Extreme-mass-ratio inspirals: Metric reconstruction and gauge transformation

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Outline

- Transforming from Regge-Wheeler to Lorenz gauge
- Odd-parity gauge generator
- Partial annihilator method
- Equations with non-compact sources
- Extended particular solutions
- Even-parity scalar equation
- Conclusions

Transforming to Lorenz gauge

- Gauge transformation from Regge-Wheeler (RW) to Lorenz (L)

$$x^\mu_{\mathrm{L}} = x^\mu_{\mathrm{RW}} + \Xi^\mu, \qquad |\Xi^\mu| \sim |p_{\mu\nu}|$$

- Metric perturbation transforms as

$$p^{\mathrm{L}}_{\mu\nu} = p^{\mathrm{RW}}_{\mu\nu} - \Xi_{\mu|\nu} - \Xi_{\nu|\mu},$$

- Demand $p^{\mathrm{L}}_{\mu\nu}$ satisfy the Lorenz gauge condition, $\bar{p}^{\mathrm{L}}_{\mu\nu}{}^{|\nu} = 0$:

$$\Box \Xi_\mu = \bar{p}^{\mathrm{RW}|\nu}_{\mu\nu}$$

- The gauge generator splits into even and odd-parity parts

$$\Xi^\mu = \Xi^\mu_{\mathrm{even}} + \Xi^\mu_{\mathrm{odd}}.$$

- We start with the odd-parity

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Odd-parity harmonic decomposition

- Odd-parity gauge generator only has θ, ϕ components

$$\Xi^{\mu}_{\text{odd}} = \left(\, 0, \; 0, \; \Xi^{\theta}, \; \Xi^{\phi} \right)$$

- Decompose Ξ^{μ} in harmonics

$$\Xi^{A}\left(x^{\mu}\right) = \sum_{\ell m} \xi_{\ell m}(t,r) X^{A}_{\ell m}(\theta,\phi), \qquad A = \{\theta, \phi\}$$

- A wave equation for each mode

$$\frac{1}{f}\left[-\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial r_*^2} - V_1(r)\right]\xi_{\ell m} = 2\frac{f}{r}h_r^{\ell m} + p_{\ell m}\delta\left[r - r_p(t)\right]$$

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- An inhomogeneous wave equation

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- Original Regge-Wheeler variable $\Psi^{\ell m}=fh_r^{\ell m}/r$
- Satisfies the equation

$$\left[-\frac{\partial^2}{\partial t^2}+\frac{\partial^2}{\partial r_*^2}-V_2\right]\Psi^{\ell m}=S_{\mathrm{RW}}$$

- Act with Regge-Wheeler wave operator on both sides

$$\left[-\frac{\partial^2}{\partial t^2}+\frac{\partial^2}{\partial r_*^2}-V_2\right]\frac{1}{f}\left[-\frac{\partial^2}{\partial t^2}+\frac{\partial^2}{\partial r_*^2}-V_1\right]\xi_{\ell m}$$
$$=2S_{\mathrm{RW}}+\text{Other singular terms}$$

- Now a 4th-order PDE, but *the source is point-singular*

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Solving the 4th-order equation

- Move to the frequency domain

$$\left[\frac{d^2}{dr_*^2} + \omega_{mn}^2 - V_2\right]\frac{1}{f}\left[\frac{d^2}{dr_*^2} + \omega_{mn}^2 - V_1\right]\tilde{\xi} = Z_\xi(r)$$

- Yields 4 linearly independent homogeneous solutions
- On the horizon side, causality dictates

$$\tilde{\xi}_2^H \sim e^{-i\omega_{mn}r_*}, \qquad \tilde{\xi}_4^H \sim f e^{-i\omega_{mn}r_*}$$

- On the spatial infinity side

$$\tilde{\xi}_2^\infty \sim e^{i\omega_{mn}r_*}, \qquad \tilde{\xi}_4^\infty \sim r e^{i\omega_{mn}r_*}$$

- Particular solution will be a linear combination of the homogeneous solutions, subject to source and boundary conditions

$$\tilde{\xi}_p^{\mathrm{Std}}(r) = c_2^H(r)\tilde{\xi}_2^H + c_4^H(r)\tilde{\xi}_4^H + c_2^\infty(r)\tilde{\xi}_2^\infty + c_4^\infty(r)\tilde{\xi}_4^\infty$$

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Finding the particular solution

- The method of variation of parameters gives the $c^n(r)$

$$c^n(r) = \int_{r_{\min}/r}^{r/r_{\max}} Z_\xi(r') \frac{W^n(r')}{fW(r')} dr'$$

- $W(r)$ is the Wronskian

$$W(r) = \begin{vmatrix} \tilde{\xi}_1^H & \tilde{\xi}_1^\infty & \tilde{\xi}_2^H & \tilde{\xi}_2^\infty \\ \tilde{\xi}_1^{H\prime} & \tilde{\xi}_1^{\infty\prime} & \tilde{\xi}_2^{H\prime} & \tilde{\xi}_2^{\infty\prime} \\ \tilde{\xi}_1^{H\prime\prime} & \tilde{\xi}_1^{\infty\prime\prime} & \tilde{\xi}_2^{H\prime\prime} & \tilde{\xi}_2^{\infty\prime\prime} \\ \tilde{\xi}_1^{H\prime\prime\prime} & \tilde{\xi}_1^{\infty\prime\prime\prime} & \tilde{\xi}_2^{H\prime\prime\prime} & \tilde{\xi}_2^{\infty\prime\prime\prime} \end{vmatrix} \qquad {}' \equiv \frac{d}{dr_*}$$

- $W^n(r)$ is the modified Wronskian. For example:

$$W^1(r) = \begin{vmatrix} 0 & \tilde{\xi}_1^\infty & \tilde{\xi}_2^H & \tilde{\xi}_2^\infty \\ 0 & \tilde{\xi}_1^{\infty\prime} & \tilde{\xi}_2^{H\prime} & \tilde{\xi}_2^{\infty\prime} \\ 0 & \tilde{\xi}_1^{\infty\prime\prime} & \tilde{\xi}_2^{H\prime\prime} & \tilde{\xi}_2^{\infty\prime\prime} \\ 1 & \tilde{\xi}_1^{\infty\prime\prime\prime} & \tilde{\xi}_2^{H\prime\prime\prime} & \tilde{\xi}_2^{\infty\prime\prime\prime} \end{vmatrix}$$

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4th-order extended homogeneous solutions

- Solve the 4th-order equation in the frequency domain by variation of parameters
- We define the frequency domain extended homogeneous solutions for all $r > 2M$

$$\tilde{\xi}^{-}(r) \equiv C_2^H \tilde{\xi}_2^H(r) + C_4^H \tilde{\xi}_4^H(r) \qquad \tilde{\xi}^{+}(r) \equiv C_2^\infty \tilde{\xi}_2^\infty(r) + C_4^\infty \tilde{\xi}_4^\infty(r)$$

- The time domain extended homogeneous solutions are

$$\xi^{\pm}(t, r) \equiv \sum_n \tilde{\xi}^{\pm}(r) e^{-i\omega_{mn} t}$$

- We claim

$$\xi(t, r) = \xi^{+}(t, r)\ \theta\left[r - r_p(t)\right] + \xi^{-}(t, r)\ \theta\left[r_p(t) - r\right]$$

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2nd-order solutions

- A method for finding solutions without relying on annihilators
- Consider again

$$\left[-\frac{\partial^2}{\partial t^2}+\frac{\partial^2}{\partial r_*^2}-V_1\right]\xi_{\ell m}=2f\Psi_{\ell m}^{\mathrm{RW}}+P_{\mathrm{Singular}}$$

- Or, in the FD:

$$\left[\frac{d^2}{dr_*^2}+\omega_{mn}^2-V_1\right]\tilde{\xi}_{\ell mn}=2fR_{\ell mn}^{\mathrm{RW}}+Z_{\mathrm{Singular}}$$

- The Z_{Singular} can always be found using EHS
- For now consider simply

$$\left[\frac{d^2}{dr_*^2}+\omega_{mn}^2-V_1\right]\tilde{\xi}_{\ell mn}=2fR_{\ell mn}^{\mathrm{RW}}$$

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$$\left[\frac{d^2}{dr_*^2}+\omega_{mn}^2-V_1\right]\tilde{\xi}_{\ell mn}=2fR^{\rm RW}_{\ell mn}+Z_{\rm Singular}$$

- The $Z_{\rm Singular}$ can always be found using EHS
- For now consider simply

$$\left[\frac{d^2}{dr_*^2}+\omega_{mn}^2-V_1\right]\tilde{\xi}_{\ell mn}=2fR^{\rm RW}_{\ell mn}$$

2nd-order solutions

- A method for finding solutions without relying on annihilators
- Consider again

$$\left[-\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial r_*^2} - V_1\right] \xi_{\ell m} = 2f\Psi_{\ell m}^{\mathrm{RW}} + P_{\mathrm{Singular}}$$

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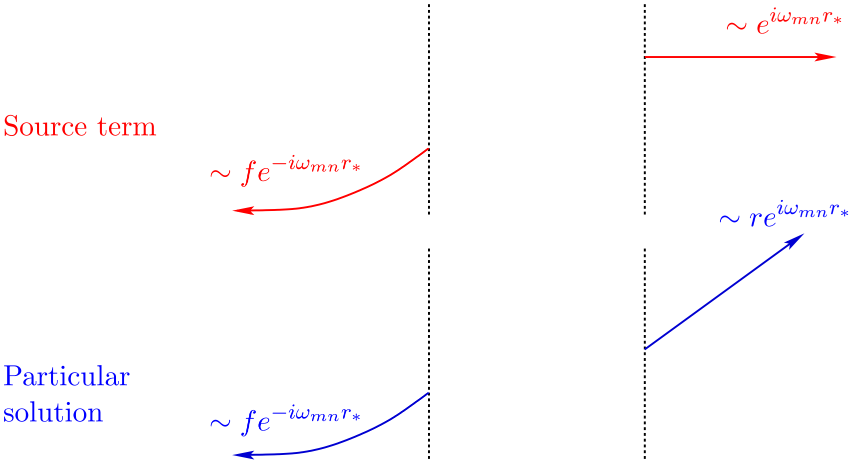
Particular solutions

$$\left[\frac{d^2}{dr_*^2} + \omega_{mn}^2 - V_1\right] \tilde{\xi}_{\ell mn} = 2 f R_{\ell mn}^{\mathrm{RW}}$$



