Is radiation reaction a quantum effect?

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In what sense is radiation reaction a quantum effect?

0. Every ‘classical phenomenon’ is an approximation to a quantum phenomenon.
1. There are indisputably-quantum forces which are of the same order in $\hbar$ as the radiation-reaction force.
2. (We argue that) the conservative part of the radiation-reaction force comes from a one-loop diagram. (The dissipative part comes from a tree diagram.)

∞. Is there an important quantum contribution to the radiation-reaction force in astrophysics that cannot be derived classically? Unlikely but not impossible?

Outline

Quantum effect at the same order in $\hbar$ as radiation reaction

ALD and anomalous-magnetic moment forces from QED?

Dissipative part of the position shift

The conservative part of the position shift

Summary

Anomalous magnetic moment and radiation reaction

The electron energy in magnetic field **H** with the spin aligned with **H**:

$$V = \frac{e\hbar}{2m_e c} gH,$$

where the *gyromagnetic ratio* is

$$g = 2\left(1 + \frac{\alpha}{2\pi} + \cdots\right).$$

$\alpha \approx 1/137$. But

$$\alpha = \frac{e^2}{\hbar c} \textit{ in cgs units}.$$

Hence the contribution of the anomalous magnetic moment to the potential energy is

$$V_{\text{am}} = \frac{e^3}{m_e c^2} H.$$

Anomalous magnetic moment and radiation reaction

If H varies in space, the force on the electron due to the anomalous magnetic moment is

$$\mathbf{F}_{\mathrm{am}} = -\frac{e^3}{m_e c^2}\nabla H.$$

The Abraham-Lorentz-Dirac force?

The acceleration of an electron in magnetic field $\mathbf{H}$:

$$\mathbf{a} = -\frac{e}{m_e c}\mathbf{v}\times\mathbf{H}.$$

Assuming that $\mathbf{H}$ does not vary rapidly in space, we have

$$\mathbf{F}_{\mathrm{ALD}} = \frac{2e^2}{3c^4}\dot{\mathbf{a}} = -\frac{2e^3}{3m_e c^4}\mathbf{a}\times\mathbf{H}.$$

Anomalous magnetic moment and radiation reaction

$$\mathbf{F}_{\text{am}} = -\frac{e^3}{m_e c^2}\nabla H, \quad \mathbf{F}_{\text{ALD}} = -\frac{2e^3}{3m_e c^4}\mathbf{a}\times\mathbf{H}.$$

If $|a| \sim v^2/r$ (circular motion of radius r) and if $|\nabla H|/H \sim \epsilon/r$, then

$$\frac{|\mathbf{F}_{\text{ALD}}|}{|\mathbf{F}_{\text{am}}|} \sim \frac{1}{\epsilon}\left(\frac{v}{c}\right)^2.$$

For non-relativistic motion

$$|\mathbf{F}_{\text{ALD}}| \ll |\mathbf{F}_{\text{am}}| \text{ if } (v/c)^2 \ll \epsilon.$$

The loop expansion is the $\hbar$-expansion only if $mc/\hbar$ is regarded as $O(\hbar^0)$. [Itzykson and Zuber, page 288]

Outline

ALD and anomalous-magnetic moment forces from QED?

Consider an electron moving in an external electromagnetic field.

Question: Do we recover the radiation-reaction force and the 'anomalous-magnetic-moment' force at order e^2 in the $\hbar \to 0$ limit of QED?

YES, if the external force is due to $\mathbf{A}(t)$, i.e. if it is a time-dependent homogeneous electric field
Giles D. R. Martin, [arXiv:0805.0666 [gr-qc]].

How do we find these forces at order e^2 in the limit $\hbar \to 0$?

The $\hbar \to 0$ limit of the position shift

We work in Minkowski spacetime.

1. We accelerate an electron wave packet by a vector potential $\mathbf{A}(t)$ in $t < 0$. This wave packet emits a photon with some probability.
2. We compare the position expectation value $\langle \mathbf{x}(0) \rangle_0$ *without the EM interaction* and $\langle \mathbf{x}(0) \rangle_e$ *with the EM interaction* and define the position shift in the limit where the wave packet is a momentum eigenstate by

$$\Delta \mathbf{x}(0) = \langle \mathbf{x}(0) \rangle_e - \langle \mathbf{x}(0) \rangle_0.$$

3. We read off the radiation-reaction force by interpreting $\Delta \mathbf{x}(0)$ to have arisen from a classical force.

