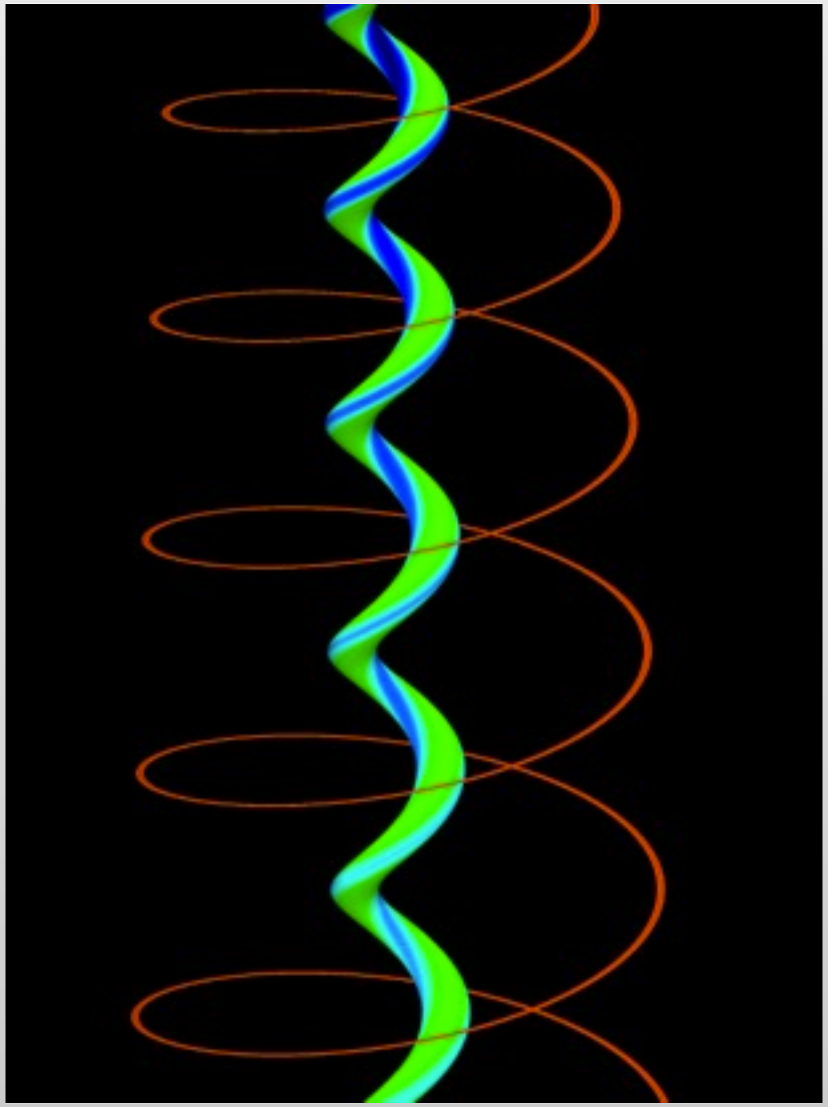
Simulations of unequal-mass black hole binaries

Harald Pfeiffer
CITA

14th CAPRA meeting
July 7 2011, Southampton



Primary NR collaborators:
Luisa Buchman, Abdul Mroue, Mark Scheel, Bela Szilagyi

Code developed by SXS collaboration
Caltech, Cornell, CITA, Washington

❖ Numerical Relativity remarkably successful w/ many applications (Talks by Damour, LeTiec, Lousto)

❖ This talk

- Explain some NR details that are often skipped
- Present non-spinning, unequal mass simulations

Outline

1. Initial data
2. Evolutions
3. Results

Initial data

Initial data

CITA–ICAT

- Metric g_{ij} and extrinsic curvature K_{ij} on hypersurface Σ.
- Must satisfy the constraints

$$R + K^2 - K_{ij}K^{ij} = 0$$
$$\nabla_j \left(K^{ij} - g^{ij} K \right) = 0$$

- Strategy: Split g_{ij} and K_{ij} into smaller pieces, some freely specifiable, the rest determined.

Choose free data $\Rightarrow$ **Solve** elliptic equations $\Rightarrow$ **Assemble** g_{ij}, K_{ij}

- Long history: Lichnerowicz, Choquet-Bruhat, York, O'Murchadha. Recently, York (1999), HP & York (2003)

Conformal methods

York 99; HP, York 03

CITA–ICAT

- **Extrisic curvature decomposition (Hamiltonian picture)**

$$\tilde\nabla^2\psi - \frac{1}{8}\psi\tilde R - \frac{1}{12}\psi^5\tau^2 + \frac{1}{8}\psi^{-7}\tilde A_{ij}\tilde A^{ij} = 0$$

$$\tilde\nabla_j\Big(\frac{1}{\tilde\sigma}(\tilde{\mathbb{L}}X)^{ij}\Big) + \tilde\nabla_j\tilde M^{ij} - \frac{2}{3}\psi^6\tilde\nabla^i\tau = 0$$

Free data $\tilde g_{ij}$, τ, $\tilde M^{ij}$ and $\tilde\sigma$

Both pictures use

$$g_{ij} = \psi^4\tilde g_{ij}$$

$$K^{ij} = \psi^{-10}\tilde A^{ij} + \frac{1}{3}g^{ij}\tau$$

- **Conformal thin sandwich eqns (Lagrangian picture)**

$$\tilde\nabla^2\psi - \frac{1}{8}\psi\tilde R - \frac{1}{12}\psi^5\tau^2 + \frac{1}{8}\psi^{-7}\tilde A_{ij}\tilde A^{ij} = 0$$

$$\tilde\nabla_j\Big(\frac{1}{2\tilde N}(\tilde{\mathbb{L}}\beta)^{ij}\Big) - \tilde\nabla_j\Big(\frac{1}{2\tilde N}\tilde u^{ij}\Big) - \frac{2}{3}\psi^6\tilde\nabla^i\tau = 0$$

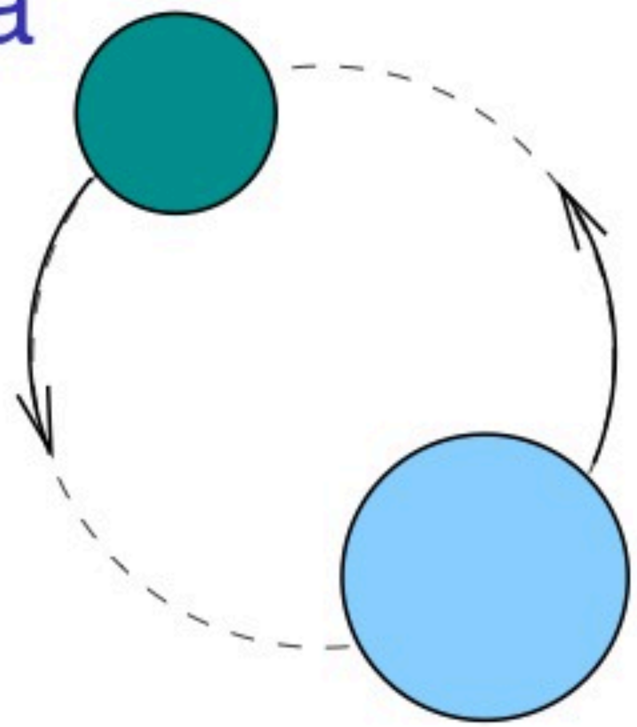
$$\tilde\nabla^2(\tilde N\psi^7) - (\tilde N\psi^7)\Big(\frac{1}{8}\tilde R + \frac{5}{12}\psi^4\tau^2 + \frac{7}{8}\psi^{-8}\tilde A_{ij}\tilde A^{ij}\Big) + \psi^5(\partial_t\tau - \beta^k\partial_k\tau) = 0$$

$$\tilde A^{ij} = \frac{1}{2\tilde N}\Big((\tilde{\mathbb{L}}\beta)^{ij} - \tilde u^{ij}\Big)$$

$$\tilde u_{ij} = \partial_t\tilde g_{ij}$$

Free data $\tilde g_{ij}$, $\partial_t\tilde g_{ij}$, τ and $\tilde N$ <u>or</u> $\partial_t\tau$

Quasi-equilibrium Binary BH initial data

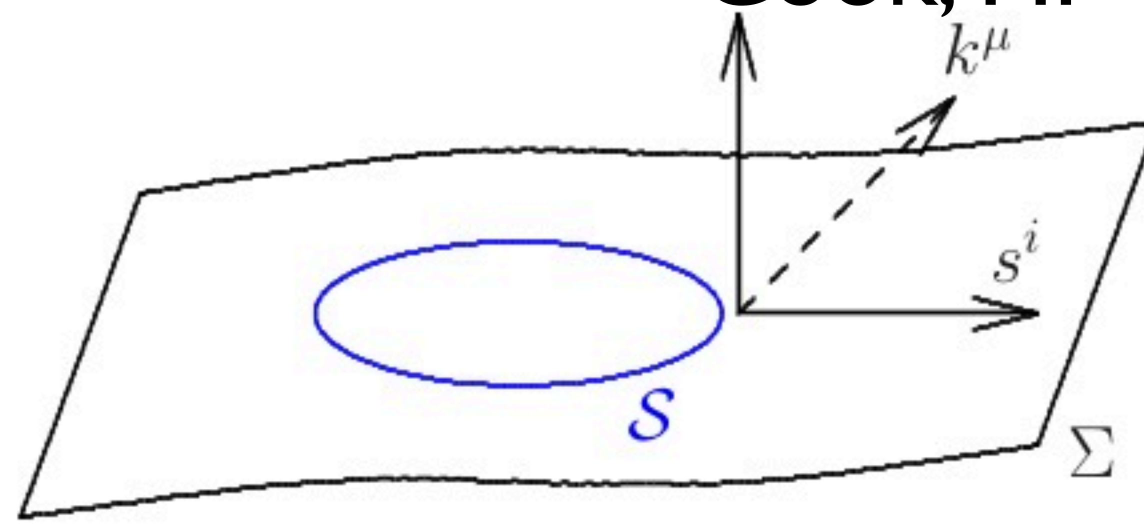


- Time-independence in corotating frame (circular orbit) GGB 2002, Cook & HP '04
- Conformal thin sandwich formalism (York '99)
  - $\partial_t \tilde{g}_{ij} = 0 = \partial_t K$
  - K gauge choice. Use $K = 0$.
  - Good choice for $\tilde{g}_{ij}$ lacking. Use conformal flatness.
- Boundary conditions at infinity

$$\psi = 1$$
$$\beta^i = (\vec{\Omega}_0 \times \vec{r})^i$$
$$N = 1$$

Quasi-equilibrium excision boundary conditions

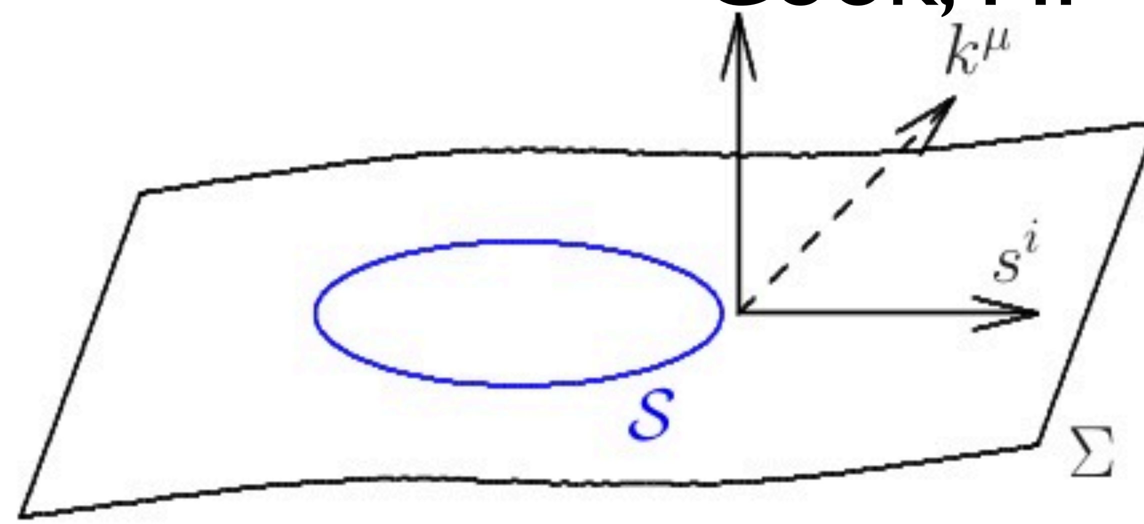
Cook, HP 04



- Excise topological spheres $\mathcal{S}$
- Require
  - $\mathcal{S}$ be apparent horizon(s)
  - Shear of k^μ vanishes (isolated horizon)
  - The AH's remain stationary in evolution

Quasi-equilibrium excision boundary conditions

Cook, HP 04



- Excise topological spheres $\mathcal{S}$
- Require
  - $\mathcal{S}$ be apparent horizon(s)
  - Shear of k^μ vanishes (isolated horizon)
  - The AH's remain stationary in evolution
- $\Rightarrow$ boundary conditions on $\mathcal{S}$

$$\partial_r \psi = \dots$$

$$\beta_\perp = N, \qquad \text{with } \beta^i = \beta_\perp s^i + \beta^i_{||}$$

$$(\tilde{\mathbb{L}}_{\mathcal{S}} \beta_{||})^{ij} = \tilde{D}^{(i}\beta^{j)}_{||} - \frac{1}{2}\tilde{h}^{ij}\tilde{D}_k \beta^k_{||} = 0$$

Initial-data procedure

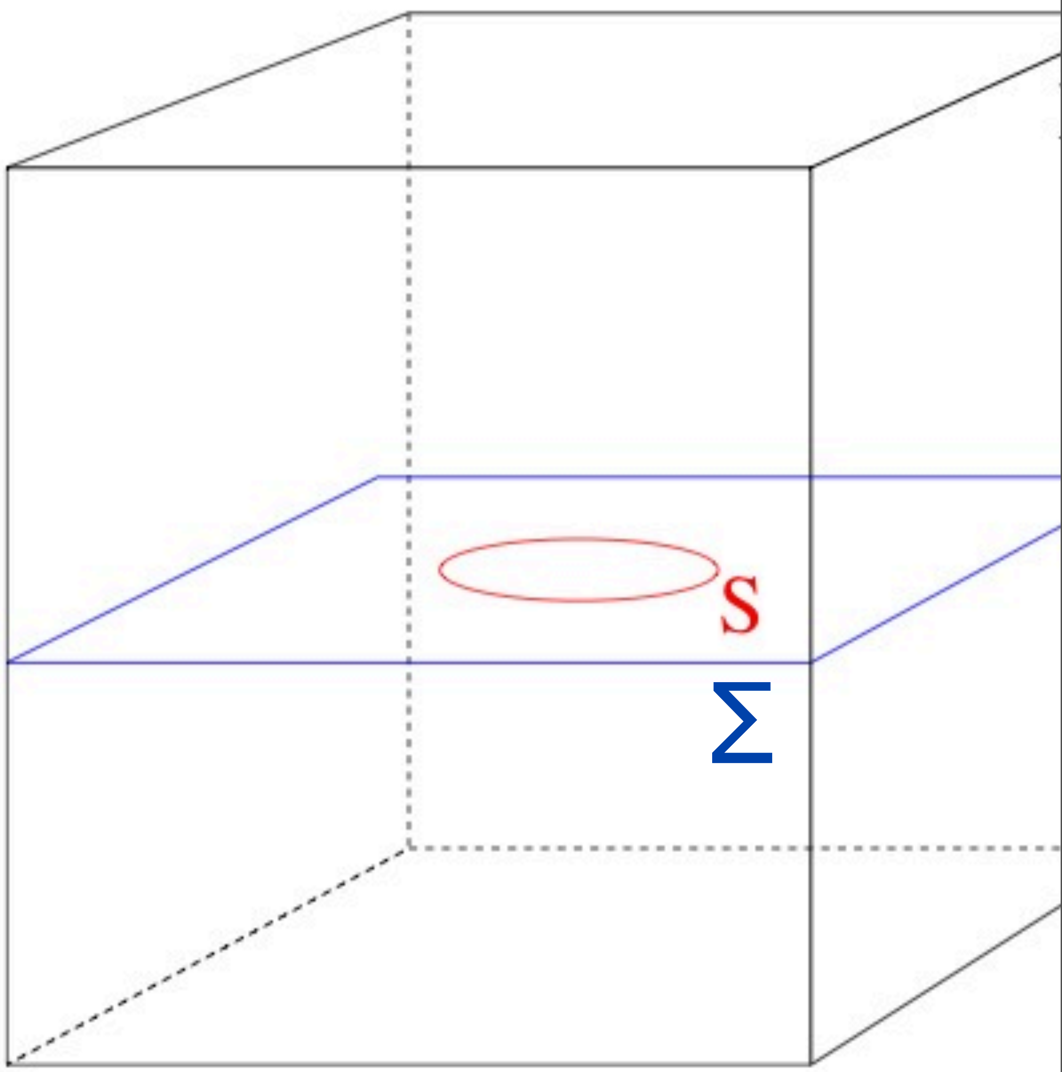
1. Choose decomposition of constraints
2. Choose parameters
   - rA, rB, D
   - OmegaA, OmegaB
   - Omega0, vr
3. Solve coupled elliptic PDE [skip]
4. Read off physical properties
   - MA, MB
   - SpinA, SpinB [next]
5. If necessary: Adjust parameters and go back to 3. [2nd next]

Black hole spin

- In axisymmetry anglar momentum rigourously defined e.g. via the Hamiltonian that generates the rotation (Brown & York, 1993; isolated/dynamical horizon framework)

$$J = \frac{1}{8\pi}\oint_{\mathcal{S}} (K_{ij} - g_{ij}K)\,\phi^i s^j\,dA$$

- ϕ^i rotational Killing vector
  s^i unit-normal to $\mathcal{S}$ in Σ
  g_{ij} metric in Σ
  K_{ij} extrinsic curvature of Σ in M
- $\mathcal{S}$ sphere at $\infty \Rightarrow$ ADM angular momentum
- $\mathcal{S}$ 2-sphere at finite distance $\Rightarrow$ quasi-local spin



Spin in non-axisymmetric spacetimes

- Would like to define "spin" in absence of axisymmetry.
- Choose "approximate Killing vector" ϕ^i; evaluate

$$J_\phi = \frac{1}{8\pi}\oint_{\mathcal{S}} (K_{ij} - g_{ij}K)\,\phi^i s^j\, dA$$

- Q: How to choose ϕ^i?
  - ϕ^i coordinate rotation, $\vec{\phi} = x\hat{e}_y - y\hat{e}_x$ (depends on coordinate system)
  - Integrate Killing transport equation (Dreyer et al, 2003) (depends on integration path; ϕ^i not smooth)
  - A variational approach

Variational approx. Killing vectors (Co

Cook & Whiting 07
Owen 07
Lovelace, Chu, HP, Owen 08

- Require $D_A\phi^A = 0 \Rightarrow \phi^A = \varepsilon^{AB}\partial_B z$ for some potential z.
(A, B: coordinates within $\mathcal{S}$, D_A derivative within $\mathcal{S}$)

- Minimize functional

$$\mathcal{I} = \oint_{\mathcal{S}} (D_{(A}\phi_{B)})(D^{(A}\phi^{B)})\, dA + \lambda\left(\oint_{\mathcal{S}} \phi_A\phi^A\, dA - N\right)$$

