The importance of including small body spin effects in the modelling of EMRIs and IMRIs

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Outline

- ❄ Motivation
- ❄ Source modelling
- ❄ Fisher Matrix analysis
- ❄ Parameter estimation results
- ❄ Model error estimates
- ❄ Conclusions and future work

Motivation

- Future low-frequency GW detectors will provide information about the astrophysical properties of compact objects through EMRI and IMRI observations (Jon's talk on LISA)

- To do this science we need to develop accurate waveform templates that can be used for source detection and data analysis

- Most astrophysical black holes will have spin $\rightarrow$ true gravitational waveforms will include small body spin effects

- Develop a 'numerical kludge (NK)' waveform model that includes small body spin effects (SBSEs), and their associated conservative self-force corrections (Jon's talk on NK models)

- Explore the regime at which SBSEs become important for parameter estimation

- Explore the extent to which the inclusion of conservative corrections reduces the magnitude of model errors

Source modelling

- Build NK model that includes SBSEs using the equations of motion of a spinning particle in the equatorial plane of a Kerr BH [Saijo et al, PRD 58, 064005, 1998]

- Include two types of corrections: i) 1st order conservative corrections to amend the orbital phase evolution; ii) 2nd order radiative corrections in $\dot{E}$, $\dot{L}_z$ to evolve the geodesic params of the inspiralling object

- Equations of motion

$$\Sigma_s \Lambda_s \frac{dt}{d\tau} = a\left(1 + \frac{3Ms^2}{p\Sigma_s}\right)\left[\tilde{J}_z - (a+s)\tilde{E}\right] + \frac{p^2 + a^2}{\Delta} P_s , \tag{1}$$

$$\Sigma_s \Lambda_s \frac{d\varphi}{d\tau} = \left(1 + \frac{3Ms^2}{p\Sigma_s}\right)\left[\tilde{J}_z - (a+s)\tilde{E}\right] + \frac{a}{\Delta} P_s , \tag{2}$$

$$\Sigma_s \Lambda_s \frac{dp}{d\tau} = \pm\sqrt{R_s} , \tag{3}$$

- where

$$
\begin{aligned}
\Sigma_s &= p^2 \left(1 - \frac{M s^2}{p^3}\right), \\
\Lambda_s &= 1 - \frac{3 M s^2 p [\tilde{J}_z - (a+s)\tilde{E}]^2}{\Sigma_s^3}, \\
R_s &= P_s^2 - \Delta \left\{ \frac{\Sigma_s^2}{p^2} + [\tilde{J}_z - (a+s)\tilde{E}]^2 \right\}, \\
P_s &= \left[(p^2 + a^2) + as \left(1 + \frac{M}{p}\right) \right] \tilde{E} - \left(a + s \frac{M}{p} \right) \tilde{J}_z, \\
\Delta &= p^2 - 2Mp + a^2,
\end{aligned} \tag{4}
$$

- These eqns of motion are valid only to linear order in the spin of the inspiralling body, hence

$$
\begin{aligned}
\frac{E}{\mu} &= \frac{r^2 - 2r \pm (q + \hat{s}/r)\sqrt{r + 3q\hat{s}/r} - 5q\hat{s}/2r}{r\sqrt{r^2 - 3r \pm (2q + \frac{3\hat{s}}{r})\sqrt{r + 3q\hat{s}/r} - 6q\hat{s}/r}}, \\
\frac{L_z}{\mu M} &= \frac{\pm\sqrt{r + 3q\hat{s}/r}\left(r^2 + q^2 + q\hat{s}(r+1)/r\right) - 2rq + \hat{s}r(r - \frac{7}{2})}{r\sqrt{r^2 - 3r \pm (2q + 3\hat{s}/r)\sqrt{r + 3q\hat{s}/r} - 6q\hat{s}/r}},
\end{aligned} \tag{5}
$$

where $r = p/M$, $\hat{s} = s/M = \eta\chi$, with χ the dimensionless spin parameter of the inspiralling BH, $\eta = \mu/M$ the mass ratio, and $q = a/M$