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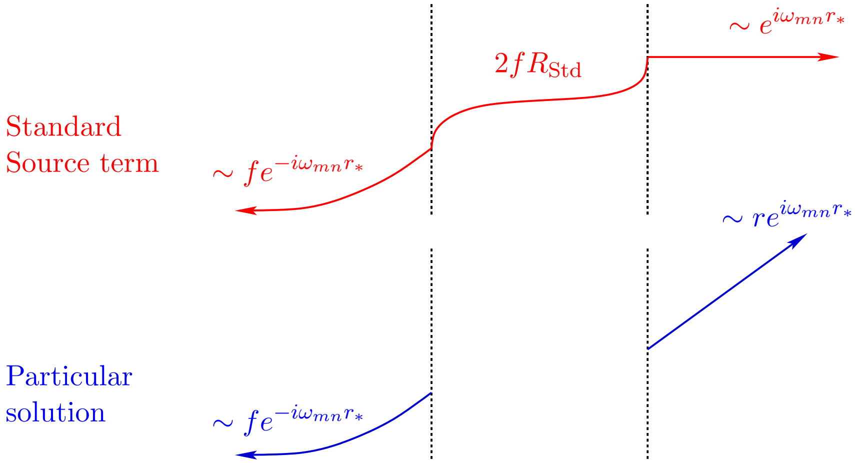

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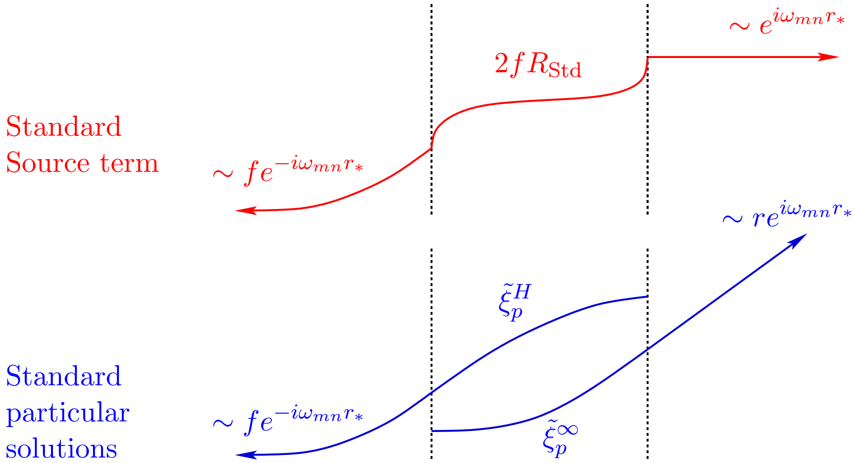
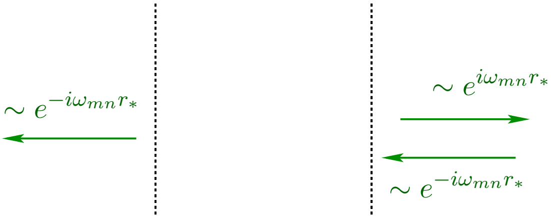


Homogeneous solutions

$$\left[\frac{d^2}{dr_*^2} + \omega_{mn}^2 - V_1\right]\tilde{\xi}_{\ell mn} = 0$$

"In" mode
homogeneous
solution



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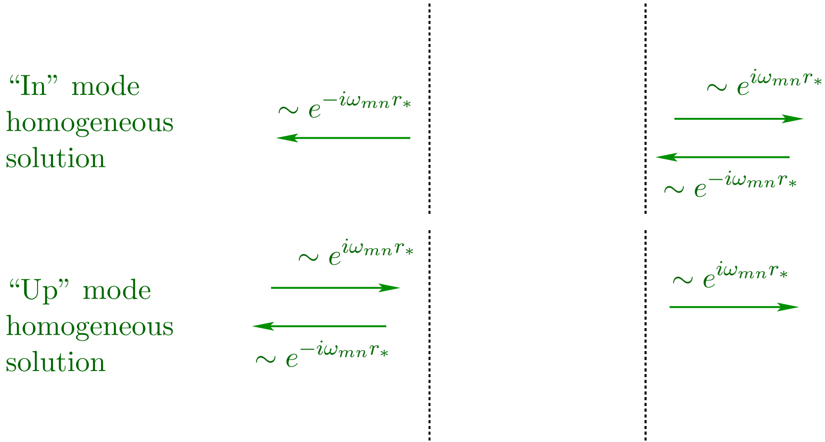
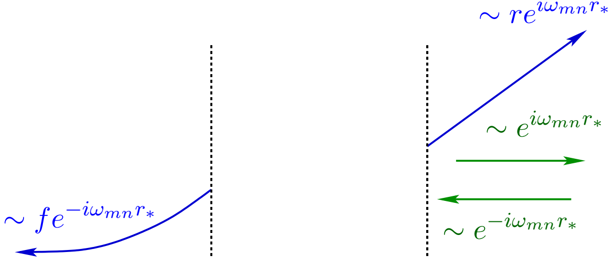


Finding causal solutions

$$\left[\frac{d^2}{dr_*^2} + \omega_{mn}^2 - V_1\right]\tilde{\xi}_{\ell mn} = 2fR_{\ell mn}^{\mathrm{RW}}$$

Integrate from
left to right



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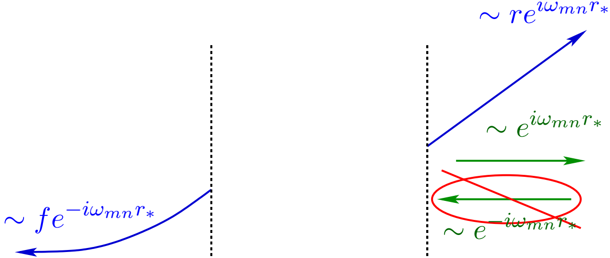
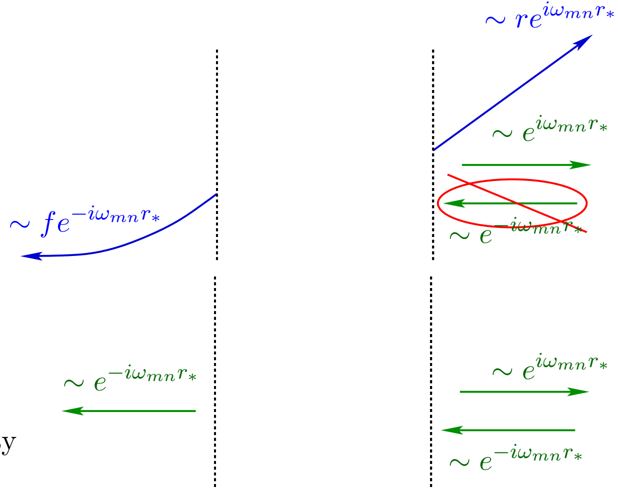


Finding causal solutions

$$\left[\frac{d^2}{dr_*^2} + \omega_{mn}^2 - V_1\right]\tilde{\xi}_{\ell mn} = 2fR_{\ell mn}^{\mathrm{RW}}$$

Integrate from left to right

Subtracte to remove acausality

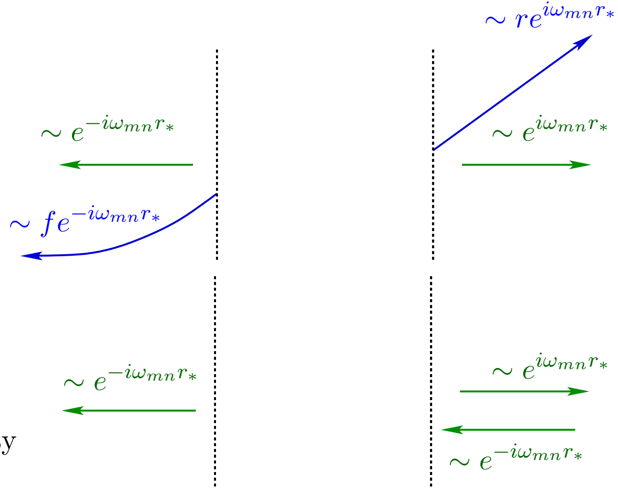


Finding causal solutions

$$\left[\frac{d^2}{dr_*^2} + \omega_{mn}^2 - V_1\right]\tilde{\xi}_{\ell mn} = 2fR^{\rm RW}_{\ell mn}$$

Causal solution remains

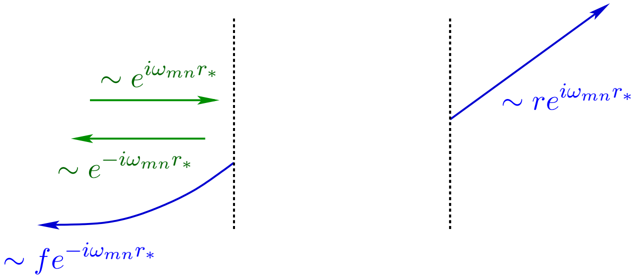
Subtracte to remove acausality



Finding causal solutions

$$\left[\frac{d^2}{dr_*^2} + \omega_{mn}^2 - V_1\right]\tilde{\xi}_{\ell mn} = 2fR^{\mathrm{RW}}_{\ell mn}$$

