2^2 \int d\tau' \, D_{\text{ret}}(x, z^{\mu'}) \int d\tau'' \, D_R(z^{\mu'}, z^{\mu''}) I_R(z^{\mu''}) \\ &\quad - \frac{1}{2} m^3 c_1^2 c_3 \int d\tau' \, D_{\text{ret}}(x, z^{\mu'}) I_R^2(z^{\mu'}) \end{aligned}}$$

- Introducing another counter term into the action allows for the divergent piece to be subtracted

$$-\frac{1}{2} \delta_2 \int d\tau \, \psi^2(z^\mu)$$

Scalar perturbations through 3rd order

- Combining all the contributions to the scalar perturbations then gives the radiative field

$$\psi_{\rm rad}(x) = \int d\tau' \, D_{\rm ret}(x, z^{\mu'}) \Big\{ - mc_1 + m^2 c_1 c_2 I_R(z^{\mu'}) - \frac{m^3 c_1^2 c_3}{2} I_R^2(z^{\mu'}) \\ -m^3 c_1 c_2^2 \int d\tau'' \, D_R(z^{\mu'}, z^{\mu''}) I_R(z^{\mu''}) + O(\varepsilon^4) \Big\}$$

Self force through 3rd order (I)

- The counter terms change the equations of motion

$$\Box\psi(x) = \int d\tau \frac{\delta^4(x-z)}{g^{1/2}}\left\{(mc_1+\delta_1)+(mc_2+\delta_2)\psi+\frac{1}{2}mc_3\psi^2+\cdots\right\}$$

$$F^\mu(\tau) = -(a^\mu + P^{\mu\nu}\nabla_\nu)\left\{(mc_1+\delta_1)\psi(z^\mu)+\frac{1}{2}(mc_2+\delta_2)\psi^2(z^\mu)+\frac{1}{6}mc_3\psi^3(z^\mu)+\cdots\right\}$$

- The radiative field solves the wave equation (obviously)

$$\psi(x) = \psi_{\rm rad}(x) = \int d\tau'\, D_{\rm ret}(x, z^{\mu'})\mathcal{S}_R(z^{\mu'})$$

- Evaluated on the worldline the radiative field has a singular piece and a regular piece

$$\psi_{\rm rad}(z^\mu) = \frac{\Lambda}{4\pi}\mathcal{S}_R(z^\mu) + \int d\tau'\, D_R(z^\mu, z^{\mu'})\mathcal{S}_R(z^\mu)$$

and defines the regular part of the field on the worldline

$$\psi_R(z^\mu) \equiv \int d\tau'\, D_R(z^\mu, z^{\mu'})\mathcal{S}_R(z^{\mu'})$$

Self force through 3rd order (2)

- Substituting the radiative field into the SF yields a regular piece and singular pieces that are proportional only to the 4-acceleration, which can be removed by adding a mass counter term into the action

$$-\delta_m \int d\tau$$

- The 3rd order SF therefore only involves the regular part of the field and we observe that the final expression can be condensed to

$$F_R^\mu(\tau) = -m(a^\mu + P^{\mu\nu}\nabla_\nu)\left\{c_1 + c_2\psi_R(z^\mu) + \frac{1}{2}c_3\psi_R^2(z^\mu)\right\} + O(\varepsilon^4)$$

- In other words, one can replace the formally divergent retarded field in the SF with the regular part of the field

Detweiler-Whiting scheme seems applicable
at higher orders

Lesson 1 for higher-order GSF:

- Renormalization via counter terms is an efficient and consistent way of removing divergences

Lesson 1 for higher-order GSF:

- ☐ Renormalization via counter terms is an efficient and consistent way of removing divergences

Lesson 2 for higher-order GSF:

- ☐ Dimensional regularization is an efficient way to evaluate power-divergent integrals

  (The divergent integral Λ evaluates to zero in dimensional regularization)

Lesson 1 for higher-order GSF:

- ☐ Renormalization via counter terms is an efficient and consistent way of removing divergences

Lesson 2 for higher-order GSF:

- ☐ Dimensional regularization is an efficient way to evaluate power-divergent integrals

(The divergent integral Λ evaluates to zero in dimensional regularization)

Lesson 3 for higher-order GSF (?):

- ☐ Detweiler-Whiting scheme should also be applicable for GSF

Non-perturbative scalar

self force effects

(in preparation)