- Use the radiation fluxes derived by [Gair et al PRD 73, 064037, 2006] augmented with accurate BH perturbation theory results that include small body spin corrections [Tanaka et al, PRD 54, 3762, 1996]

$$\begin{aligned}
\dot{E} &= -\frac{32}{5}\frac{\mu^2}{M}\left(\frac{1}{r}\right)^5 \Bigg\{ 1 - \frac{1247}{336}\left(\frac{1}{r}\right) + \left(4\pi - \frac{73}{12}q - \frac{25}{4}\eta\chi\right)\left(\frac{1}{r}\right)^{3/2} + \\
&\quad \left(-\frac{44711}{9072} + \frac{33}{16}q^2 + \frac{71}{8}q\eta\chi\right)\left(\frac{1}{r}\right)^2 + \text{higher order Teukolsky fits} \Bigg\}, \\
\dot{L}_z &= -\frac{32}{5}\frac{\mu^2}{M}\left(\frac{1}{r}\right)^{7/2} \Bigg\{ 1 - \frac{1247}{336}\left(\frac{1}{r}\right) + \left(4\pi - \frac{61}{12}q - \frac{19}{4}\eta\chi\right)\left(\frac{1}{r}\right)^{3/2} + \\
&\quad \left(-\frac{44711}{9072} + \frac{33}{16}q^2 + \frac{59}{8}q\eta\chi\right)\left(\frac{1}{r}\right)^2 + \text{higher order Teukolsky fits} \Bigg\}.
\end{aligned} \tag{6}$$

- Using these fluxes we can evolve a circular orbit using the ‘circular goes to circular rule’ [Tanaka et al, PRD 54, 3762, 1996]

$$\dot{E}(r) = \frac{1}{r^{3/2} + q}\left(1 - \frac{3}{2}\eta\chi\frac{\sqrt{r} - q}{r^2 + q\sqrt{r}}\right)\dot{L}_z(r). \tag{7}$$

- Furthermore, the evolution in time of the radial coordinate is given by

$$\dot{r} = \frac{\mathrm{d}r}{\mathrm{d}E}\dot{E} = \frac{\mathrm{d}r}{\mathrm{d}L_z}\dot{L}_z. \tag{8}$$

- This model only includes the radiative piece of the self-force, which drives the evolution of the shape constants. We should also include the conservative component, which leads to an accumulation of a phase error over time. We include this effect by amending the evolution of the ϕ frequency as follows,

$$\frac{\mathrm{d}\phi}{\mathrm{d}t} = \left(\frac{\mathrm{d}\phi}{\mathrm{d}t}\right)_{\mathrm{geo}}\left(1 + \delta\Omega\right). \tag{9}$$

- Self-force program has made great progress in the past few years (Monday & Tuesday talks), and the computation of the gravitational self-force for Kerr inspirals is under way [Dolan et al, arXiv:1107.0012]. We require gravitational self-force corrections for the inspirals of spinning black holes into Kerr black holes. Not available at present.
- Use available post-Newtonian (PN) results to correct the NK model
- Ensure that asymptotic observables are consistent with PN results in the weak field (see [Babak et al, PRD 75, 024005, 2007] and [Huerta&Gair, PRD, 79, 084021, 2009])

- Re-write the orbital frequency at 2PN order

$$
\begin{aligned}
\Omega &= \frac{1}{M}\left(\frac{1}{r}\right)^{3/2}\left(1-\left(q+\frac{3}{2}\eta\chi\right)\left(\frac{1}{r}\right)^{3/2}+\frac{3}{2}q\eta\chi\left(\frac{1}{r}\right)^{2}\right)\left(1+\delta\Omega\right), \\
&= \frac{1}{M}\left(\frac{1}{r}\right)^{3/2}\left(1-\left(q+\frac{3}{2}\eta\chi\right)\left(\frac{1}{r}\right)^{3/2}+\frac{3}{2}q\eta\chi\left(\frac{1}{r}\right)^{2}\right)\Bigg\{1+ \\
&+ \eta\left(d_0+d_1\left(\frac{1}{r}\right)+\left(d_{1.5}+q\,f_{1.5}+\chi\,g_{1.5}\right)\left(\frac{1}{r}\right)^{3/2}+\left(d_2+k_2\,q\chi\right)\left(\frac{1}{r}\right)^{2}\right)\Bigg\}.
\end{aligned}
\tag{10}
$$