- Results in generalized Eigenvalue problem

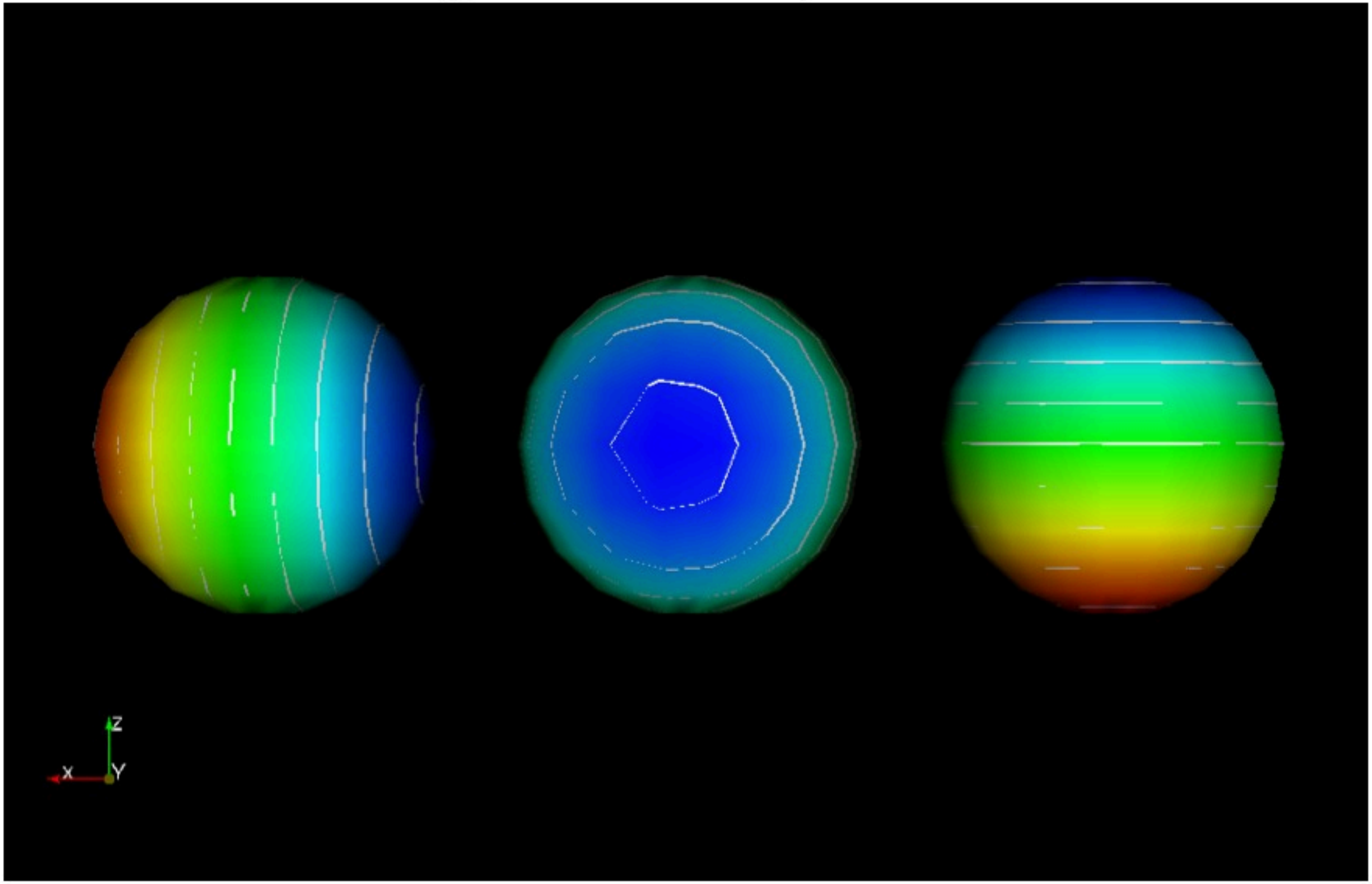
$$\left(D^2 + {}^2R\right) D^2 z + (D_A{}^2R)D^A z = \lambda D^2 z$$

- Spectral expansion $z(\theta,\varphi) = \sum_{l=1}^{L}\sum_{|m|\le l} A^{lm} Y_{lm}(\theta,\varphi)$ results in matrix-equation for coefficients A^{lm}:

$$\Rightarrow M^{lm}{}_{l'm'}A^{l'm'} = \lambda N^{lm}{}_{l'm'}A^{l'm'}$$

Example

- The three smallest eigenvalues correspond to rotations



Adjusting ID parameters (part 1)

- Fix D, Ω_0, v_r
- Choose parameters $\lambda \equiv \{r_A, r_B, \vec{\Omega}_A, \vec{\Omega}_B, c_x, c_y\}$ such that

$$F(\lambda) \equiv \{M_A, M_B, \vec{S}_A, \vec{S}_B, P_x^{\mathrm{ADM}}, P_y^{\mathrm{ADM}}\} = F_{\mathrm{target}}$$

  - 8-dim rootfinding.
  - Each function evaluation $F(\lambda)$ requires complete constraint solve!

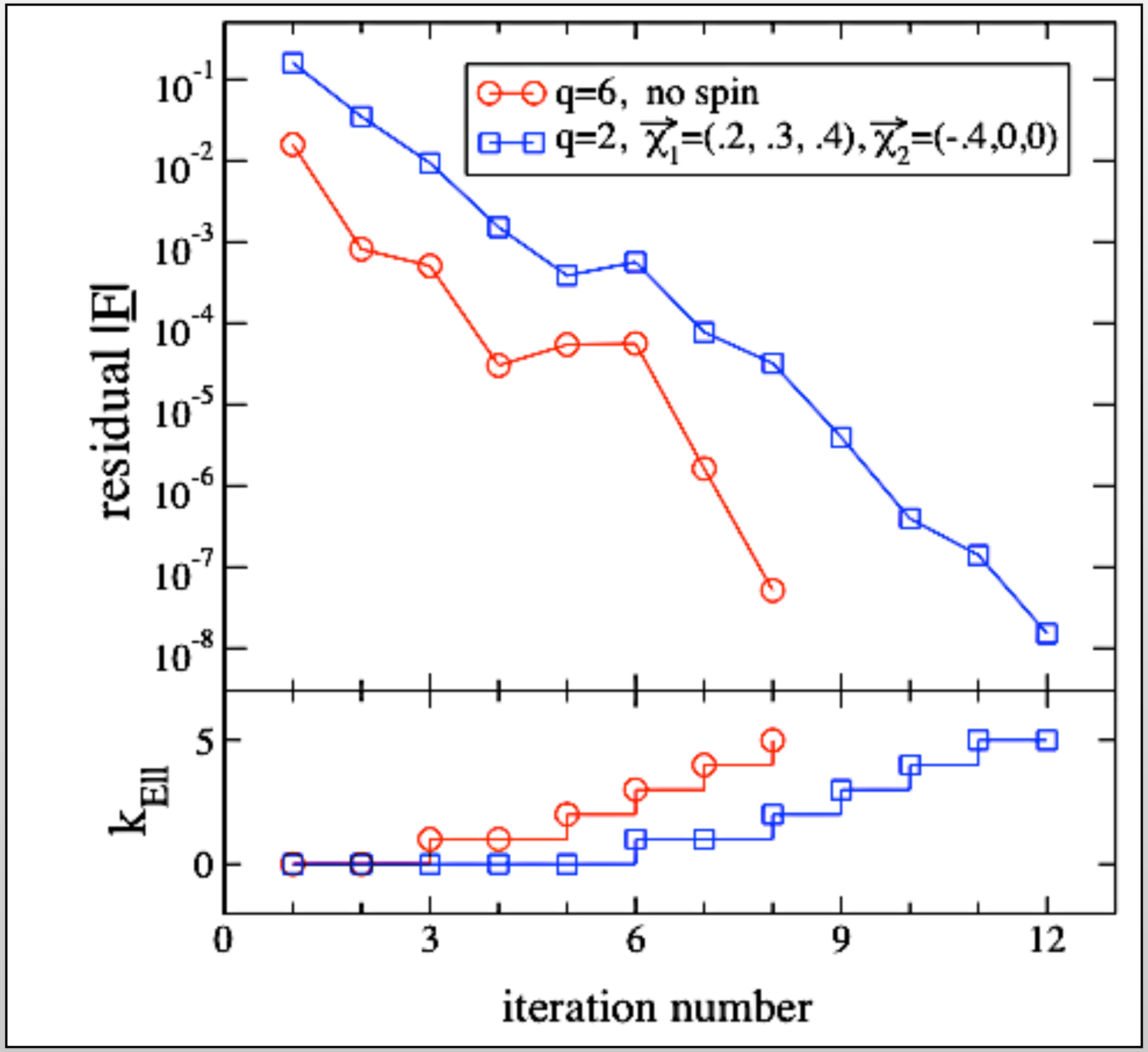
- Newton-Raphson

$$\lambda^{(n+1)} = -\left(\frac{\partial F}{\partial \lambda}\right)^{-1} F(\lambda^{(n)}) \approx \mathcal{J}^{-1} F(\lambda^{(n)})$$

  - Approximate Jacobian J from Newtonian & single BH information (Buchman, HP, Scheel, Szilagyi in prep)

Efficiency of root-finding

- ❖ A dozen function evaluations, only 1-2 at high resolution!



(Buchman, HP, Scheel, Szilagyi in prep)

Evolution

Techniques I: Generalized Harmonic

- Einstein's equations

$$0 = R_{ab}[g_{ab}] = -\frac{1}{2}\Box g_{ab} + \nabla_{(a}\Gamma_{b)} + \text{lower order terms}, \qquad \Gamma_a = -g_{ab}\Box x^b.$$

- Generalized harmonic coordinates $g_{ab}\Box x^b \equiv H_a(x^a, g_{ab})$
  (Friedrich 1985, Pretorius 2005; $H = 0$ used since 1920's)

$$\Box g_{ab} = \text{lower order terms.}$$

$\Rightarrow$ Constraint $C_a \equiv H_a - g_{ab}\Box x^b = 0$

- Constraint damping (Gundlach, et *al.*, Pretorius, 2005)

$$\Box g_{ab} = \gamma\left[t_{(a}C_{b)} - \frac{1}{2}g_{ab}t^c C_c\right] + \text{lower order terms}$$

$$\partial_t C_a \sim -\gamma C_a.$$

Techniqes II: Spectral methods

- Expand in basis-functions, solve for coefficients

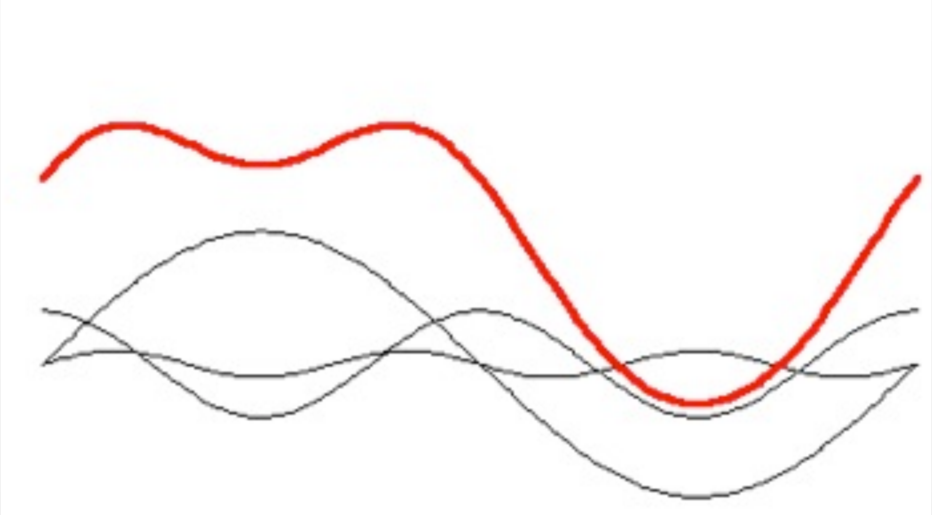
$$u(x,t) = \sum_{k=1}^{N} \tilde{u}_k(t)\Phi_k(x)$$

- Compute derivatives analytically

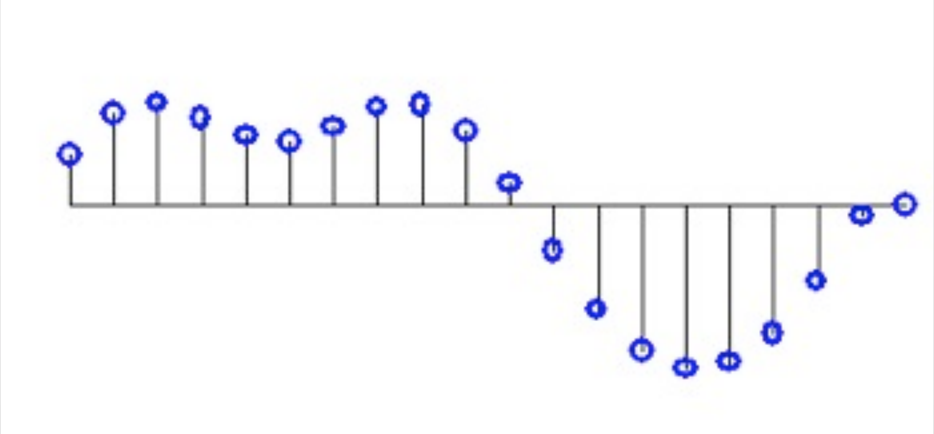
$$u'(x,t) = \sum_{k=1}^{N} \tilde{u}_k(t)\Phi'_k(x)$$

- Compute nonlinearities in physical space

Spectral

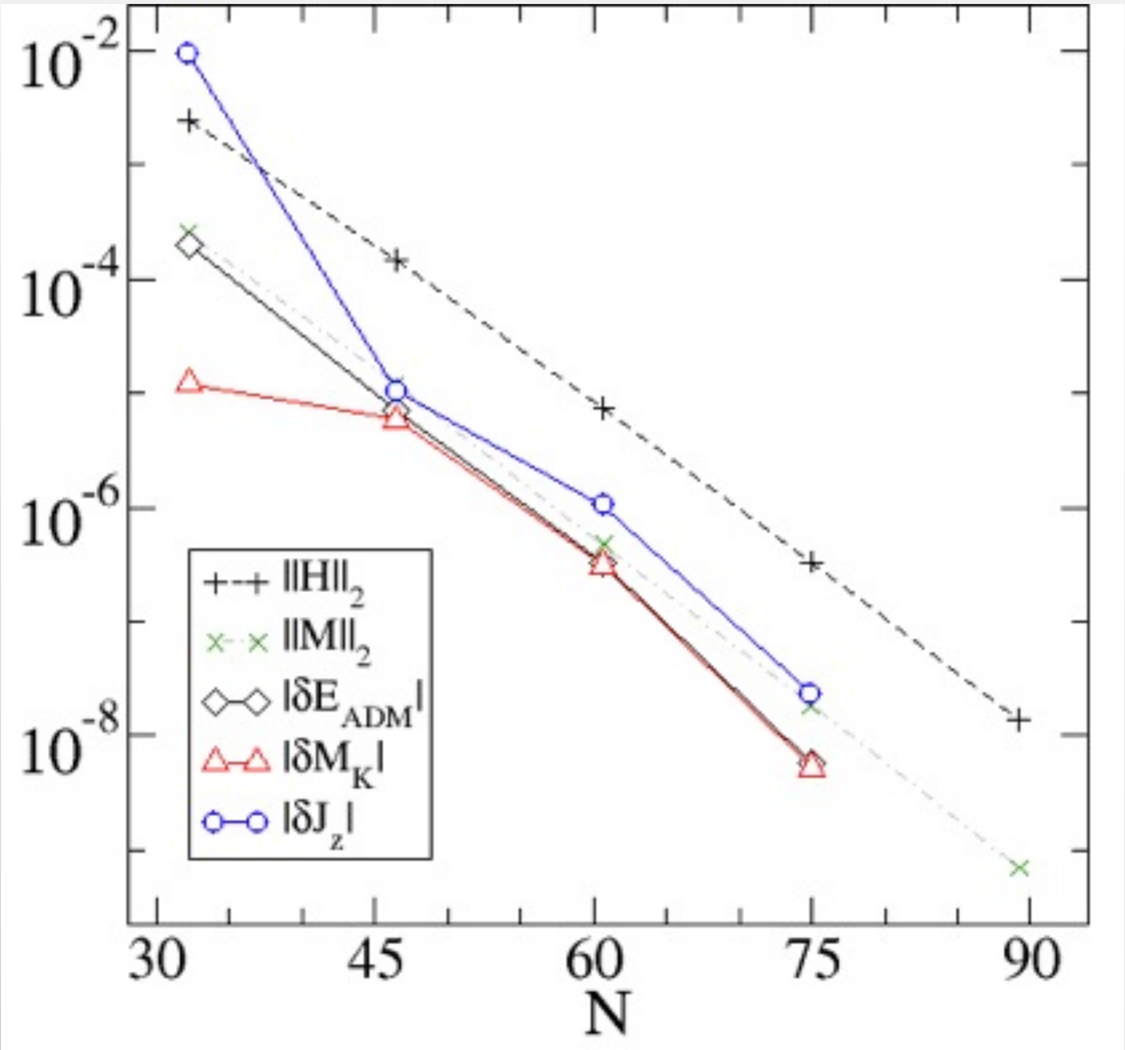


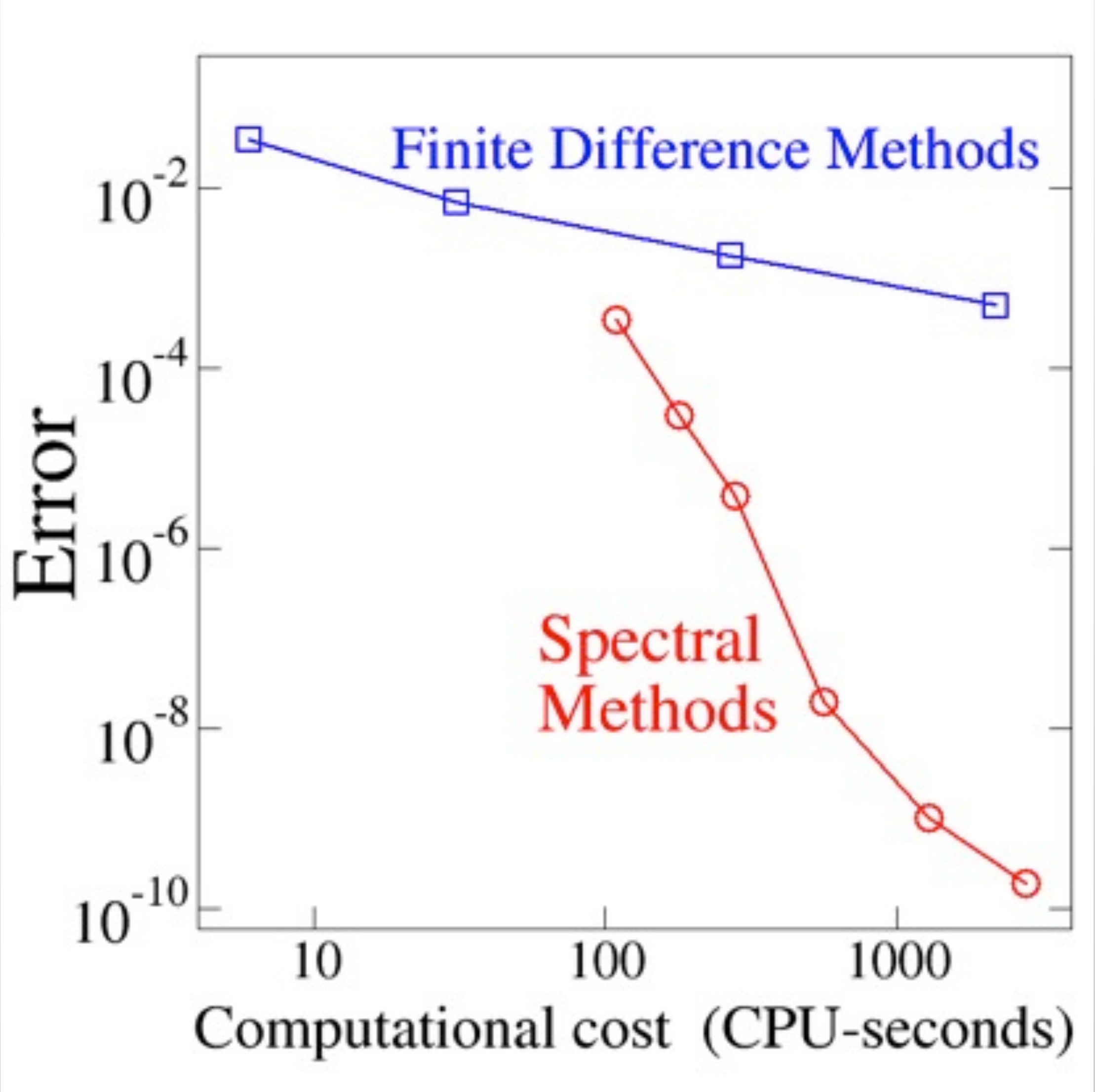
Finite differences



Why spectral methods?