Assumptions

- $$S[z^\mu,\psi] = -\frac{1}{2}\int_x \psi_{,\alpha}\psi^{,\alpha} - m\int d\tau\left(1 + c_1\psi(z) + \frac{1}{2}c_2\psi^2(z)\right)$$
- Ignore dissipative effects, finite size effects, and spin

Consequence (1) $\Box\psi(x) = m\int d\tau\,(c_1 + c_2\psi(z))$

- Formally singular solution

$$\psi(x) = -mc_1\int d\tau\, D_{\rm ret}(x,z^\mu) - mc_2\int d\tau\, D_{\rm ret}(x,z^\mu)\psi(z^\mu)$$

- Evaluating the field on the worldline and evaluating the singular integrals in dimensional regularization (which then vanish)

$$\psi_R(z^\mu) = -mc_1 I_R(z^\mu) - mc_2\int d\tau'\, D_R(z^\mu, z^{\mu'})\psi_R(z^{\mu'})$$

- For circular geodesics in Schwarzschild, the regular part of the field is constant in proper time [Diaz-Rivera + (2004)]

$$\boxed{\psi_R(r_o) = -\frac{mc_1 I_R(r_o)}{1 + mc_2 I_R(r_o)}}$$

Nonperturbative orbital quantities (I)

- Using the regular part of the field, it is straightforward to use the components of the worldline equations of motion for circular orbits to derive non-perturbative expressions for:

Effective potential

$$V(r_o) = f(r_o)\frac{1 + r_o\partial_r \ln C(r_o)}{1 - r_o\partial_r \ln f^{1/2}(r_o)}$$

Orbital frequency

$$\Omega^2(r_o) = \frac{f(r_o)}{r_o}\frac{\partial_r(f^{1/2}(r_o)C(r_o))}{1 + r_o\partial_r \ln C(r_o)}$$

u^t

$$u^t = f^{-1/2}(r_o)\sqrt{\frac{1 + r_o\partial_r C(r_o)}{1 - r_o\partial_r \ln(f^{1/2}(r_o)C(r_o))}}$$

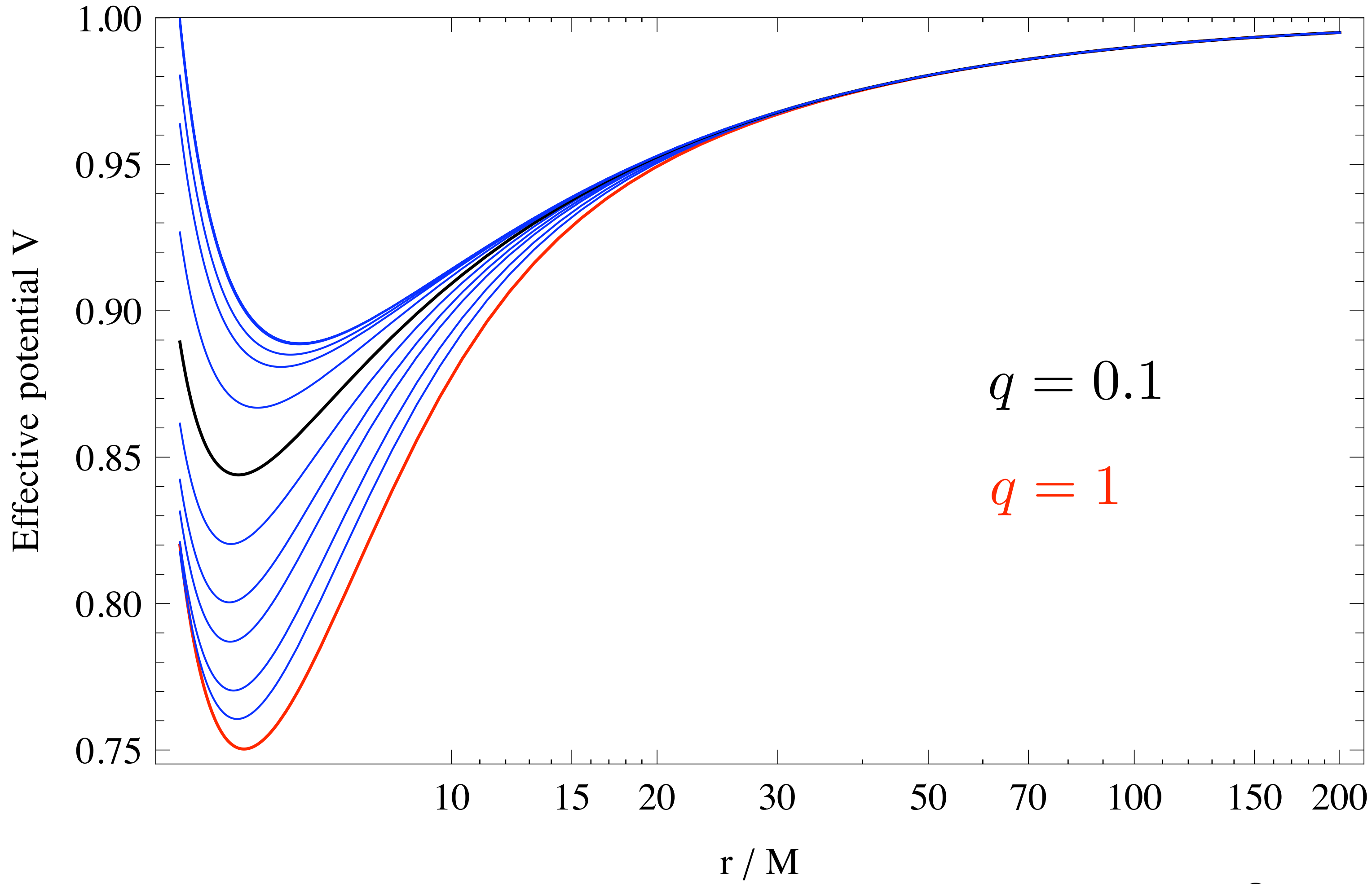
etc.