Integrate from right to left



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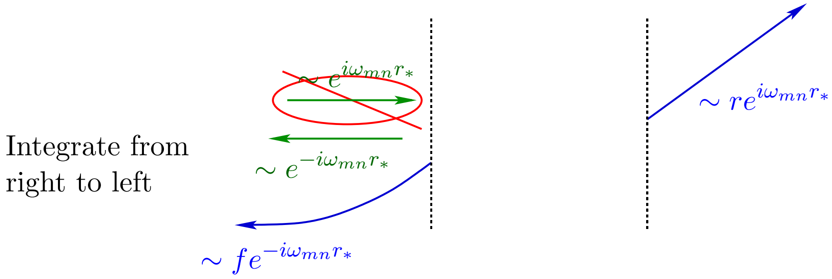

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Subtract to remove acausality

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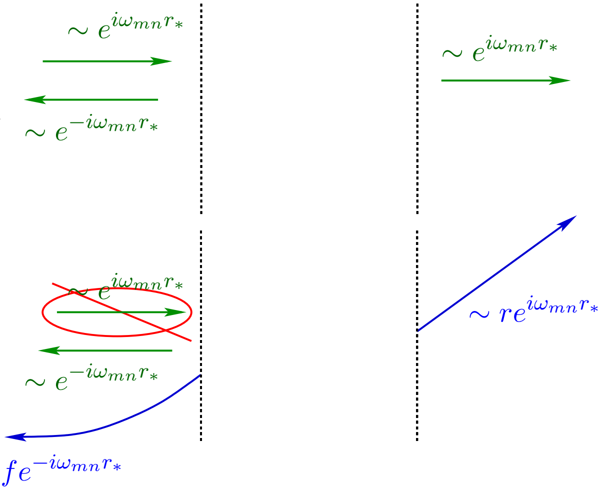

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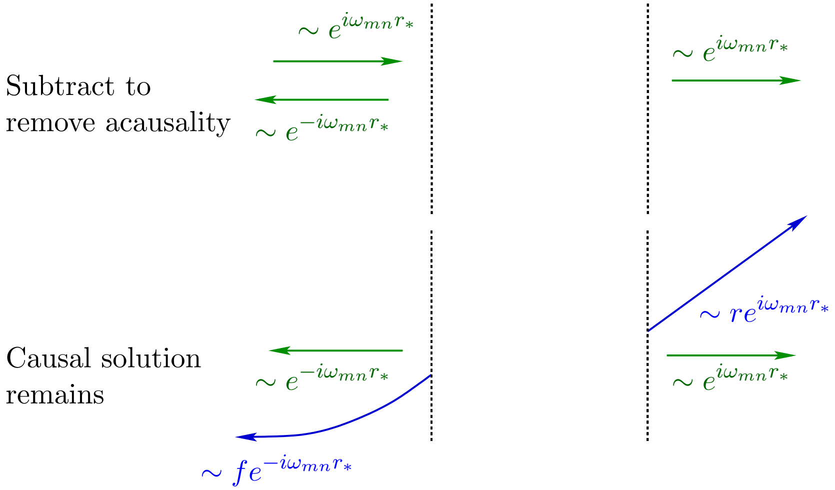
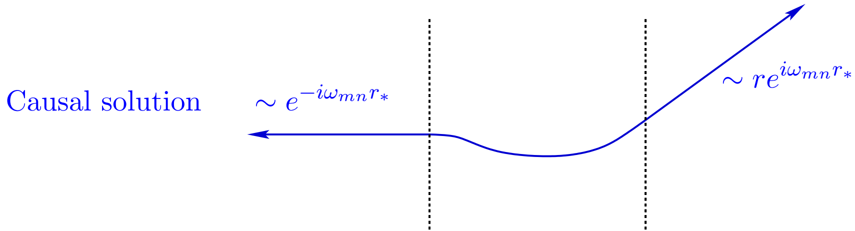


Time domain reconstruction

$$\left[\frac{d^2}{dr_*^2} + \omega_{mn}^2 - V_1\right]\tilde{\xi}_{\ell mn} = 2fR_{\ell mn}^{\mathrm{RW}}$$



- TD reconstruction

$$\xi(t,r) = \sum_n \tilde{\xi}(r)e^{-i\omega_{mn}t}$$

- The TD source is discontinuous (C^{-1}), so the convergence is algebraic $\sim 1/n^3$ at the particle.
- We would like exponential convergence.

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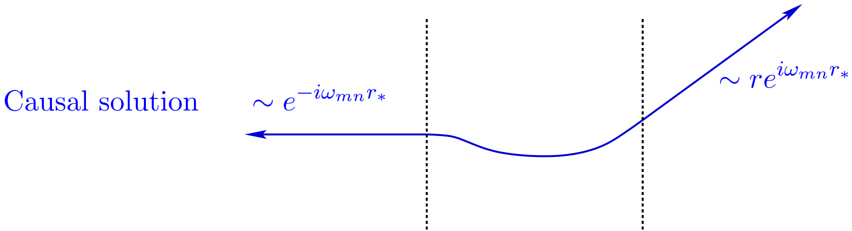

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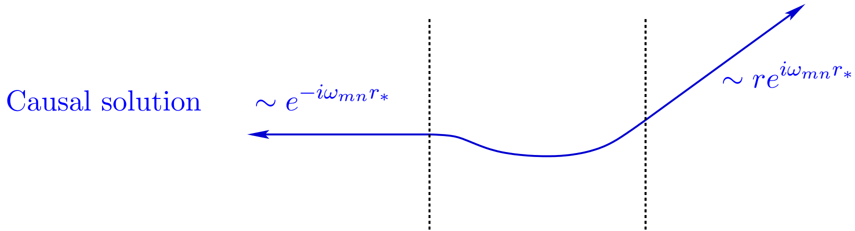

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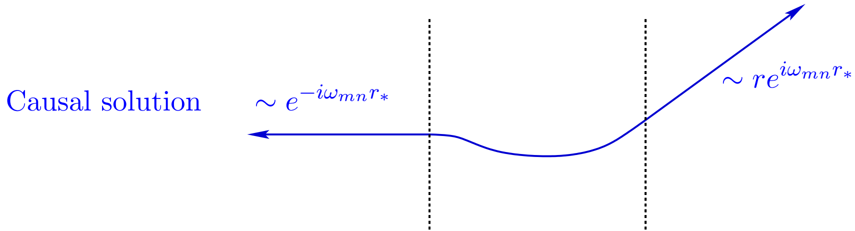

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Extended particular solutions

- We look for a time domain solution of the form

$$\xi(t,r) = \xi^{+}(t,r)\ \theta\left[r - r_p(t)\right] + \xi^{-}(t,r)\ \theta\left[r_p(t) - r\right]$$

- Where

$$\xi^{\pm}(t,r) = \xi_p^{\pm}(t,r) + \xi_h^{\pm}(t,r)$$

- Particular solution
- Homogeneous solution
- Defined for $r > 2M$

$$\xi_p^{\pm}(t,r) \equiv \sum_n \tilde{\xi}_p^{\pm}(r) e^{-i\omega_{mn} t}, \qquad \xi_h^{\pm}(t,r) \equiv \sum_n \tilde{\xi}_h^{\pm}(r) e^{-i\omega_{mn} t}$$

- How do we find $\tilde{\xi}_p^{\pm}(r)$ and $\tilde{\xi}_h^{\pm}(r)$?

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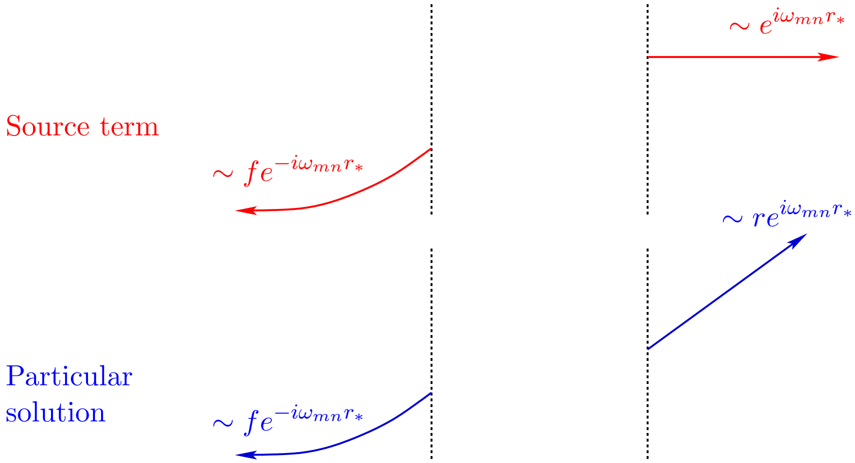

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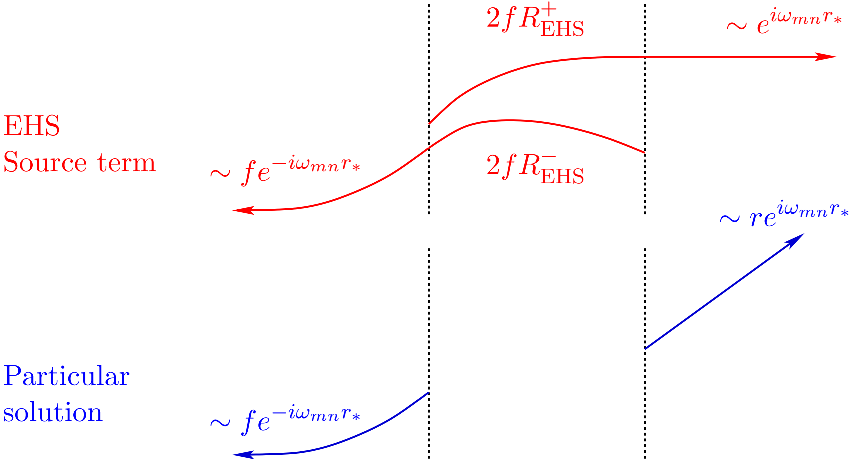