- Compute $\dot{\Omega}=(d\Omega/dr)\dot{r}$, using eqns. (10), and $\dot{r}=\frac{\mathrm{d}r}{\mathrm{d}L_z}\dot{L}_z$ at 2PN order.
- The PN expressions for $\Omega_{\rm PN}$ and $\dot{\Omega}_{\rm PN}$ are given by

$$
\begin{aligned}
\Omega_{\rm PN}^2 &= \frac{m_{\rm T}}{R^3}\Bigg\{1-\frac{m_{\rm T}}{R}(3-\eta)-\left(\frac{m_{\rm T}}{R}\right)^{3/2}\sum_i\left(2\left(\frac{m_i}{m_{\rm T}}\right)^2+3\eta\right)\hat{\boldsymbol{L}}\cdot\boldsymbol{\chi_i} \\
&+ \left(\frac{m_{\rm T}}{R}\right)^2\left(6+\frac{41}{4}\eta-\frac{3\eta}{2}\left(\boldsymbol{\chi_1}\cdot\boldsymbol{\chi_2}-3\hat{\boldsymbol{L}}\cdot\boldsymbol{\chi_1}\hat{\boldsymbol{L}}\cdot\boldsymbol{\chi_2}\right)\right)\Bigg\},
\end{aligned}
\tag{11}
$$

where $m_{\rm T}=M+m$, $\hat{\boldsymbol{L}}$ is a unit vector directed along the orbital momentum, $\boldsymbol{\chi}=\boldsymbol{\chi_1}=\boldsymbol{S_1}/\mu^2$, $\boldsymbol{q}=\boldsymbol{\chi_2}=\boldsymbol{S_2}/M^2$.

* Additionally,

$$\dot{\Omega}_{\rm PN} = \frac{96}{5}\eta m_{\rm T}^{5/3}\omega^{11/3}\left\{1-\left(\frac{743}{336}+\frac{11}{4}\eta\right)(m_{\rm T}\omega)^{2/3}+(4\pi-\beta)(m_{\rm T}\omega)\right.$$
$$\left.+\left(\frac{34103}{18144}+\frac{81}{16}q^2+\sigma+\eta\left(\frac{13661}{2016}+\zeta q^2\right)\right)(m_{\rm T}\omega)^{4/3}\right\}. \quad (12)$$

* The spin–orbit β and spin–spin parameters σ are given by

$$\beta = \frac{1}{12}\sum_i\left(113\frac{m_i^2}{m_{\rm T}^2}+75\eta\right)\boldsymbol{\hat{L}}\cdot\boldsymbol{\chi_i},$$
$$\sigma = \frac{\eta}{48}\left(-247\boldsymbol{\chi_1}\cdot\boldsymbol{\chi_2}+721\boldsymbol{\hat{L}}\cdot\boldsymbol{\chi_1}\boldsymbol{\hat{L}}\cdot\boldsymbol{\chi_2}\right). \quad (13)$$

* Find a coordinate transformation to relate the NK coordinates with those of the PN formalism, i.e., $r \to r(R/M)$

- Compare the expressions for the orbital frequencies and their first time derivatives between the kludge and the PN formalism and find

$$\Omega \quad = \quad \frac{1}{r^{3/2}+q}\left(1-\frac{3}{2}\eta\chi\frac{\sqrt{r}-q}{r^2+q\sqrt{r}}\right)\Bigg\{1+ \\ \eta\left(\frac{1}{8}+\frac{1975}{896}\left(\frac{1}{r}\right)-\left(\frac{27\pi}{10}+q\,\frac{191}{160}\right)\left(\frac{1}{r}\right)^{3/2}+\frac{1152343}{451584}\left(\frac{1}{r}\right)^2\right)\Bigg\}. \tag{14}$$