$$C(z^\mu) \equiv 1 + c_1\psi_R(z^\mu) + \frac{1}{2}c_2\psi_R^2(z^\mu)$$

Nonperturbative orbital quantities (2)

- The values of (4 pi M $I_R(r_o)$), which are independent of the mass ratio, were calculated already in Diaz-Rivera + (2004) so one can simply use those values to plot how these quantities vary over the whole range of mass ratios

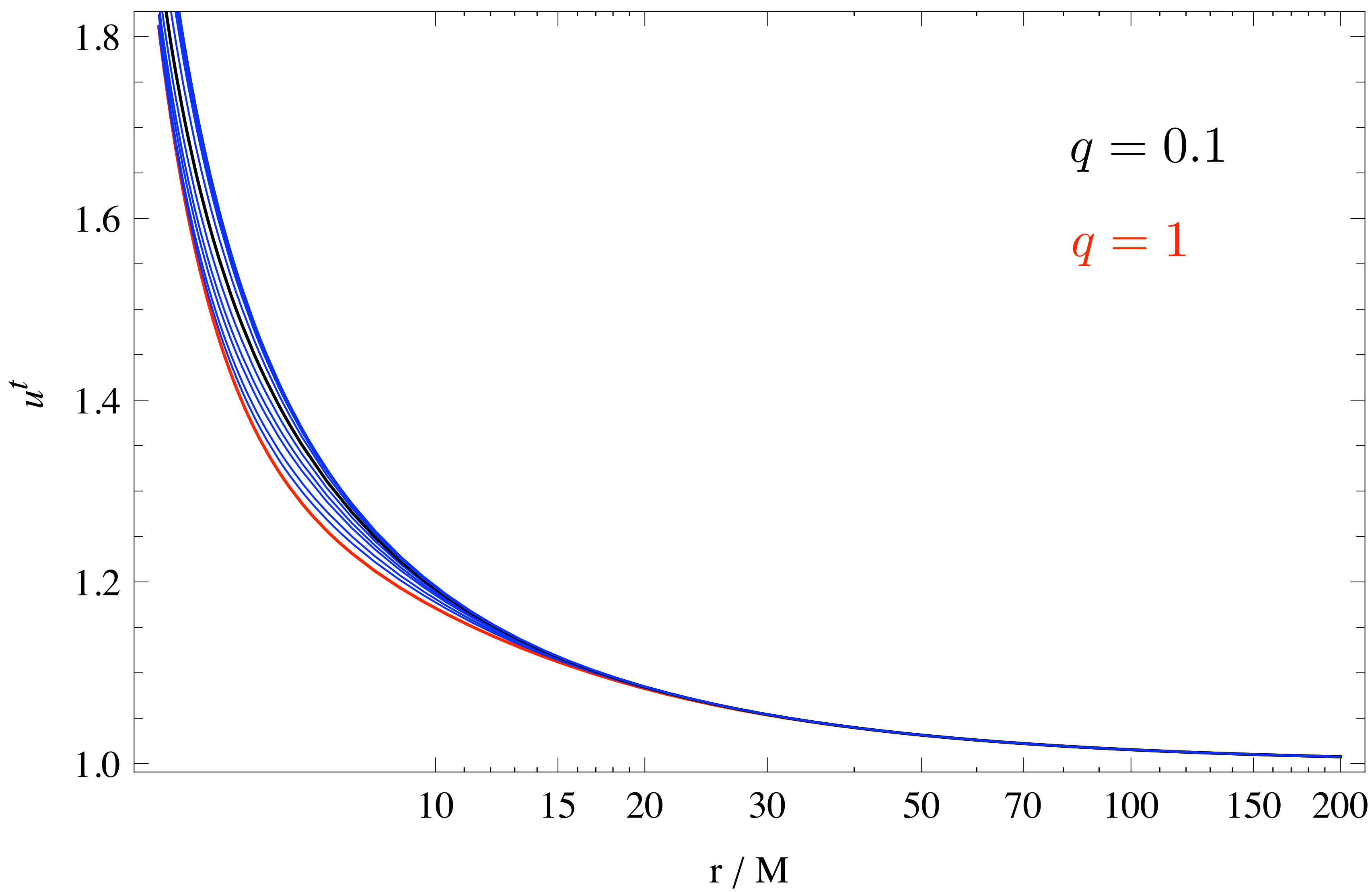
  e.g., Effective potential



$c_1 = 2, \; c_2 = 0.1$

Nonperturbative orbital quantities (3)

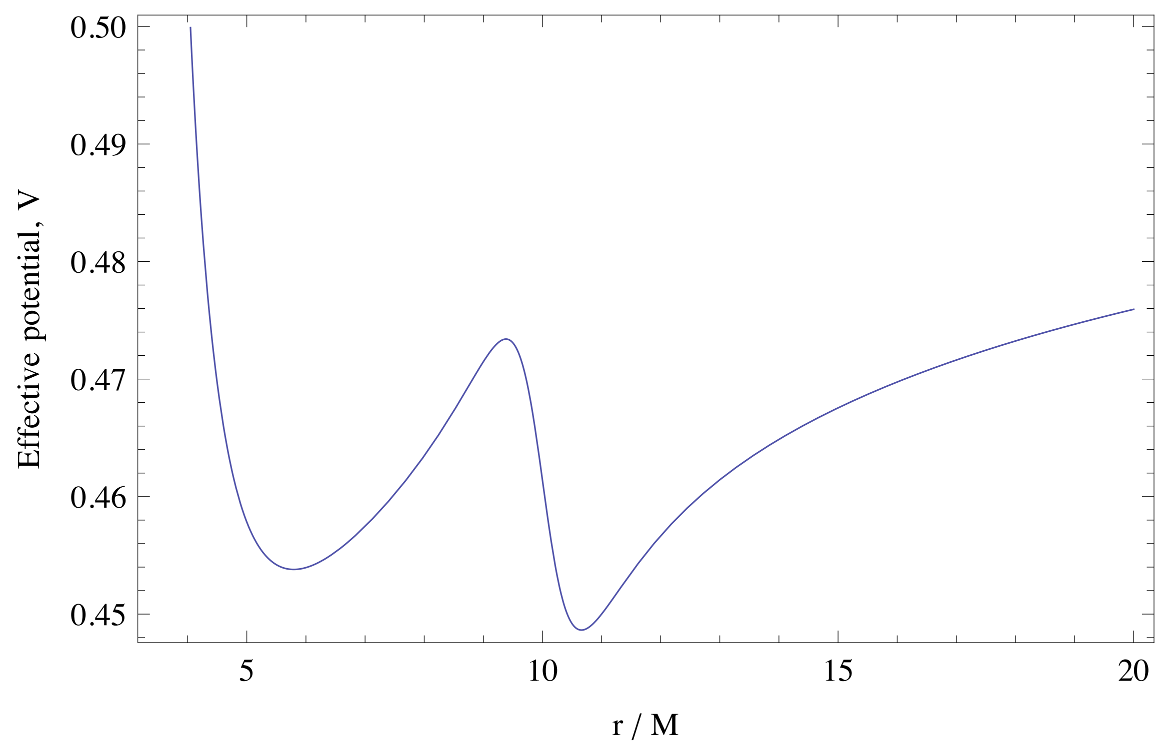
e.g., u^t "gauge invariant"