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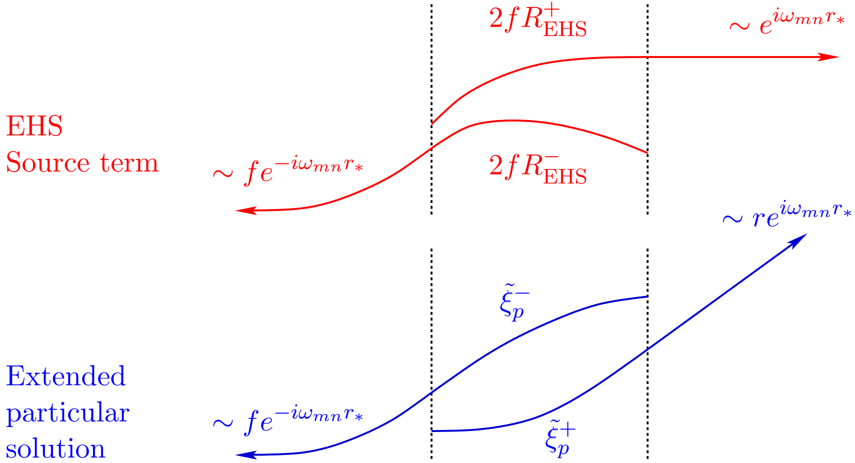
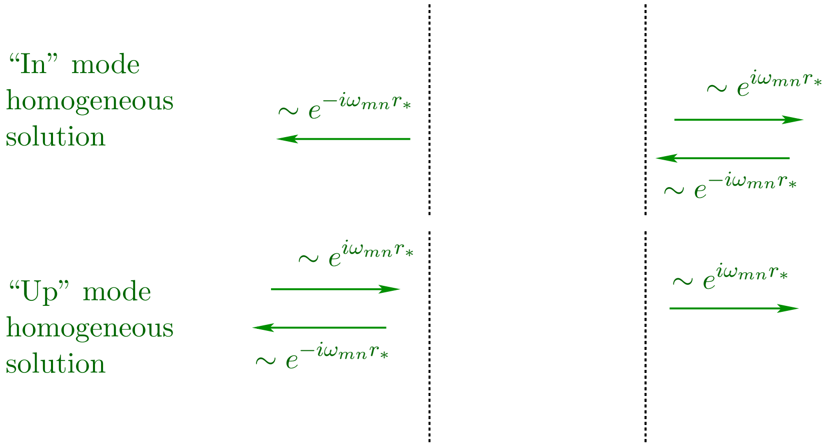


Homogeneous solutions

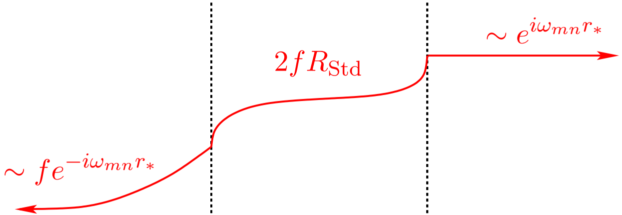
$$\left[\frac{d^2}{dr_*^2} + \omega_{mn}^2 - V_1\right]\tilde{\xi}_{\ell mn} = 0$$

- Same as in standard case, scaled to remove acausal behavior



2nd-order EPS summary

- Std. source
- Std. particular solutions: $\tilde{\xi}_p^{\infty/H}$
- Causality gives homog. sols: $\tilde{\xi}_h^{\pm}$
- EHS source
- Extended particular solutions: $\tilde{\xi}_p^{\pm}$
- Use same homog. sols: $\tilde{\xi}_h^{\pm}$



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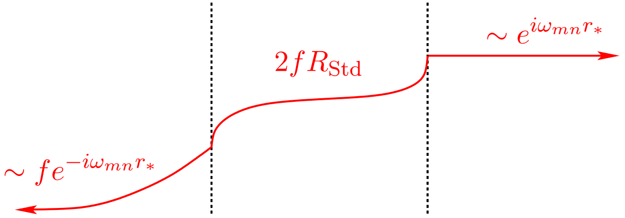

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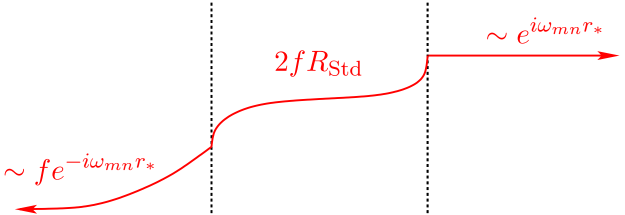

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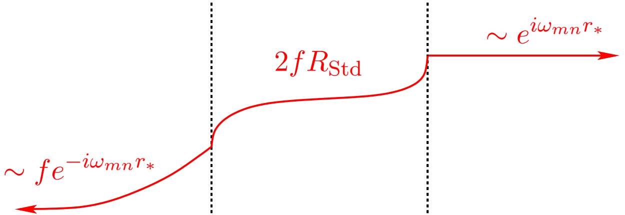

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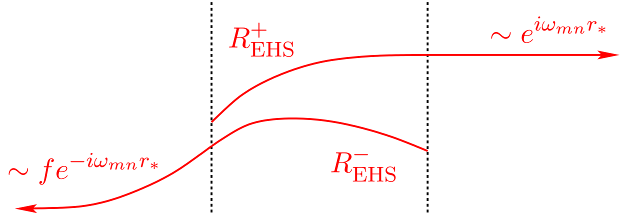

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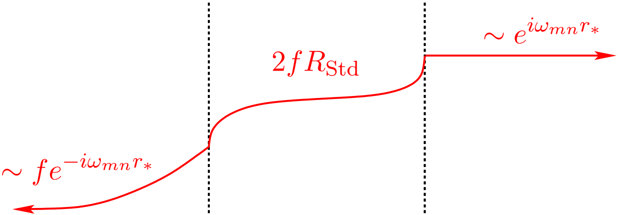

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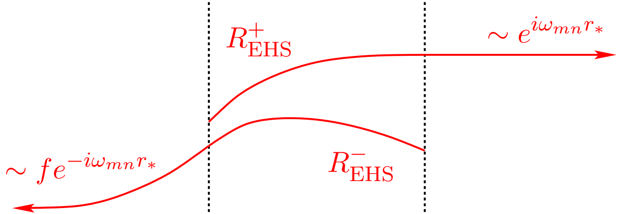

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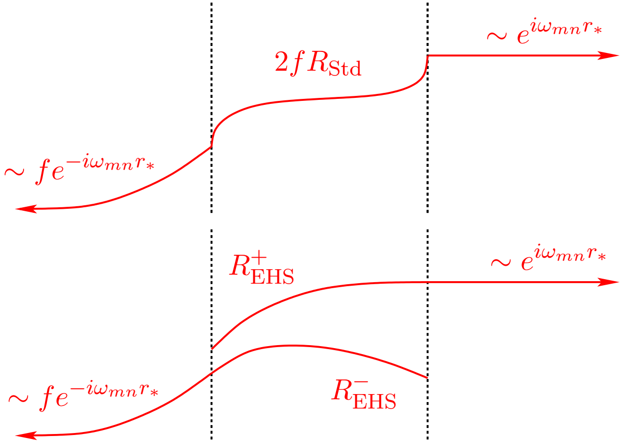

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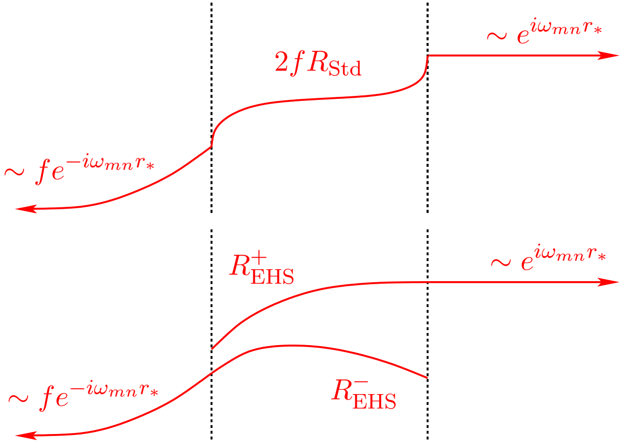

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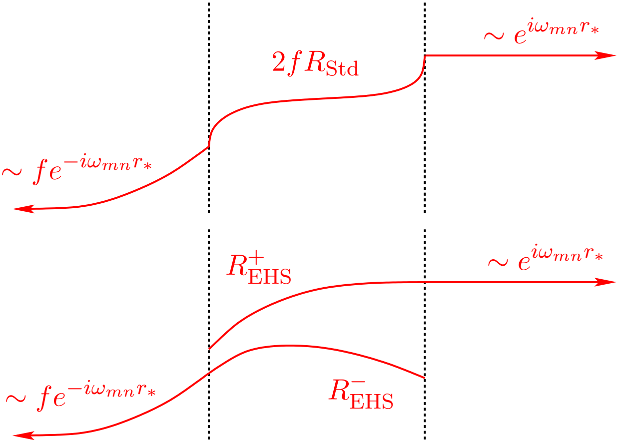

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2nd-order EPS $\Longleftrightarrow$ 4th-order EHS

- 4th-order equation had 4 homogeneous solutions

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- Same as the 2 homogeneous solutions to the 2nd-order equation
- Constants from variation of params same we get from causality
- Same as the particular solutions to the 2nd-order equation
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$$\xi(t,r) = \xi^{+}(t,r)\ \theta\left[r - r_p(t)\right] + \xi^{-}(t,r)\ \theta\left[r_p(t) - r\right]$$

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How to check the solution

- Given the metric perturbation transforms as

$$p^{\mathrm{L}}_{\mu\nu} = p^{\mathrm{RW}}_{\mu\nu} - \Xi_{\mu|\nu} - \Xi_{\nu|\mu},$$