- Use this relation for Ω and $\dot{r}=\frac{\mathrm{d}r}{\mathrm{d}L_z}\dot{L}_z$ including SBSEs and high-order Teukolsky fits to generate the inspiral trajectory.
- Build waveform using a flat-space quadrupole generation formula applied to the trajectory of the inspiralling body in Boyer-Lindquist coordinates, which we identify with spherical-polar coordnates in a flat-space
- Implement LISA's response function to obtain [Barack et al, PRD 69, 082005, 2004]

$$h_\alpha(t)=\frac{\sqrt{3}}{2D}\left[F_\alpha^+(t)A^+(t)+F_\alpha^\times(t)A^\times(t)\right], \tag{15}$$

where $\alpha=I,II$ refers to the two independent Michelson-like detectors that constitute the LISA response at low frequencies. The functions $A^{+,\times}(t)$, $F_\alpha^{+\times}$ are the polarization coefficients and the antenna pattern functions, respectively

- Include Doppler phase modulation in the detector response

$$\Phi(t) \to \Phi(t) + 2\frac{\mathrm{d}\phi}{\mathrm{d}t} R \sin\theta_S \cos[2\pi(t/T) - \phi_S], \tag{16}$$

  with $\Phi(t)$ the phase of the waveform, $R = 1\mathrm{AU}/\mathrm{c} = 499.00478\mathrm{s}$, $\mathrm{d}\phi/\mathrm{d}t$ the azimuthal velocity of the orbit of the inspiralling object, and (θ_S, ϕ_S) the source's location in the sky
- Construct noise-weighted waveforms using the total LISA noise, $S_h(f)$, which includes instrumental noise, confusion noise from short-period galactic binaries, and confusion noise from extragalactic binaries [Barack et al, PRD 69, 082005, 2004]

$$\hat{h}_\alpha(t) \equiv \frac{h_\alpha(t)}{\sqrt{S_h(f(t))}}, \qquad f(t) = \frac{1}{\pi}\frac{\mathrm{d}\phi}{\mathrm{d}t}, \tag{17}$$

- Consider 11-D parameter space $h = h(t, \theta^i)$, 5 intrinsic: $\ln m, \ln M, q, \chi, p_0$, 6 phase parameters
- For high signal-to-noise ratio (SNR), the expectation value of the errors $\Delta\theta^i$ is given by

$$\left\langle \Delta\theta^i \Delta\theta^j \right\rangle = (\Gamma^{-1})^{ij} + \mathcal{O}(\mathrm{SNR})^{-1}. \tag{18}$$

- The Fisher matrix is given by [Barack et al, PRD 69, 082005, 2004]

$$\Gamma_{ab} = 2\sum_\alpha \int_0^T \partial_a \hat{h}_\alpha(t) \partial_b \hat{h}_\alpha(t) \mathrm{d}t\,. \tag{19}$$