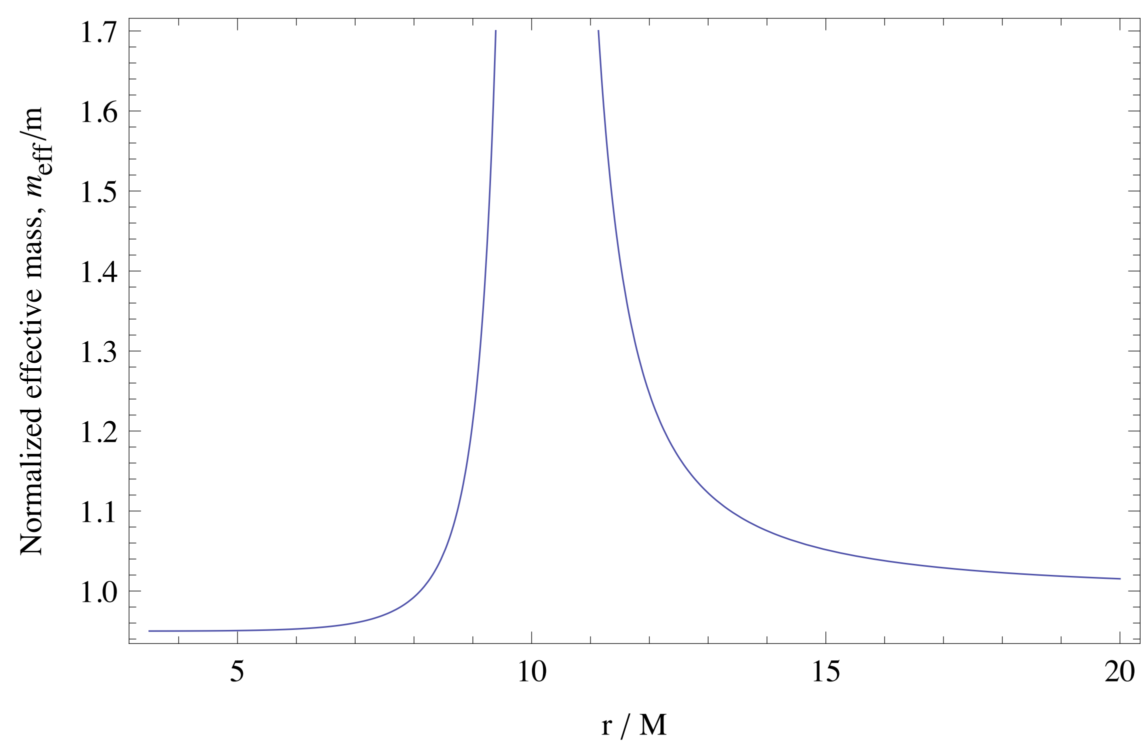


$c_1 = 2, \; c_2 = 0.1$

Rich structures

- If the c_2 parameter is much larger than c_1 than one finds rich structure in the non-perturbative orbital quantities





$$q = 1$$

$$c_1 = 1, \ c_2 = 10$$

An ambiguity

- Worldline equations of motion

$$ma^\mu = -m(a^\mu + P^{\mu\nu}\nabla_\nu)C(z)$$

- Collect 4-acceleration to one side

$$mC(z)a^\mu = -mP^{\mu\nu}\nabla_\nu C(z)$$

An ambiguity

- Worldline equations of motion

$$ma^\mu = -m(a^\mu + P^{\mu\nu}\nabla_\nu)C(z)$$

- Collect 4-acceleration to one side

$$mC(z)a^\mu = -mP^{\mu\nu}\nabla_\nu C(z)$$

- Two equivalent interpretations:

1) Particle carries an effective mass

$$m_{\rm eff} = mC(z)$$

2) Inertial mass but an effective self force

$$F^\mu_{\rm eff}(\tau) = \frac{F^\mu_R(\tau)}{C(z)} = -mP^{\mu\nu}\nabla_\nu \ln C(z)$$

An improved perturbation theory (1)

- In general, one cannot resum the field to get exact, non-perturbative expressions. For example, at first order the effective action approach gives

$$ma^\mu = -mc_1(\underline{a^\mu} + P^{\mu\nu}\nabla_\nu)\psi_R(z)$$

- Typically, the 1st term on the right is ignored as being a higher order correction. But it can be regarded as part of the "energy" at 1st order since it came from the variation of a first order effective action.