- The metric perturbation amplitudes are pushed via

$$h_r^{\ell m,\mathrm{L}} = h_r^{\ell m,\mathrm{RW}} - \frac{\partial}{\partial r}\xi_{\ell m} + \frac{2}{r}\xi_{\ell m}$$
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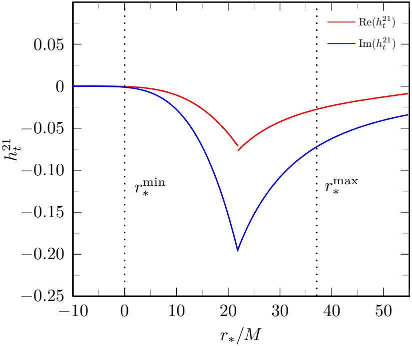
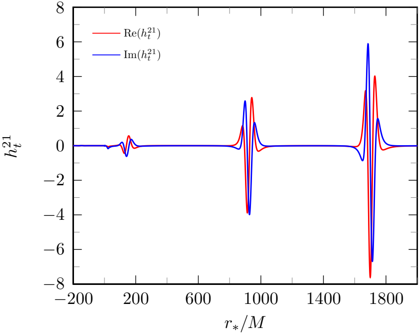
$h_t^{\ell m}$ in Regge-Wheeler gauge

$h_t^{21}(t_\circ, r_*)$ locally



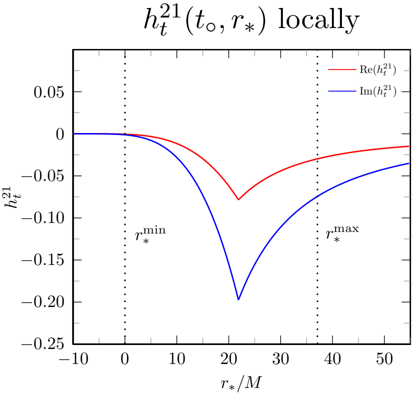
$$p = 8.75455$$
$$e = 0.764124$$
$$t_\circ = 143.45M$$
$$-50 \le n \le 50$$

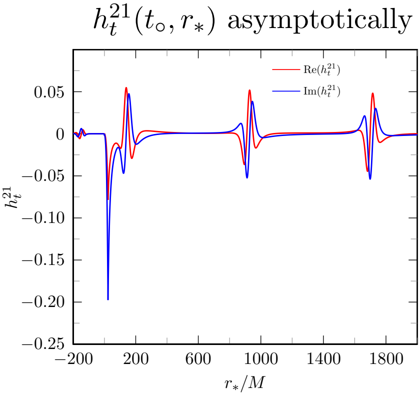
$h_t^{21}(t_\circ, r_*)$ asymptotically



- Now C^{-1} at the particle
- Asymptotically grows

$h_t^{\ell m}$ in Lorenz gauge





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$h_r^{\ell m}$ in Regge-Wheeler gauge

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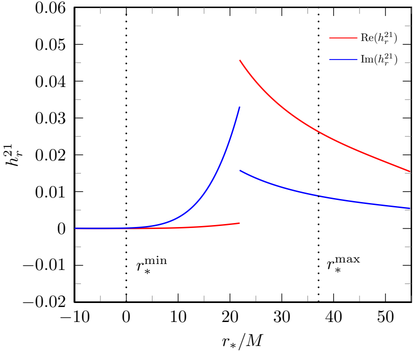
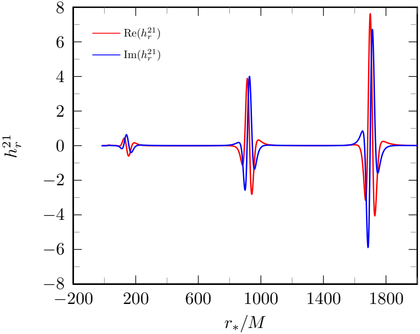


$h_r^{21}(t_\circ, r_*)$ asymptotically



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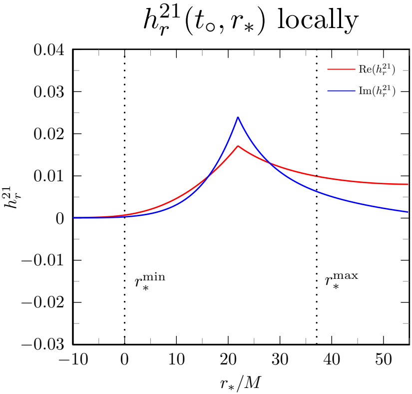

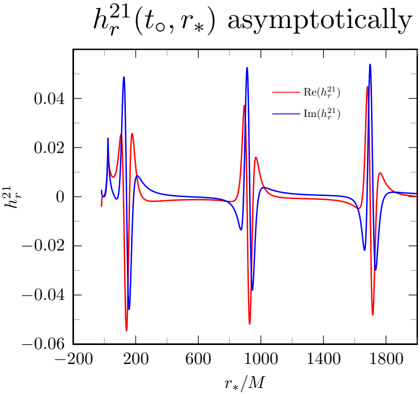

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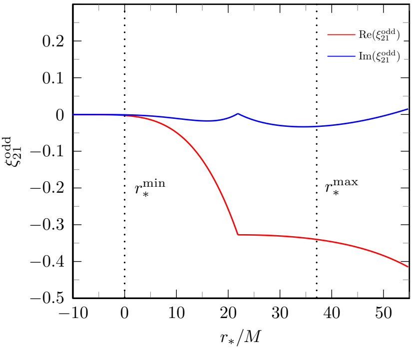
$\xi^{\text{odd}}_{\ell m}$ - numerical results

$\xi_{21}(t_\circ, r_*)$ locally



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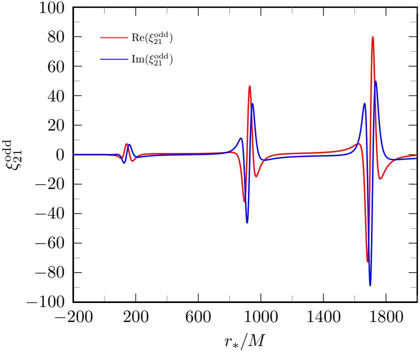

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- We see the expected local and asymptotic behavior following the partial sum.

Even-parity scalar equation

- The even-parity splits into scalar and divergence free vector parts

$$\Xi^{\mu}_{\text{even}} = \Xi_{(s)}{}^{|\mu} + \Xi^{\mu}_{(v)}$$

- A harmonic decomposition of the scalar part yields

$$\left[-\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial r_*^2} - V_0\right] \frac{1}{rf} \left[-\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial r_*^2} - V_0\right] \xi^{\text{s}} = \alpha(r)\Psi_{\text{ZM}} + \beta(r)\frac{d\Psi_{\text{ZM}}}{dr_*} + S_{\text{Singular}}$$

- In the FD

$$\left[\frac{d^2}{dr_*^2} + \omega_{mn}^2 - V_0\right] \frac{1}{rf} \left[\frac{d^2}{dr_*^2} + \omega_{mn}^2 - V_0\right] \tilde{\xi}^{\text{s}} = \alpha(r)R_{\text{ZM}} + \beta(r)\frac{dR_{\text{ZM}}}{dr_*} + S_{\text{Singular}}$$

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- The singular part can be found through EHS.
- We are concerned with

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- Where

$$Z_\xi \equiv \alpha(r)R_{\mathrm{ZM}}+\beta(r)\frac{dR_{\mathrm{ZM}}}{dr_*}$$

$$\alpha(r)\equiv-\frac{2f}{r^6\Lambda}\Big(\lambda(\lambda+1)r^3+\lambda M\left(\lambda+1\right)r^2+3M^2\left(3\lambda-1\right)r+24M^3\Big)$$

$$\beta(r)\equiv-2\frac{f^2}{r^3}\Big(r\left(\lambda+1\right)+4M\Big),\qquad \lambda\equiv\frac{1}{2}\left(\ell-1\right)\left(\ell+2\right)$$

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- We use extended particular solutions instead

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Particular solutions

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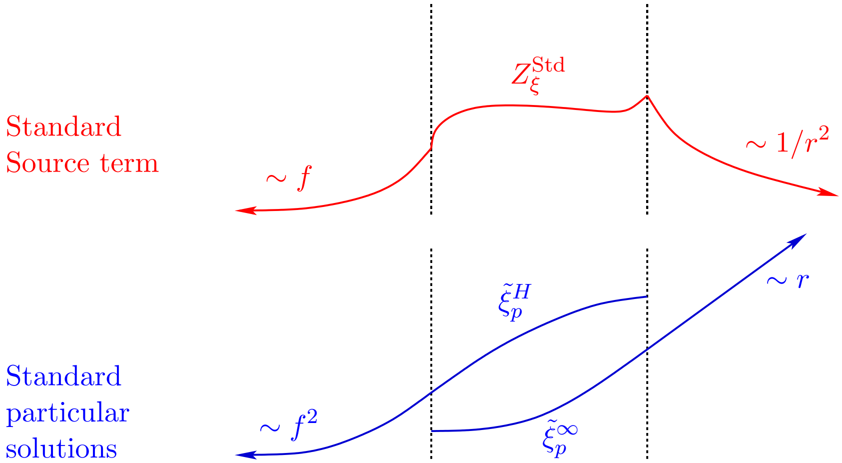

Homogeneous solutions

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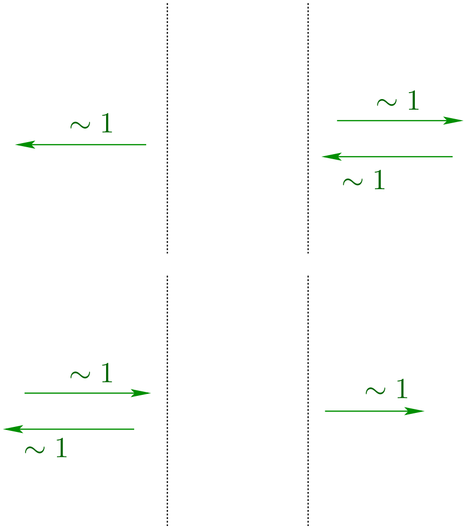