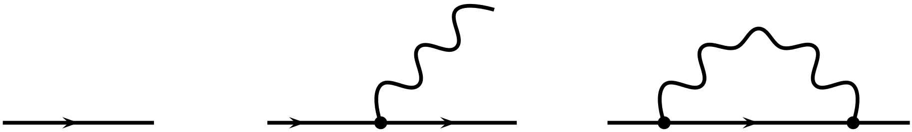
In this talk I assume that the initial and final momenta are the same for simplicity.

The $\hbar \to 0$ limit of the position shift

Suppose the electron of momentum $\mathbf{p}$ denoted $|\mathbf{p}\rangle$ evolves as

$$
\begin{aligned}
|\mathbf{p}\rangle \quad \to \quad & \left[1 + \frac{i}{\hbar}\mathcal{F}(\mathbf{p})\right]|\mathbf{p}\rangle \\
& + \frac{i}{\hbar}|\mathbf{p}\rangle \otimes \int \frac{d^3\mathbf{k}}{(2\pi)^3 2|\mathbf{k}|}\mathcal{A}^{\mu}(\mathbf{p},\mathbf{k})a^{\dagger}_{\mu}(\mathbf{k})|0\rangle,
\end{aligned}
$$

$\mathcal{F}(\mathbf{p})$: forward-scattering amplitude of order e^2 (conservative), $\mathcal{A}^{\mu}(\mathbf{p},\mathbf{k})$: one-photon emission amplitude of order e (dissipative); $\mathbf{k}$ is the wave number of the photon.



one photon emission

forward scattering

The $\hbar \to 0$ limit of the position shift

electron: $\mathbf{p}$ (classical) $\mathbf{p}/\hbar$ (not classical)
photon: $\hbar\mathbf{k}$ (not classical) $\mathbf{k}$ (classical)

We find

$$\Delta\mathbf{x}(0) = \Delta\mathbf{x}(0)_{\text{diss}} + \Delta\mathbf{x}(0)_{\text{cons}},$$

where

$$\begin{aligned}
\Delta\mathbf{x}(0)_{\text{diss}} &= -\frac{i}{2}\int \frac{d^3\mathbf{k}}{(2\pi)^3 2|\mathbf{k}|}\overline{\mathcal{A}_\mu(\mathbf{p},\mathbf{k})}\,\frac{\overleftrightarrow{\partial}}{\partial\mathbf{p}}\,\mathcal{A}^\mu(\mathbf{p},\mathbf{k}), \\
\Delta\mathbf{x}(0)_{\text{cons}} &= -\frac{\partial}{\partial\mathbf{p}}\mathrm{Re}\,\mathcal{F}(\mathbf{p}).
\end{aligned}$$

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Classical emission process

The emission process is essentially classical in the limit $\hbar \to 0$.

Reminder

If $\mathbf{x}_\mathbf{p}(t)$ is classical position of the electron of momentum $\mathbf{p}$ under the electric force $\mathbf{E}(t) = -\dot{\mathbf{A}}(t)$, then

$$
\begin{aligned}
\mathbf{j}_{(\mathbf{p})}(\mathbf{x}, t) &= e\dot{\mathbf{x}}_\mathbf{p}(t)\delta^3(\mathbf{x} - \mathbf{x}_\mathbf{p}(t)), \\
j^0_{(\mathbf{p})}(\mathbf{x}, t) &= e\delta^3(\mathbf{x} - \mathbf{x}_\mathbf{p}(t)).
\end{aligned}
$$

If $G^{\text{ret}}_{\mu\mu'}(x, x')$ is the retarded Green's function in the Feynman/Lorenz gauge, then

$$
A^{\text{ret}}_{(\mathbf{p})\mu}(x) = \int d^4x' G^{\text{ret}}_{\mu\mu'}(x, x') j^{\mu'}_{(\mathbf{p})}(x').
$$

$(x = (t, \mathbf{x}),\, x' = (t', \mathbf{x}'))$

The dissipative part of the position shift

$$\Delta\mathbf{x}(0)_{\text{diss}} = -\frac{i}{2}\int \frac{d^3\mathbf{k}}{(2\pi)^3 2|\mathbf{k}|}\overline{\mathcal{A}_\mu(\mathbf{p},\mathbf{k})}\,\frac{\overleftrightarrow{\partial}}{\partial\mathbf{p}}\,\mathcal{A}^\mu(\mathbf{p},\mathbf{k}).$$

One finds (with j^μ adiabatically turned off)

$$\mathcal{A}_\mu(\mathbf{p},\mathbf{k}) = \lim_{T\to\infty}\int_{t=T} d^3\mathbf{x}\, e^{ik\cdot x}\,\overleftrightarrow{\partial_t}\, A^{\text{ret}}_{(\mathbf{p})\mu}(x).$$