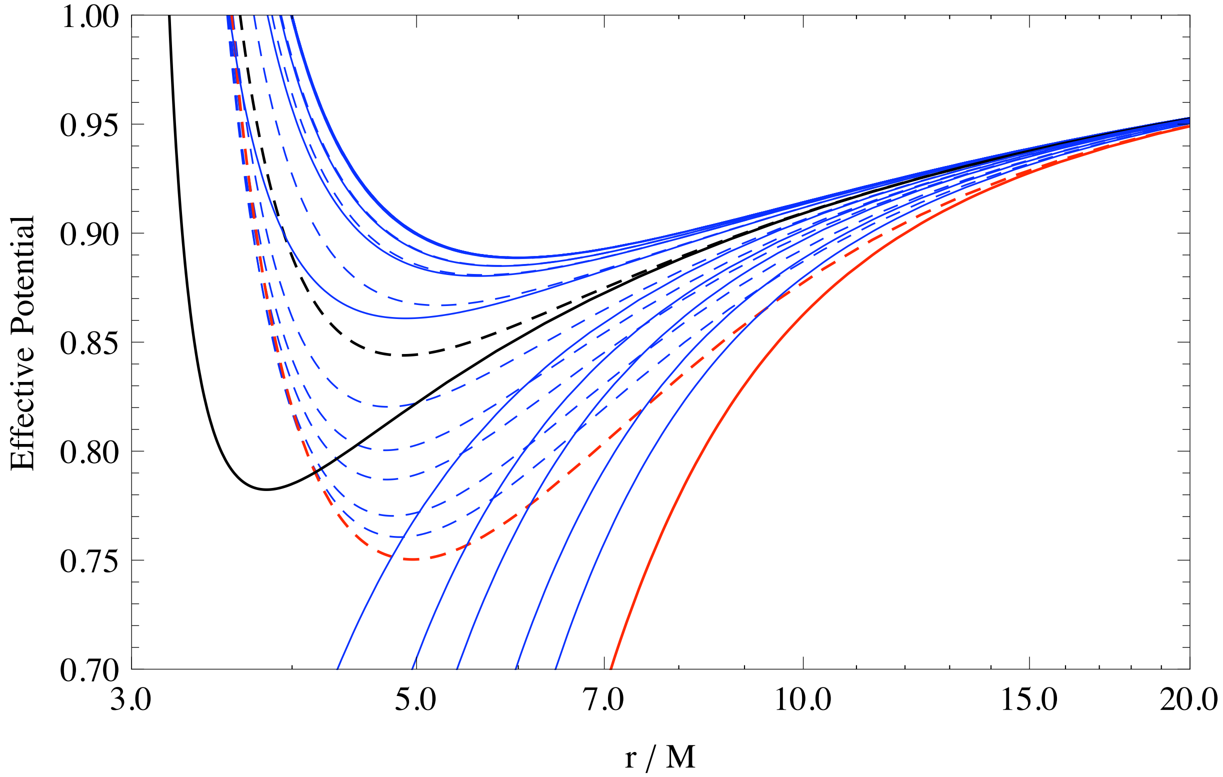
  Keeping that term implies an effective SF given by

$$ma^\mu = -\frac{mc_1 P^{\mu\nu}\nabla_\nu\psi_R(z)}{1 + c_1\psi_R(z)}$$

- What's the implication for the 1st order potential, say?