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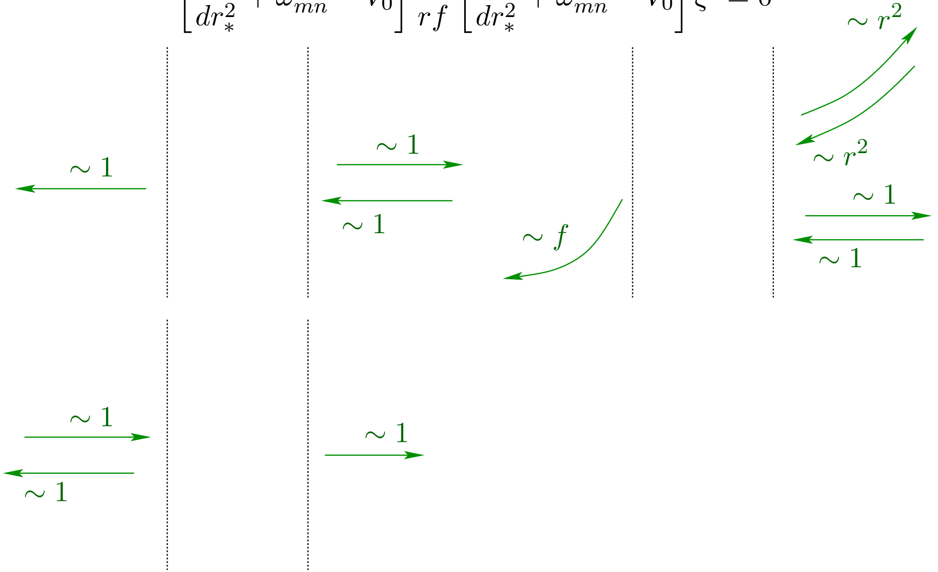

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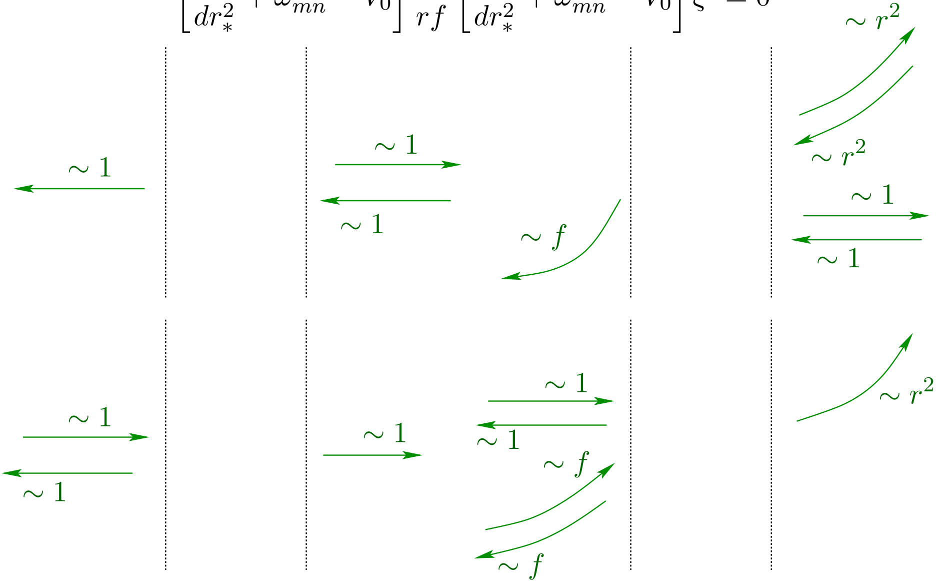

Finding causal solutions

$$\left[\frac{d^2}{dr_*^2} + \omega_{mn}^2 - V_0\right] \frac{1}{rf} \left[\frac{d^2}{dr_*^2} + \omega_{mn}^2 - V_0\right] \tilde{\xi}^{\mathrm{s}} = Z_\xi$$

- Integrate from left to right
- Note acausal behavior
- Subtract off $\sim r^2 e^{-i\omega_{mn} r_*}$ part
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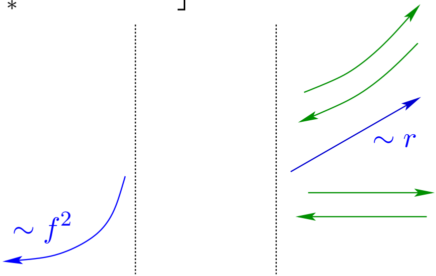
- Causal solution remains



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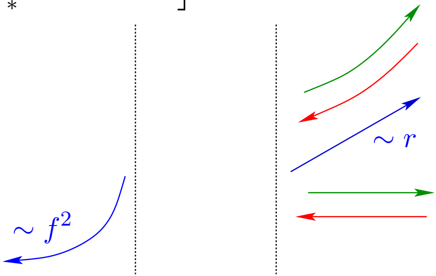
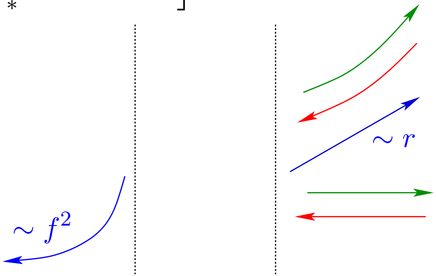
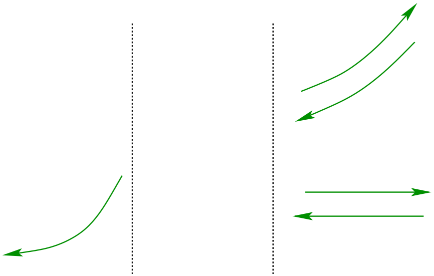


Finding causal solutions

$$\left[\frac{d^2}{dr_*^2} + \omega_{mn}^2 - V_0\right] \frac{1}{rf} \left[\frac{d^2}{dr_*^2} + \omega_{mn}^2 - V_0\right] \tilde{\xi}^{\mathrm{s}} = Z_\xi$$

- Integrate from left to right
- Note acausal behavior
- Subtract off $\sim r^2 e^{-i\omega_{mn} r_*}$ part
- Subtract off $\sim e^{-i\omega_{mn} r_*}$
- Causal solution remains





Finding causal solutions

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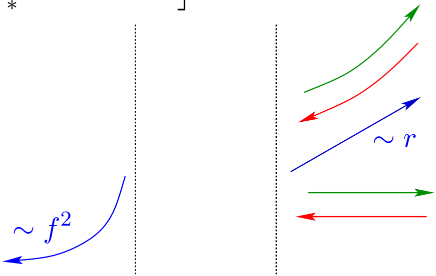

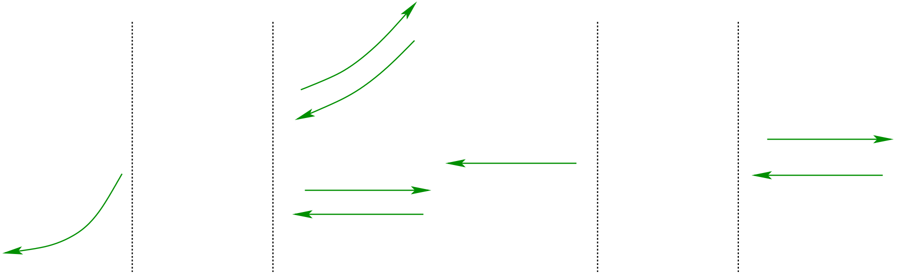

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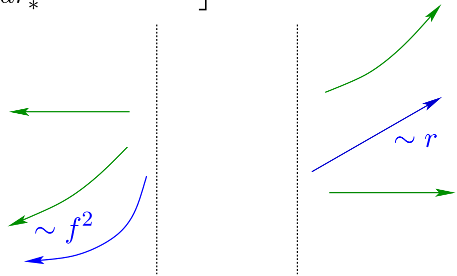

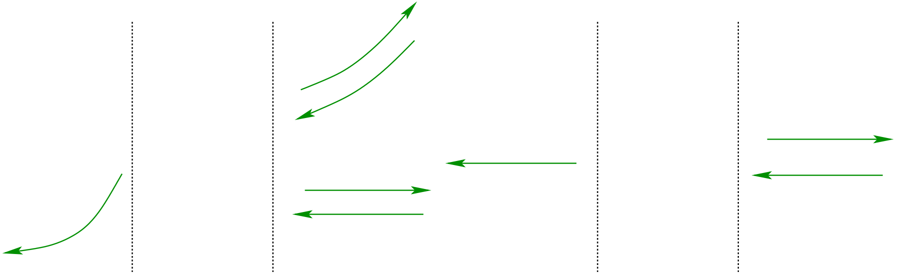

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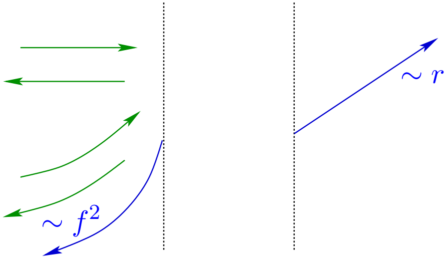

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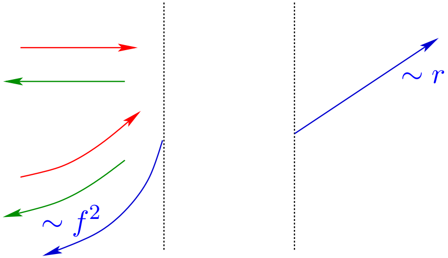

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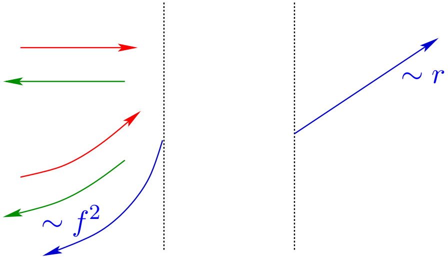

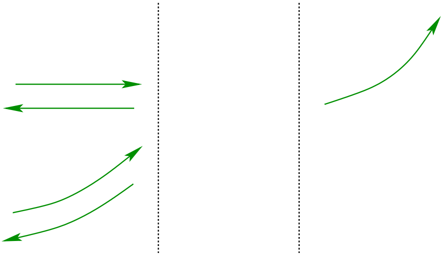

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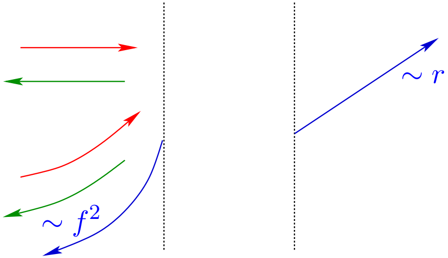