- Smooth solutions ⇒ exponential convergence

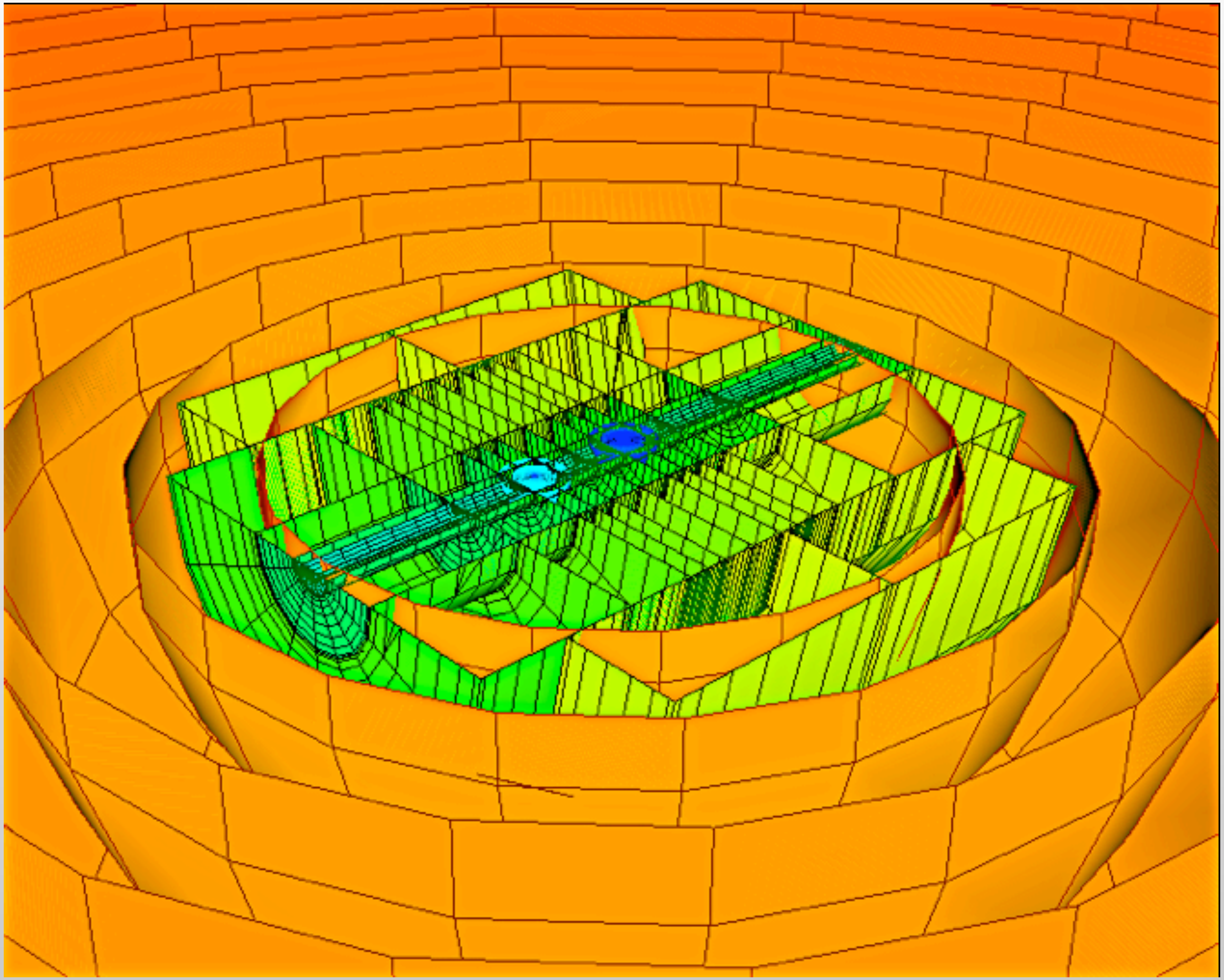




HP et al, 2002

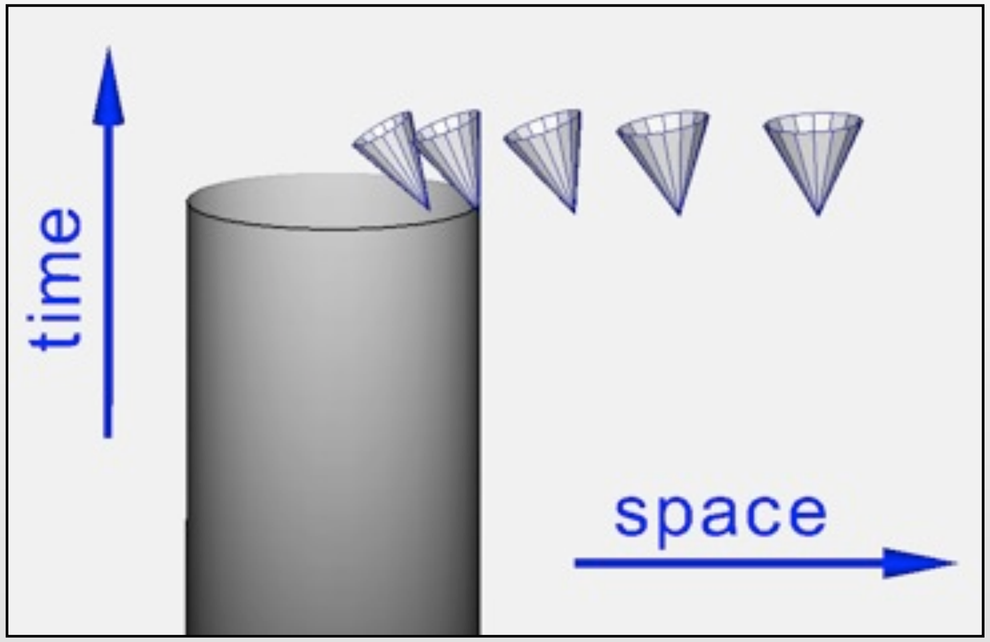
... but more difficult than finite differences

Techniques III: Domain-decomposition



Spectral Einstein Code *SpEC* (Caltech-Cornell-CITA)
http://www.black-holes.org/SpEC.html

More Technical details



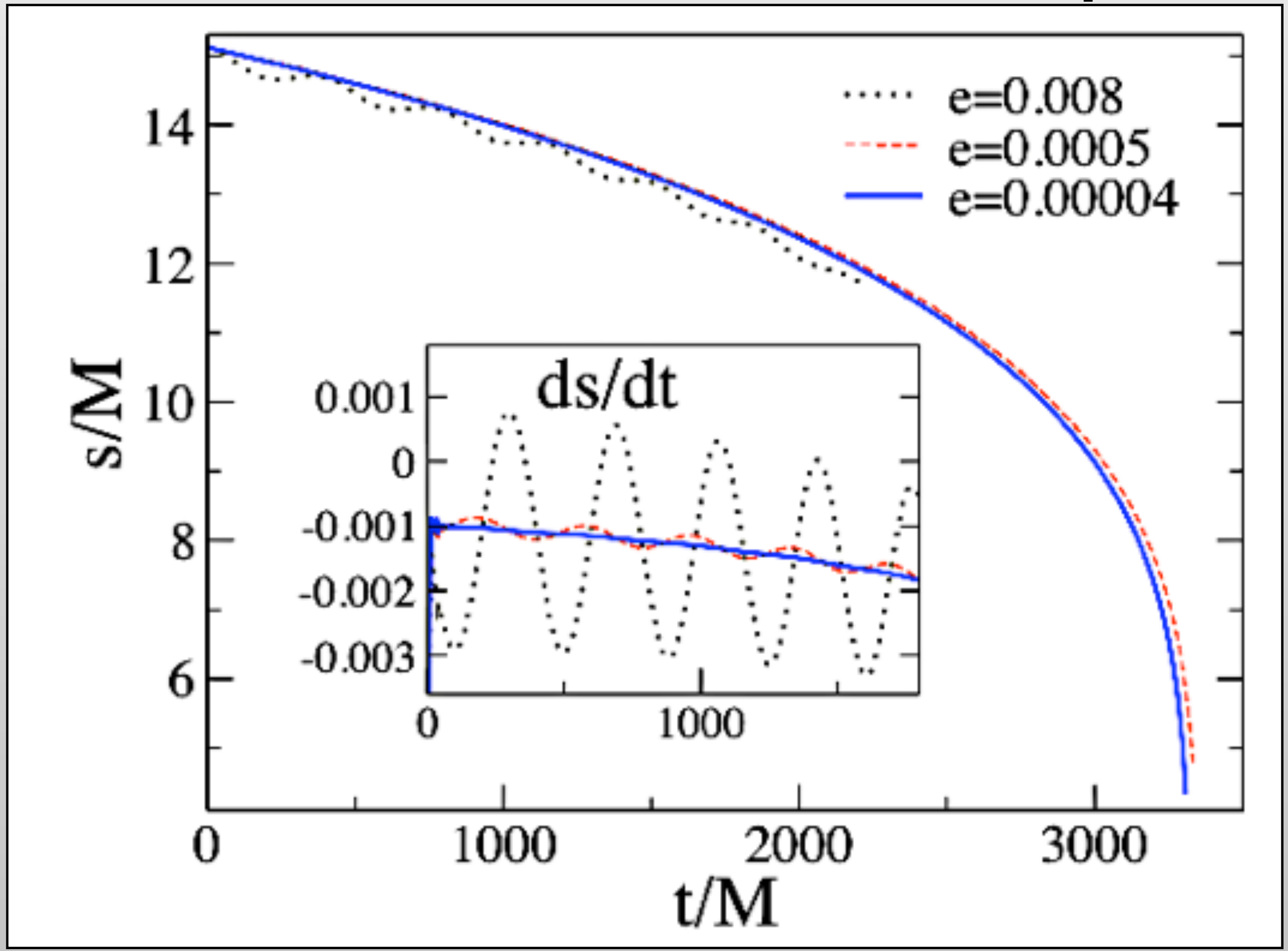
- ❖ BH excision (no inner BCs)
- ❖ Non-reflective outer BCs (Lindblom, Rinne et al. 06)
- ❖ Wave-extraction & extrapolation (Boyle et al 07, Boyle & Mroue, 09)
- ❖ Coordinate conditions (Pretorius; Lindblom & Szilagyi, 09)
- ❖ Domain-decomposition follows BHs (Scheel, et al., 06)

Adjusting ID parameters (part 2)

- Remaining parameters D, Ω_0, v_r
  - Choose separation D for desired time-to-coalescence
  - Ω_0, v_r affect eccentricity & position of periastron
- Goal: Determine Ω_0, v_r such that e=0

1. Pick PN values Ω_0, v_r
2. Evolve ~2 orbits
3. Analyze. If e small done.
4. Adjust Ω_0, v_r such that Newtonian orbit would become circular.
5. Goto 2.