An improved perturbation theory (2)

1st order

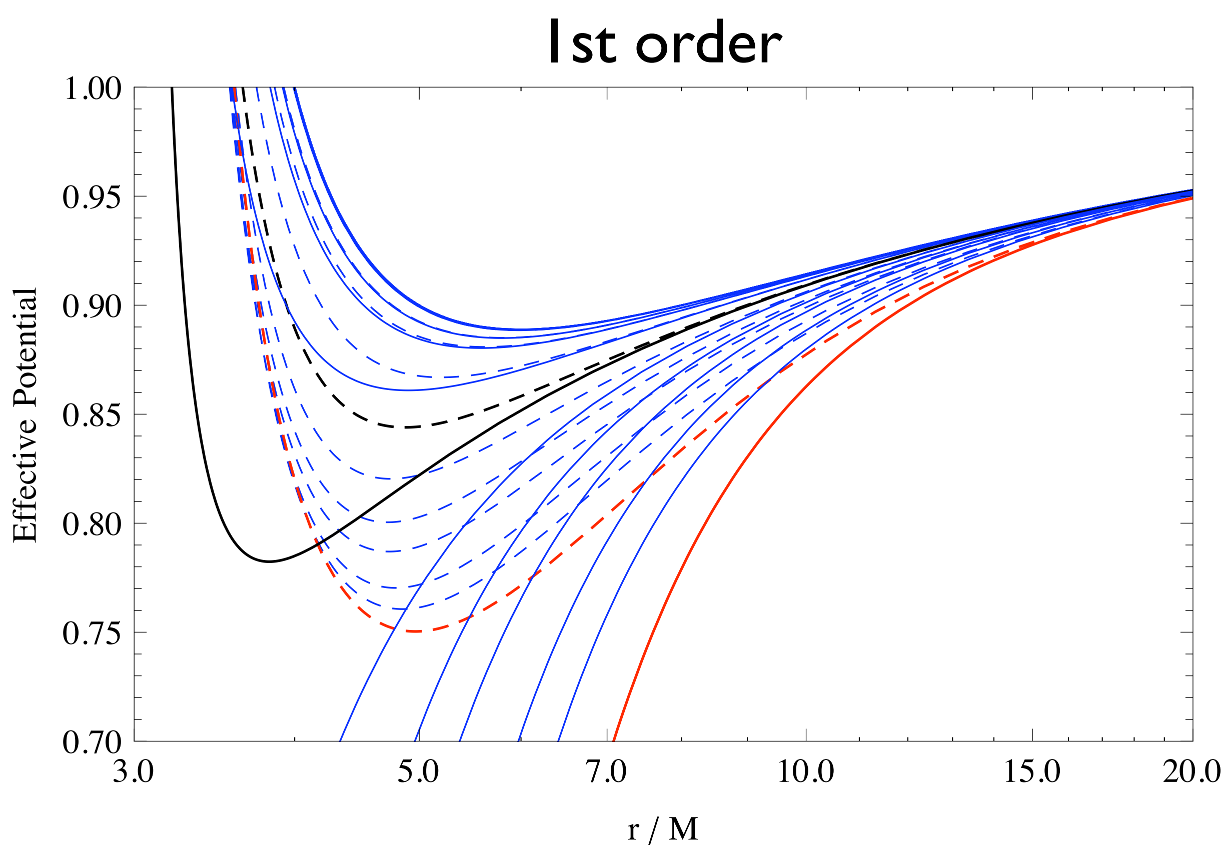


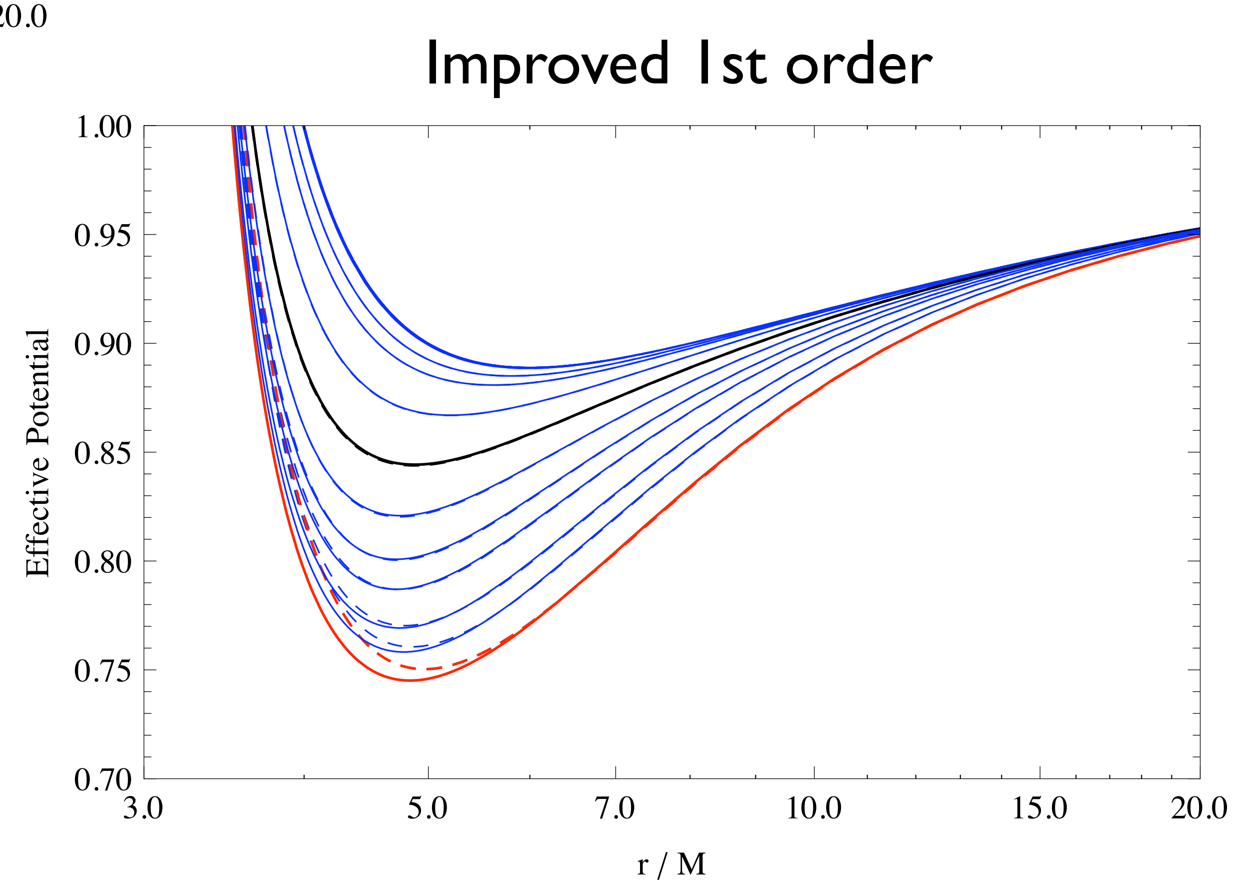
$q = 0.1$

$q = 1$

$c_1 = 2,\ c_2 = 0.1$

An improved perturbation theory (2)





$q = 0.1$

$q = 1$

$c_1 = 2, \; c_2 = 0.1$

Lesson 4 for higher-order GSF?:

- First order GSF from an action also gives rise to a naively 2nd order contribution proportional to

$$-16\pi G m\left(\frac{1}{2}a^{\mu}u^{\alpha}u^{\beta}+P^{\mu(\alpha}a^{\beta)}\right)h^{R}_{\alpha}(z)$$

It would be worthwhile to see if first order GSF predictions for ISCO shifts can be improved by using the corresponding effective GSF

On a circular geodesic

$$-16\pi m a^{r}\left(\frac{1}{2}u^{\alpha}u^{\beta}h^{R}_{\alpha\beta}(r_o)+g^{rr}(r_o)h^{R}_{rr}(r_o)\right)$$

- The corresponding 1st order effective GSF (radial component) would then be

$$F^{r}_{\rm eff}(r_o)=\frac{16\pi m P^{\mu\alpha\beta\nu}\nabla_{\nu}h^{R}_{\alpha\beta}(r_o)}{1+8\pi(h^{R}_{uu}(r_o)+g^{rr}(r_o)h^{R}_{rr}(r_o))}+O(\varepsilon^{2})$$

How might this compare with NR LSO's for comparable mass binaries?

Summary

- We constructed a class of nonlinear scalar models analogous to the perturbative description of EMRIs in GR

- Calculated the scalar perturbations and SF through 3rd order

- Explicitly showed that DW scheme is valid at higher orders

- A subclass of these models can be resummed exactly to yield non-perturbative expressions in the mass ratio

- Showed how various orbital quantities vary with full mass ratio range

- Perturbative SF can be improved by retaining the "effective mass" piece