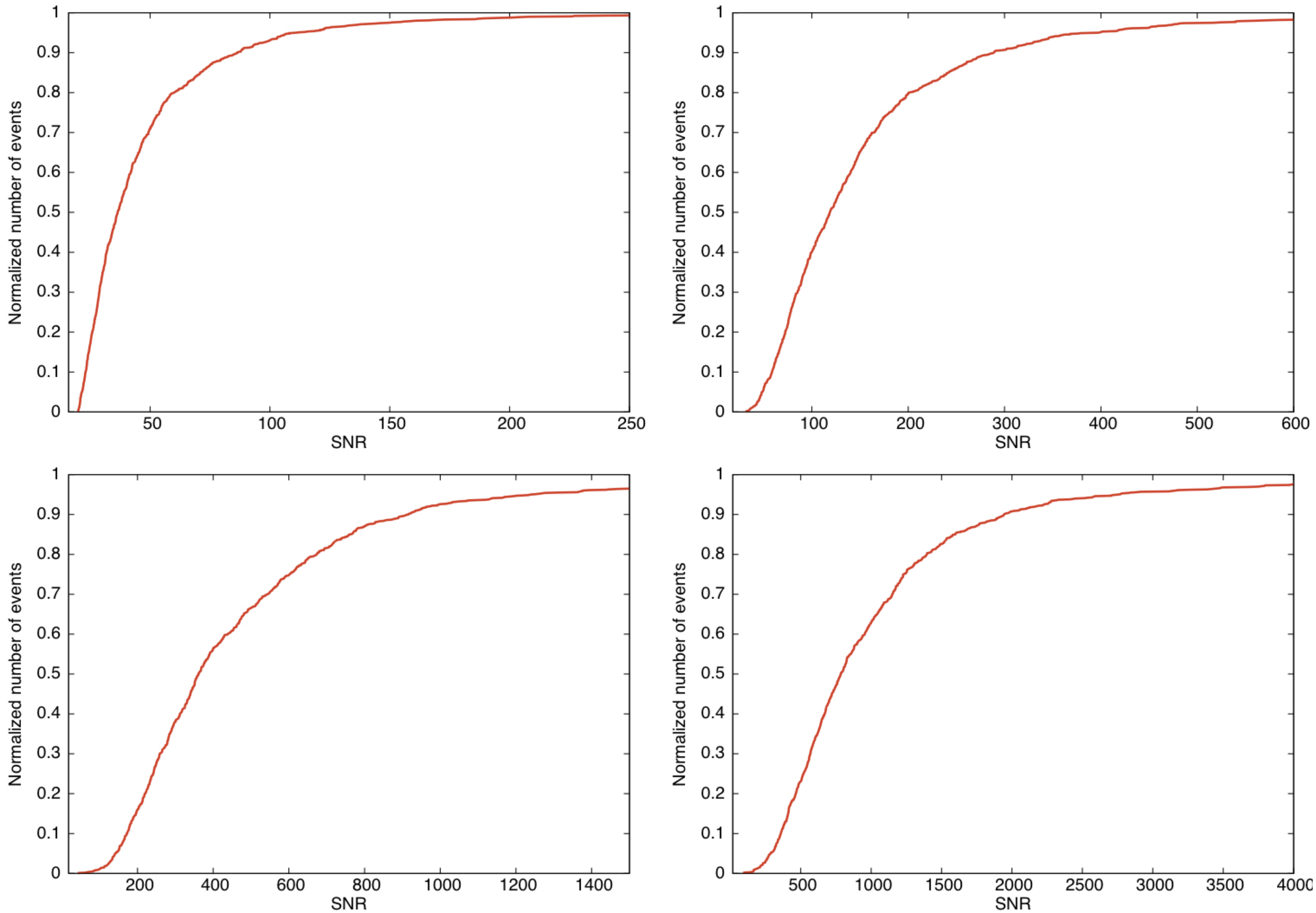
Parameter estimation results

- Estimate noise-induced errors for fixed values of the intrinsic params of the source, but with a Monte Carlo over possible values of the extrinsic params
- Compute the Fisher Matrix for a source at $D = 1$Gpc, and the corresponding SNR using the expression

$$\mathrm{SNR}^2 = 2 \sum_{\alpha=I,II} \int_{t_{\mathrm{init}}}^{t_{\mathrm{LSO}}} \hat{h}_\alpha^2(t)dt. \tag{20}$$

- Renormalise the results to a fixed 'typical' SNR, which is chosen separately for each of the systems considered.
- 'Typical' SNR: Monte Carlo simulation in which the extrinsic parameters of each source are chosen randomly. Consider events distributed uniformly out to a redshift of $z = 1$, and which are detected in a certain time window at the detector. Look at how the SNRs of these detected events are distributed.
- Consider a cosmological population of binary systems with central BHs of redshifted mass $10^6 M_\odot$, and spin parameter $q = 0.9$. The inspiralling BHs have specific spin parameter $\chi = 0.9$, and redshifted masses $\mu = 10 M_\odot$, $\mu = 10^2 M_\odot$, $\mu = 10^3 M_\odot$, and $\mu = 5 \times 10^3 M_\odot$

SNR cumulative distributions



- Results for EMRIs: i) including the spin of the small CO for EMRIs with mass ratios $\eta \lesssim 10^{-5}$ will not significantly affect parameter determination or detection; ii) GW observations will not be able to constrain at all the spin parameter of the inspiralling BH
- At fixed SNR of 400, GW observations will be able to determine the spin parameter of inspiralling BHs for systems with component masses $10^3 M_\odot + 10^6 M_\odot$ to an accuracy better than $\sim 28\%$.
- For an inspiralling BH with mass $\mu = 5 \times 10^3 M_\odot$, at a fixed SNR of 1000