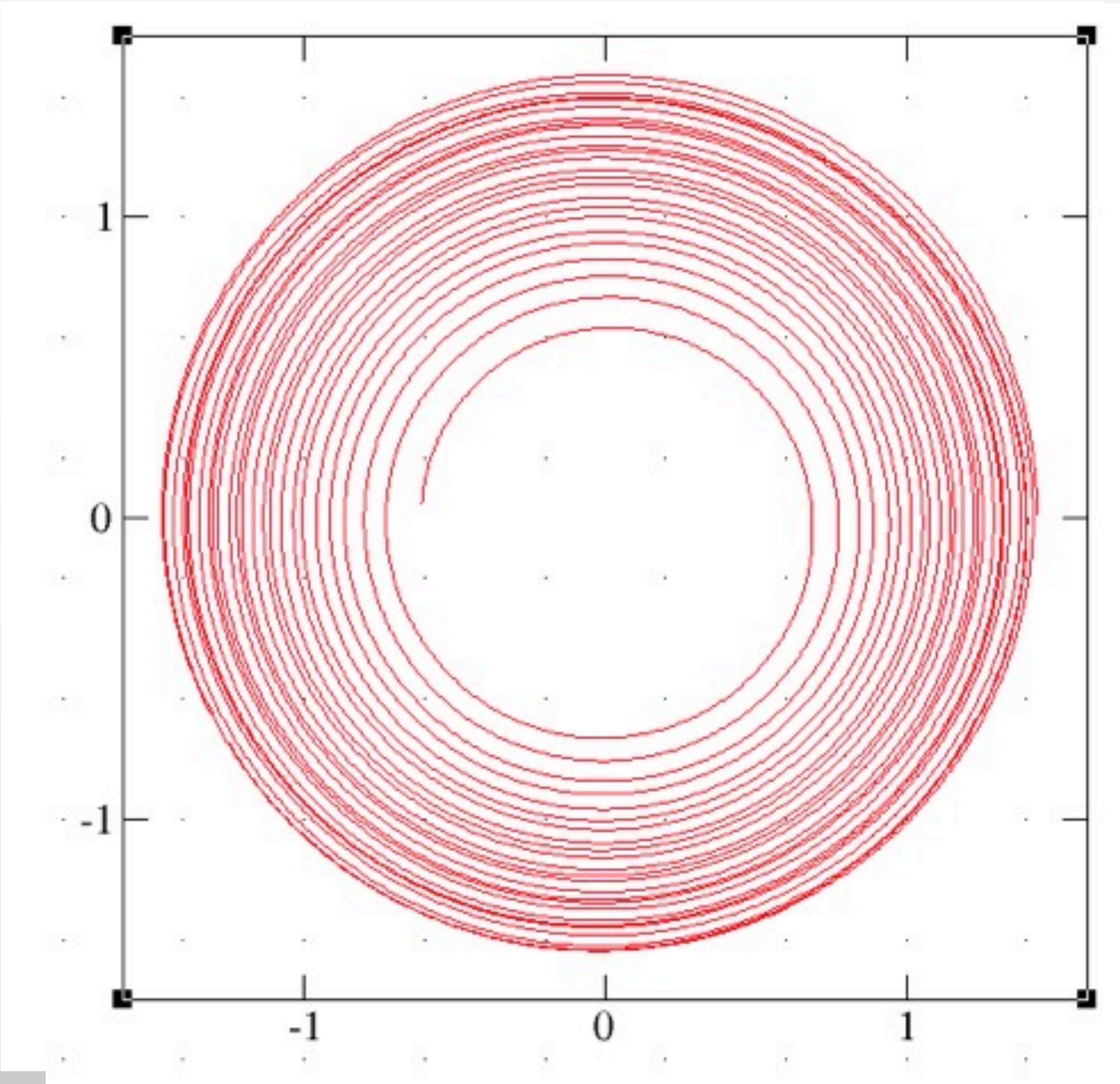
q=4

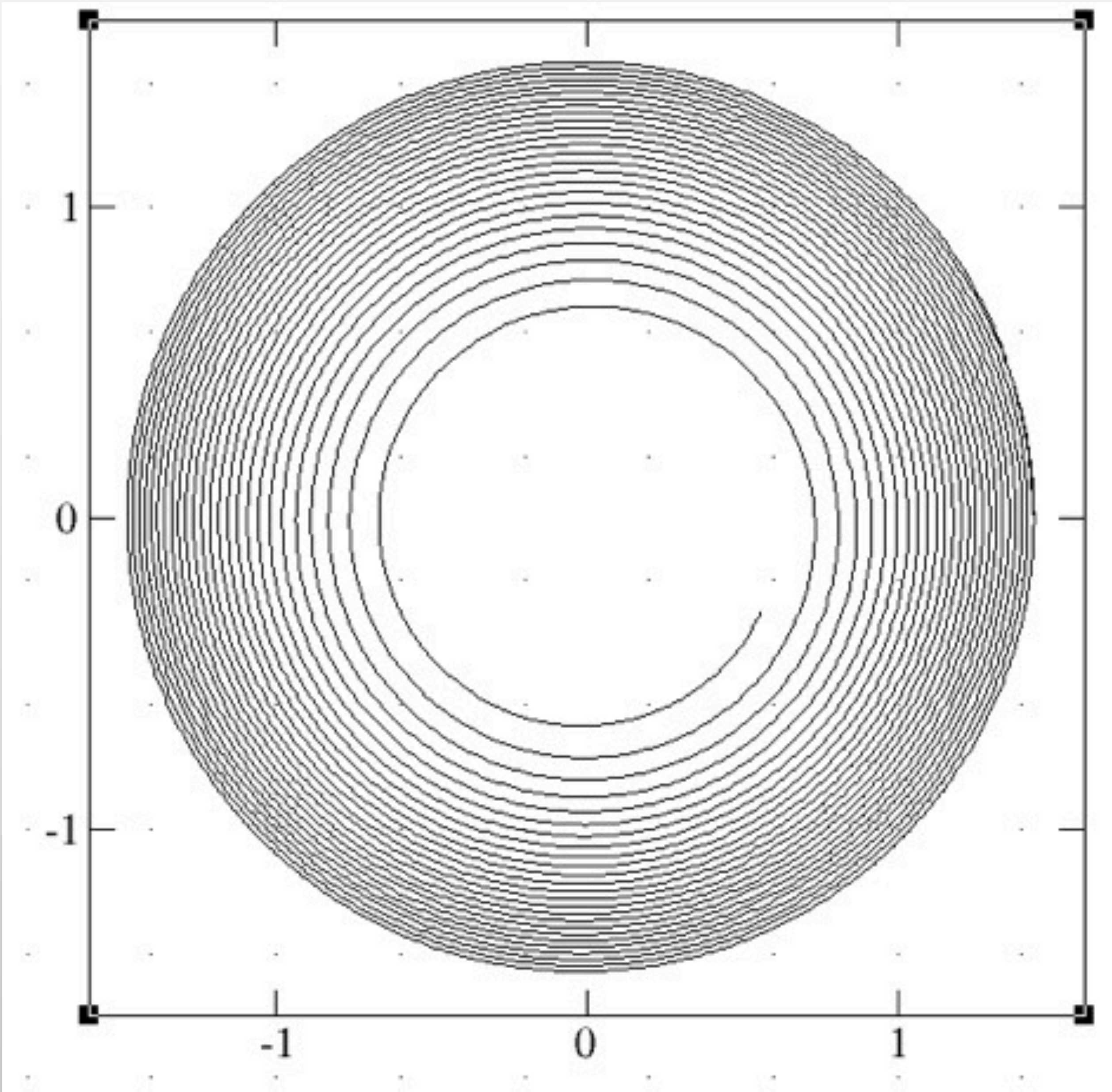


Eccentricity removal: Results

Before: e=0.01

After: e=5e-5



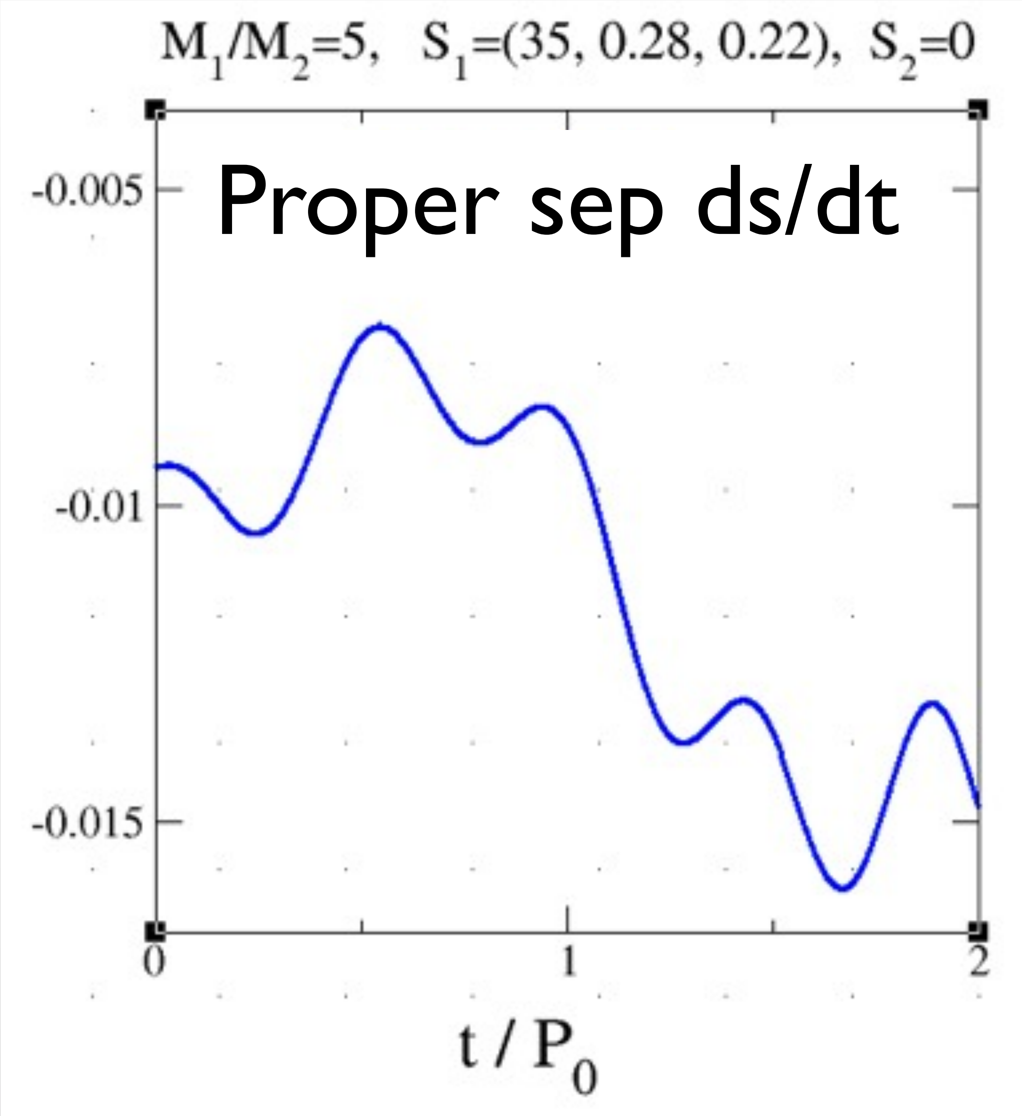


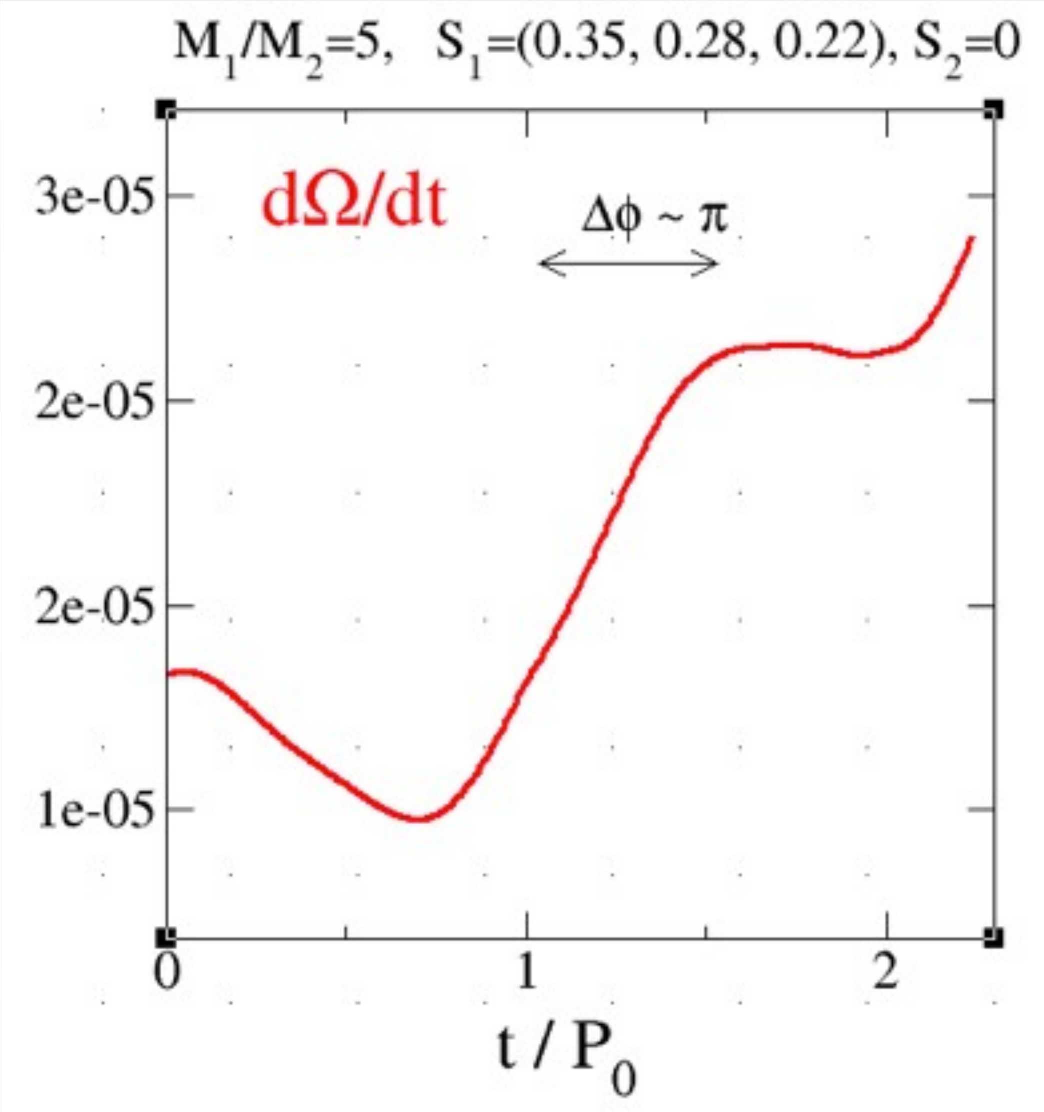
q=8

Mroue

Interlude: Precessing binaries

Buonnano, Kidder, Mroue, HP, Tarraccini, 2010





❖ contamination by periodicity P/2

Post-Newtonian Analysis

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- Spin-induced oscillations at **P/2**

$$\delta\dot{r} = B\sin(\omega t+\phi) - \frac{\omega}{2M^2 r} S_{0\perp}^2 \sin(2\omega t+\gamma)$$

$$\delta\dot{\Omega} = B\sin(\omega t+\phi) - \frac{\omega^2}{2M^2 r} S_{0\perp}^2 \sin(2\omega t+\gamma)$$

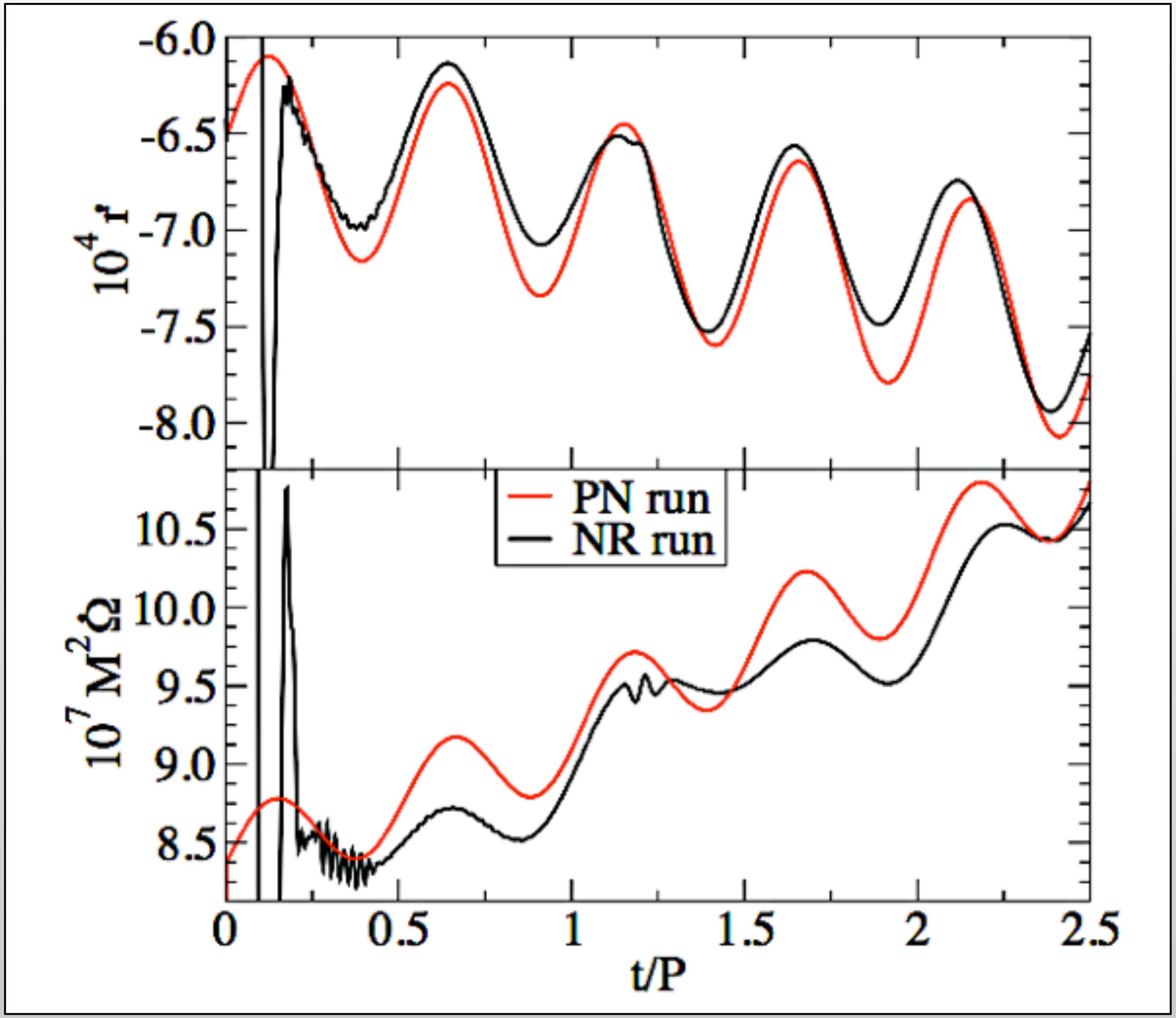
- Dominate orbital eccentricity for

$$e \lesssim 10^{-3}\left(\frac{S_{0\perp}}{M^2}\right)^2\left(\frac{r}{15M}\right)^{-2},$$

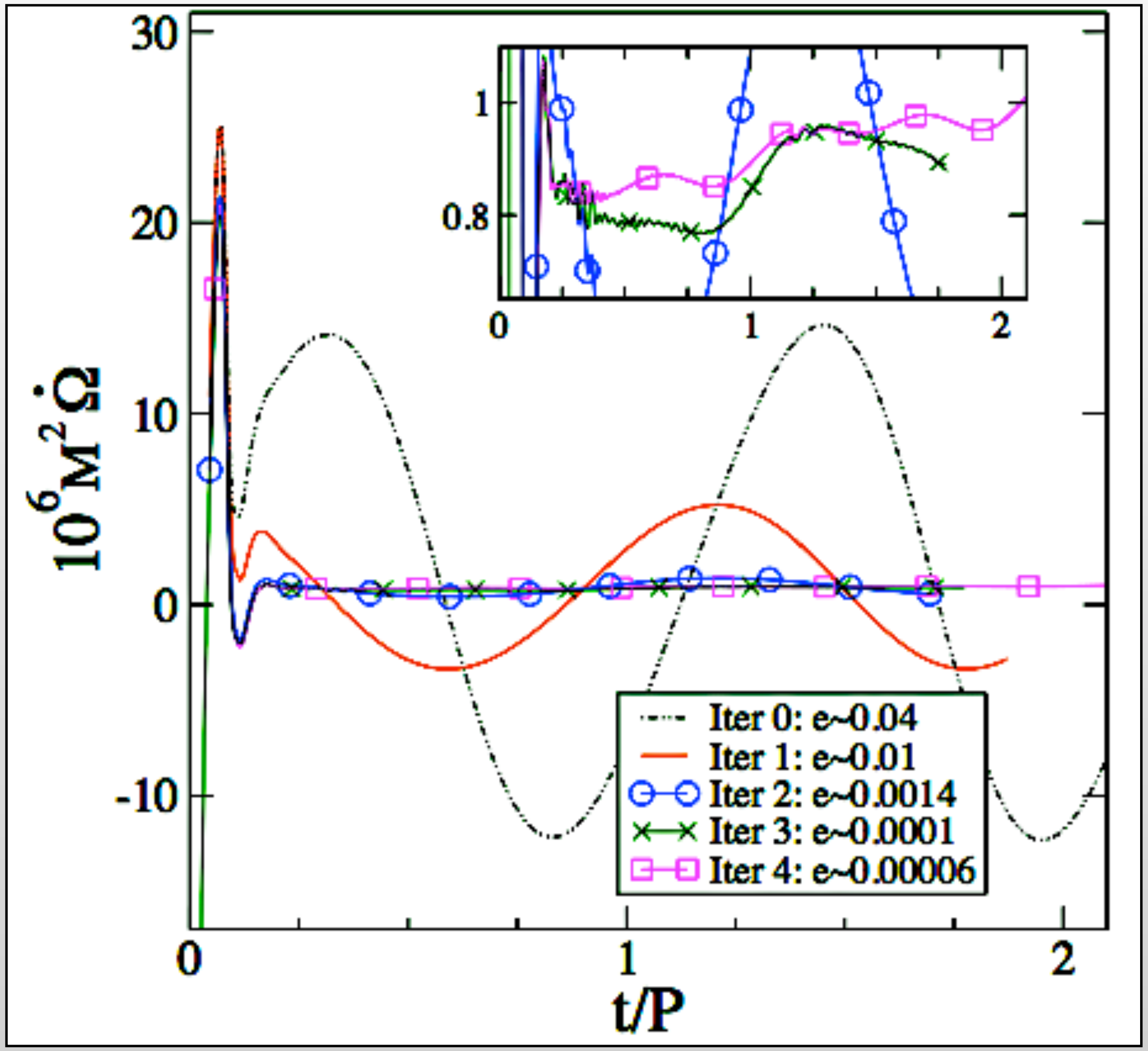
$$\frac{\mathbf{S}_0}{M^2} = \frac{m_2}{M}\frac{\mathbf{S}_1}{m_1^2} + \frac{m_1}{M}\frac{\mathbf{S}_2}{m_2^2}$$

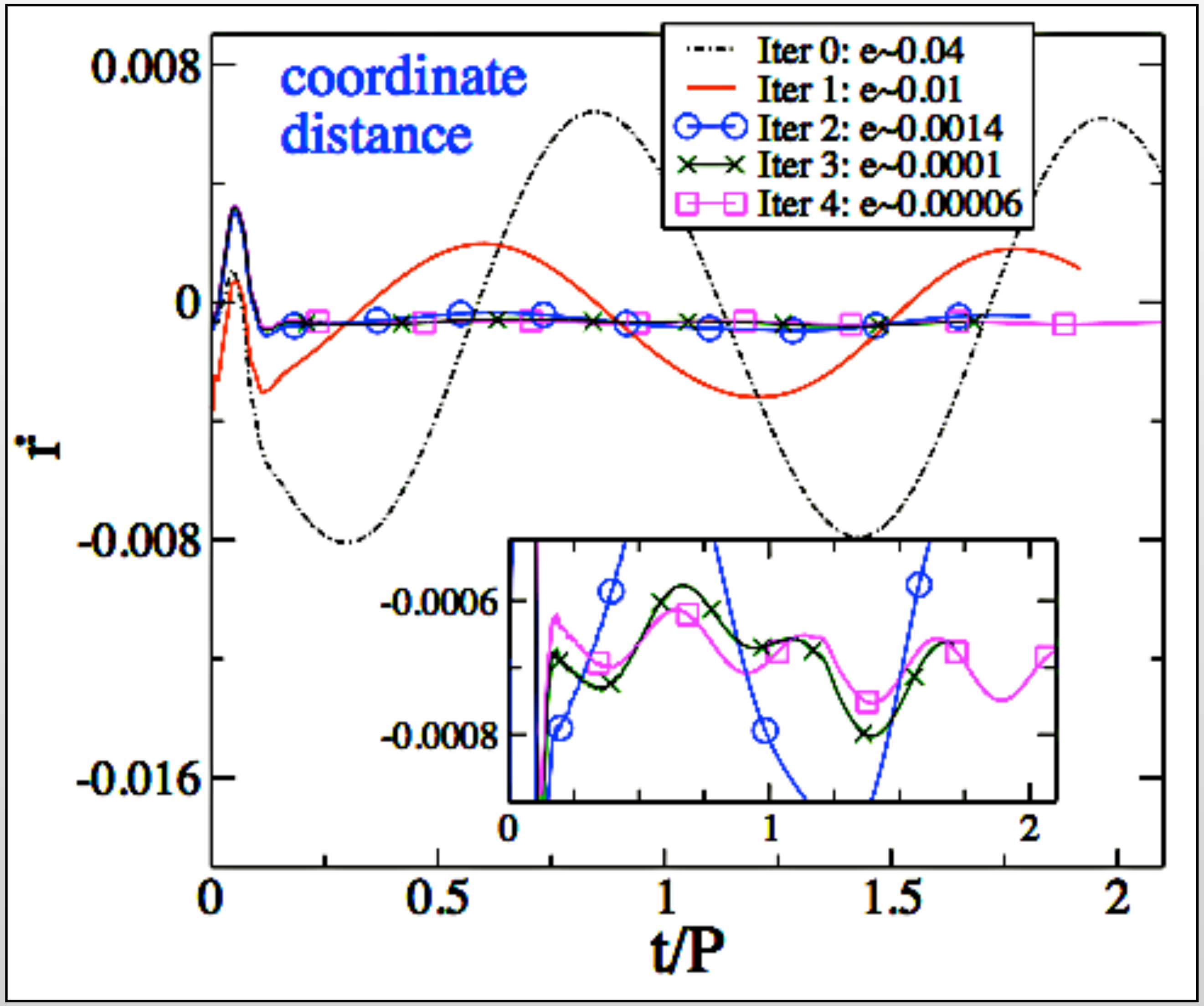
- Orbital frequency less affected (by factor 2)

Comparison PN-NR



NR simulations (precessing)