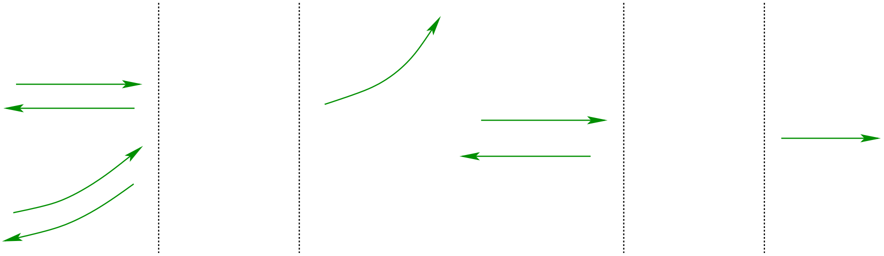

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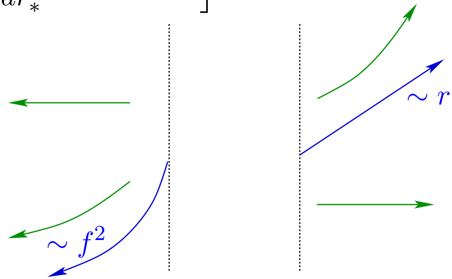
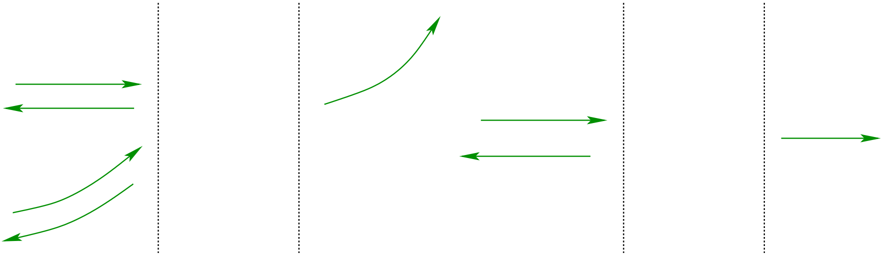
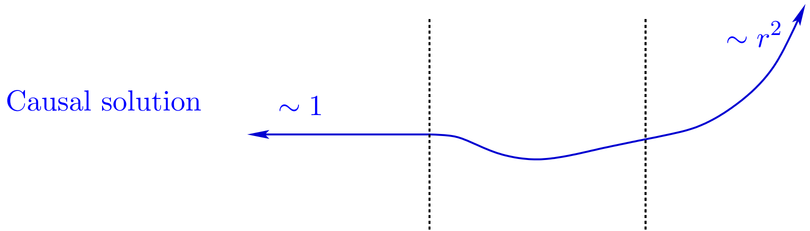




Time domain reconstruction

$$\left[\frac{d^2}{dr_*^2} + \omega_{mn}^2 - V_0\right] \frac{1}{rf} \left[\frac{d^2}{dr_*^2} + \omega_{mn}^2 - V_0\right] \tilde{\xi}^{\mathrm{s}} = Z_\xi$$



- TD reconstruction

$$\xi(t,r) = \sum_n \tilde{\xi}(r) e^{-i\omega_{mn} t}$$

- TD source is singular, $\delta(z)$, so the convergence is algebraic $\sim 1/n^2$.
- We seek exponential convergence through EPS.

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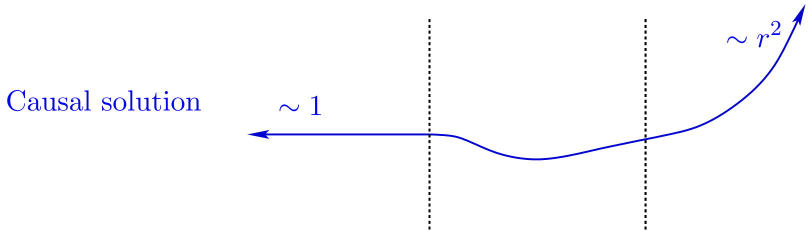

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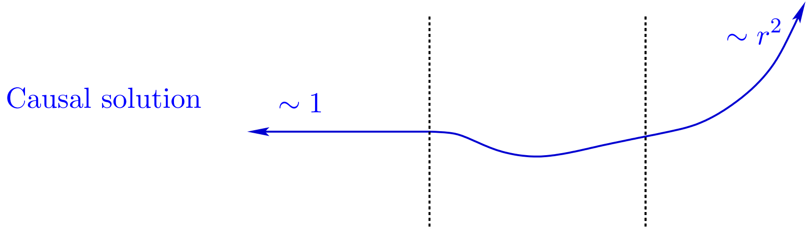

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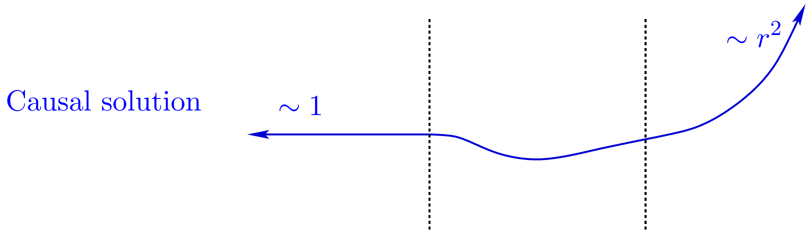

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Extended particular solutions

- We look for a time domain solution of the form

$$\xi(t,r) = \xi^{+}(t,r)\ \theta\left[r - r_p(t)\right] + \xi^{-}(t,r)\ \theta\left[r_p(t) - r\right]$$

- Where

$$\xi^{\pm}(t,r) = \xi_p^{\pm}(t,r) + \xi_{h2}^{\pm}(t,r) + \xi_{h4}^{\pm}(t,r)$$

- Defined through

$$\xi_p^{\pm}(t,r) \equiv \sum_n \tilde{\xi}_p^{\pm}(r) e^{-i\omega_{mn}t},$$

- And

$$\xi_{h2}^{\pm}(t,r) \equiv \sum_n \tilde{\xi}_{h2}^{\pm}(r) e^{-i\omega_{mn}t} \qquad \xi_{h4}^{\pm}(t,r) \equiv \sum_n \tilde{\xi}_{h4}^{\pm}(r) e^{-i\omega_{mn}t}$$

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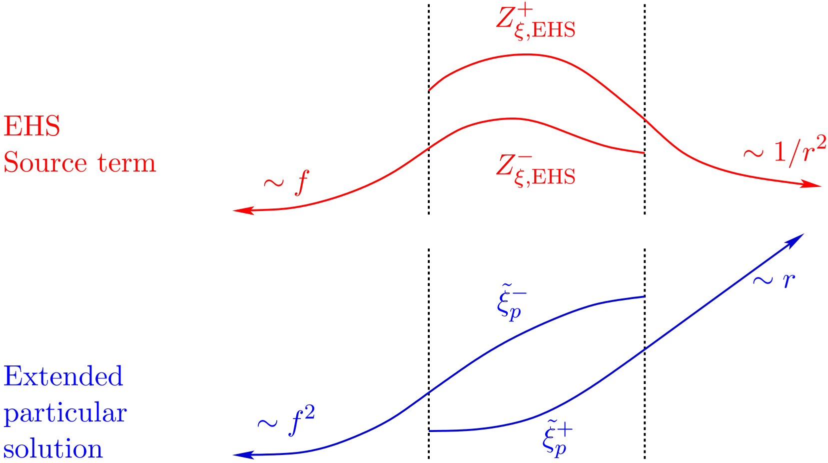
Particular solutions

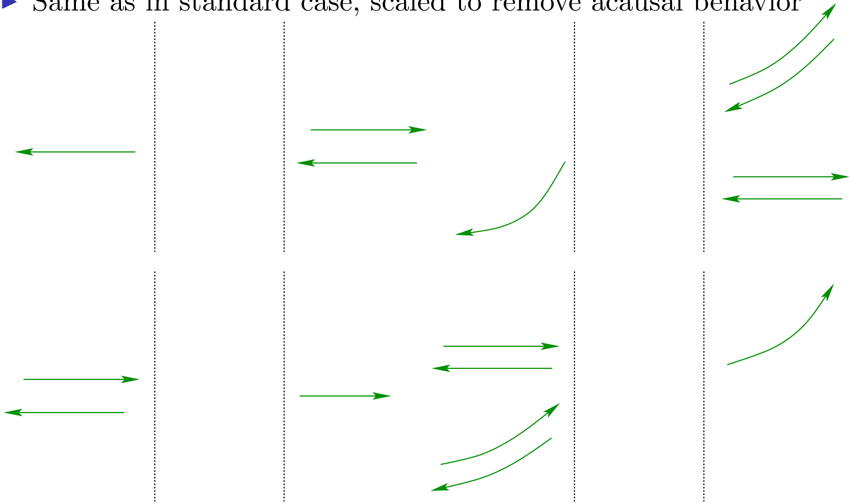
$$\left[\frac{d^2}{dr_*^2}+\omega_{mn}^2-V_0\right]\frac{1}{rf}\left[\frac{d^2}{dr_*^2}+\omega_{mn}^2-V_0\right]\tilde{\xi}^{\mathrm{s}}=Z_\xi$$



Homogeneous solutions

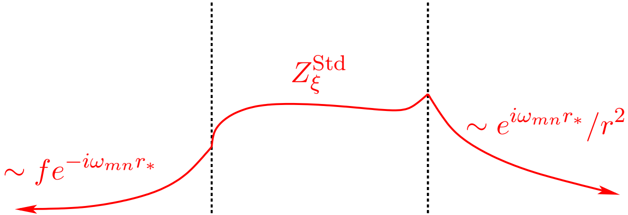
$$\left[\frac{d^2}{dr_*^2} + \omega_{mn}^2 - V_0\right] \frac{1}{rf} \left[\frac{d^2}{dr_*^2} + \omega_{mn}^2 - V_0\right] \tilde{\xi}^{\mathrm{s}} = 0$$