Model		Distribution of $\log_{10}(\Delta X)$ in error, ΔX, for parameter $X =$										
		$\ln(m)$	$\ln(M)$	q	χ	p_0	ϕ_0	θ_S	ϕ_S	θ_K	ϕ_K	$\ln(D)$
	Mean	-3.12	-2.94	-4.38	-1.07	-1.60	-1.48	-2.06	-2.07	-1.85	-1.71	-1.90
$q = 0.9$	St. Dev.	0.089	0.089	0.083	0.081	0.089	0.417	0.379	0.391	0.434	0.468	0.326
	L. Qt.	-3.14	-2.99	-4.40	-1.09	-1.65	-1.91	-2.39	-2.23	-2.13	-2.00	-2.10
$\chi = 0.9$	Med.	-3.13	-2.94	-4.38	-1.06	-1.64	-1.72	-2.07	-2.02	-1.89	-1.73	-1.95
	U. Qt.	-3.03	-2.84	-4.37	-1.02	-1.54	-1.42	-1.87	-1.74	-1.62	-1.53	-1.71
	Mean	-2.62	-2.44	-3.92	-0.60	-1.19	-1.49	-2.09	-2.04	-1.75	-1.62	-1.81
$q = 0.9$	St. Dev.	0.181	0.185	0.176	0.172	0.205	0.359	0.440	0.365	0.398	0.444	0.411
	L. Qt.	-2.77	-2.59	-4.07	-0.70	-1.31	-1.96	-2.44	-2.32	-2.12	-2.09	-2.10
$\chi = 0.1$	Med.	-2.60	-2.41	-3.90	-0.61	-1.21	-1.66	-2.06	-2.03	-1.82	-1.77	-1.89
	U. Qt.	-2.46	-2.29	-3.77	-0.44	-1.09	-1.36	-1.78	-1.69	-1.60	-1.41	-1.67

- We have explored the trend with the spin of the small/big body for the ‘best case’ scenario

- Results for a slowly rotating central BH with spin parameter $q = 0.1$ at a fixed SNR of 500

Model		Distribution of $\log_{10}(\Delta X)$ in error, ΔX, for parameter $X =$										
		$\ln(m)$	$\ln(M)$	q	χ	p_0	ϕ_0	θ_S	ϕ_S	θ_K	ϕ_K	$\ln(D)$
$q = 0.1$ $\chi = 0.9$	Mean	-3.09	-2.92	-2.61	-0.15	-1.62	-1.58	-1.99	-1.89	-1.75	-1.63	-1.79
	St. Dev.	0.071	0.072	0.064	0.062	0.072	0.365	0.401	0.416	0.387	0.416	0.283
	L. Qt.	-3.16	-2.99	-2.66	-0.20	-1.68	-1.82	-2.25	-2.15	-2.00	-1.92	-1.98
	Med.	-3.09	-2.93	-2.62	-0.16	-1.62	-1.66	-1.87	-1.85	-1.77	-1.64	-1.83
	U. Qt.	-3.01	-2.85	-2.56	-0.11	-1.56	-1.41	-1.67	-1.64	-1.54	-1.38	-1.62
$q = 0.1$ $\chi = 0.1$	Mean	-3.09	-2.92	-2.61	-0.15	-1.62	-1.68	-1.99	-1.90	-1.85	-1.71	-1.86
	St. Dev.	0.069	0.070	0.064	0.060	0.070	0.282	0.397	0.444	0.348	0.385	0.254
	L. Qt.	-3.15	-2.98	-2.66	-0.20	-1.68	-1.86	-1.21	-2.12	-2.07	-1.96	-2.01
	Med.	-3.09	-2.92	-2.61	-0.16	-1.62	-1.68	-1.89	-1.83	-1.83	-1.71	-1.89
	U. Qt.	-3.03	-2.86	-2.56	-0.11	-1.56	-1.54	-1.68	-1.59	-1.61	-1.48	-1.72

- For binaries with $\eta \gtrsim 10^{-3}$, the determination of the intrinsic parameters is best accomplished when both binary components are rapidly rotating
- If the central BH is slowly rotating i) the accuracy with which the intrinsic parameters of the system can be determined is not very sensitive to the spin of the inspiralling body; ii) GW observations will not provide an accurate measurement of the spin of the inspiralling object