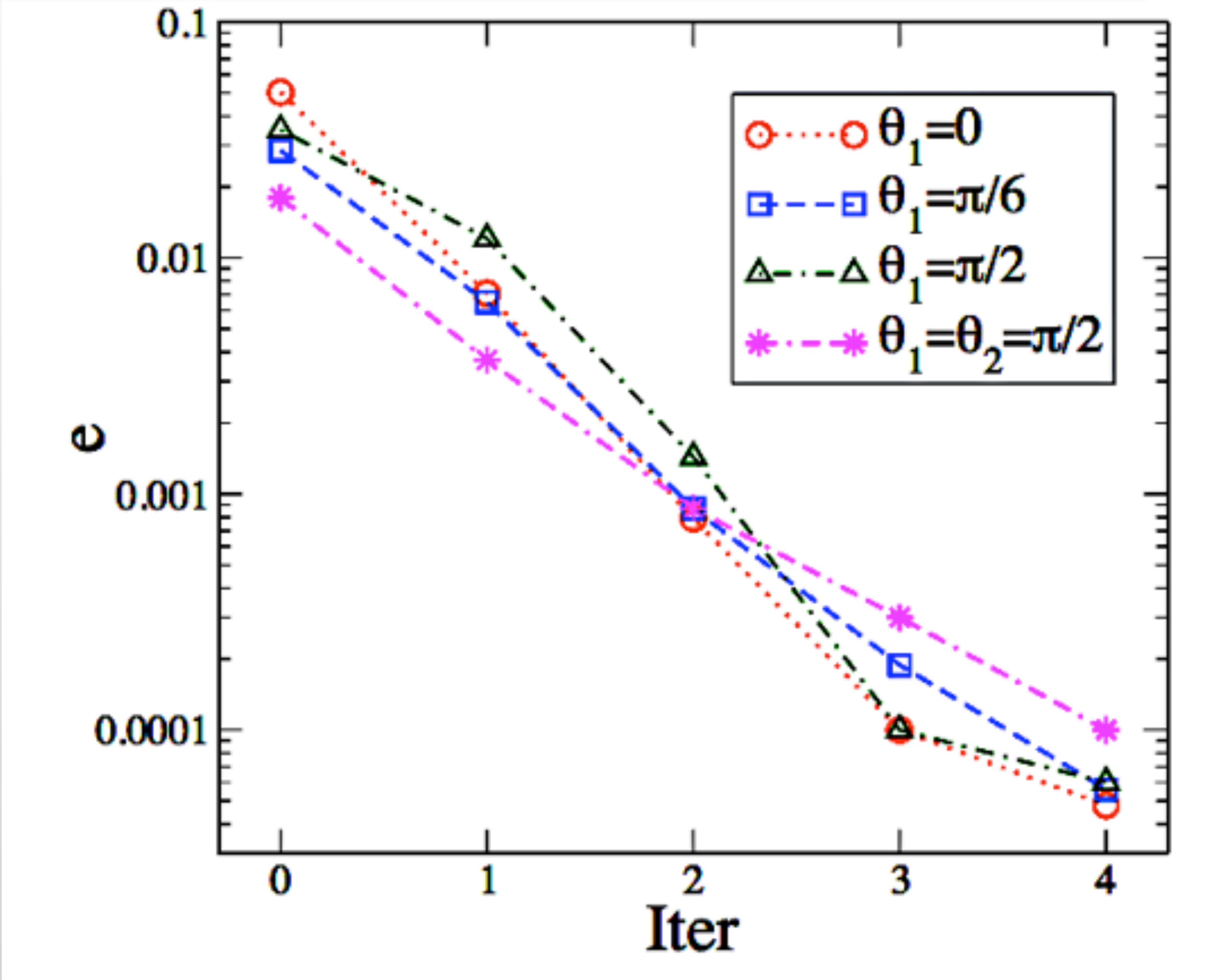


$S_1/M_1^2=0.5$, $S_2=0$, reach $e<10^{-4}$

Generic configurations

❖ BH 1:
- S/M^2=0.5

❖ BH 2:
- S/M^2=0
- S/M^2=0.5

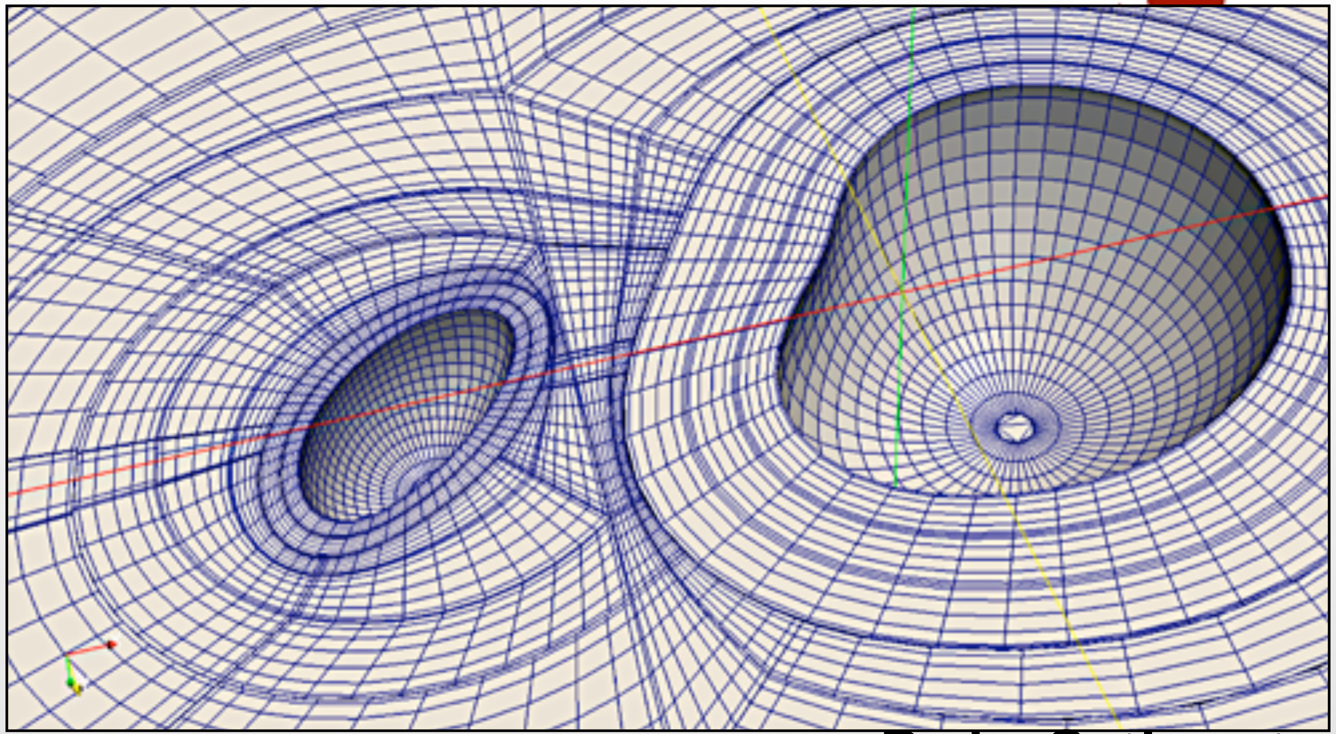


Eccentricity removal possible

Merger & Ringdown

❖ ***Mark Scheel, Bela Szilagyi***

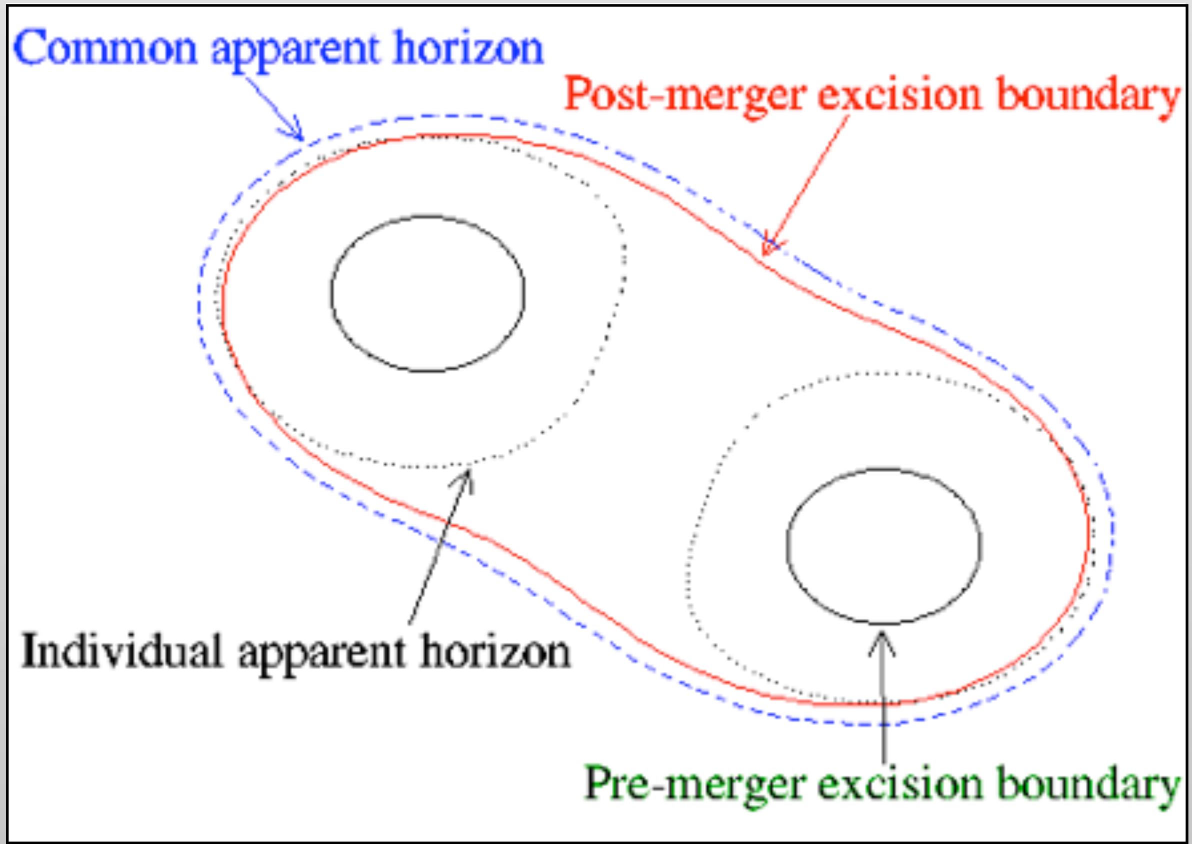
Szilagyi, Lindblom, Scheel 08.
Many additions since then



Bela Szilagyi

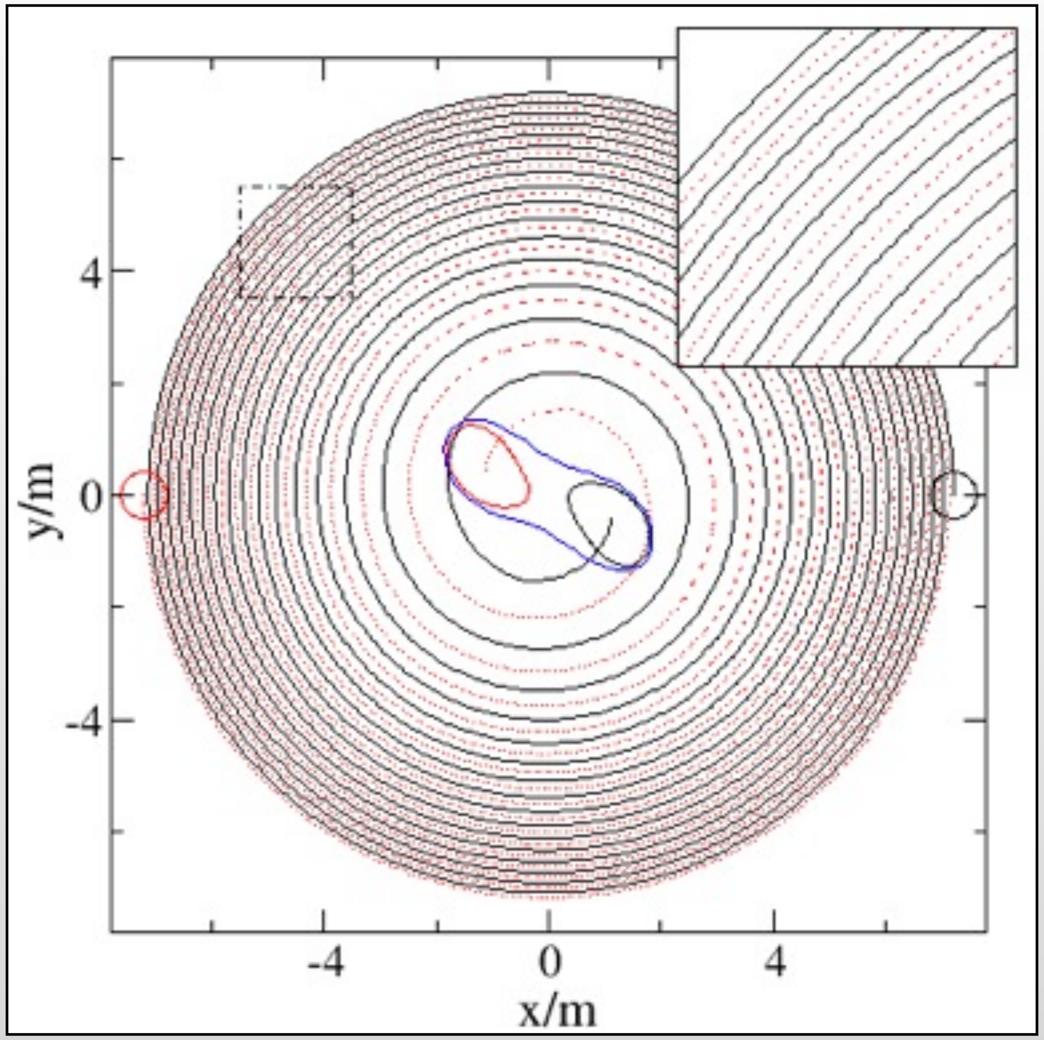
❖ Close to merger
- Switch domain-decomposition
- Active gauge conditions
- Adaptive Mesh Refinement

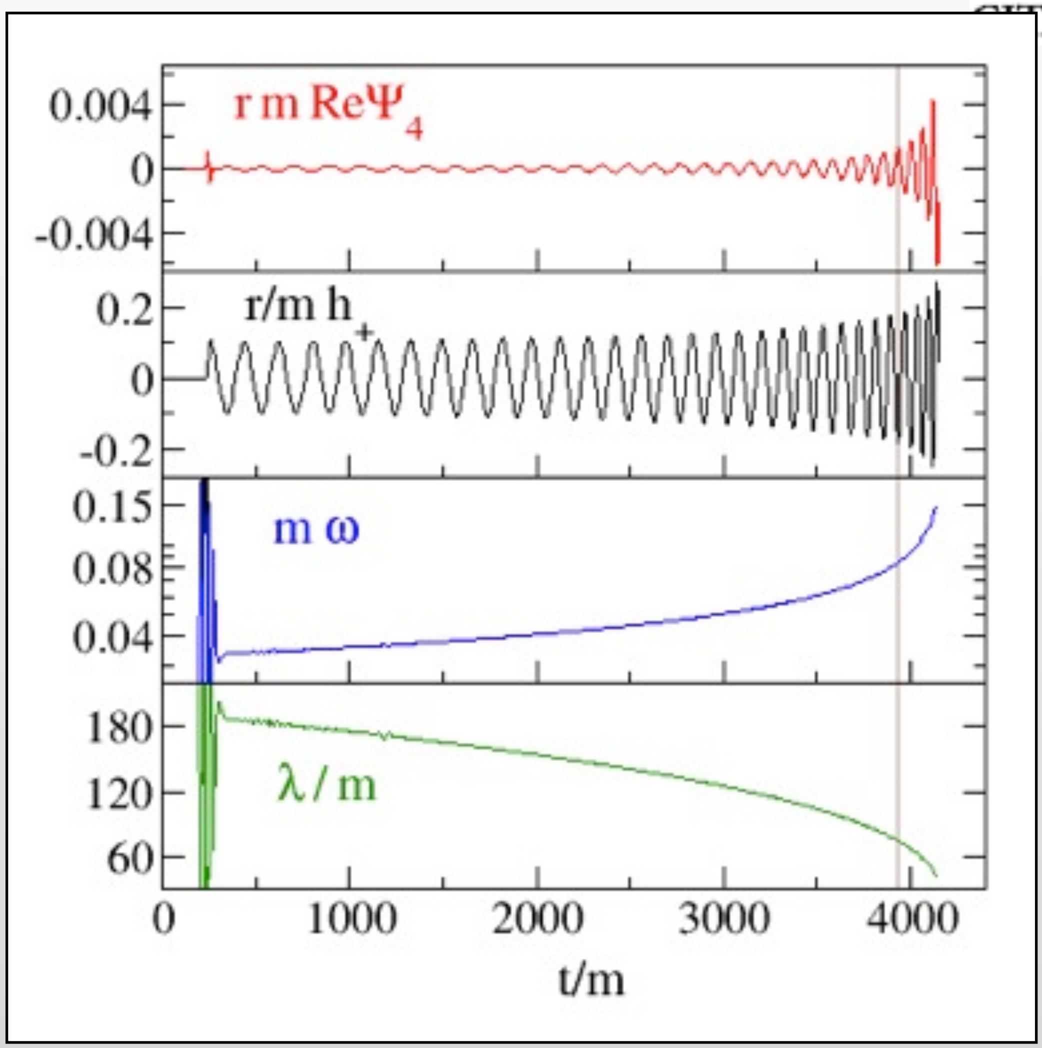
❖ After common horizon
- Switch to distorted concentric shells

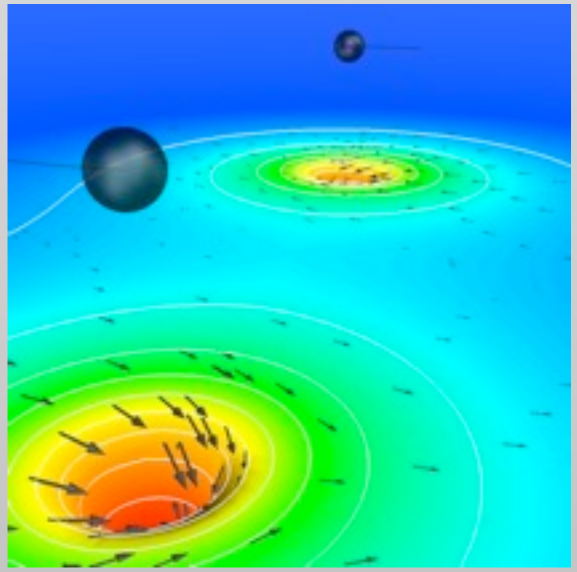


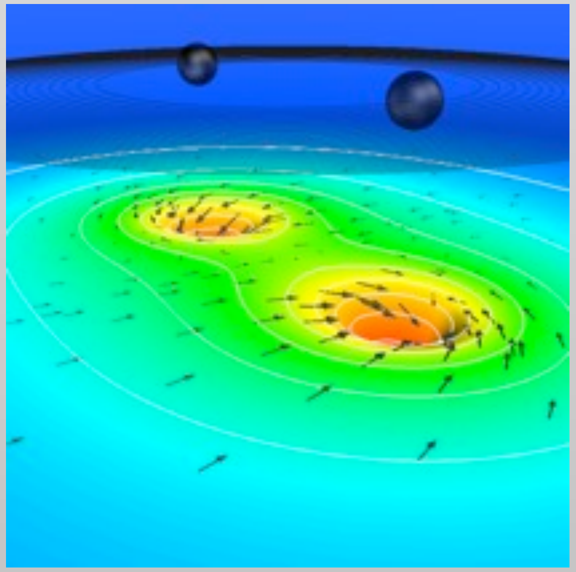
Mark Scheel

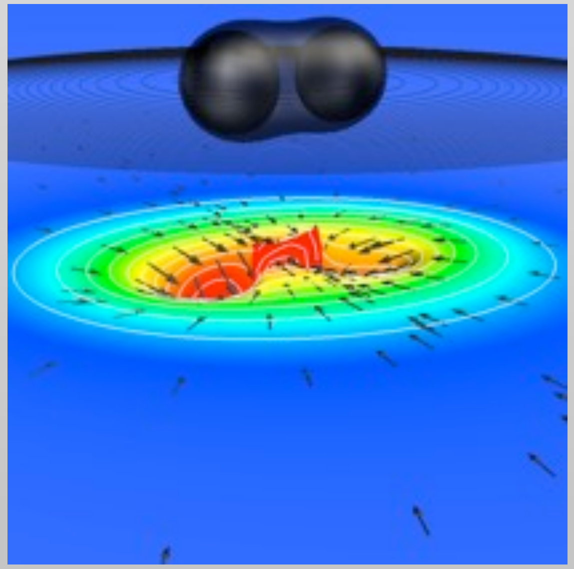
Equal-mass, zero spin (HP et al '07, Scheel et al '07, '09)