- Same as in standard case, scaled to remove acausal behavior



4nd-order EPS summary

- Std. source
- Std. particular solutions: $\tilde{\xi}_p^{\infty/H}$
- Causality gives homog. sols: $\tilde{\xi}_h^{\pm}$



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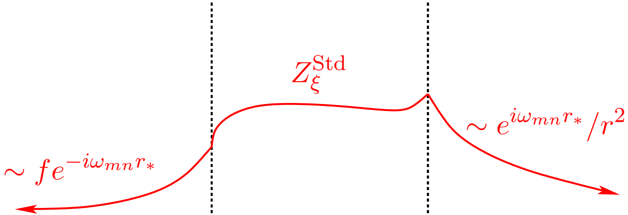

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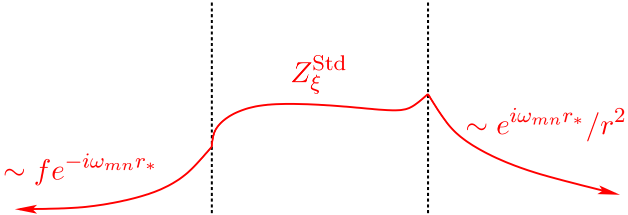

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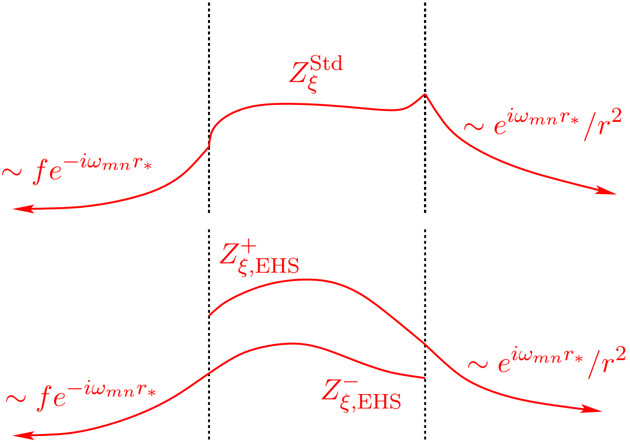

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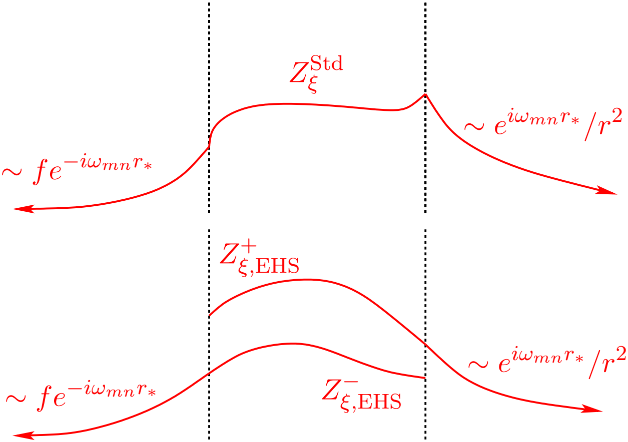

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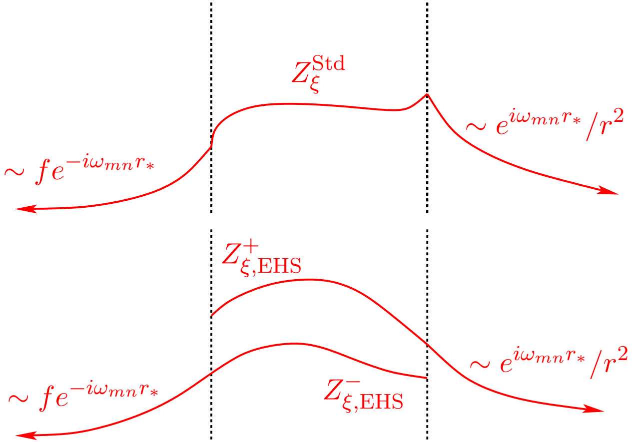

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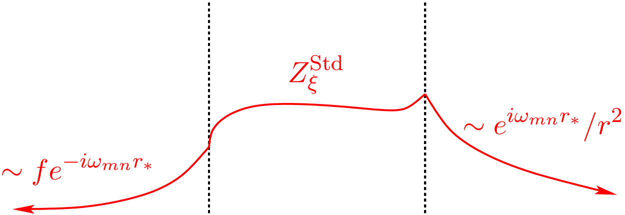

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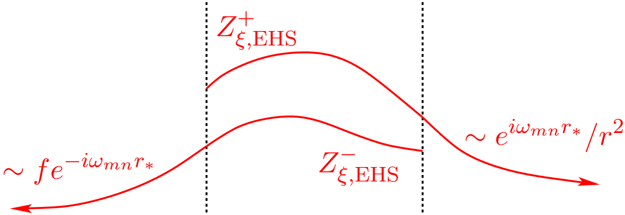

$$\xi_p^{\pm} \equiv \sum \tilde{\xi}_p^{\pm} e^{-i\omega_{mn} t}, \quad \xi_{h2}^{\pm} \equiv \sum \tilde{\xi}_{h2}^{\pm} e^{-i\omega_{mn} t} \quad \xi_{h4}^{\pm} \equiv \sum \tilde{\xi}_{h4}^{\pm} e^{-i\omega_{mn} t}$$

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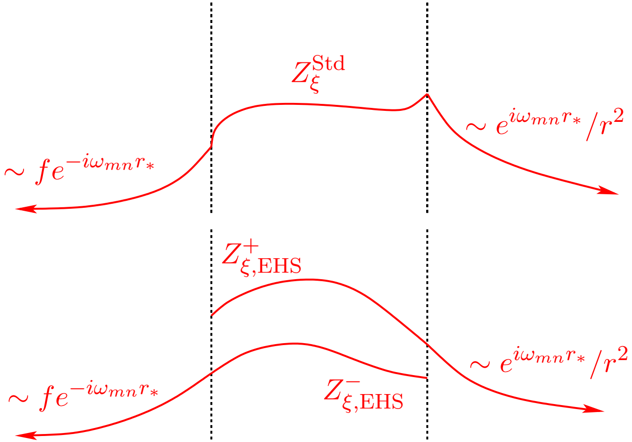

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$$\xi^\pm(t,r) = \xi_p^\pm(t,r) + \xi_{h2}^\pm(t,r) + \xi_{h4}^\pm(t,r)$$

Even-parity scalar solution

$$\xi_p^\pm \equiv \sum \tilde{\xi}_p^\pm e^{-i\omega_{mn}t}, \quad \xi_{h2}^\pm \equiv \sum \tilde{\xi}_{h2}^\pm e^{-i\omega_{mn}t} \quad \xi_{h4}^\pm \equiv \sum \tilde{\xi}_{h4}^\pm e^{-i\omega_{mn}t}$$

$$\xi^\pm(t,r) = \xi_p^\pm(t,r) + \xi_{h2}^\pm(t,r) + \xi_{h4}^\pm(t,r)$$

- By the same argument of analyticity, we claim

$$\xi(t,r) = \xi^+(t,r)\ \theta\left[r - r_p(t)\right] + \xi^-(t,r)\ \theta\left[r_p(t) - r\right]$$

solves

$$\left[-\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial r_*^2} - V_0\right] \frac{1}{rf} \left[-\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial r_*^2} - V_0\right] \xi^{\mathrm{s}} = \alpha(r)\Psi_{\mathrm{ZM}} + \beta(r)\frac{d\Psi_{\mathrm{ZM}}}{dr_*}$$

- This works, but I'm not going to show you ...

Even-parity scalar solution

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$$\xi^\pm(t,r) = \xi_p^\pm(t,r) + \xi_{h2}^\pm(t,r) + \xi_{h4}^\pm(t,r)$$

- By the same argument of analyticity, we claim

$$\xi(t,r) = \xi^+(t,r)\ \theta\left[r - r_p(t)\right] + \xi^-(t,r)\ \theta\left[r_p(t) - r\right]$$

solves

$$\left[-\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial r_*^2} - V_0\right] \frac{1}{rf} \left[-\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial r_*^2} - V_0\right] \xi^{\mathrm{s}}$$

$$= \alpha(r)\Psi_{\mathrm{ZM}} + \beta(r)\frac{d\Psi_{\mathrm{ZM}}}{dr_*}$$

- This works, but I'm not going to show you ...

Even-parity scalar solution

$$\xi_p^\pm \equiv \sum \tilde{\xi}_p^\pm e^{-i\omega_{mn}t}, \quad \xi_{h2}^\pm \equiv \sum \tilde{\xi}_{h2}^\pm e^{-i\omega_{mn}t} \quad \xi_{h4}^\pm \equiv \sum \tilde{\xi}_{h4}^\pm e^{-i\omega_{mn}t}$$

$$\xi^\pm(t,r) = \xi_p^\pm(t,r) + \xi_{h2}^\pm(t,r) + \xi_{h4}^\pm(t,r)$$

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$$\left[-\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial r_*^2} - V_0\right]\frac{1}{rf}\left[-\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial r_*^2} - V_0\right]\xi^{\rm s} = \alpha(r)\Psi_{\rm ZM} + \beta(r)\frac{d\Psi_{\rm ZM}}{dr_*}$$

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Conclusions

- We consider eccentric orbits on Schwarzschild
- Highly accurate metric perturbation in Regge-Wheeler gauge at location of particle
- Performed the odd-parity gauge transformation taking the metric perturbation from Regge-Wheeler to Lorenz gauge
- Partial annihilators and extended particular solutions give the same result
- Even-parity scalar part calculated using EPS
- Even-parity vector part remains. Equations are of a similar form.
- Allows for mode-sum regularization
- Will allow for high accuracy calculation of the self-force

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