$(k\cdot x = |\mathbf{k}|t - \mathbf{k}\cdot\mathbf{x})$. Then

$$\Delta\mathbf{x}(0)_{\text{diss}} = \int d^4x\,\partial_{\mathbf{p}} j^\nu_{(\mathbf{p})}(x) A^{\text{rad}\,\nu}(x),$$

where

$$A^{\text{rad}}_\mu(x) = \frac{1}{2}(A^{\text{ret}}_\mu(x) - A^{\text{adv}}_\mu(x)).$$

The dissipative part of the position shift

$$\Delta \mathbf{x}(0)_{\text{diss}} = \int d^4x \, \partial_{\mathbf{p}} j^{\nu}_{(\mathbf{p})}(x) A^{\text{rad}\,\nu}(x).$$

This can be shown to equal the position shift from the Lorentz force due to the radiation field

$$F^{\text{rad}}_{\mu\nu}(x) = \nabla_\mu A^{\text{rad}}_\nu(x) - \nabla_\nu A^{\text{rad}}_\mu(x).$$

This agrees with the classical dissipative force.

This result can be generalised to an electron moving in time-dependent metric $g_{\mu\nu}(t)$.
P. J. Walker, PhD thesis

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The conservative part of the position shift

$$\Delta\mathbf{x}(0)_{\text{cons}} = -\frac{\partial}{\partial\mathbf{p}}\mathrm{Re}\,\mathcal{F}(\mathbf{p}).$$

If $J_Q^\mu(x)$ is the quantum current operator $\overline{\psi}(x)\gamma^\mu\psi(x)$, then

$$\mathcal{F}(\mathbf{p}) = \int d^4x \int d^4x' \langle\mathbf{p}|T[J_Q^\mu(x)J_Q^{\mu'}(x')]|\mathbf{p}\rangle G^{\mathrm{F}}_{\mu\mu'}(x,x'),$$

where $|\mathbf{p}\rangle$ is the electron state with momentum $\mathbf{p}$ and where $G^{\mathrm{F}}_{\mu\mu'}(x,x')$ is the Feynman propagator satisfying

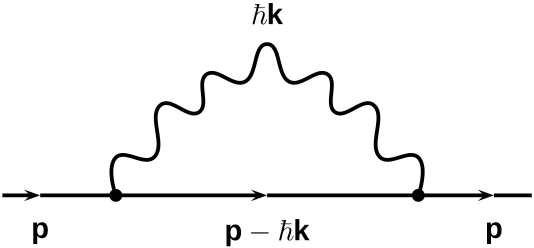
$$-\mathrm{Re}\,G^{\mathrm{F}}_{\mu\mu'}(x,x') = G^{\text{sing}}_{\mu\mu'}(x,x') = \frac{1}{2}(G^{\text{ret}}_{\mu\mu'}(x,x') + G^{\text{adv}}_{\mu\mu'}(x,x')).$$

We (i) identify the infinite contribution to the mass in $\mathcal{F}(\mathbf{p})$ and subtract it (mass renormalization) and (ii) take the limit $\hbar \to 0$.

The conservative part of the position shift

$$\mathcal{F}(\mathbf{p}) = \int d^4x \int d^4x' \langle \mathbf{p}|T[J_Q^{\mu}(x)J_Q^{\mu'}(x')]|\mathbf{p}\rangle G^{\mathrm{F}}_{\mu\mu'}(x,x').$$

This can be analysed in the momentum space: the momentum is conserved because the external field $\mathbf{A}(t)$ depends only on t in our model. $m_e(d\mathbf{x}/d\tau) = \mathbf{p} - e\mathbf{A}(t)$



We cannot say e.g. $\hbar|\mathbf{k}| \ll m_e c^2$ because $0 < \hbar|\mathbf{k}| < \infty$. So we first try the $\hbar$-expansion with $\mathbf{K} = \hbar\mathbf{k}$ regarded to be $O(\hbar^0)$.

The conservative part of the position shift

The $\hbar$-expansion with $\mathbf{K} = \hbar\mathbf{k}$ regarded as $O(\hbar^0)$:

$$\int \frac{d^3\mathbf{k}}{(2\pi)^3 2|\mathbf{k}|} = \frac{1}{\hbar^2}\int \frac{d^3\mathbf{K}}{(2\pi)^3 2|\mathbf{K}|}.$$

- $O(\hbar^{-1}) \sim m_e^2 \log(\Lambda/m_e)$ ("$|\mathbf{K}| \leq \Lambda$") cancelled by the mass counterterm. (The mass renormalization is at order $\hbar^{-1}$.)
- $O(\hbar^0) \sim (e^2/m_e)\mathbf{s}\cdot(\mathbf{p}\times\dot{\mathbf{p}})$, ("anomalous-magnetic moment".)
- $O(\hbar)$ and higher: $\hbar$-expansion breaks down due to small $\mathbf{K}$ contribution. (The nth order term $\sim \hbar^n K_{\text{min}}^{1-n}$ for $n \geq 2$.)