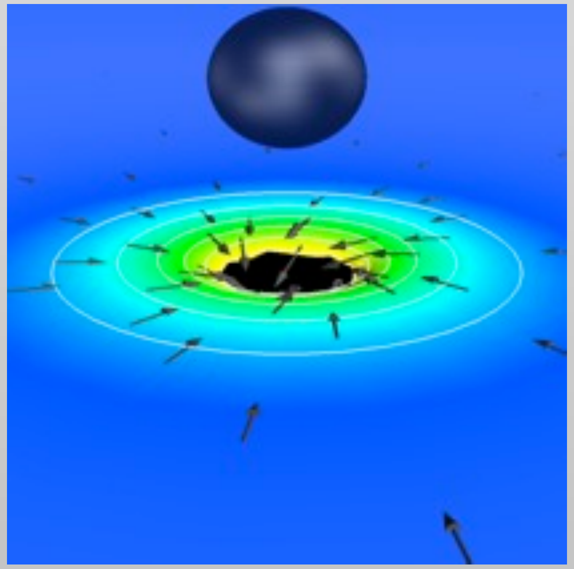


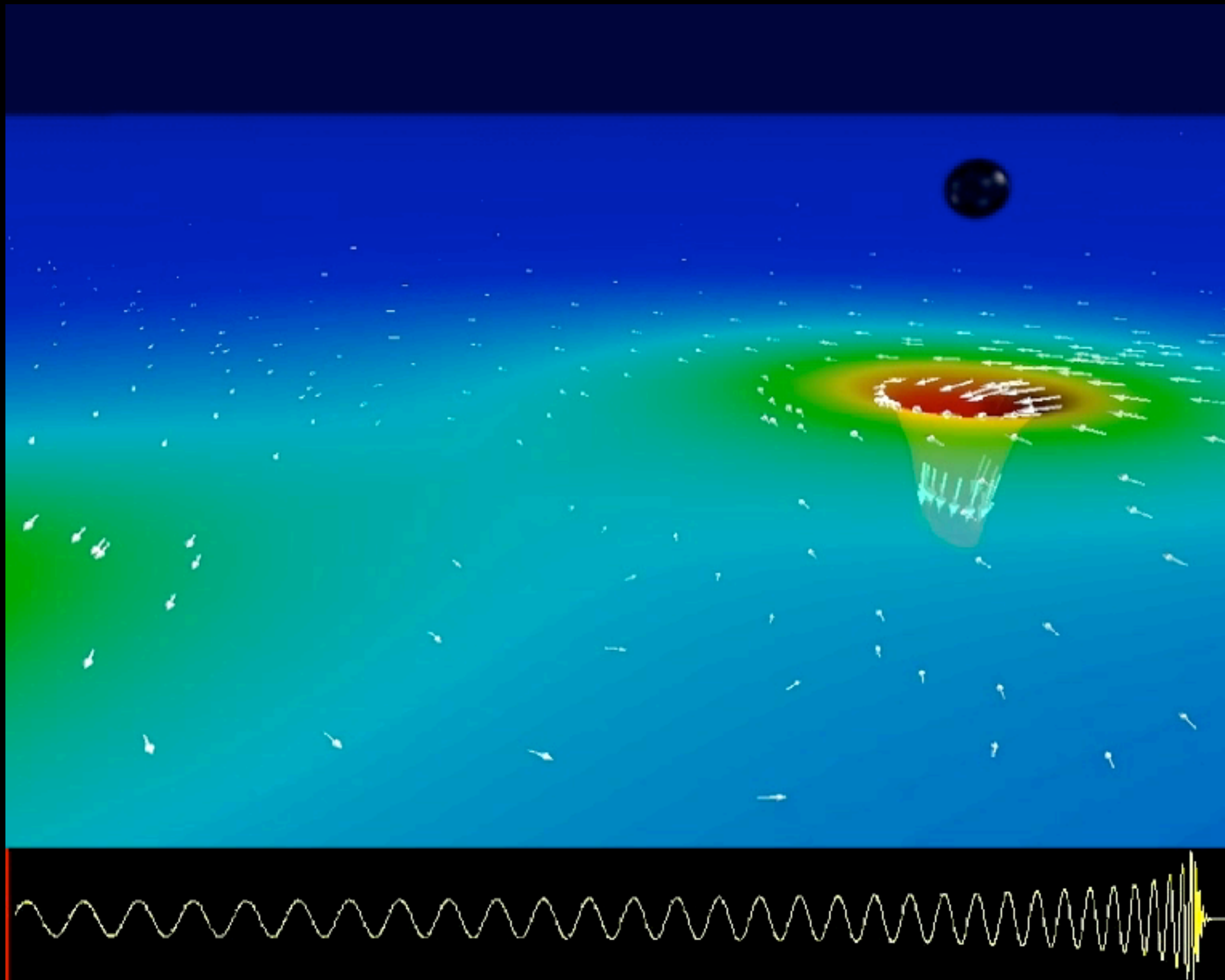








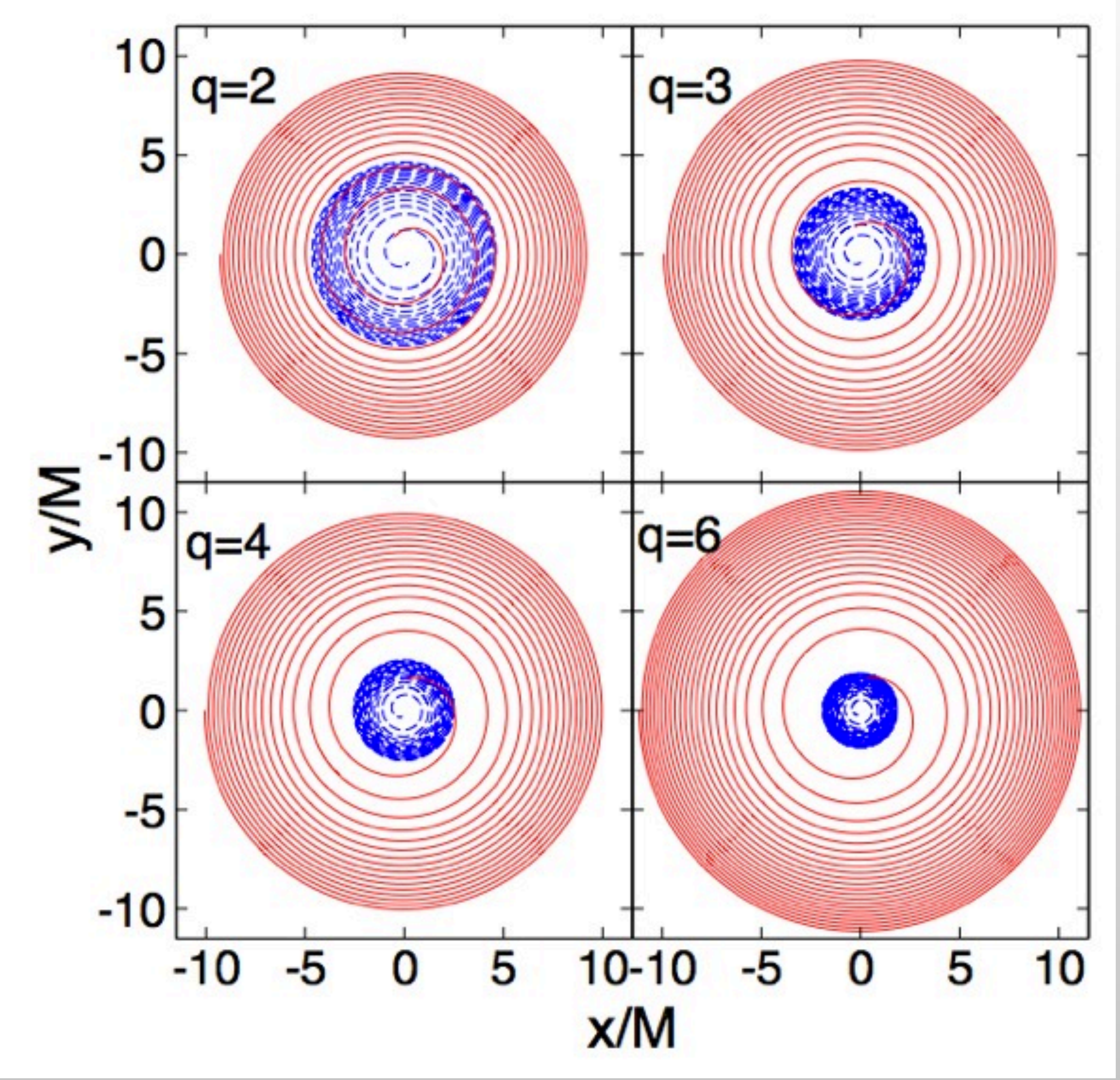




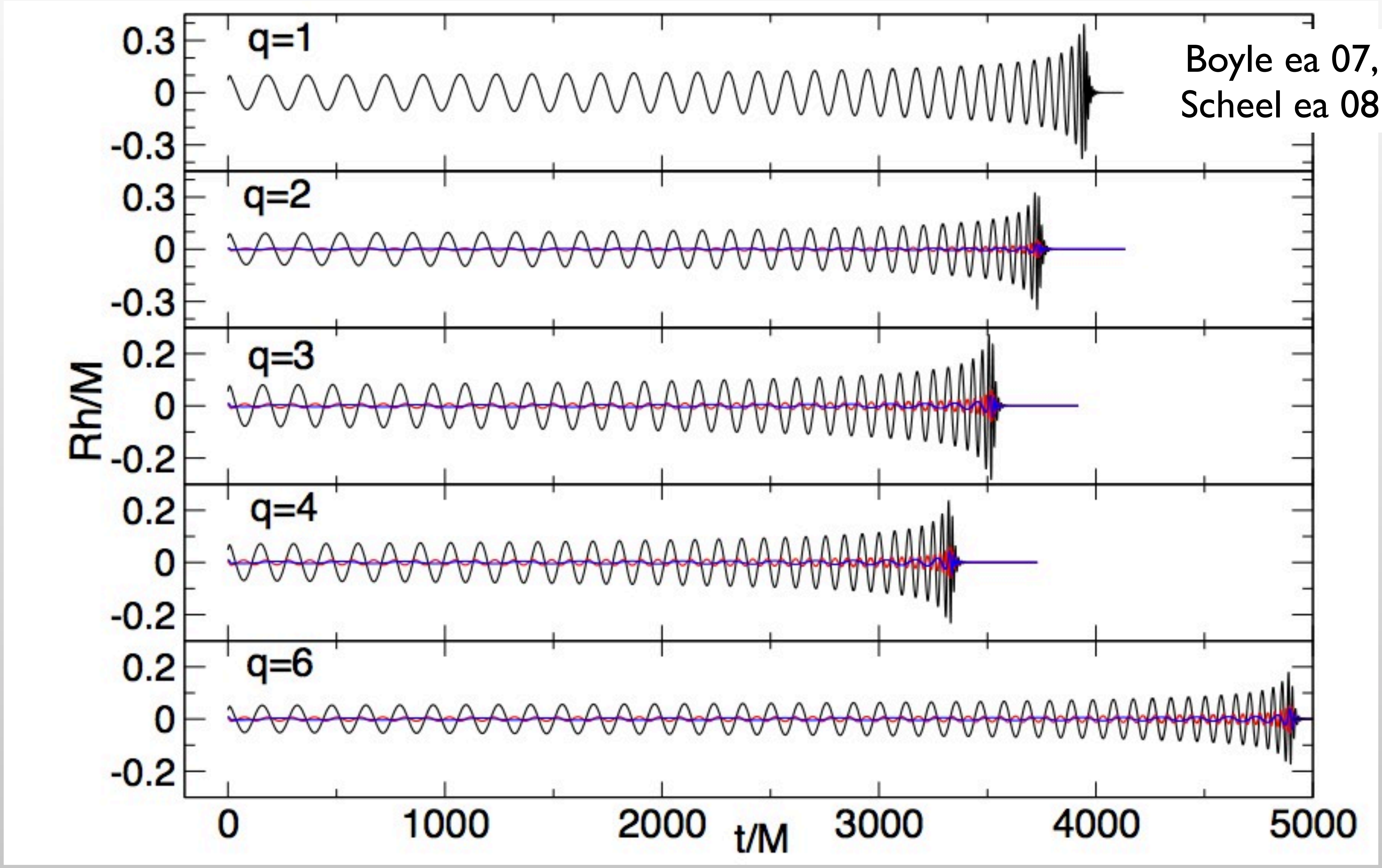
Results

Non-equal mass, non-spinning BBH

- Series 1
  (Buchman, HP, Scheel, Szilagyi in prep)
- q=1/2, 1/3, 1/4, 15 orbits
- q=1/6, 21 orbits
- Eccentricity few 10^{-5}

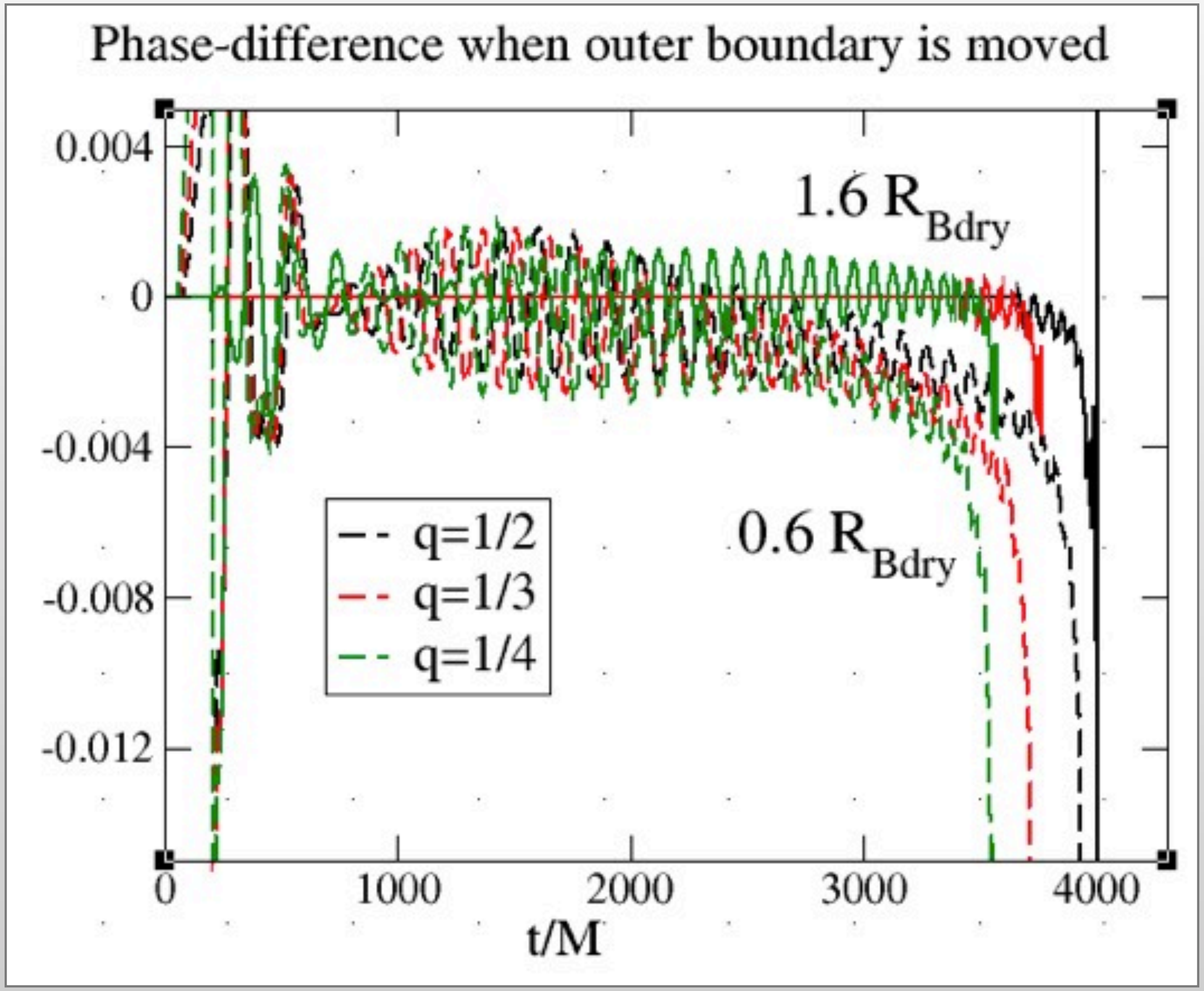


Waveforms (2,2) (3,3) (2,1)



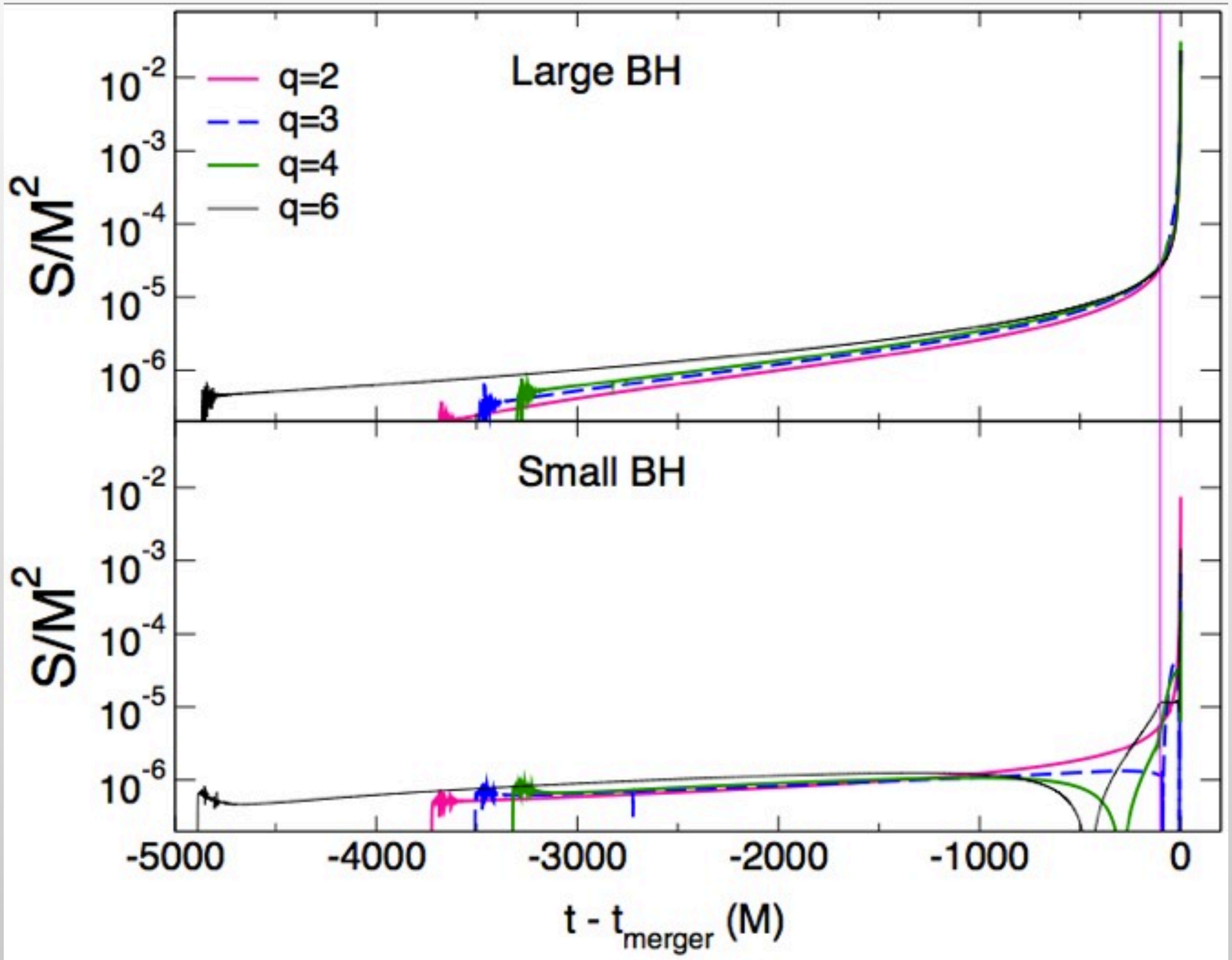
Effect of outer boundary

- Outer BC:
  - non-reflecting (almost)
  - Constraint preserving
  - Lindblom ea 06, Rinne ea 06,
- Boundary in causal contact RBdry~500M, but little effect



Black hole spins

- ❖ Relaxation of initial-data causes non-zero spins early in evolution
- ❖ Slow increase during inspiral (tidal spin-up)
- ❖ Quick increase just before merger (Spin defn?)