Model error results

- Model errors arise from the approximate nature of the waveform model
- Use the scheme developed by Cutler & Vallisneri [PRD 76, 104018, 2004] to estimate the magnitude of the model errors, $\Delta_{\rm th}\theta^i$, that could arise in the EMRIs and IMRIs in which the inspiral has significant spin, i.e.,

$$\Delta_{\rm th}\theta^i = \left(\Gamma^{-1}(\theta)\right)^{ij}\left(\partial_j \mathbf{h}_{\rm AP}(\theta)\middle|\left(\mathbf{h}_{\rm GR}(\theta) - \mathbf{h}_{\rm AP}(\theta)\right)\right) \tag{21}$$

- $\mathbf{h}_{\rm GR}(\theta)$ refers to templates that include conservative correction at 2PN order; $\mathbf{h}_{\rm AP}(\theta)$ includes non or only part of the conservative corrections
- This scheme provides reliable results if the waveform is written in an amplitude-phase form

$$\Delta_{\rm th}\theta^i \approx \left(\Gamma^{-1}(\theta)\right)^{ij}\left(\underbrace{\left[\Delta\mathbf{A} + i\mathbf{A}\Delta\mathbf{\Psi}\right]e^{i\mathbf{\Psi}}}_{\text{at }\theta}\middle|\partial_j \mathbf{h}_{\rm AP}(\theta)\right) \tag{22}$$

- Compute the ratio of the systematic error to the noise-induced error $\mathcal{R}$
- If $\mathcal{R} \lesssim 1$ then the estimates obtained from a model that ignores the conservative piece should still be reliable; if $\mathcal{R} >> 1$ then we must include the conservative corrections.
- The vast majority of EMRI sources fulfill $\mathcal{R} \lesssim 1$ [Huerta&Gair, PRD 79, 084021, 2009]

- For more massive binaries model errors are likely to be larger than statistical errors (see [Cutler et al, PRD 76, 104018, 2007])
- For a spinning BH with mass $\mu = 10^3 M_\odot$, and noise–induced errors quoted at a fixed SNR= 400, the error ratio $\log_{10} \mathcal{R}$ is

Model		Distribution of $\log_{10}(\Delta X)$ in error, ΔX, for parameter $X =$										
		$\ln(m)$	$\ln(M)$	q	χ	p_0	ϕ_0	θ_S	ϕ_S	θ_K	ϕ_K	$\ln(D)$
2PN vs 0PN	Mean	1.28	1.30	1.50	1.38	1.30	1.81	1.63	1.68	1.90	1.91	1.90
	St. Dev.	0.599	0.651	0.494	0.651	0.563	0.715	0.725	0.643	0.664	0.673	0.725
	L. Qt.	0.89	0.92	1.22	0.98	0.93	1.24	1.15	1.21	1.26	1.24	1.21
	Med.	1.30	1.40	1.47	1.51	1.40	1.79	1.69	1.76	1.96	1.99	1.98
	U. Qt.	1.75	1.75	1.80	1.88	1.76	2.52	2.23	2.20	2.67	2.69	2.75
2PN vs 1.5PN	Mean	0.24	0.24	0.58	0.44	0.25	0.66	0.77	0.69	0.80	0.84	0.81
	St. Dev.	0.629	0.614	0.519	0.611	0.596	0.759	0.724	0.708	0.740	0.691	0.737
	L. Qt.	-0.25	-0.24	0.35	-0.04	-0.27	0.04	0.05	0.09	-0.03	0.03	-0.01
	Med.	0.41	0.40	0.57	0.53	0.44	0.70	0.91	0.73	0.94	0.95	0.92
	U. Qt.	0.79	0.77	0.87	0.97	0.79	1.36	1.61	1.44	1.65	1.75	1.74

- For a spinning BH with mass $\mu = 5 \times 10^3 M_\odot$, and noise–induced errors quoted at a fixed SNR$= 1000$, we find