If we let $|\mathbf{K}| \geq K_{\text{min}} = \lambda\hbar^\beta$ with $\frac{3}{4} < \beta < 1$. Then

$$\hbar^n K_{\text{min}}^{1-n} = \lambda^{1-n}\hbar^{\beta+n(1-\beta)} \to 0 \text{ as } \hbar \to 0.$$

Only the mass-renormalisation and anomalous-magnetic-moment contributions remain.

The conservative part of the position shift

For the contribution from $|\mathbf{K}| \leq \lambda\hbar^\beta$ we change back to $\mathbf{k}$.

$$0 < |\mathbf{K}| < \lambda\hbar^\beta \leftrightarrow 0 < |\mathbf{k}| < \lambda\hbar^{\beta-1}.$$

Since $0 < \beta < 1$, $\lambda\hbar^\beta \to 0$ but $\lambda\hbar^{\beta-1} \to \infty$ as $\hbar \to 0$.

We find this contribution to be

$$-\mathrm{Re}\,\mathcal{F}^<(\mathbf{p}) = \frac{1}{2}\int d^4x d^4x' j^\mu_{(\mathbf{p})}(x)\tilde{G}^{\text{sing}}_{\mu\mu'}(x,x')j^{\mu'}_{(\mathbf{p})}(x'),$$

where

$$\tilde{G}^{\text{sing}}_{\mu\mu'}(x,x') = ig_{\mu\mu'}\mathrm{sgn}(t-t')\int_{|\mathbf{k}|\leq\lambda\hbar^{\beta-1}} \frac{d^3\mathbf{k}}{(2\pi)^3 2|\mathbf{k}|}\left(e^{-ik\cdot x} - e^{ik\cdot x}\right)$$

is the regularized singular Green's function,

$$G^{\text{sing}}_{\mu\mu'}(x,x') = \frac{1}{2}(G^{\text{ret}}_{\mu\mu'}(x,x') + G^{\text{adv}}_{\mu\mu'}(x,x')).$$

The conservative part of the position shift

The contribution to Re $\mathcal{F}(\mathbf{p})$ from $|\mathbf{K}| \leq \lambda\hbar^{\beta}$ ($|\mathbf{k}| \leq \lambda\hbar^{\beta-1}$):

$$-\mathrm{Re}\,\mathcal{F}^{<}(\mathbf{p}) = \frac{1}{2}\int d^4x d^4x' j^{\mu}_{(\mathbf{p})}(x)\tilde{G}^{\mathrm{sing}}_{\mu\mu'}(x,x')j^{\mu'}_{(\mathbf{p})}(x').$$

This is infinite. After the subtraction of the contribution to the mass counterterm from $|\mathbf{K}| \leq \lambda\hbar^{\beta}$, if we can write

$$-\mathrm{Re}\,\mathcal{F}^{\mathrm{ref},<}(\mathbf{p}) = \frac{1}{2}\int d^4x d^4x' j^{\mu}_{(\mathbf{p})}(x)G^{\mathrm{reg}}_{\mu\mu'}(x,x')j^{\mu'}_{(\mathbf{p})}(x'),$$

Then the corresponding conservative radiation-reaction force is the Lorentz force with

$$A^{\mathrm{reg}}_{\mu}(x) = \int d^4x' G^{\mathrm{reg}}_{\mu\mu'}(x,x')j^{\mu'}_{(\mathbf{p})}(x').$$

In our model (flat space, external $\mathbf{A}(t)$), $G^{\mathrm{reg}}_{\mu\mu'}(x,x') = 0$.

Outline

Summary

For an electron accelerated by $\mathbf{A}(t)$ in Minkowski spacetime,

- The dissipative part of radiation reaction is from the one-photon emission process in QED.
- The conservative part, which vanishes, comes from the one-loop diagram.

This is likely to be true in general.

The one-loop correction to the external potential $\mathbf{A}(t)$:

- $|\mathbf{K}| \geq \lambda\hbar^{\beta}$ (particle-like virtual photon):
  (infinite) renormalization of the mass and what is regarded as a quantum effect (anomalous magnetic moment)
- $|\mathbf{K}| \leq \lambda\hbar^{\beta}$ (wave-like virtual photon):
  the conservative part of radiation reaction force.

Does this clear split of the quantum and classical parts of the one-loop contribution persist in curved spacetime; for gravitational radiation reaction?