Remnant properties

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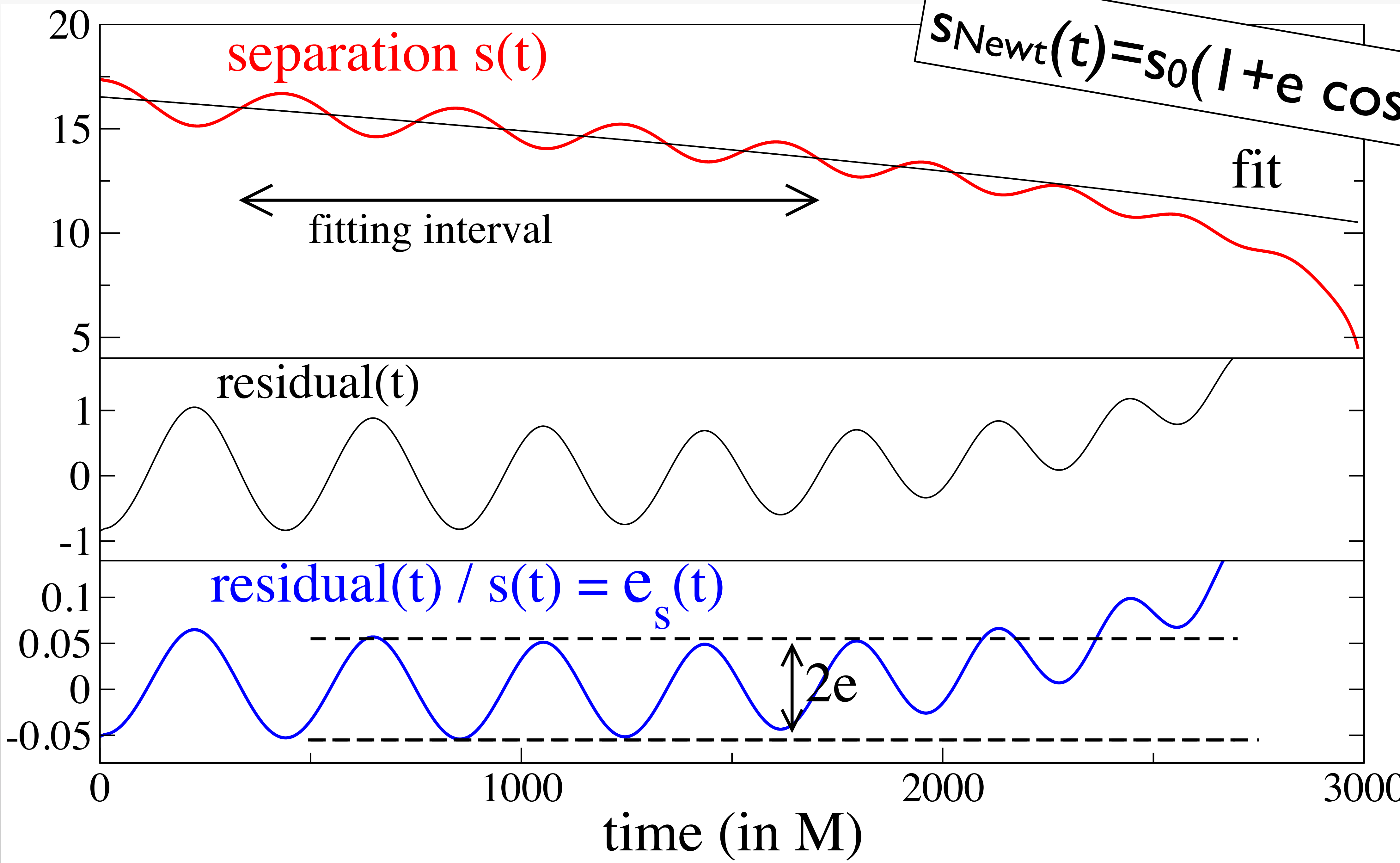
q	Initial Data			Inspiral			Merger & Ringdown	
	$10^3\Omega_0$	$10^6\dot{a}_0$	D_0	$10^5\varepsilon_{ds/dt}$	N	T^a	$M_{c,f}$	$S_f/(M_{c,f})^2$
2	17.6711	-62.53	13.8738	3	15	3290	0.96125 ± 5e-5	0.623435 ± 5e-6
3	18.9994	-63.63	13.1767	2	15	2770	0.971273 ± 5e-6	0.54059 ± 3e-5
4	20.3077	-66.08	12.5652	3	15	2949	0.977922 ± 6e-6	0.47159 ± 3e-5
6	29.5914	-150.1	9.58293	8	8	984	0.985470± 4e-6	0.37245 ± 1e-5

(q=6 from earlier, shorter run)

Application:
Eccentricity decay & periastron advance

Mroue, HP, Kidder, Teukolsky 2010
LeTiec, Mroue, Barack, Buonnano, HP, Sago, Taracchini 2011

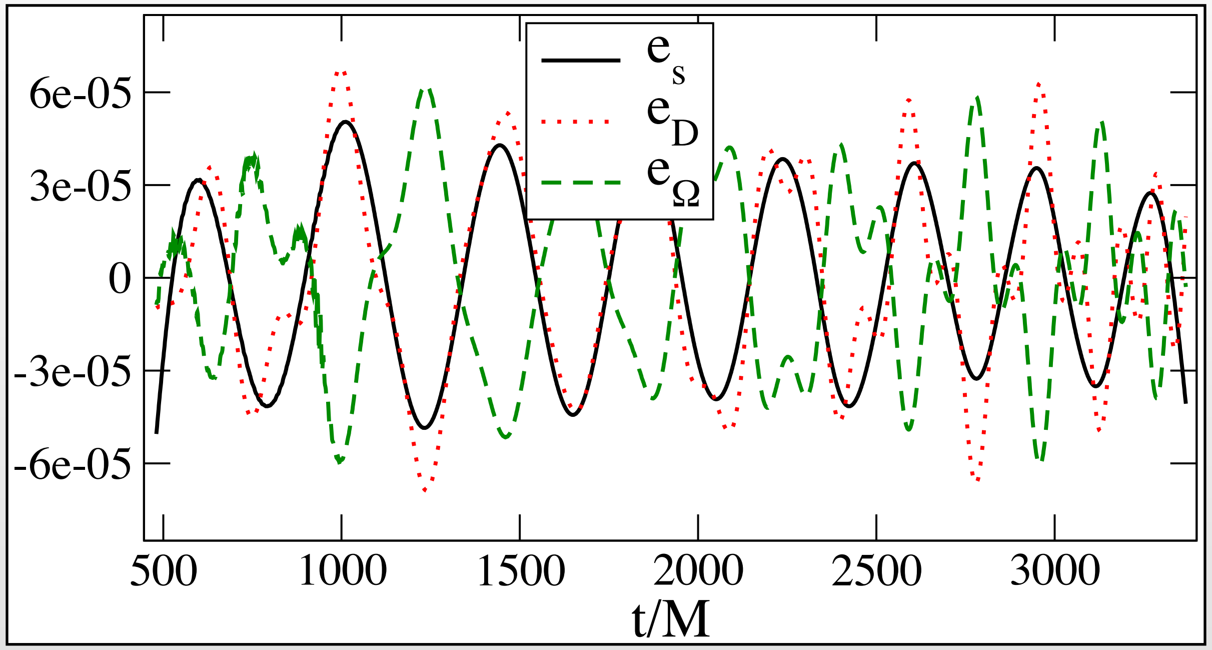
Measuring eccentricity

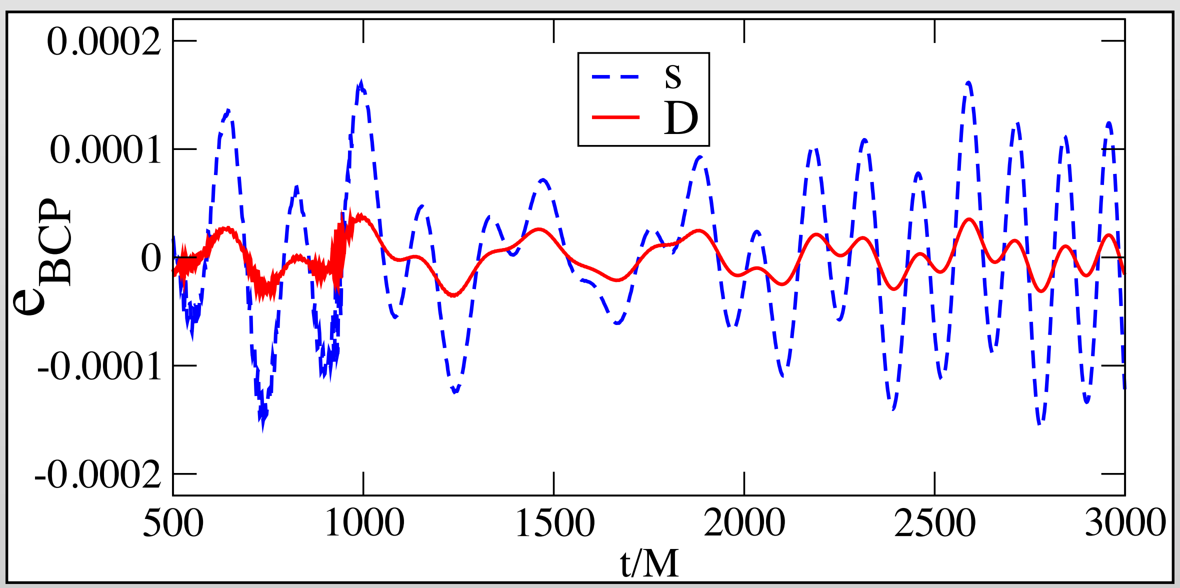


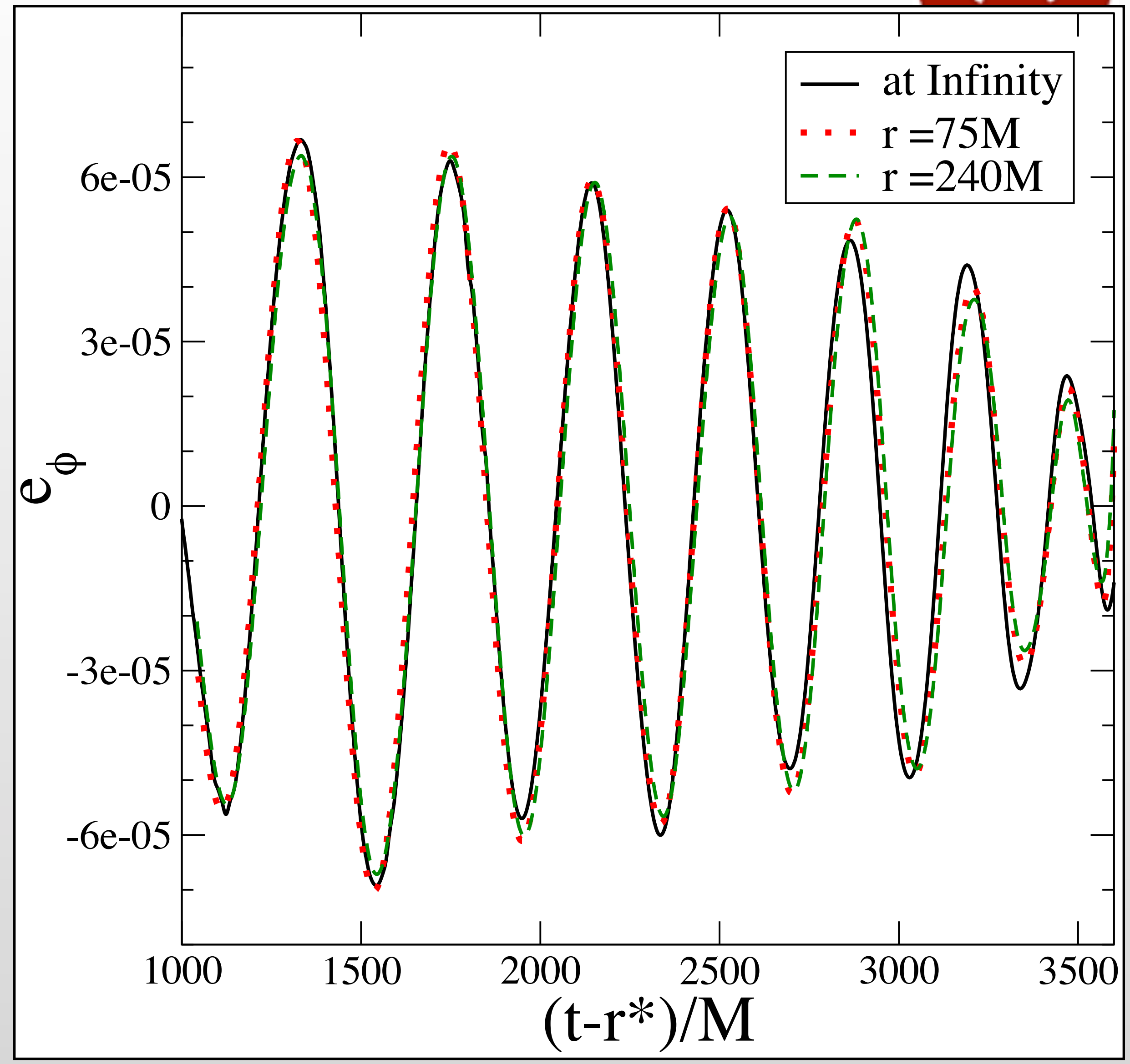
$s_{Newt}(t) = s_0(1 + e \cos \Omega t)$

❖ Eccentricity estimator $e_s(t)$: Oscillating w/ amplitude e

Many possibilities

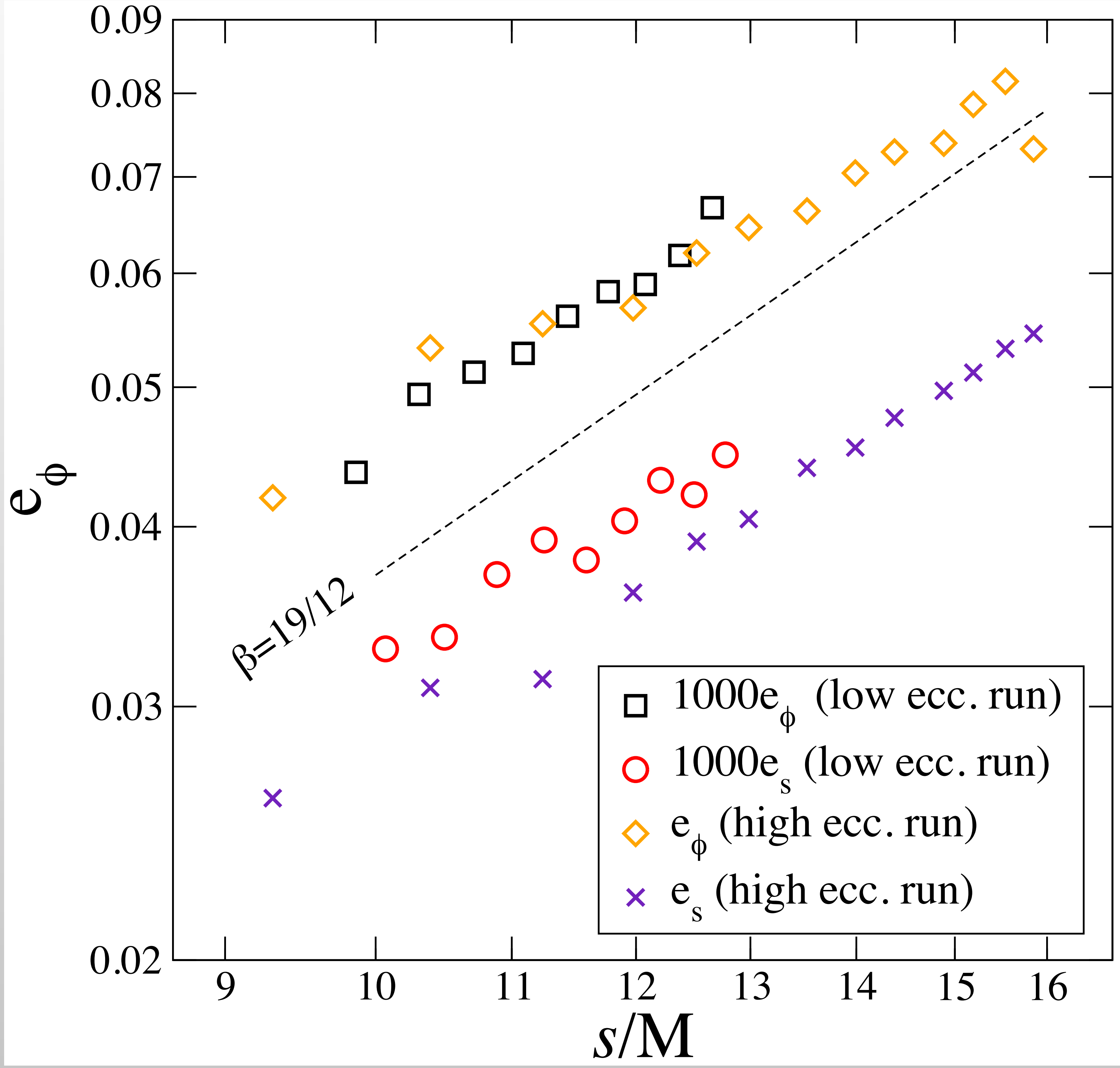






- ❖ Most eccentricity estimators agree
- ❖ Gravitational wave phase cleanest (least gauge dependence)

Eccentricity decay



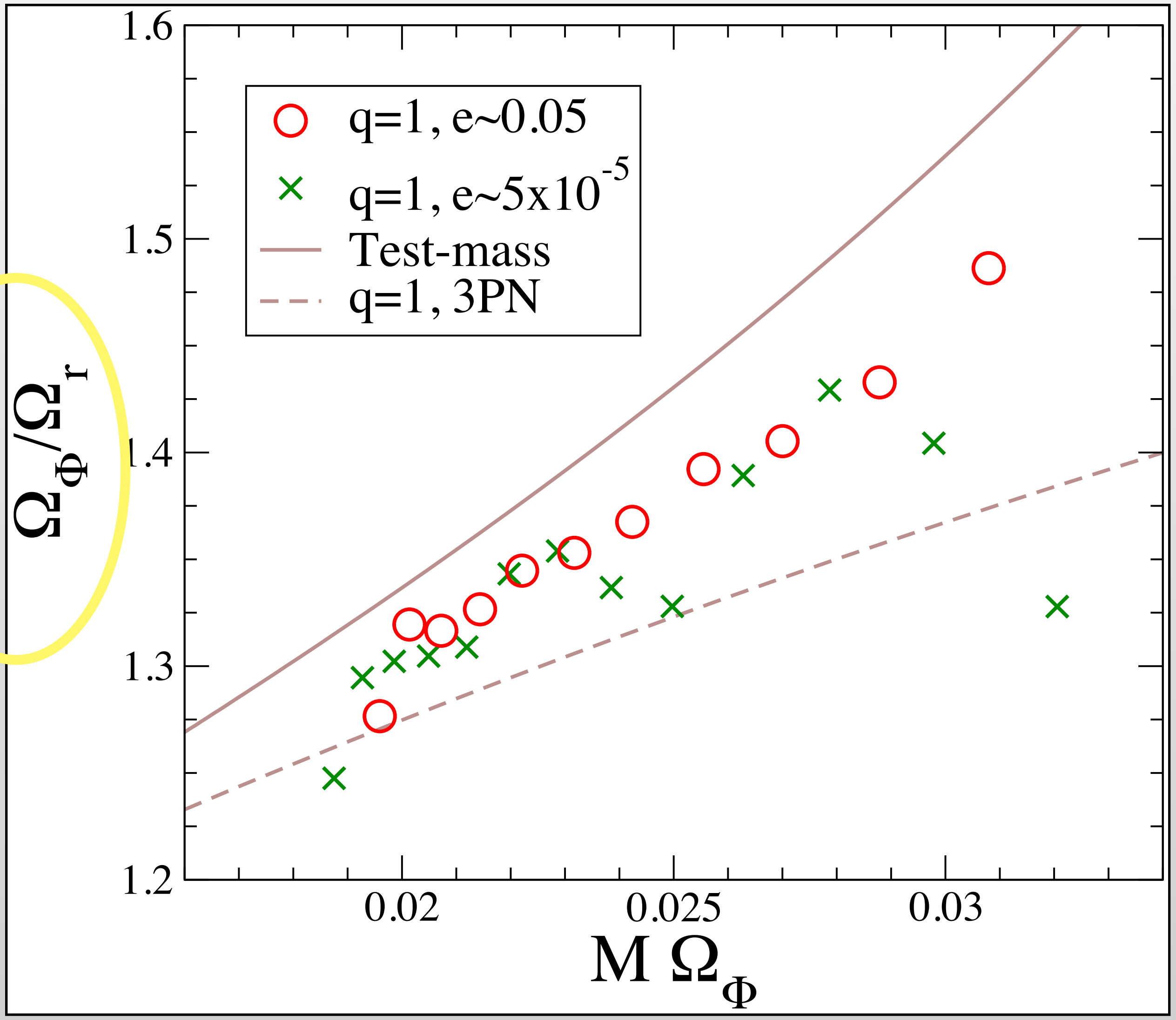
- Consistent with Peters (1965)