Model		Distribution of $\log_{10}(\Delta X)$ in error, ΔX, for parameter $X =$										
		$\ln(m)$	$\ln(M)$	q	χ	p_0	ϕ_0	θ_S	ϕ_S	θ_K	ϕ_K	$\ln(D)$
2PN vs 0PN	Mean	1.72	1.73	1.82	1.83	1.73	2.47	2.24	2.22	2.54	2.54	2.56
	St. Dev.	0.709	0.686	0.597	0.614	0.677	0.708	0.679	0.731	0.752	0.789	0.826
	L. Qt.	1.29	1.25	1.52	1.35	1.29	1.92	1.74	1.80	1.90	1.90	1.91
$\chi = 0.9$	Med.	1.81	1.78	1.85	1.92	1.77	2.43	2.24	2.33	2.56	2.56	2.54
	U. Qt.	2.21	2.21	2.18	2.32	2.21	3.04	2.64	2.81	3.29	3.34	3.35
2PN vs 1.5PN	Mean	0.38	0.39	0.76	0.49	0.39	1.04	0.93	0.90	1.04	1.11	1.10
	St. Dev.	0.719	0.604	0.490	0.585	0.594	0.720	0.715	0.637	0.799	0.735	0.816
	L. Qt.	-0.18	-0.16	0.56	-0.11	-0.18	0.40	0.38	0.30	0.39	-0.02	0.35
$\chi = 0.9$	Med.	0.47	0.45	0.82	0.64	0.51	0.96	1.00	0.98	1.08	1.17	1.11
	U. Qt.	0.90	0.87	1.04	1.04	0.84	1.77	1.54	1.45	1.78	1.93	1.94
2PN vs 1.5PN	Mean	0.50	0.50	0.77	0.63	0.50	1.06	0.97	0.92	1.11	1.14	1.10
	St. Dev.	0.708	0.622	0.405	0.612	0.621	0.731	0.632	0.680	0.701	0.703	0.795
	L. Qt.	0.02	0.01	0.58	0.14	0.03	0.39	0.38	0.28	0.34	0.36	0.30
$\chi = 0.1$	Med.	0.63	0.63	0.78	0.77	0.62	1.00	1.00	0.97	1.12	1.16	1.18
	U. Qt.	0.98	0.98	1.02	1.11	0.97	1.73	1.68	1.60	2.00	2.01	1.99

- $\mathcal{R}$ is smaller when $\mathbf{h}_{\rm AP}$ tends to $\mathbf{h}_{\rm GR}$
- The relative importance of the 1.5PN $\rightarrow$ 2PN change is small, $\mathcal{R} \lesssim 4$, even for the most massive systems
- Including conservative corrections to 2PN order may be sufficient to reduce systematic errors to an acceptable level
- Explore how much the parameter estimation would be degraded if we did not include the spin of the inspiralling body in the waveform template, $\mathbf{h}_{\rm AP}$, but it was included in the "true" waveform, $\mathbf{h}_{\rm GR}$
- In the best case scenario, we found that the model errors associated with the CO mass, SMBH mass and SMBH spin are a factor of ~ 6, 18, 11, bigger than the noise-induced errors, respectively

Conclusions & future work

- ☆ LISA observations will not be able to measure the spin of stellar mass COs inspiralling into a SMBH
- ☆ SBSEs are relevant for detection and parameter estimation for binaries with mass ratio $\eta \gtrsim 10^{-3}$
- ☆ At a fixed SNR of 1000, a LISA observation of a binary with masses $5 \times 10^3 M_\odot + 10^6 M_\odot$ whose components have specific spin parameter $q = \chi = 0.9$, will be able to determine the CO and SMBH masses, the SMBH spin magnitude and the inspiralling BH spin magnitude, χ, to within fractional errors of $\sim 10^{-3}$, 10^{-3}, 10^{-4}, $\sim 10\%$, and the location of the source in the sky and the SMBH spin orientation to within $\sim 10^{-4}$ steradians.
- ☆ SBSEs will be more noticeable when both components of a massive binary are rapidly rotating, but not when the central SMBH is slowly rotating.
- ☆ Including conservative corrections up to 2PN order may be sufficient to reduce these systematic errors to an acceptable level for IMRIs
- ☆ Include SBSEs in IMRI models in which the inspiralling body is not spinning [Huerta&Gair PRD 83, 044020; 83, 044021, 2011] to assess the importance and measurability of these effects in such systems