$e \sim a^{19/12}$

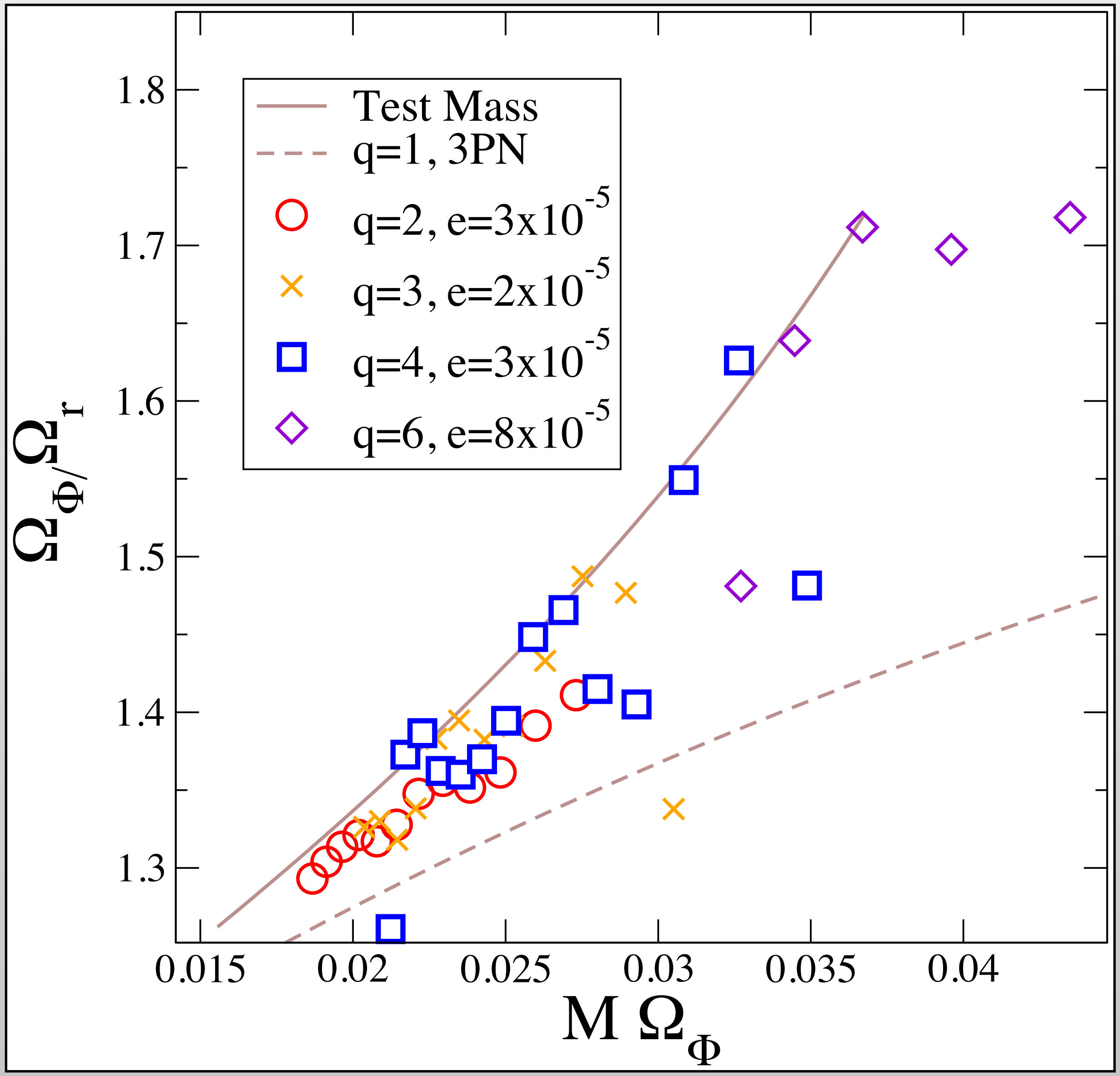
Periastron advance (first analysis)

Mroue, HP, Kidder, Teukolsky, PRD 2010

Equal mass BBH



Mass-ratio 2 - 6

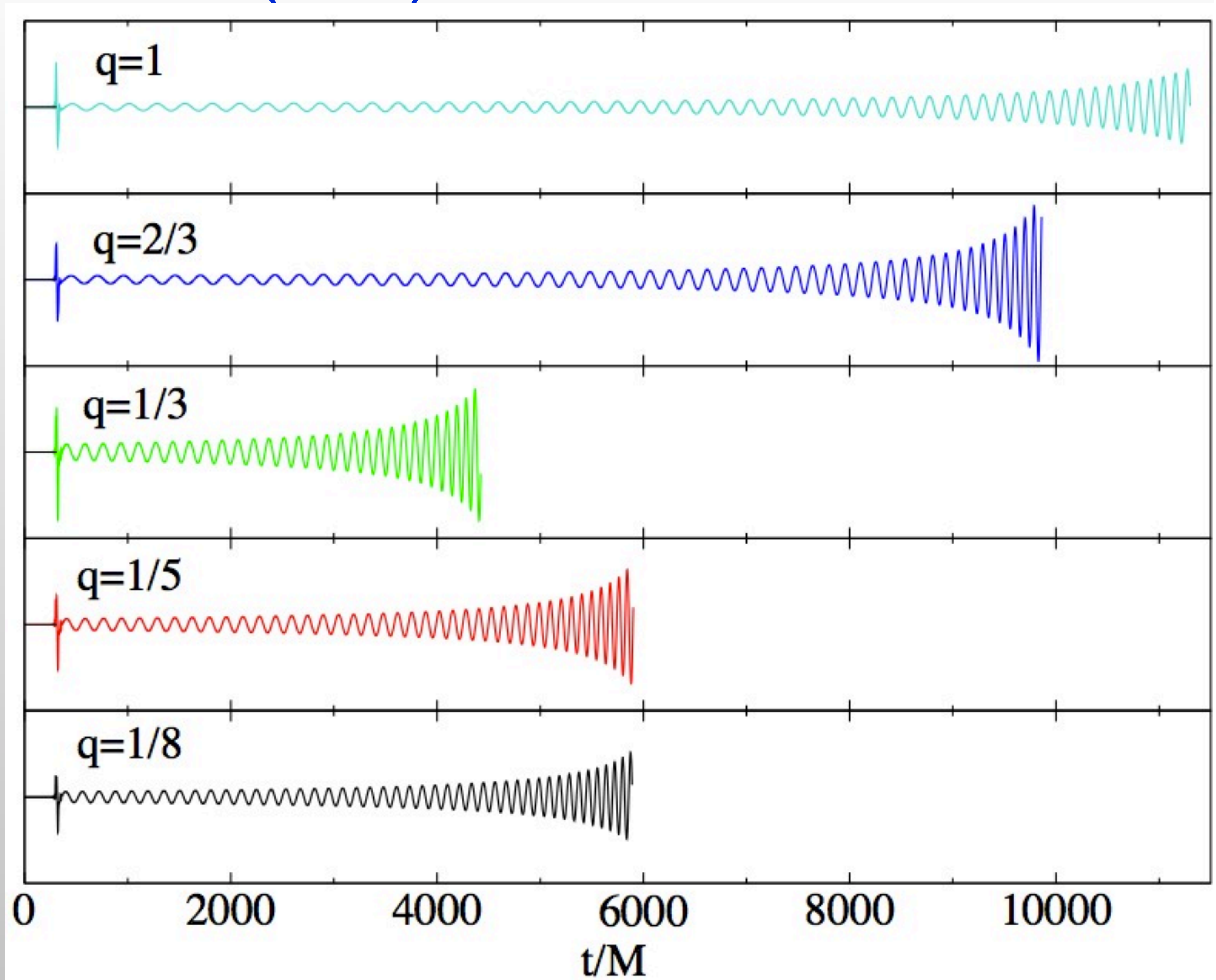


Periastron advance: Second try

CITA–ICAT

- ❖ Original analysis difficult
  - eccentricity too small
  - simplistic analysis
    - just look for peaks in $e_s(t)$
- ❖ New simulations
  - Abdul Mroue (CITA)
    - q=1, 2/3, 1/3, 1/5, 1/8
    - Even longer, for better contact with PN
    - Slightly eccentric (e~0.01)
    - Only inspiral presently

Waveforms (Psi4)



Periastron advance: Second try

❖ Better analysis

- Local fits around time T covering ~3 periods

$$\Omega(t) = p_0(p_1 - t)^{p_2} + p_3 \cos\left[p_4 + p_5(t-T) + p_6(t-T)^2\right]$$

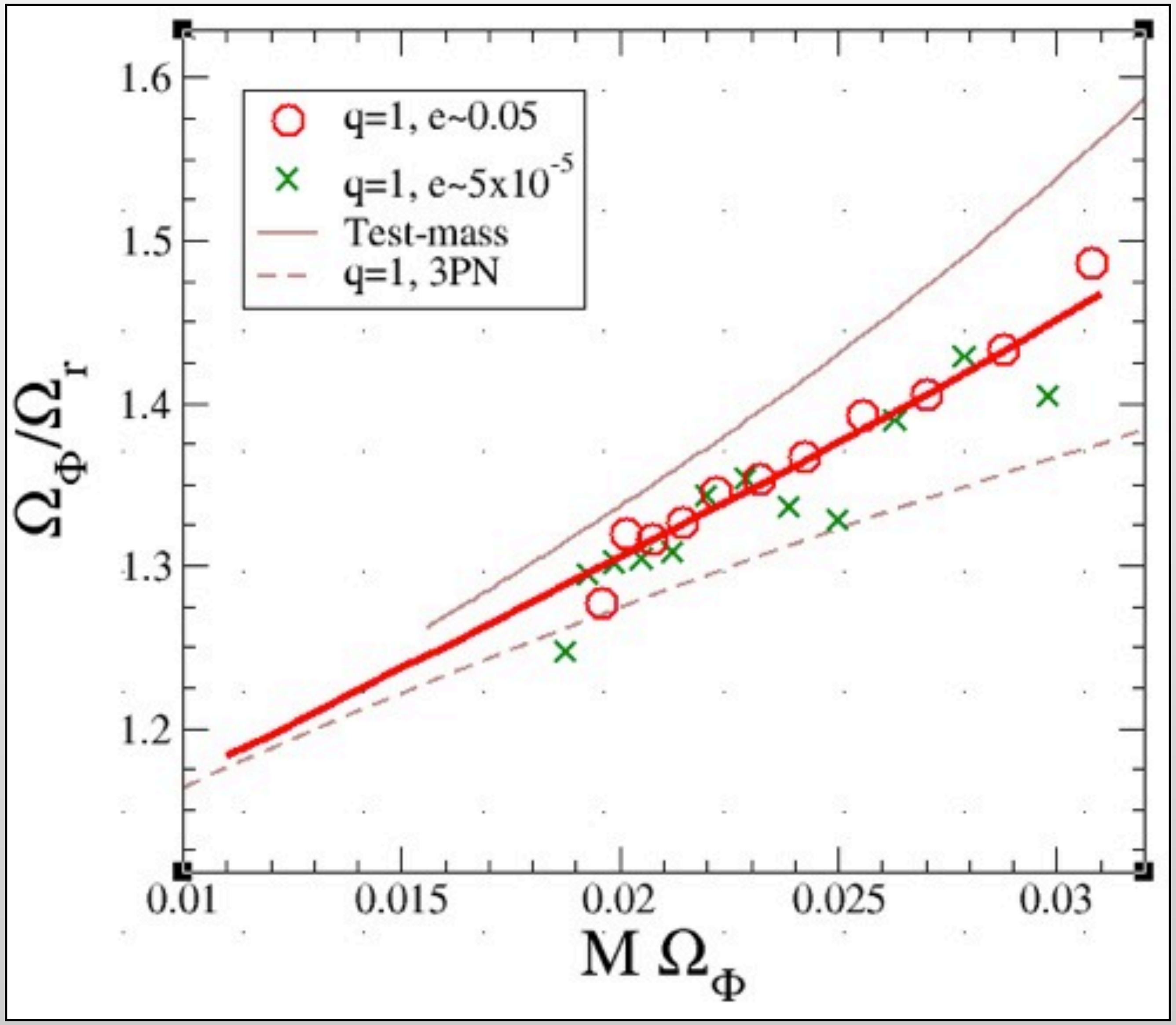
- Non-oscillatory piece gives $\Omega_\phi(T) = p_0(p_1 - T)_2^p$

- Oscillatory piece gives $\Omega_r(T) = p_5$

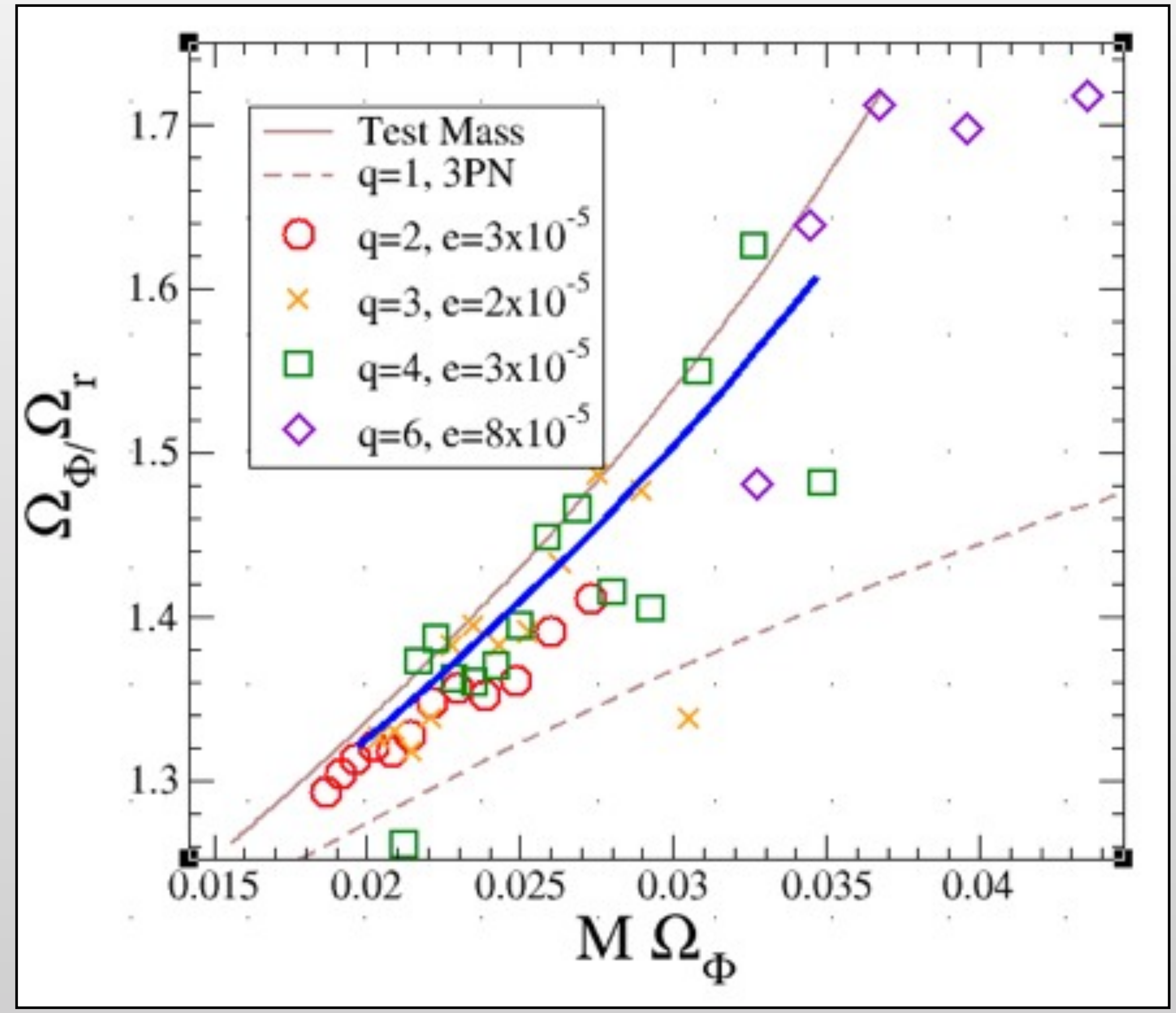
- Repeat for many T

Periastron advance (new data)

New q=1



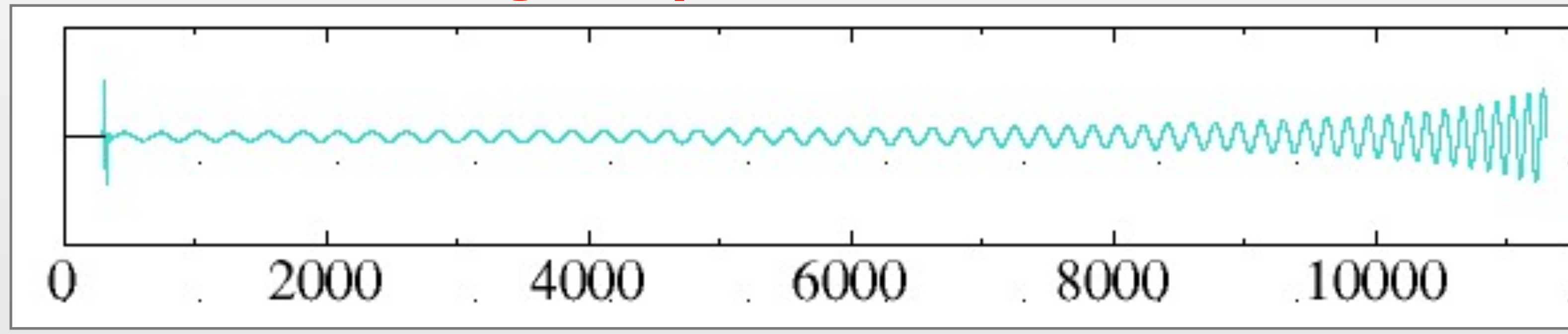
New q=8



Alex LeTiec (next talk): application of these data

Summary

- ❖ Numerical relativity has come a long way



- ❖ Some (often neglected) details of simulations
  - Measure spins, root-finding in ID
- ❖ complete simulations for q=1, 1/2, 1/3, 1/4, 1/6
- ❖ inspiral simulations for q=1, 2/3, 1/3, 1/5, 1/8
- ❖ Extraction of periastron-advance